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Compressibility–Pressure Gradient and the Constitution of the Earth's Outer Core

K. E. Bullen

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Summary

Re-interpretation of seismic data used by Birch in respect of the Earth's outer core indicates that his finite-strain theory, while serviceable to a useful first approximation, is probably not adequate for drawing fine conclusions on the variation of the incompressibility with pressure in the core. On currently available seismic data, there is a strong suggestion of mild inhomogeneity inside the outermost 700 km of the core. The whole region from 2700 to 3600 km depth seems to be slightly abnormal, with a faint suggestion of continuous change of phase in the outer part of the core. The conclusions are sensitive to the values in determinations of seismic P velocity gradients in the outer core, and it is desirable that some renewed attention be given to estimating the precision of the determinations.

1. Introduction

Various approaches yield estimates of the same order of magnitude for the incompressibility k and its gradient with respect to the pressure p in the Earth's deep interior. In particular, the degree of agreement between approaches of the writer and colleagues (Bullen 1942, 1950, 1965a, 1965b) and of Birch (1952) gives good mutual support up to a point. Throughout this paper, the units for k and p will be 10^{12} dyn cm^{-2}. The data used by the writer consist principally of bodily wave seismic data and evidence on the variation of the density ρ with p and the depth z in the Earth. Birch's approach uses his adaptation of Murnaghan's finite-strain theory. In the finer detail there are, however, some significant discordancies between results on the approaches over the pressure range $0{\cdot}4 < p < 4{\cdot}0$, approximately, i.e. below a depth of 1000 km.

Difficulties in reconciling discordancies in estimates of the jump Δk in k at the the Earth's mantle–core boundary N have already been discussed in some detail (Bullen 1968a, 1968b; Bullen & Haddon 1968). The writer's approach leads to a preferred value of zero, within 1 per cent, for the proportionate jump in k at N, as against a value not less than $+6$ per cent on Birch's theory as normally interpreted. (It is to be noted that the preferred result $\Delta k \approx 0$ has arisen as a consequence of a revision of the earlier Earth model A (Bullen 1965a), for which Δk is significantly negative, taking account of recent evidence on the Earth's moment of inertia and other factors. This result $\Delta k \approx 0$ is derived quite independently of the writer's $k-p$ hypothesis (Bullen 1946, 1949a).)

The purpose of the present paper is to analyse apparent discordancies between estimates of k and dk/dp inside the Earth's core, especially the outer core E' (Bullen, 1965b). Although the evidence does not yield unique conclusions, it will be seen that the possibilities are limited and that considerations relating to dk/dp lead to interesting suggestions about the constitution of the outermost several hundred kilometres of the core.

In line with Bolt's analysis (1964) of the distribution of the seismic P wave velocity α, the outer core E' will be taken as extending from N to about 4500 km depth. Below 4500 km, the analysis gives a marked reduction in $d\alpha/dz$, pointing (Bullen, 1965b) to a significant deviation from chemical homogeneity, with $d\rho/dz$ having three to four times the normal value for a range of depth. It will be convenient for present purposes to sub-divide E' into the regions $E_1(2900 < z < 3600 \text{ km})$ and $E_2(3600 < z < 4500 \text{ km})$.

Unprimed symbols will relate to properties at depth z in the Earth. The symbol d/dp will mean $(dp/dz)^{-1} d/dz$. Single and double primes will relate to adiabatic and isothermal conditions, respectively. Birch's work indicates that $(\partial k'/\partial p)' < \partial k''/\partial p)''$ and that differences between $(\partial^2 k'/\partial p^2)'$ and $(\partial^2 k''/\partial p^2)''$ may be neglected in the present context.

Use will be made of the formula (Bullen 1963)

$$\frac{dk}{dp} = \eta + g^{-1}\frac{d\phi}{dz}, \tag{1}$$

which applies to a region of the Earth where changes, if any, in the chemical composition and phase are continuous. In (1), $\phi = \alpha^2 - 4\beta^2/3$, β is the seismic S velocity, g is the gravitational intensity, and $\eta - 1$ is a measure of the departure from uniform composition and phase and adiabatic temperature gradient.

2. Features of determinations of k-p relations in the Earth

For the outer core, determinations by the writer of ρ and p have mostly assumed uniform composition and phase and neglected deviations from an adiabatic temperature gradient. The P velocity distribution derived by Jeffreys (1939) and retained by Bolt has been used, and the S velocity treated as negligible. Values of k have been derived using $k = \rho\alpha^2$.

