

Compressional-wave velocities in attenuating media: A laboratory physical model study

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ABSTRACT

Elastic-wave velocities are often determined by picking the time of a certain feature of a propagating pulse, such as the first amplitude maximum. However, attenuation and dispersion conspire to change the shape of a propagating wave, making determination of a physically meaningful velocity problematic. As a consequence, the velocities so determined are not necessarily representative of the material's intrinsic wave phase and group velocities. These phase and group velocities are found experimentally in a highly attenuating medium consisting of glycerol-saturated, unconsolidated, random packs of glass beads and quartz sand. Our results show that the quality factor Q varies between 2 and 6 over the useful frequency band in these experiments from ~200 to 600 kHz. The fundamental velocities are compared to more common and simple velocity estimates. In general, the simpler methods estimate the group velocity at the predominant frequency with a 3% discrepancy but are in poor agreement with the corresponding phase velocity. Wave velocities determined from the time at which the pulse is first detected (signal velocity) differ from the predominant group velocity by up to 12%. At best, the onset wave velocity arguably provides a lower bound for the high-frequency limit of the phase velocity in a material where wave velocity increases with frequency. Each method of time picking, however, is self-consistent, as indicated by the high quality of linear regressions of observed arrival times versus propagation distance.

INTRODUCTION

Although seemingly trivial, it is worthwhile to consider how the velocities of elastic waves propagating in real attenuating media should be determined experimentally. On the one hand, theoretical analyses that link elastic properties to wave

propagation phenomena are almost exclusively based on the motion of monochromatic, elementary plane waves travelling at the frequency-dependent phase velocity. On the other hand, in many real laboratory and field situations wave velocities are determined by charting the picked transit times versus propagation distance of a band-limited wave packet, the general assumption being that such methods yield group velocity. However, phase and group velocities only coincide with certainty when there is no wave-velocity dispersion.

Attenuation is intrinsically linked to dispersion (Aki and Richards, 1980). The evolution of a propagating wave packet is related to the distance traveled and the attenuative properties of the medium. To determine a velocity in practice, one would plot the transit time of a certain feature of the pulse, such as the first amplitude extremum, against propagation distance. However, if the shape of the pulse is continually evolving, there is no reason to suspect a priori that the moveout of such a feature necessarily provides a good measure of the wave velocity.

It is not immediately obvious what a more definitive measure of wave velocity might be. Perhaps most fundamental are the phase and group velocities, the dispersion of which can provide important insight into mechanisms influencing attenuation and hence the petrophysical properties of the material (Winkler, 1983; Jones, 1986; Brown and Seifert, 1997). The determination of these wave velocities is nontrivial and requires the initial phase information of the outgoing pulse, which is often absent in everyday field practice. As a result, transit-time determinations such as picking the first peak or zero crossing of amplitude are often used instead when calculating wave velocities.

In the absence of attenuation, the propagation velocity of the amplitude envelope maximum is equivalent to the group velocity (e.g., Merkulova, 1967; Sachse and Pao, 1978; Raggolino, 1981). However, the definition of the group velocity in attenuating media is less clear as the propagating wave packet shape evolves with distance (e.g., Hines, 1951).

Although a complete physical description of a material requires that the velocity dispersion be determined, picking

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a single traveltime in a pulse transit experiment will probably remain common practice. Here, the group and phase velocities of a highly attenuating medium are compared directly to those determined using simpler time-picking criteria. The intent is to qualitatively describe the ambiguities that may occur when determining velocities in attenuating media and to suggest simple estimators of the group and phase velocities from recorded propagated signals. Further research is required to apply these findings quantitatively to specific cases such as seismic and sonic data. We take an experimental physical model approach much along the lines of that first used by Wuenschel (1965). The advantage of this approach is that waves propagate through a real material whose attenuative character has not been predetermined by many of the simplifying assumptions brought to bear in the study of attenuation. The results have particular implications to the interpretation of laboratory physical property determinations, sonic logging, crosswell tomography, and general seismology.

