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COMPRESSION FAILURE OF QUASIBRITTLE MATERIALS AND SIZE EFFECT

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ABSTRACT

The paper presents a model for size effect in the failure of columns or other compression made members made of quasibrittle materials such as concretes, rocks, or composites. The size effect is explained by energy release due to transverse propagation of a band of axial splitting cracks. The microslabs of the material between the splitting cracks are considered to buckle and undergo post-critical deflections. The failure condition is based on the equality of the energy released from the uncracked elastic member and the energy consumed by the axial splitting cracks in the band. The analysis leads to a closed-form expression relating the nominal strength of the structure and the structures characteristic size. The resulting formulation is compared to the test results on model reinforced concrete columns reported previously by Bažant and Kwon (1994). Satisfactory match of the test data is achieved.

INTRODUCTION

In a number of previous studies, it has been shown that structures made of quasibrittle materials such as concrete, rock or composites, exhibit a significant size effect (Bažant, 1993a). The nominal strength of structure is not constant, as predicted for materials following yield or strength failure criteria, but decreases with an increasing size of structure. The size effect is due to fracture or damage phenomena, particularly the fact that these modes of material failure are governed by energy release and consume a certain amount of energy which is a material property. Previous work has demonstrated that such size effect exists in various tensile and shear failures, including diagonal shear failure of reinforced concrete beams, torsional failure, punching shear failure, pullout of bars and anchors, and others. In a recent work (Bažant, 1993b), a theoretical argument has been presented showing that a deterministic size effect due to energy release should also exist in compression failures in which the energy release depends on the structure size, for example reinforced concrete columns. The general formulation of the size effect for compression failures has been presented, however, detailed formulas for the size effect have not been derived and comparisons with test results have not been made. To do that, is the objective of the present brief conference paper. The detailed analysis will be presented in a forthcoming report and journal article (Bažant and Xiang, 1994).

Compression failure of brittle ~~of~~ or quasibrittle materials has been studied extensively and important results have been achieved (Biot, 1965; Ashby and Hallam, 1986; Batto and Schulson, 1993; Horii and Nemat-Nasser, 1985, 1986; Kendall, 1978; Bažant, 1967; Bažant et al. 1991; Sammis and Ashby, 1986; Shette et al., 1986). However, attention has been focused primarily on the microscopic mechanisms that initiate the compression failure rather than on the final global mode of failure and the size effect.

ANALYSIS OF ENERGY RELEASE

Consider a prismatic compression member (a column) shown in Fig. 1, having length L, width D (taken as the characteristic dimension), and unit thickness $b = 1$. One end cross section is fixed. The other is subjected to axial displacement u and rotation θ and is loaded by axial compressive force P of eccentricity e. The initial normal stress in the cross sections before any fracturing is

$$\sigma_0(x) = -\frac{E}{L}\left[u + \theta\left(\frac{D}{2} - x\right)\right] \tag{1}$$

where E = Young's elastic modulus, and x = transverse coordinate measured from the compressed face (Fig. 1a). We now assume that, at a certain moment of loading, axial cracks of spacing s and length h, forming a band as shown in Fig. 1a,b,c, suddenly appear and the microslabs of the material between the axial cracks, behaving as beams of depth s, lose stability and buckle. This can happen in any one of the three mechanisms shown in Fig. 1a,b,c. For all of them the present type of approximate solution turns out to be identical if the length of the cracks in the pair of inclined bands in Fig. 1c is denoted as $h/2$. The critical stress for the microslab buckling shown in Fig. 1a,b,c is, in all cases,

$$\sigma_{cr} = -\frac{\pi^2 E s^2}{3h^2} \tag{2}$$

The key idea now is the calculation of the change in stored strain energy caused by buckling (Bažant, 1993b). On the side of the crack band, there is obviously a zone in which the initial stress σ_0 is reduced. For the sake of simplified analysis we assume that the stress in the shaded triangular areas of Fig. 1a,b,c is reduced all the way to σ_{cr} while outside these areas the initial stress does not change. The triangular areas are limited by the so-called "stress diffusion lines" of slope k, whose magnitude is close to 1 (the effective approximate value of k we determine empirically). For the analysis of size effect the important fact is that k is a constant if geometrically similar columns are considered. In the shaded triangular stress-relief zones, the strain energy density before and after fracture is given by the areas of the triangles 0120 and 0340 in Fig. 1d, and so the loss of strain energy density along a vertical line of horizontal coordinate x (Fig. 1a) is

$$\Delta\bar{\Pi}_r = \frac{\sigma_0^2(x)}{2E} - \frac{\sigma_{cr}^2(x)}{2E} \tag{3}$$