The most recent results are represented (Bullen 1968c) to 2 per cent accuracy by the relation

$$k = 2{\cdot}34 + 3{\cdot}00p + 0{\cdot}10p^2, \tag{2}$$

which applies to the whole lower mantle and core. To nearly the same accuracy the linear relations

$$k = 2{\cdot}29 + 3{\cdot}16p, \tag{3}$$

$$k = 1{\cdot}84 + 3{\cdot}44p, \tag{4}$$

represent the results for the lower mantle and core separately. The form (2) gives k continuous at N, with dk/dp increasing from 3·27 at N to 3·58 at $z = 4500$ km. The forms (3) and (4) give $\Delta k \approx -0{\cdot}1$ at N, and equation (4) gives $dk/dp = 3{\cdot}44$ throughout the core.

For a material of uniform chemical composition and phase, Birch's theory gives

$$3\left(\frac{\partial k''}{\partial p}\right)'' = \frac{12+49f}{1+7f} = 7 + \frac{5}{1+7f}, \tag{5}$$

where f denotes the compression. The formula (5) gives $(\partial k''/\partial p)''$ diminishing as f and p increase, and always less than 4. Minimum values of f of interest in the outer core, derived using the relation (Bullen, 1968d)

$$f(3k - 7p) = p, \tag{6}$$

are $f = 0{\cdot}13$, $0{\cdot}17$, $0{\cdot}18$, $0{\cdot}19$ at N and at $z = 3600$, 4000, 4500, respectively. The corresponding values of $(\partial k''/\partial p)''$ derived from equation (5) are 3·21, 3·09, 3·07, 3·05, respectively. The values of $(\partial k'/\partial p)'$ would, if significantly different, be expected to be smaller.

Inside E', to good approximation, conditions may reasonably be taken as adiabatic. In respect of both the mean value of dk/dp and its rate of change, it follows that Birch's formula (5) gives results discordant with equations (2) and (4). Birch (1952, pp. 266–7) considered that (5) could be reconciled with selected seismic data and accordingly inferred compatibility with uniform composition in the outer core. As will be seen, in the light of later developments the selection of data for this purpose does not, however, now seem justified, and the seismic background will be scrutinized in the next section.

3. The seismic and related evidence

In respect of dk/dp in the outer core, the writer's earlier determinations referred to in Section 2 are equivalent to using (1) with $\eta = 1$. Tables of Bullen & Haddon (1967, Tables 1 and 3) for Earth models with central densities ρ_0 of 13 and 15 g cm^{-3} show differences in g less than 2 per cent down to $z = 4000$ km, and less than 4 per cent down to $z = 4500$ km. If one assumes, following Birch (1963), that ρ_0 does not significantly exceed 13 g cm^{-3}, or even if one permits a moderate excess, it follows that g is closely determined throughout E'. Hence uncertainties in the estimation of $dk/dp - \eta$ through equation (1) arise mainly from observational uncertainties on $d\phi/dz$.

Birch (1952, p. 267) gave a table showing estimates of $g^{-1}\,d\phi/dz$ derived using data of Jeffreys (J) and Gutenberg (G) on α for the range $2900 < z < 5000$ km, approximately. For the J data, Birch's table shows $g^{-1}\,d\phi/dz$ increasing on the whole with z (and therefore p), the entries being equivalent to 2·1, 2·6 and 3·1 at $z = 3100$, 4500, 4800 km, respectively. For the G data, $g^{-1}\,d\phi/dz$ is shown as increasing from 2·1 at $z = 3200$ to 2·5 at 4100 km, but falling to 1·3 at $z = 4600$ and 5000 km, approximately. Birch inferred mean values of 2·6, 2·0 for $g^{-1}\,d\phi/dz$ in the outer core on the J, G values, respectively, and, noting an apparently considerable observational scatter, took as test point the G mean value of 2·0. Since this value gives $dk/dp = 3{\cdot}0$ on using equation (1) with $\eta = 1$, he concluded that his equation of state (5) agrees reasonably with the G data, assuming a homogeneous outer core. (He also noted the deviation from the J data, which he appears to have attributed to lack of high observational precision for the core.)