DESCRIPTION OF EXPERIMENTS

The experiments were carried out using a pulse transmission technique. Transmitting and receiving transducers were mounted on aluminum buffer caps at opposing ends of the cylindrical samples with lengths from 10 to 50 mm. The attenuating media consisted of glycerol-saturated packs, contained within an acrylic tube (Figure 1), of either glass beads or quartz sand (Table 1). Glycerol was used as a saturant because its high

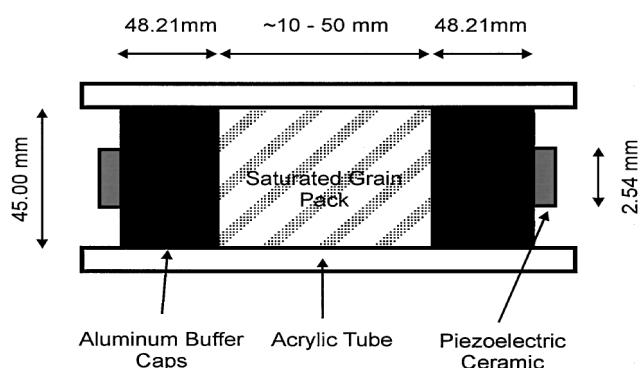


FIG. 1. Sample configuration. A signal is recorded through several lengths of identically prepared saturated bead packs.

wave velocity minimized the potential for contamination of the signal attributable to head waves through the acrylic container. The pack lengths were changed to provide measures of the evolution of the wave packet with distance propagated.

The packs were prepared by first pouring glycerol into the acrylic tube was closed at the bottom with an aluminum buffer cap. Grains were slowly dropped into the fluid while the tube was forcibly shaken to ensure consistent packing of the grains with lengths determined to ± 0.1 mm by a vernier caliper. The observed porosities of 36% and 38% (Table 2) agreed well with that expected for a random pack of spheres (Bourbie et al., 1987).

Piezoelectric ceramics (BaTiO_3 , 25.41-mm diameter, 1-MHz resonant frequency) were mounted to the ends of the aluminum buffer rods (45.00 mm in diameter, 48.21 mm in length). The transmitter was activated with a square-wave input voltage. The resulting pulse propagated through the aluminum buffer caps, the saturated grain pack, and the second aluminum

Table 2. Comparison of measured and estimated wave speeds.

Sample	#3 Quartz	#3 Glass	#7 Glass
Porosity	0.36 ± 0.04	0.38 ± 0.02	0.38 ± 0.01
Wood's estimate (km/s)	2.2	2.3	2.3
V_g (km/s)	2.24 ± 0.04 (0.998)	2.42 ± 0.02 (0.999)	2.30 ± 0.04 (0.999)
V_p (km/s)	2.00 ± 0.13 (0.975)	2.48 ± 0.06 (0.987)	2.37 ± 0.03 (0.999)
V_s (km/s)	2.52 ± 0.10 (0.997)	2.58 ± 0.03 (1.000)	2.58 ± 0.01 (1.000)
V_x (km/s)	2.29 ± 0.04 (0.998)	2.46 ± 0.02 (0.999)	2.37 ± 0.03 (0.999)
V_m (km/s)	2.31 ± 0.03 (1.000)	2.46 ± 0.01 (1.000)	2.35 ± 0.02 (1.000)
V_H (km/s)	2.19 ± 0.02 (1.000)	2.39 ± 0.02 (1.000)	2.38 ± 0.03 (1.000)
V_{gmax} (km/s)	2.24 ± 0.04 (360 kHz)	2.43 ± 0.02 (150 kHz)	2.58 ± 0.05 (800 kHz)
V_{pmax} (km/s)	2.11 ± 0.07 (740 kHz)	2.49 ± 0.08 (350 kHz)	2.44 ± 0.03 (100 kHz)

Note: For V_{gmax} and V_{pmax} the values in brackets are the frequency at which these maximum values of velocity are observed. In all others, the values in brackets are the coefficient of correlation of the least squares linear fit of the observed transit times versus relative lengths as in Figure 4.

Table 1. Physical properties of pack constituents.

	#3 Ottawa sand	#3 Glass beads	#7 Glass beads	Glycerol
Material	Quartz	Soda-lime glass	Soda-lime glass	$\text{C}_3\text{H}_8\text{O}_3$
Mean diameter (mm)	0.81 ± 0.14	0.67 ± 0.05	0.22 ± 0.04	NA
Aspect ratio	0.76 ± 0.18	1	1	NA
Density (g/cm^3)	2.648^2	2.48 ± 0.01^3	2.47 ± 0.01^2	1.26 ± 0.01^2
Isentropic bulk modulus (GPa)	37.8^1	37.8^4	37.8^3	4.67^3
Shear modulus (Gpa)	44.4^1	25.4^5	25.4^4	0
Shear wave speed (km/s)	4.092^6	$\sim 3.2^6$	$\sim 3.2^5$	0
Compressional wave speed (km/s)	6.047^1	$\sim 5.4^5$	$\sim 5.4^3$	1.926 ± 0.001^2

¹Bass (1995).