The situation is more complicated in the crack band. The microslabs buckle, and the energy associated with the postbuckling behavior must be taken into account, which is a key idea of the present approach (Bažant, 1993b). The strain energy density before the buckling of microslabs is given by the area 0120 in Fig. 1e. The analysis of postbuckling behavior of columns (Bažant and Cedolin, 1991, Sec. 1.9 and 5.9) indicates that the stress in the axis of the microslabs follows, after the attainment of the critical load, the straight line 35 which has a very small positive slope (precisely equal to $\sigma_{cr}/2$). This slope is far smaller than the slope E before buckling and can therefore be neglected. So the postbuckling behavior is approximately a horizontal plateau 35 in Fig. 1e (however, this is not the same as plastic behavior because unloading proceeds along the path 530). Because the microslabs remain elastic during buckling, the stress-strain diagram

035 is fully reversible and the energy under this diagram is the stored elastic strain energy. The triangular area 0340 in Fig. 1e represents the axial strain energy density of the microslabs and the rectangular area 35643 represents the bending energy density. The change in strain energy density in the microslabs is the difference of areas 0120 and 03560 in Fig. 1e, that is,

$$\Delta\bar{\Pi}_c = \frac{\sigma_0^2(x)}{2E} - \left[\sigma_{cr}(x)\epsilon_c(x) - \frac{\sigma_{cr}^2(x)}{2E}\right] \tag{4}$$

where ϵ_c is the axial strain of the microslabs in the crack band after buckling (it is important that it is generally not equal to 04 or 02 in Fig. 1e).

Integration of (3) and (4) yields the total loss of potential energy at constant u and θ:

$$\Delta\bar{\Pi} = \int_0^a \left(\frac{\sigma_0^2(x)}{2E} - \frac{\sigma_{cr}^2(x)}{2E}\right) 2k(a-x)dx + \int_0^a \left\{\frac{\sigma_0^2(x)}{2E}\left[\sigma_{cr}(x)\epsilon_c(x) - \frac{\sigma_{cr}^2(x)}{2E}\right]\right\} h\,dx \tag{5}$$

where a = horizontal length of the crack band (Fig. 1a,b,c). This energy must be equal to the energy consumed by the formation of the surfaces of all the axial splitting cracks. Assuming that there is no other energy dissipation but fracturing, we may write the energy balance criterion of fracture mechanics as:

$$-\left[\frac{\partial\Delta\Pi}{\partial a}\right]_{\theta,u} = \frac{\partial}{\partial a}\left(G_f h\frac{a}{s}\right) = \frac{h}{s}G_f \tag{6}$$

where G_f is the fracture energy of the axial splitting cracks, assumed to be a material property.

The axial strain in the crack band can be determined from the compatibility condition. Because the end cross sections are assumed to be fixed during buckling (i.e., u, θ = constant), the stress in the blank areas of the column in Fig. 1a,b,c remains constant, and so the line segment GJ in Fig. 1a at any x does not change length. Expressing the change of length of this segment on the basis of σ_{cr}, ϵ_c and σ_0 and setting this change equal to zero, one obtains the following compatibility condition

$$\epsilon_c(x) = \frac{\sigma_0(x)}{Eh}[h + 2k(a-x)] - \frac{2k}{h}(a-x)\frac{\sigma_{cr}(x)}{E} \tag{7}$$

It can be shown (Bažant, 1993b) that the foregoing equations yield a failure criterion of the form

$$f_c(k, a, s, h, G_f, \sigma_N) = 0, \qquad \sigma_N = \frac{P}{bD}\left(1 + \frac{6e}{D}\right) \qquad (b = 1) \tag{8}$$

in which P = maximum load, and σ_N = nominal strength of the compression member. The unknown spacing of the axial cracks, s, can be determined from the condition that load P is minimized, which leads to the condition $\partial\delta f_c/\partial\delta_s = 0$. The value of the diffusion slope k can approximately be further estimated as the k−value which gives the exact energy release rate for an edge-cracked tensile fracture specimen according to linear elastic fracture mechanics. It can further be shown that this condition indicates that s increases with size D as $D^{-2/5}$. In case of slender columns which undergo global buckling, the effect of slenderness on σ_N can be taken into account by one the two concepts proposed in Bažant (1993, Eqs. 45–50). The simpler of these two concepts is based on modifying eccentricity e in Eq. 8 on the basis of the magnification factor for global buckling of columns (e.g., Bažant and Cedolin, 1991, Chapter 1).