At the time of Birch's discussion, the outer core, as delineated by the writer (see Bullen 1942) and followed by Birch, extended to nearly 5000 km depth. A subsequent raising of the estimated depth of the lower boundary to about 4500 km (1965b), however, makes the data quoted by Birch on $d\phi/dz$ below 4500 km inapplicable to the present problem. In particular, the G data leading to the abnormally small estimate of 1·3 for $g^{-1}\,d\phi/dz$ at $z = 4600$, 5000 km must be rejected as irrelevant to the constitution of the outer core proper. This rejection results in the estimated mean value of $g^{-1}\,d\phi/dz$ being raised on the G data to at least 2·3. It also results in a significant increase of $g^{-1}\,d\phi/dz$ with z and p throughout E' on both the J and G data, contrary to the trend indicated by equation (5) if $\eta = 1$. Thus if E' is uniform in composition and phase, Birch's formula (5) is actually significantly discordant with his quoted data of both Jeffreys and Gutenberg, and further investigation is needed of possible ways of reconciliation.

Rather than use the limited entries in Birch's table, we shall now refer back to the data on α as given in the original papers of Jeffreys (1939) and Gutenbery (1951). Values of g will be taken from an appropriate Bullen–Haddon model (1967, Table 3)

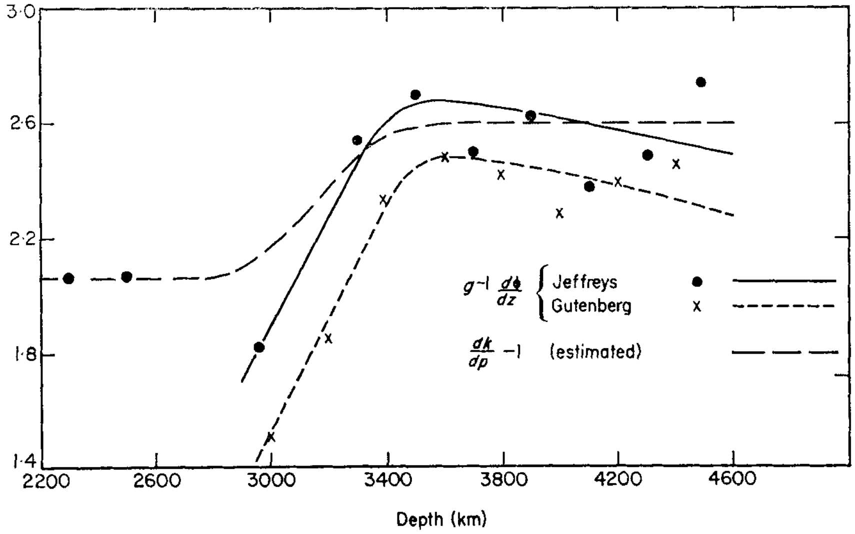

Fig. 1. Values of $g^{-1}\, d\phi/dz$ derived using seismic data of Jeffreys and Gutenberg. The figure also shows a curve giving preferred estimates of $dk/dp-1$ over the range 2300–4500 km depth.

with $\rho_0 = 13\,\mathrm{g\,cm}^3$, and β will continue to be taken as zero in E'. Fig. 1 shows values of $g^{-1}\,d\phi/dz$ thus derived on the J and G data. The figure shows plotted points and smoothed curves drawn through them. The G points are less reliably determined than the J points because the original G data on α are presented in the form of a curve from which it is difficult to derive gradients accurately.

The two curves in Fig. 1 are nearly parallel, the J curve giving slightly greater values at practically all depths in the outer core. The curves agree in three significant respects. They both show: (a) a steady increase in $g^{-1}\,d\phi/dz$ in the outermost 700 km of the outer core, i.e. in the region E_1, amounting to 0·8—0·9; (b) a mean value of $g^{-1}\,d\phi/dz$ in the outer core markedly greater than that in the lower part of the mantle; (c) approximately constant values, equal to about 2·6 and 2·4 on the J and G data, of $g^{-1}\,d\phi/dz$ in the region E_2 of the outer core. (Both sets of data suggest a slight decrease of $g^{-1}\,d\phi/dz$ with z inside R_2, but not significantly enough to warrant attention for the present.)

The features (a) and (b), at least, are supported in more recent seismic investigations of the outer core. For example, Ergin (1967), who gives a rather complicated P velocity distribution for the whole core, shows $d\alpha/dz$ practically constant for $2900 < z < 4000$ km; on his results, $g^{-1}\,d\alpha^2/dz$ increases steadily from about 1·9 at N to 3·1 at 4000 km. Some questions have been raised concerning aspects of Ergin's distribution at greater depths, and it is indeed very desirable that further observational checks should be made on $d\alpha/dz$ in the outer core, taking account of seismic phases such as *SKS* as well as *PKP*, etc. The overall evidence favouring (a), (b) and (c) is, however, sufficiently suggestive to warrant a study of its implications.

