Computational Complexity of Relay Placement in Sensor Networks

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Wireless Sensor Networks

- *Sensor nodes* are small and inexpensive *computers* which are equipped with *sensors* and *wireless communication* capabilities
- Sensor nodes may be deployed manually or even dropped from an aeroplane
- After deployment, sensor nodes form an *ad-hoc network* which will *route* data from sensor nodes towards a *sink* node
- *Energy consumption* must be very low: nodes may need to operate for years without anyone changing or recharging batteries
- *Possible uses* include environmental and weather monitoring; home automation; agriculture; tracking goods in commerce and industry; monitoring machines; health care and medical diagnostics; security systems; and military applications

Optimising Sensor Networks (1)

Possible target functions:

- Lifetime before batteries are drained
- Amount of data gathered during lifetime
- Quality of data gathered:
  - coverage: space, time
  - accuracy of data
  - probability of detecting or missing events

We focus on *balanced data gathering*: $\lambda \min q_\eta + (1 - \lambda) \operatorname{avg} q_\eta$.

- Not only lot of data but also some data from all nodes
- Formulated by Falck *et al.* (2004)

Optimising Sensor Networks (2)

Possible variables:

- Node hardware and software
- Node placement
- Scheduling node activity
- Routing
- Aggregating, summarising, and buffering data

We combine both *node placement* and *routing* issues.

Relay Placement Problem (1)

Problem:

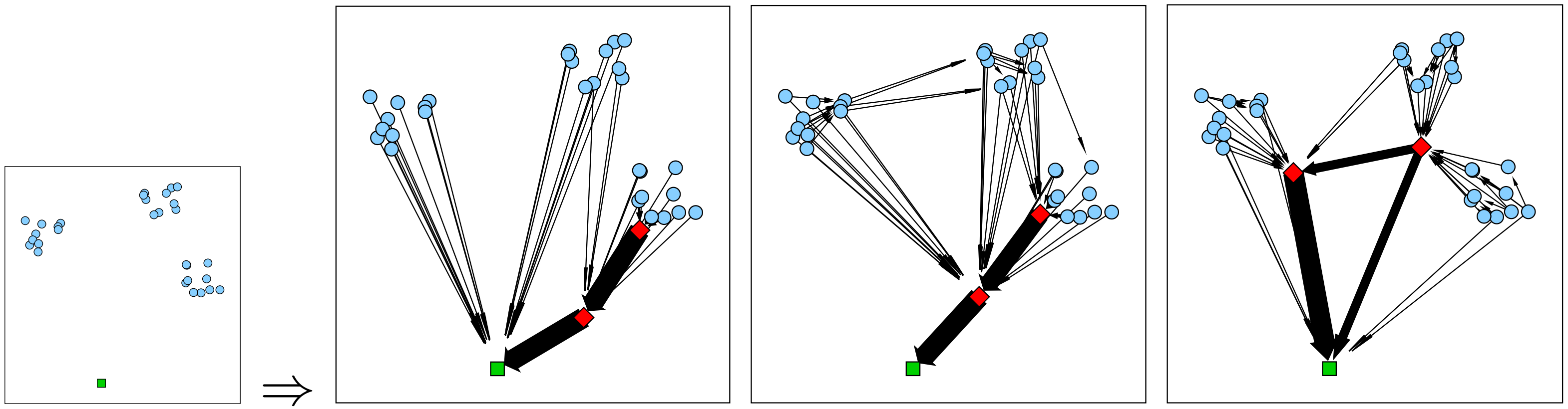
- Given a *deployed sensor network*,
- add a small number of new *relay nodes*
- in order to maximise *balanced data gathering*

Typically, the relay nodes would be more expensive devices with larger batteries. Relays do not sense, they only forward data.

If we can afford a few relay nodes, where should we put them?