²Measured in this study.

³Calculated: $K = \rho(V_p^2 - 4V_s^2/3)$.

⁴Calculated: $\mu = \rho V_s^2$.

⁵Estimated from glass 1 of Carmichael (1982).

⁶Soga (1968).

buffer cap and were recorded by the piezoelectric receiver at a 1-GHz sampling rate. The first arrivals were finally isolated with a Gaussian window.

Phase velocities were determined by calculating the phase difference $\Delta\phi(\omega) = (\phi_2(\omega) - \phi_1(\omega))$ from the unwrapped phase spectrum of two recorded waveforms produced by propagation through different lengths of pack (Figure 2). Such a relative measurement obviates measurement of the initial phase of the source. The phase lag time between observations was $\Delta t_p(\omega) = \Delta\phi(\omega)/\omega$, with the phase velocities equal to $(x_2 - x_1)/\Delta t_p(\omega)$. Similarly, the group delay times $\Delta t_g(\omega) = d(\Delta\phi(\omega))/d\omega$ led to the group velocity $V_g(\omega) = (x_2 - x_1)/\Delta t_g(\omega)$ (e.g., Winkler and Plona, 1982; Droin et al., 1998).

These phase and group velocities were compared to more conventional measures determined from picking the first onset of energy (Molyneux and Schmitt, 1999), the first amplitude extremum (King, 1966), the first peak of the Hilbert transform amplitude envelope (Taner et al., 1979), and crosscorrelation lags (Lu et al., 1990). These velocity values are referred to as the signal (V_s), the first extremum (V_m), the peak of the amplitude envelope (V_H), and the crosscorrelation (V_x) velocities, respectively.

Attenuation is intrinsically linked to wave-velocity dispersion by the requirement that a real pulse propagating in an attenuating medium must be causal (e.g., Futterman, 1962). As a consequence, although this study focuses on wave-velocity determination, it is useful to characterize the corresponding attenuation because this is relevant to the evolution of the pulse. Attenuation coefficients are found by first noting that

the absolute Fourier amplitude spectrum of a pulse observed for bead pack length x_1 is (after Yin, 1993)

$$A(\omega, x_1) = S(\omega)D(\omega)e^{-\alpha(\omega)x_1}(1 - R^2)G(\omega, x_1), \quad (1)$$

where $S(\omega)$ is the amplitude spectrum of the outgoing pulse produced by the transmitter, $D(\omega)$ is the amplitude spectrum of the detecting transducer's response, $\alpha(\omega)$ is the frequency-dependent attenuation of the sample to be determined, R is the reflection coefficient from the aluminum buffer to the sample, and $G(\omega, x_1)$ is the diffraction loss resulting from the geometry of the sample. Upon rearrangement, the ratio of the spectra for two subsequent waveforms obtained through lengths x_2 and x_1 is then used to find $\alpha(\omega)$:

$$\alpha(\omega) = -\ln \left[\frac{\left(\frac{G(\omega, x_1)}{A(\omega, x_1)} \right) \left(\frac{A(\omega, x_2)}{G(\omega, x_2)} \right)}{(x_2 - x_1)} \right]. \quad (2)$$

In such a comparison, the source and detector responses and the influence of reflectivity between the sample and the aluminum buffers is eliminated substantially, simplifying the determination of $\alpha(\omega)$. The $G(\omega, x)$ was determined by measuring the decay of amplitude with distance through the setup when it was filled only with glycerol; this function was found to be independent of frequency and was best described empirically by an exponential decay $G(x) = e^{-\beta x}$ with $\beta = 0.0067 \text{ mm}^{-1} \pm 0.0006 \text{ mm}^{-1}$. Finally, the attenuation, $\alpha(\omega)$, was converted to a quality factor via the relationship $Q = \omega/2\alpha(\omega)V_p(\omega)$.