4. Implications on finite-strain theory

The seismic details give grounds for suggesting that Birch's finite-strain formulae fall short of adequacy for the materials of the Earth's core. First, the magnitude of $g^{-1}\,d\phi/dz$ in E_2 confirms the incompatibility of the formula (5) with the pertinent seismic data. On the evidence in Section 2 indicating minimum values of f between 0·17 and 0·19 in E_2 (these values have been derived using the finite-strain theory), equation (5) gives maximum values of dk/dp ranging from 3·09 to 3·05. On applying equation (1), the J and G data referred to under (c) above give $dk/dp - \eta = 2{\cdot}6$, 2·4, respectively, in E_2. Thus the seismic data and the finite-strain theory would together entail $\eta \leqslant 0{\cdot}7$ in E_2. A value of η as small as this would demand improbable thermodynamical conditions inside the outer core.

Next, it will be shown in Section 5 that η is probably significantly greater than unity near the top of E_1, a result which would entail a slight reduction in previous estimates of ρ just below N, and hence a reduction in k, since $k = \rho\alpha^2$. The estimated jump Δk would then also be reduced, thereby accentuating the difficulties already referred to in Section 1 of fitting Birch's finite-strain formulae for k.

It seems therefore that Birch's finite-strain formulae need some significant modifications before they can be applied in full detail in the Earth's core. In view, however, of the degree of strength of the evidence adduced by Birch in constructing his theory, it is a desirable guiding principle when a choice of hypotheses has to be made to deviate no further than necessary from the results of the theory. Detail in the present paper incidentally suggests the order of magnitude and direction of the amendments that may be required.

A further point is that curves presented by Birch (1963) using shock-wave data indicate values of order 6 units for $\partial k/\partial p$ for iron at the pressures prevailing in the Earth's inner core. These curves correspond to Hugoniot thermodynamical conditions which of course are markedly different from adiabatic and isothermal conditions. It is a question, however, as to whether the adjustments required to give the isothermal gradient $(\partial k''/\partial p)''$ would permit a reduction from 6 to 3 units, which is approximately the maximum value of $(\partial k''/\partial p)''$ as given by equation (5) near the Earth's centre.

5. Implications on outer core structure

Orson Anderson (1968) has inferred experimentally that for a wide range of metals and rocks $(\partial^2 k''/\partial p^2)''$ is undetectably different from zero. The same would be expected to apply to $(\partial^2 k'/\partial p^2)'$ to good approximation. If, taking note of Anderson's findings, we postulate $d^2k/dp^2 = 0$ in the outer core, equation (1) gives

$$\frac{d\eta}{dz} = -\frac{d}{dz}\left(g^{-1}\frac{d\phi}{dz}\right). \tag{7}$$

With the J and G data as represented in Fig. 1, the result (a) and equation (7) would give η decreasing by 0·8 – 0·9 inside E_1. It is implausible that a variation in η of this size could be accounted for solely by temperature effects. Thus the seismic data and an assumed constancy of dk/dp would require the chemical composition or phase to change continuously inside E_1, the departure from uniform conditions being greatest at the top and diminishing with depth throughout E_1. Since it is expected that $\eta \geqslant 1$ everywhere in the core, the minimum value of η indicated at the top of the core would be 1·8–1·9. If, on the other hand, d^2k/dp^2 deviates significantly from zero inside E_1, Anderson's results would again imply that E_1 is not homogeneous. So in either case, some degree of change of composition or phase throughout E_1 seems indicated.

For the region E_2, the postulate $d^2k/dp^2 = 0$ and the data of Fig. 1 give η approximately constant. The greater η is in E_2, the greater is the value of dk/dp given by equation (1), and hence the greater is the discrepancy from Birch's theory. On this and general grounds, the region E_2 is probably nearly uniform in composition and phase, with η equal to about unity, and we shall from now on assume this to be the case. The most probable average value of dk/dp in E_2 is thus about 3·6 on the *J* data, 3·4 on the G.

In seeking to estimate the distributions of η and dk/dp inside E_1, we have to refer back to (1) without now assuming $d^2k/dp^2 = 0$. With the data in Fig. 1, (1) is now a single equation of condition on the two variables dk/dp and η, and there is no obvious specific evidence to determine how the observational variation of $g^{-1}d\phi/dz$ inside E_1 should be distributed between dk/dp and η. It is, however, possible to arrive at a reasonable assessment as follows.

