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Computation of electric fields and potential on polluted insulators using a boundary element method

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COMPUTATION OF ELECTRIC FIELDS AND POTENTIAL ON POLLUTED INSULATORS USING A BOUNDARY ELEMENT METHOD

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***Abstract*–This paper presents a calculation method for electric fields and potential usable in the case of polluted insulator. This method based on boundary integral equations is very suitable for three-dimensional geometries and polluted layers. Results given by this method are compared with analytical solutions (potential, currents) and with measured values (potential, leakage current).**

INTRODUCTION

The presence of pollution layer on HV insulators is very frequent in industrial and coastal regions. This pollution layer when combined with moisture becomes conductive and a leakage current flows through it. Dry bands can appear and the distributions of electric fields and potential become distorted and flashovers can occur. Therefore, it is important to compute electric fields and potential in studying the behaviour of polluted insulators.

Several numerical computation methods were developed for solving this problem. They are based on domain methods: finite differences[1,2] and finite element methods[3,4,5], charge simulation methods [6,7] and boundary methods[8,9]. Most of these methods are conceived for plane or axisymetrical arrangements. In this paper we present a boundary integral equation method which can solve three-dimensional arrangements (geometry and pollution). It is a continuation of the work presented by HUANG in [12,13]. This method presents several advantages over the former ones (reduced number of unknowns, these being V and $\Psi(=\epsilon_r E_n)$ at the insulator surfaces, taking account of infinite domains).

THEORETICAL FORMULATION

Boundary integral equation method

The governing equation for potential distribution inside a domain Ω with a zero charge density obeys Laplace's equation

$$\Delta V = 0 \qquad \text{in } \Omega \quad (1)$$

Applying Green's theorem, we can express the potential V on all points P of Ω in terms of V and the variable $\Psi=\epsilon_r\frac{\partial V}{\partial n}$ on the boundary Σ of Ω as follows:

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$$cV(P) = -\int_{\Sigma} (V\frac{\partial G}{\partial \mathbf{n}} - \frac{1}{\epsilon_r} G\Psi)ds \qquad (2)$$

In this equation G is the Green's function, **n** is the unit normal outward vector to Σ, c = A/4π where A is the solid angle under which the point P sees the oriented surface Σ and ϵ_r is the relative permittivity of Ω.

Case of polluted insulator

In the case of polluted insulator, Fig. 1, we assume that the charge density is zero in the air domain Ω_1 whose relative permittivity is $\epsilon_{r1}=1$ and in insulating material domain Ω_2 which has a relative permittivity ϵ_{r2}. The polluted area S of the insulator is characterized by its surface conductivity σ_s.

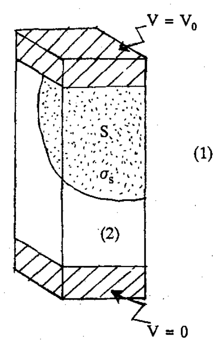

Fig. 1. Polluted insulator.

We write (2) for the air domain whose frontier is S_1 and for the insulating material domain whose frontier is S_2, we obtain for air domain,

$$cV = -\int_{S_1} (V\frac{\partial G}{\partial \mathbf{n}} - \frac{1}{\epsilon_{r1}} G\Psi_1)ds \qquad (3)$$

and for insulating material domain,

$$cV = -\int_{S_2} (V\frac{\partial G}{\partial \mathbf{n}} - \frac{1}{\epsilon_{r2}} G\Psi_2)ds \qquad (4)$$

In these equations Ψ_1 represents the value of Ψ on the air side ans Ψ_2 that of insulating material side: Fig. 2.

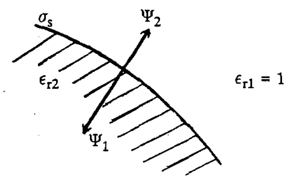

Fig. 2. Definition of Ψ_1 and Ψ_2.