Relay Placement Problem (2)

Example:



problem instance ⇒

$\lambda = 0.0$ maximise average

$\lambda = 0.5$

$\lambda = 1.0$ maximise minimum

(1.25-approximate solutions illustrated)

Problem Classes (1)

The general relay placement problem needs to be restricted; otherwise we do not even have a finite parametrisation of a problem instance. We consider restrictions in the following five dimensions.

1. Type:	Decision
	Relay-constrained optimal
	Relay-constrained k-optimal
	Utility-constrained optimal
	Utility-constrained k-optimal
2. Utility:	Balanced data gathering

Problem Classes (2)

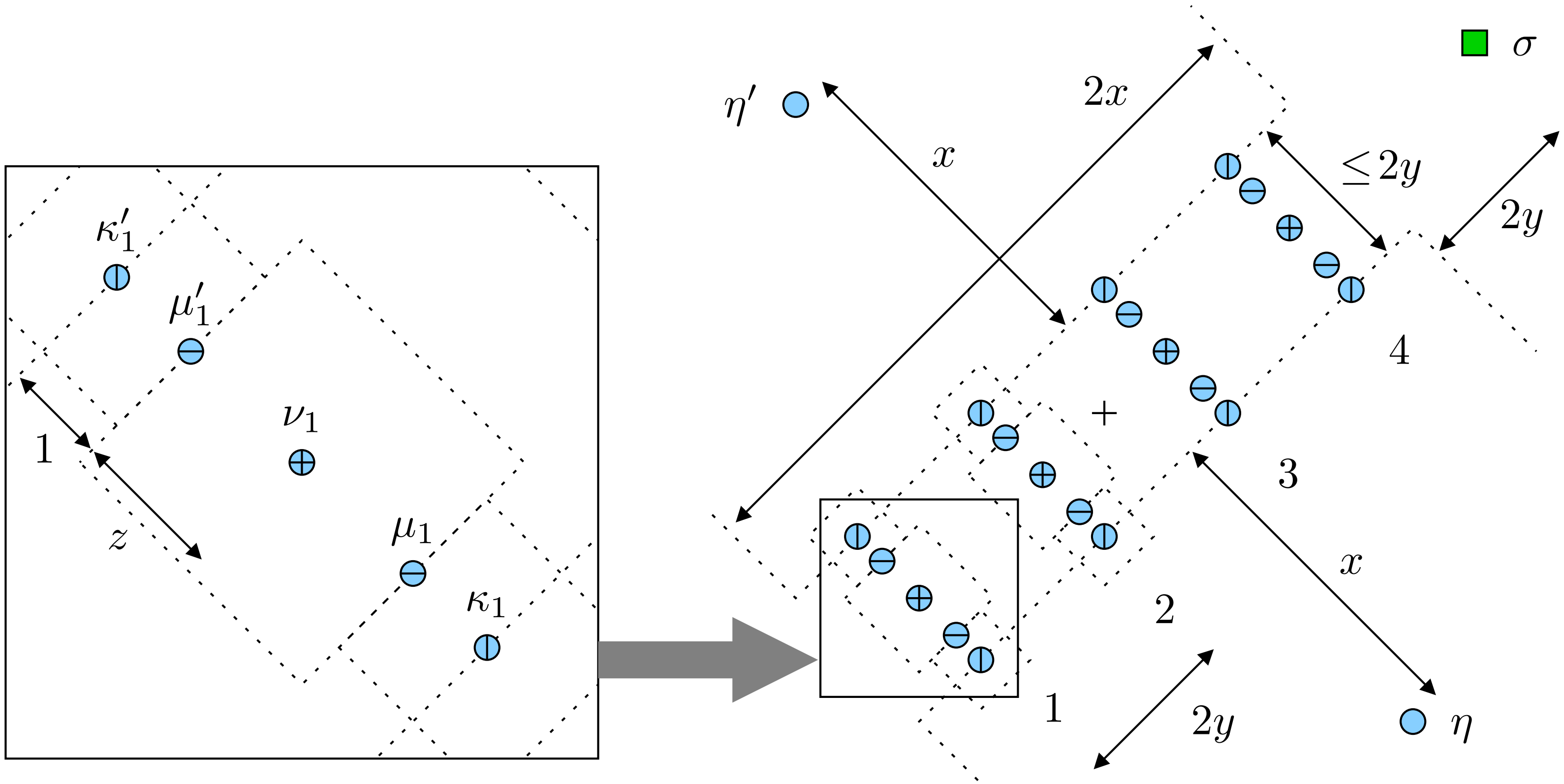
3. Possible relay locations:	Unrestricted – Planar – Finite set – Sensor upgrade