Since some inhomogeneity is indicated inside E_1, it is necessary to have $\eta > 1$ in at least part of E_1. The overall evidence indicates that the departures from homogeneity are not great—taking $\eta = 1$ throughout the outer core has proved in the past to be a fairly satisfactory first approximation. On the present seismic evidence, we have seen that formally taking $d^2k/dp^2 = 0$ in E_1 would require $\eta = 1{\cdot}8$–$1{\cdot}9$ at the top of E_1. (Taking Birch's formula (5), which gives $d^2k/dp^2 < 0$, would require η to be still larger.) Allowing dk/dp to increase inside E_1 by an amount between zero and 0·8 would, however, allow η to be less than 1·8 and greater than unity at the top of E_1. Inside the mantle at $z = 2700$ km, the J curve gives $g^{-1}d\phi/dz = 2{\cdot}07$ and hence, assuming $\eta = 1$, $dk/dp = 3{\cdot}07$. (This value is a little less than the value given in the writer's original investigation (1949a) because of revisions in some of the data used, including the revised moment of inertia of the Earth.) On evidence previously given (Bullen, 1949a), the most probable value of dk/dp at the bottom of the mantle is equal to the value 3·1 at 2700 km. Formal assumption of continuity of dk/dp at the mantle–core boundary would thus give, at the top of E_1, $dk/dp = 3{\cdot}1$ and, on the J data, $\eta = 1{\cdot}4$. This value happens to be about midway between the bounds 1·0 and 1·8–1·9 between which η is expected to lie at the top of the core, and at the same time results in the least overall deviation from the formula (5) over the whole range $2700 < z < 3600$ km. Thus on the J data, the most probable values of η are about 1·4 at the top of E_1, and 1·0 at $z = 3600$ km, the corresponding dk/dp being about 3·1 and 3·6. The G data would give a slightly smaller value of dk/dp at $z = 3600$ km, but about the same range of values of η. Fig. 1 shows the variation of $dk/dp - 1$ in the outer core as yielded on the above procedure.

It would incidentally appear that the whole range of depth from 2700 to 3600 km, which embraces regions on both sides of the mantle–core boundary N, is somewhat abnormal. Just below N, the excess of the implied rate of density increase above that due to normal compression would be about 40 per cent, or one-fifth of the excess rate indicated (1949a) in the region D'' ($2700 < z < 2900$ km). There is also the suggestion that a modified version of the mantle–core phase-change hypothesis earlier considered by Ramsey (1948) and the writer (1949b), extended to cover a finite range of depth, might be relevant to this part of the Earth.

Irrespective of the mantle–core phase-change theory, it would be of interest to discriminate if possible between the alternatives of composition change and phase change inside E_1. Although the indicated increase of dk/dp from 3·1 to 3·6 inside E_1 is not unduly great, it gives d^2k/dp^2 rather greater inside E_1 than inside either the lower mantle or E_2. It is possible that a continuous change of phase could account for the effect more readily than change of chemical composition. Further, if the increase of dk/dp inside E_1 should be solely due to compositional changes, the changes might be enough to inhibit convection currents and confine the main convection currents in the core to the range $3600 < z < 4500$ km. A range of depth of the order of only 1000 km might well be too narrow to meet requirements of the dynamo theory of the Earth's main magnetic field, so that evidence from geomagnetism might assist in the discrimination. There is thus a faint suggestion of continuous phase change near the top of the core, the rate of change diminishing with depth.

A formal possibility not considered above is that the S velocity β reaches significant values in the outer core. Through the relation $\phi = \alpha^2 - 4\beta^2/3$, this possibility would enable the assessments of $g^{-1}\,d\phi/dz$ to be readily reduced and could in fact remove all difficulties of reconciliation with Birch's results. However, the calculations of Takeuchi and Molodenski on an upper bound to rigidity in the outer core would seem to preclude this possibility. Alternatively, a parameter representing dissipation in the transmission of seismic wave energy could cause modifications to estimates of $d\phi/dz$. But any significant effect would be expected to be accompanied by greater attenuation of core P waves than has been observed. Thus escape in these ways from the conclusions drawn does not seem plausible at present.

6. Conclusions

The main conclusions of this paper are that there is a strong suggestion of slight inhomogeneity inside the outermost 700 km of the core and that Birch's finite-strain results are not finely applicable to the core. There is a slight suggestion that the inhomogeneity in the outer core may be connected principally with changes of phase rather than changes of chemical composition. The possibility of phase or composition changes near the top of the core has so many important implications that is very desirable to test further the reliability of the pertinent seismic data. It is suggested that some special attention be given to estimating the precision of determinations of $d\alpha/dz$ over the range $2900 < z < 4500$ km.

The University of Sydney,
Australia.

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