Assuming that the ratio a/D for failures of compression members of various sizes is the same, the following relationship between characteristic dimension D and the nominal strength σ_N can be deduced from the foregoing formulation.

$$D = k\frac{(\sigma_0 - \sigma_{cr} + \sigma_N)(\bar{\sigma} + \sigma_{cr} - \sigma_N)}{(\sigma_N - \sigma_{cr})}, \qquad \sigma_N = \frac{P}{bD} \qquad (b = 1) \tag{9}$$

in which σ_0, σ_{cr} = constants, σ_{cr} = critical normal compressive stress for the microslab buckling = intrinsic compression strength of the material.

COMPARISON TO TEST RESULTS

The present formulation has been compared and calibrated with the test data for failure of reduced-scale reinforced concrete columns reported in Bažant and Kwon (1994); see Fig. 2. The data points are the experimental results for various column slendernesses $\lambda = r/\ell$ where r = radius of gyration of the cross section and ℓ = effective length of the columns. The predictions according to the present theory are indicated by the curves. The horizontal asymptote of the size effect curve according to the strength theory, and the inclined asymptote according to linear elastic fracture mechanics (which has the slope $-2/5$) are also marked in the figures. As one can see, reasonable agreement with the test results can be achieved. It should be noted that the existing code procedures for concrete structures give no size effect, which is in disagreement with the present test results.

CONCLUSION

Compression failure of quasibrittle materials can be described as the propagation of a band of axial microcracks. Assuming the axial stress transmitted by the band to be limited by buckling of the microslabs of the material between the axial splitting cracks, the failure loads can be calculated on the basis of the energy release. This calculation predicts a size effect which is in reasonable agreement with available test results.

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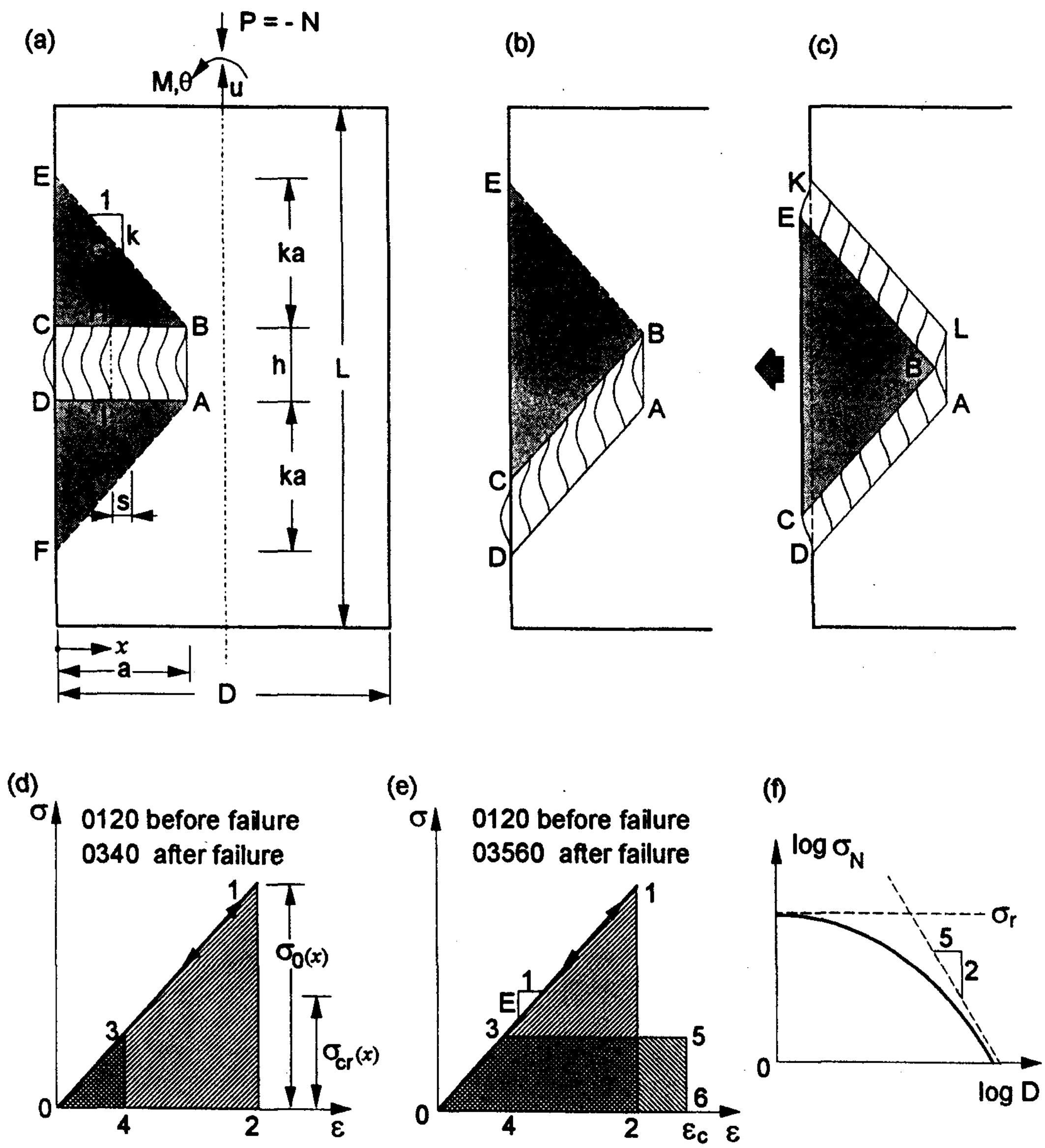