For clean areas, we have $\Psi_1 + \Psi_2 = 0$ and for polluted ones we use the conservation of charge in taking account of the surface current $\mathbf{J}_s = -\sigma_s \mathbf{grad}_s V$. The conservation of charge is written as[10]

$$\mathrm{div}_s \mathbf{J}_s + \frac{\partial(\mathbf{D.n})}{\partial t} = 0 \qquad \text{on S} \quad (5)$$

where $\mathbf{D}$ is the electric displacement and t is the time. The subscript s indicates a surfacic operator. If we express $\mathbf{D.n}$ in terms of Ψ_1 and Ψ_2 and if we use a sinusoidal source with an angular frequency ω ($\frac{\partial}{\partial t} = j\omega$ with $j = \sqrt{-1}$), (5) becomes

$$\mathrm{div}_s(-\sigma_s \mathbf{grad}_s V) + j\omega\epsilon_0(\Psi_1 + \Psi_2) = 0 \qquad \text{on S} \quad (6)$$

where ϵ_0 denotes the free space permittivity. We introduce an auxiliary unknown Θ defined by $\Theta = \Psi_1 + \Psi_2$, so (6) becomes

$$\mathrm{div}_s(-\sigma_s \mathbf{grad}_s V) + j\omega\epsilon_0\Theta = 0 \qquad \text{on S} \quad (7)$$

We solve this equation by using a weighted residual method with the following boundary conditions, Fig. 3:

$$V = V_0 \qquad \text{on } \Gamma_1$$
$$\frac{\partial V}{\partial \nu} = 0 \qquad \text{on } \Gamma_2$$

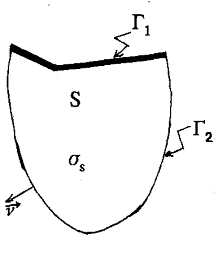

Fig. 3. Boundary conditions on the polluted area.

then we obtain

$$\int_S \sigma_s \mathbf{grad}_s w \; \mathbf{grad}_s V ds - \int_{\Gamma_1} \sigma_s w \frac{\partial V}{\partial \nu} d\Gamma + j\omega\epsilon_0 \int_S w\Theta ds = 0 \quad (8)$$

where w is the weighting functiốn.
If we now express Ψ_2 in terms of Θ and Ψ_1 and if we insert this expression into (4), we obtain

$$cV = -\int_{S_2} \left(V\frac{\partial G}{\partial \mathbf{n}} + \frac{1}{\epsilon_{r2}} G\Psi_1 - \frac{1}{\epsilon_{r2}} G\Theta\right) ds \qquad (4')$$

The set of equation that we have to solve is formed by (3), (8) and (4').
We divide the different boundaries S_1, S_2 into finite elements and on each finite element we interpolate V, $\Psi = \Psi_1$ and Θ with the corresponding nodal values V_i, Ψ_i, Θ_i:

$$V = \Sigma\alpha_i V_i$$
$$\Psi = \Sigma\alpha_i \Psi_i$$
$$\Theta = \Sigma\alpha_i \Theta_i$$

where α_i are the interpolating functions. In our discretization we use an element with eight nodes ($1 \leq i \leq 8$). After discretizing and assembling, we obtain a system of complex linear equations, which is solved by using gaussian elimination.

RESULTS

In this part, we report computations results which we compare with analytical and measured ones.

Comparison with analytical solutions:

The example we use here for comparison is extracted from [3]. The geometry is reproduced in Fig. 4 with $\epsilon_{r2} = 4$, $\sigma_s = 10^{-9}$ S and $V_0 = 100$ V. The potential V(x) at a point x is given by

$$V(x) = V_0 \frac{\cosh[(1+j)k(l-x)]}{\cosh[(1+j)kl]}$$

where l is the lenght of coating, $k = \sqrt{\frac{\omega\epsilon_0\epsilon_{r2}}{2\sigma_s a}}$ with a the thickness of the insulating material and $\omega = 100\pi$. The modulus of $\Psi(x)$ and the tangential field are derived from V(x). For solving this problem we add a width of 60 mm, so we obtain a three-dimensional geometry, Fig. 5. We compare the potential values along the line OA, Fig. 6-7. Also comparisons between analytical and computed values of the modulus of Ψ and tangential electric fields show good similarities [11].

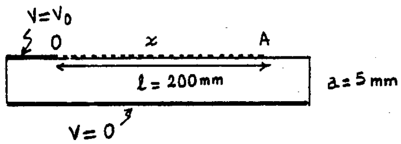

Fig. 4. Two dimensional problem.

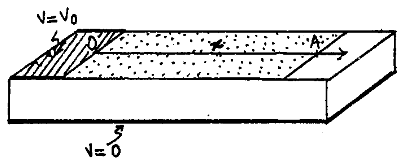

Fig. 5. Three dimensional problem.