RESULTS

The waveforms obtained in the tests noticeably broaden with propagation length, indicative of dispersion (Figure 3). From the Fourier transforms, both group and phase delay times are calculated at each frequency over the range from 0 to 0.8 MHz

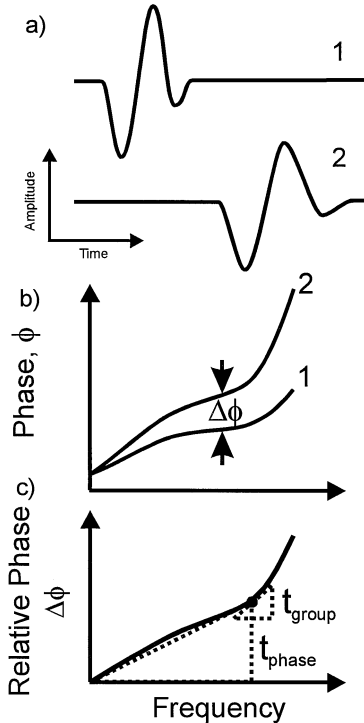


FIG. 2. Illustration of Fourier method for determining phase and group delay. (a) Elastic wave pulses 1 and 2 observed through two lengths x_1 and x_2 of the bead pack, respectively. (b) Unwrapped phase of the Fourier transforms of pulses 1 and 2; $\Delta\phi(\omega)$ is the relative phase difference. (c) Determination of the frequency-dependent time phase and group time lags.

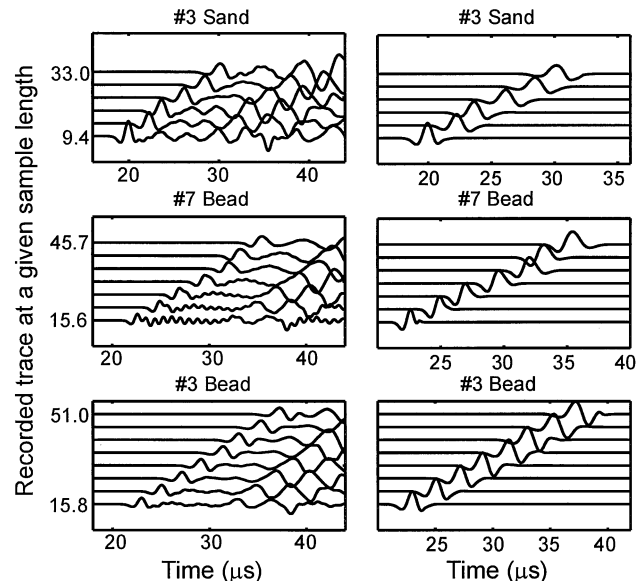


FIG. 3. Observed (left panels) and isolated (right panels) waveform amplitudes with propagation distance through the three different saturated packs.

for every combination of the traces acquired for a given sample. The redundancy of measurement enables statistical error estimation on the determination of $V_p(\omega)$ and $V_g(\omega)$. One such series of relative group time lags for the #7 glass bead pack at a frequency of 0.38 MHz is shown in Figure 4. The 21 group delay times are plotted versus the relative length for the seven grain pack lengths tested. The slope of a line fit to these data provides the group wave velocity at this frequency. The final

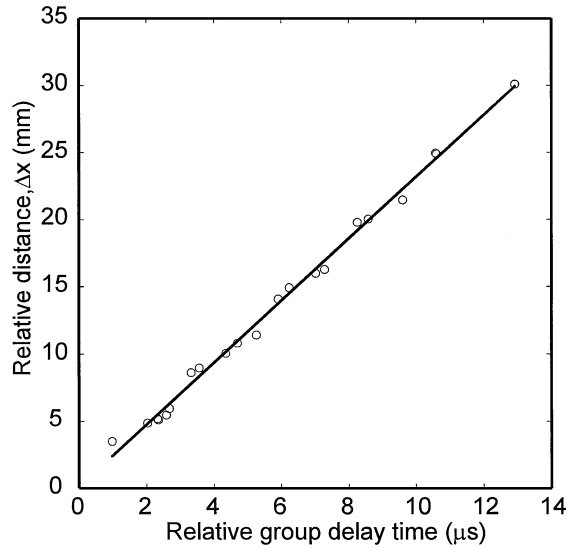


FIG. 4. Relative distance ($x_2 - x_1$) versus relative group delay time ($t_2 - t_1$) for #7 glass beads at 0.38 MHz. Line represents result of least squares linear regression, the slope of which gives the value of the group velocity.

phase and group velocities are summarized in Figure 5. The wave-velocity estimates provided by the conventional travel-time picking methods are determined using the same differential procedure (Table 2). Uncertainties are based solely on the first standard deviation of slope that exists in the distance-time scatter plots. The various measures of velocity for each of the packs are compared in Table 2. Phase and group velocities are provided at the peak amplitude, or predominant, frequencies of 0.33 and 0.38 MHz for the sand and the glass bead packs, respectively.