Fig.1 (a-c) compression splitting bands, (d,e) energy release, (f) size effect

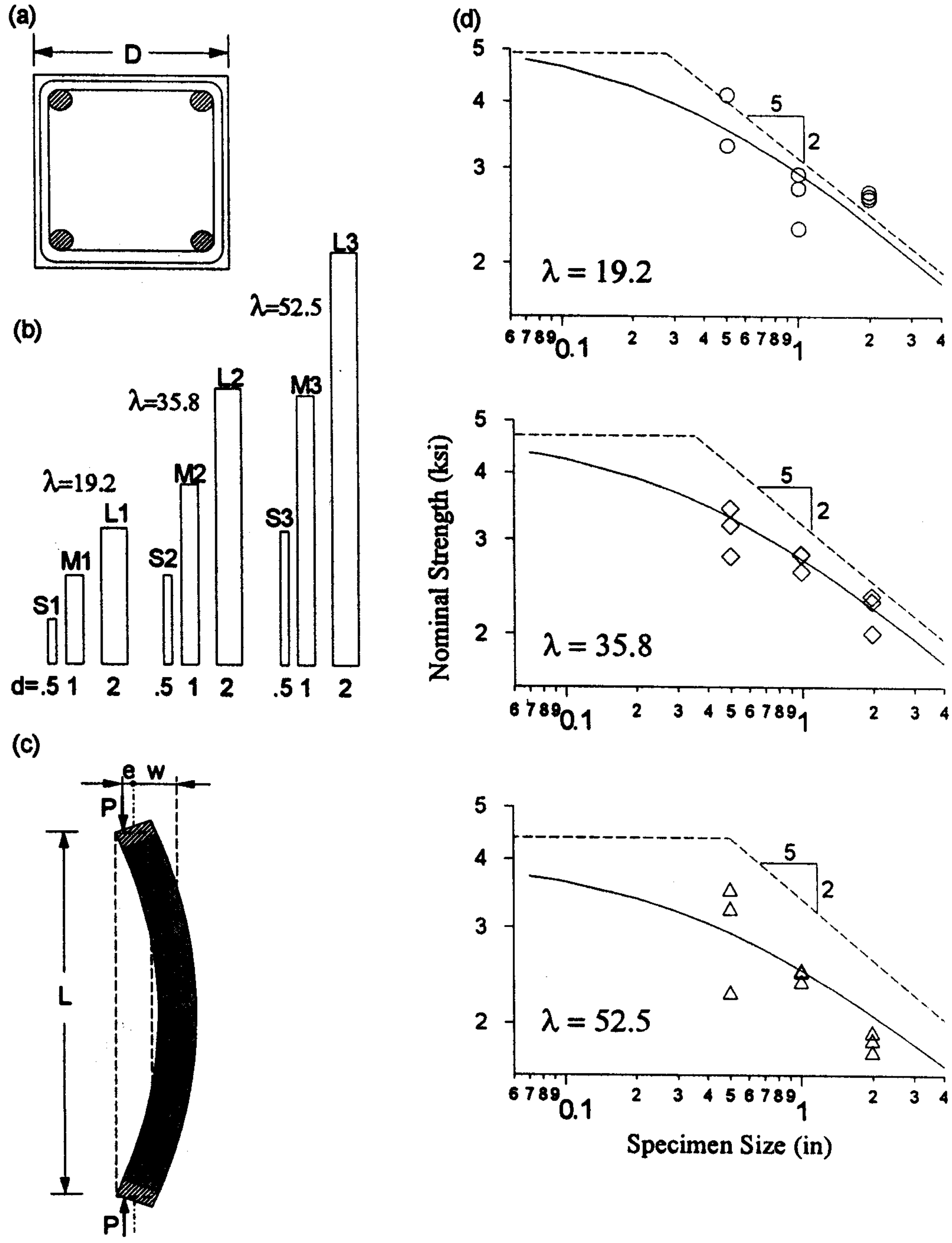

Fig.2 Reduced scale columns tested and size effect observed, with comparison to theory (solid lines, coef. of variation of deviation ω =11% and a/D=.22, h= .93in., G_f= .078 lb/in)