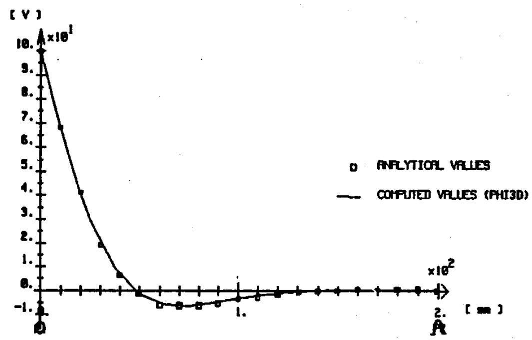

Fig. 6. Real part of V.

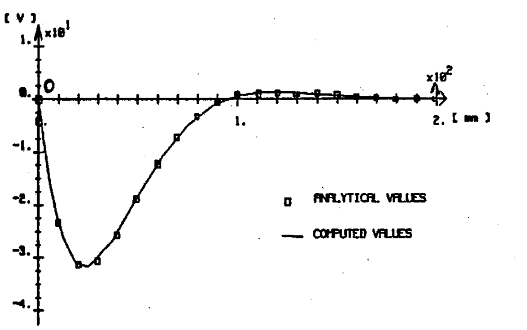

Fig. 7. Imaginary part of V.

These figures show that our results agree with analytical solutions. For the current values (capacitive and conductive), the comparisons are presented in Table I. We note a good agreement with analytical values.

TABLE I
COMPARISON OF ANALYTICAL AND COMPUTED CURRENTS

Capacitive current values (A)		Conductive current values (A)	
theoretical	computed	theoretical	computed
$j2.\ 10^{-7}$	$-2.\ 10^{-9}+j2.02\ 10^{-7}$	$1.\ 10^{-7} + j1.\ 10^{-7}$	$1.\ 10^{-7}+j.99\ 10^{-7}$

Comparison with measured values:

For this sort of comparison the geometries used for the measurements are axisymetric, therefore we used only 1/12 of the real geometries in our calculation.

For the leakage currents, a sample insulator was sprayed with salt fog. Four experiments were done and for each experiment the leakage current was measured. The corresponding surface conductivity was calculated and this value was introduced in the computation. The computed leakage currents are compared with the measured values [12]: Table II.

TABLE II
COMPARISON OF MEASURED AND COMPUTED CURRENTS

$V_0(V_{rms})$	$\sigma_s(\mu S)$	measured values(mA)	computed values(mA)
830	1.077	2.12	2.13
1070	1.03	2.62	2.64
600	3.54	5.02	5.07
1000	3.81	9.05	9.11

For the potential, the measurement was done along a portion A'C of the pin-insulator (generatrix). This is reported with its mesh in Fig. 8. The part ABB'A' was coated with a semiconducting layer ($\sigma_s = 1.4\ 10^{-8}$ S). The computed (PHI3D) and measured values [5] are indicated in Fig. 9. Clearly, the values do agree.

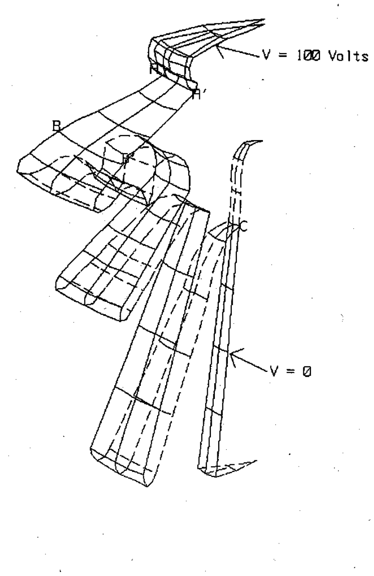

Fig. 8. Geometry used for the pin insulator.

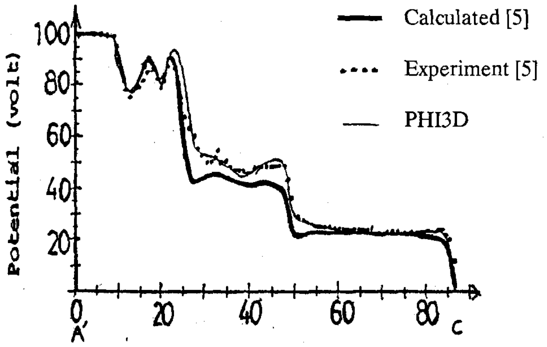

Fig. 9. Potential values along A'B'C.

CONCLUSION

We have presented a boundary integral equation method which presents several advantages over the existing ones in terms of the reduced number of unknowns called into play, the taking account of the infinite domain and the obtaining of direct values of potential and electric fields at the insulator surfaces: V and $\Psi(=\epsilon_r E_n)$. The two kinds of comparison show that the proposed method is appropriate for three dimensional arrangements.

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