The frequency-dependent attenuation for each pack is determined from equation (2), with the differential comparisons of up to 28 independent measurements. At each frequency the resulting attenuation values α are widely distributed, as observed by the large single standard deviation error bars (Figure 5b). This is common with attenuation measurements, and uncertainties of 100% are not unusual (e.g., White, 1992). This merely reflects uncertainties in the spectral decomposition and windowing of the signal.

DISCUSSION

The primary objective of this paper is to find the most representative relationships between the fundamental material properties of group and phase velocity and those velocities determined by common traveltime picking methods. Since this may depend on how the waves are attenuated, the character of the attenuation is examined first.

Character of the attenuation

The obvious decay in amplitude and spreading in time of the first arriving pulses (Figure 5) indicate that these saturated

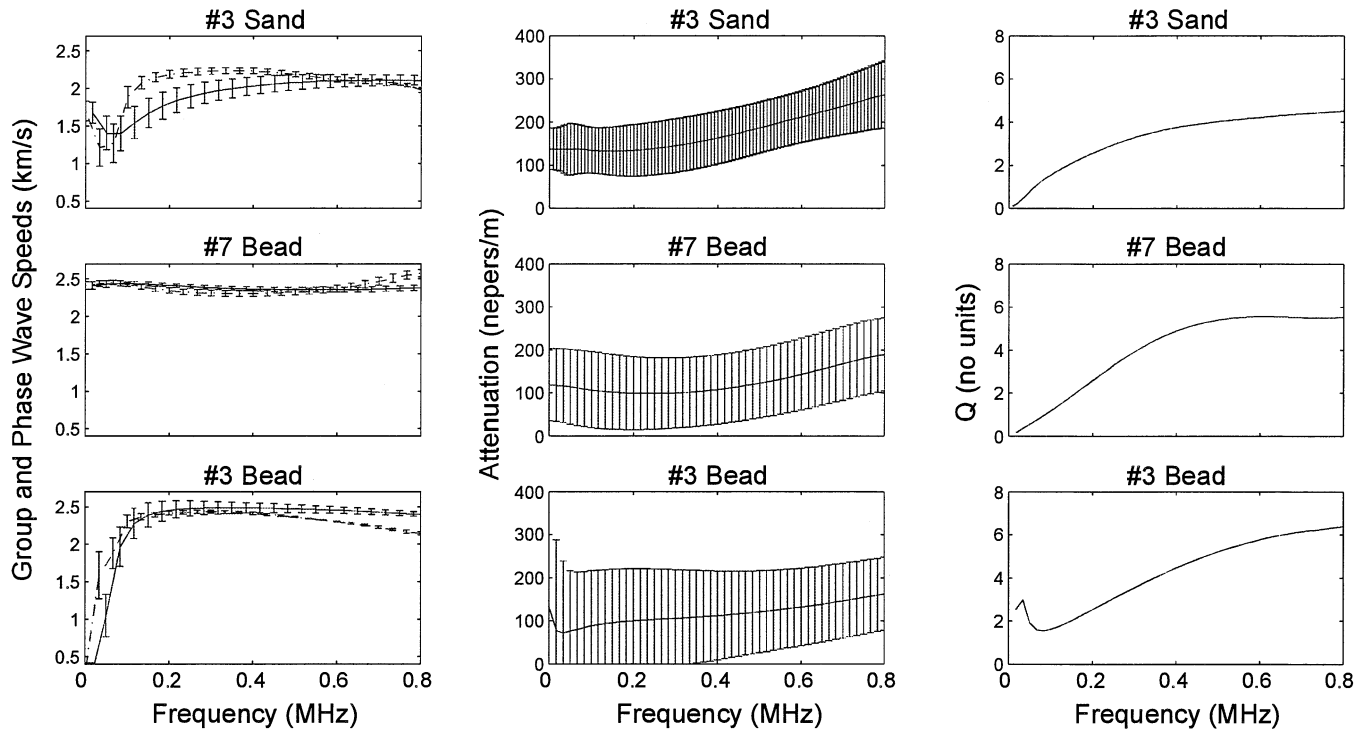


FIG. 5. Left panels: Group (dashed line) and phase (solid line) versus frequency for the packs. Center panels: Attenuation coefficient α versus frequency for the packs. Envelope of values represents the uncertainties in α . Right panels: Quality factor Q calculated for the packs.

glass bead packs successfully produced strongly attenuating media. This is further supported by the high values of attenuation (~ 100 nepers/m) and corresponding low Q (~ 4) derived directly from the waveforms in Figure 5.