4. Transmission costs:	Unrestricted
	– Location dependent
	    – Line-of-sight
	        – Free space

5. Batteries:	Unrestricted – Identical

All Classes Are NP-hard (1)

Reduction from PARTITION:

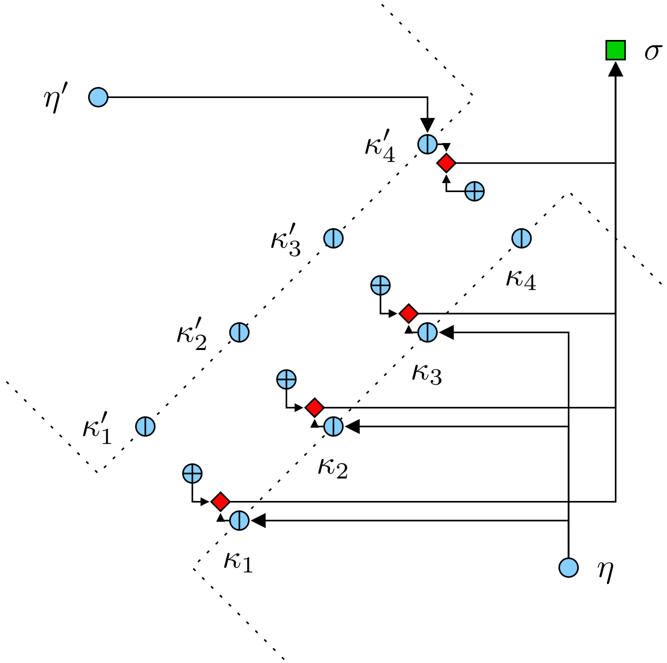


All Classes Are NP-hard (2)

Battery capacities of nodes κ_i correspond to values a_i in the Partition problem.

All available data in η (η') can be transmitted to the sink via some of the nodes κ_i (κ'_i) and the corresponding relays iff the relays are placed according to the solution of the partition problem.

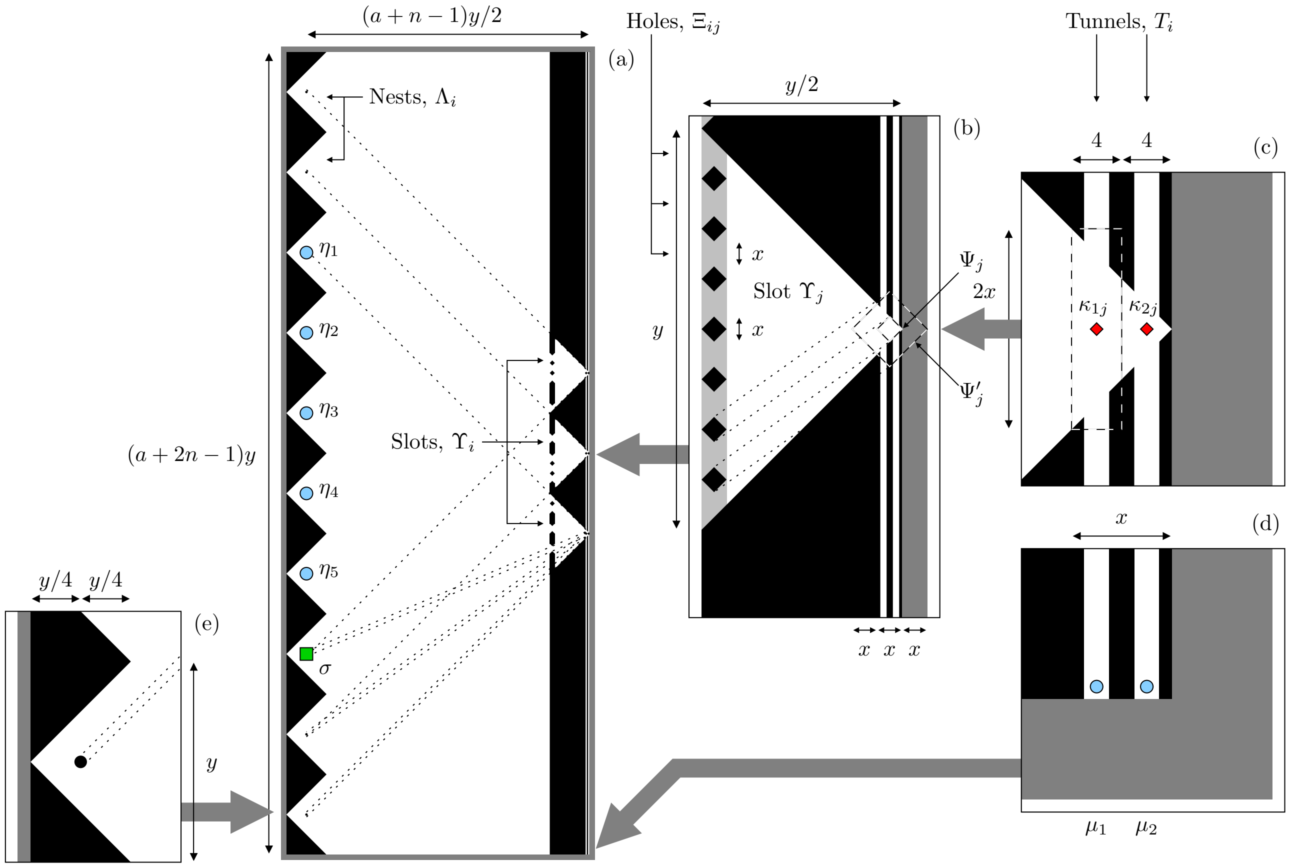
A solution is optimal iff all available data is gathered.



Here $X = \{1, 2, 3\}$ is a solution to the Partition problem: $a_1 + a_2 + a_3 = a_4$.

With Obstacles, Approximation Is NP-hard (1)

Reduction from SET COVERING:



With Obstacles, Approximation Is NP-hard (2)

Reduction from SET COVERING:

- Placing m relays corresponds to choosing m subsets.
- For each i, there is a transmission path $\eta_i \to$ relay $\to$ sink iff there is a subset that contains i.
- Choose $\lambda = 1$, optimise minimum. The utility is zero unless some data is gathered from each node.
- Finding a solution with *any* positive utility is at least as hard as solving SET COVERING exactly.
- Any approximation algorithm for relay placement would provide an exact solution to SET COVERING.

Papers

- J. Suomela: Computational complexity of relay placement in sensor networks. *Proc. SOFSEM 2006*.
- P. Floréen, P. Kaski, J. Kohonen and P. Orponen: Exact and approximate balanced data gathering in energy-constrained sensor networks. *Theoretical Computer Science* 344 (2005).
- E. Falck, P. Floréen, P. Kaski, J. Kohonen and P. Orponen: Balanced data gathering in energy-constrained sensor networks. *Proc. Algosensors 2004*.

Software

- Source code for k-optimal relay placement is freely available.

Summary

- How to optimise data gathering in wireless sensor networks by adding a small number of new relay nodes

Future Research

- More on approximability and inapproximability
- Focus on the amount of new relevant information instead of the amount of raw sensor readings
- Not only relay placement and routing but also sensor placement and data aggregation

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