It is not clear what the major source of the attenuation might be in these bead packs, and it is likely that a number of mechanisms such as grain–grain sliding, scattering, mode conversion, bead resonance, and differential fluid motion all contribute to some degree. A detailed discussion of the mechanisms is delayed for a later contribution.

A plot of attenuation versus frequency would have a constant slope in a constant Q medium; Wuenschel (1965) observes a nearly linear attenuation–frequency plot in polymethyl methacrylate over the bank below 150 kHz, as do Sachse and Pao (1978) in 6061-T6 aluminum alloy. Such linearity contrasts with the results of Figure 5, which shows that the slope generally increases with frequency.

Wave-velocity comparisons

Comparisons of the various measures of wave velocity are made in Table 2. The group velocity is used as the reference because we usually take for granted that this is what wave-velocity measurements derived from traveltimes determine. Further, it is more stable than determining the other intrinsic property of phase velocity, which is subject to the previously mentioned unwrapping errors.

There is no requirement that V_p and V_g should have the same magnitude, except in the absence of attenuation (and anisotropy). That they are not the same in such highly attenuating media is not surprising. The greatest discrepancy is observed for the #3 sand where, at the dominant frequency, the group velocity is about 11% greater than the phase velocity. In the other samples, the group velocities are less than the phase velocity by about 3%. These discrepancies suggest that care should be taken when attempting, in lossy materials, to use an estimate of group velocity to further gain an estimate of the phase velocity.

It is worth noting that all velocities for the quartz sand are lower than the corresponding measure in the glass bead packs. At first examination this appears unusual because of the higher elastic moduli of quartz (Table 1); but such effects are more than offset by the greater quartz density, as indicated by simple modeling using Wood's formula (Berryman, 1995) given in Table 2.

The values V_x , V_m , and V_H provide reasonably good estimates of the group velocity to better than 3% at the predominant frequency in these bead packs. For the two glass bead packs, both V_x and V_m are in good agreement with V_p at the predominant frequency, but this may be coincidental.

In contrast, V_s is always greater than V_g by up to 19%, i.e., the relative signal-time delays are smaller than the respective group delays. Theoretically, small amplitude arrivals with a discrete onset can occur well in advance of the main signal in attenuating media (Strick, 1970), but this is generally obscured by noise. Molyneux and Schmitt (1999) note that the discrepancy between signal velocity measurements and other common timing measurements is negligible when $Q > 30$.

In highly attenuating media, the first arriving energy arguably appears at a time that cannot be earlier than that dictated by the greatest phase speed. In earth materials, this

phase speed is the limiting velocity at infinitely high frequency. Such high-frequency components are strongly attenuated and are not detectable, whereas the lower frequency, lower phase-velocity components retain observable amplitude. Strick (1970) refers to these undetectable precursors as the pedestal to the observable waveform. Therefore, in highly attenuating media the best physical interpretation of the signal velocity is that it provides a lower bound estimate of the limiting high-frequency phase velocity.

Although the various measures of transit time differ, they remain individually consistent. The times picked in the same way are always related linearly to propagation distance, i.e., each type of time-picking measurement is surprisingly self-consistent despite the fact that the waveform shapes continue to evolve.

CONCLUSION

Ultrasonic wave pulses were propagated through attenuative media to observe the evolution of the first arriving pulse with distance traveled. Frequency-dependent group and phase velocities were compared to velocities derived using standard traveltimes picking procedures. Group and phase velocities generally are not the same and differ at the predominant frequency by up to 12%. Wave velocities estimated from picking the time of the signal's first amplitude extremum, from a crosscorrelation procedure, and from the peak of the amplitude envelope generally agree well with the group wave velocity at the predominant frequency to better than 3%. As such, these simple measures appear to provide reasonable estimates of the group velocity at the predominant frequency.

In contrast, the signal velocity V_s , determined from the picked time of the pulse's onset is as much as 19% greater than the other speed measures. The meaning of a velocity determined from the onset time is not clear, but this velocity can probably be taken as a lower bound estimate to the limiting phase velocity at high frequency in materials where phase velocity increases with frequency.

The results of this study suggest that some care needs to be exercised in wave-velocity determinations that use pulse transmission methods. The saturated pack media used here are an extreme case to highlight the effects of attenuation and dispersion. However, waves are strongly attenuated in many situations of geophysical interest. Determining phase- and group-velocity frequency dispersion in such cases can aid in more detailed characterization of the material.

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