COMPUTATIONAL CONTINUUM MECHANICS

This book presents the nonlinear theory of continuum mechanics and demonstrates its use in developing nonlinear computer formulations for large displacement dynamic analysis. Basic concepts used in continuum mechanics are presented and used to develop nonlinear general finite element formulations that can be effectively used in large displacement analysis. The book considers two nonlinear finite element dynamic formulations: a general large-deformation finite element formulation and then a formulation that can efficiently solve small deformation problems that characterize very and moderately stiff structures. The book presents material clearly and systematically, assuming the reader has only basic knowledge in matrix and vector algebra and dynamics. The book is designed for use by advanced undergraduates and first-year graduate students. It is also a reference for researchers, practicing engineers, and scientists working in computational mechanics, bio-mechanics, computational biology, multibody system dynamics, and other fields of science and engineering using the general continuum mechanics theory.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521885690

First published 2008

Printed in the United States of America.

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Shabana, Ahmed A.,
Computational continuum mechanics / Ahmed A. Shabana.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-88569-0
1. Continuum mechanics. 2. Engineering mathematics. I. Title.
QA808.2.S46 2008
531–dc22 2007038135

ISBN 978-0-521-88569-0 hardback

Cambridge University Press
978-0-521-88569-0 - Computational Continuum Mechanics
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Preface

Nonlinear continuum mechanics is one of the fundamental subjects that form the foundation of modern computational mechanics. The study of the motion and behavior of materials under different loading conditions requires understanding of basic, general, and nonlinear, kinematic and dynamic relationships that are covered in continuum mechanics courses. The finite element method, on the other hand, has emerged as a powerful tool for solving many problems in engineering and physics. The finite element method became a popular and widely used computational approach because of its versatility and generality in solving large-scale and complex physics and engineering problems. Nonetheless, the success of using the continuum-mechanics-based finite element method in the analysis of the motion of bodies that experience general displacements, including arbitrary large rotations, has been limited. The solution to this problem requires resorting to some of the basic concepts in continuum mechanics and putting the emphasis on developing sound formulations that satisfy the principles of mechanics. Some researchers, however, have tried to solve fundamental formulation problems using numerical techniques that lead to approximations. Although numerical methods are an integral part of modern computational algorithms and can be effectively used in some applications to obtain efficient and accurate solutions, it is the opinion of many researchers that numerical methods should only be used as a last resort to fix formulation problems. Sound formulations must be first developed and tested to make sure that these formulations satisfy the basic principles of mechanics. The equations that result from the use of the analytically correct formulations can then be solved using numerical methods.

This book is focused on presenting the nonlinear theory of continuum mechanics and demonstrating its use in developing nonlinear computer formulations that can be used in the large displacement dynamic analysis. To this end, the basic concepts used in continuum mechanics are first presented and then used to develop nonlinear general finite element formulations that can be effectively used in the large displacement analysis. Two nonlinear finite element dynamic formulations will be considered in this book. The first is a general large-deformation finite element formulation, whereas the second is a formulation that can be used efficiently to solve small-deformation problems that characterize very and moderately stiff structures.

In this latter case, an elaborate method for eliminating the unnecessary degrees of freedom must be used in order to be able to efficiently obtain a numerical solution. An attempt has been made to present the materials in a clear and systematic manner with the assumption that the reader has only basic knowledge in matrix and vector algebra as well as basic knowledge of dynamics. The book is designed for a course at the senior undergraduate and first-year graduate level. It can also be used as a reference for researchers and practicing engineers and scientists who are working in the areas of computational mechanics, biomechanics, computational biology, multibody system dynamics, and other fields of science and engineering that are based on the general continuum mechanics theory.

In **Chapter 1** of this book, matrix, vector, and tensor notations are introduced. These notations will be repeatedly used in all chapters of the book, and, therefore, it is necessary that the reader reviews this chapter in order to be able to follow the presentation in subsequent chapters. The polar decomposition theorem, which is fundamental in continuum and computational mechanics, is also presented in this chapter. D'Alembert's principle and the principle of virtual work can be used to systematically derive the equations of motion of physical systems. These two important principles are discussed, and the relationship between them is explained. The use of a finite dimensional model to describe the continuum motion is also discussed in Section 8; whereas in Section 9, the procedure for developing the discrete equations of motion is outlined. In Section 10, the principles of momentum and principle of work and energy are presented. In this section, the problems associated with some of the finite element formulations that violate these analytical mechanics principles are discussed. Section 11 of Chapter 1 is devoted to a discussion on the definitions of the gradient vectors that are used in continuum mechanics to define the strain components.

In **Chapter 2**, the general kinematic displacement equations of a continuum are developed. These equations are used to define the strain components. The Green–Lagrange strains and the Almansi or Eulerian strains are introduced. The Green–Lagrange strains are defined in the reference configuration, whereas the Almansi or Eulerian strains are defined in the current deformed configuration. The relationships between these strain components are established and used to shed light on the physical meaning of the strain components. Other deformation measures as well as the velocity and acceleration equations are also defined in this chapter. The important issue of objectivity that must be considered when large deformations and inelastic formulations are used is discussed. The equations that govern the change of volume and area, the conservation of mass, and examples of deformation modes are also presented in this chapter.

Forces and stresses are discussed in **Chapter 3**. Equilibrium of forces acting on an infinitesimal material element is used to define the Cauchy stresses, which are used to develop the partial differential equations of equilibrium. The transformation of the stress components and the symmetry of the Cauchy stress tensor are among the topics discussed in this chapter. The virtual work of the forces due to the change of the shape of the continuum is defined. The deviatoric stresses, stress objectivity, and energy balance equations are also discussed in Chapter 3.

The definition of the strain and stress components is not sufficient to describe the motion of a continuum. One must define the relationship between the stresses and strains using the constitutive equations that are discussed in **Chapter 4**. In Chapter 4, the generalized Hooke's law is introduced, and the assumptions used in the definition of homogeneous isotropic materials are outlined. The principal strain invariants and special large-deformation material models are discussed. The linear and nonlinear viscoelastic material behavior is also discussed in Chapter 4.

In many engineering applications, plastic deformations occur due to excessive forces and impact as well as thermal loads. Several plasticity formulations are presented in **Chapter 5**. First, a one-dimensional theory is used in order to discuss the main concepts and solution procedures used in the plasticity analysis. The theory is then generalized to the three-dimensional analysis for the case of small strains. Large strain nonlinear plasticity formulations as well as the J_2 flow theory are among the topics discussed in Chapter 5. This chapter can be skipped in its entirety because it has no effect on the continuity of the presentation, and the developments in subsequent chapters do not depend on the theory of plasticity in particular.

Nonlinear finite element formulations are discussed in Chapter 6 and 7. Two formulations are discussed in these two chapters. The first is a large-deformation finite element formulation, which is discussed in **Chapter 6**. This formulation, called the absolute nodal coordinate formulation, is based on a continuum mechanics theory and employs displacement gradients as coordinates. It leads to a unique displacement and rotation fields and imposes no restrictions on the amount of rotation or deformation within the finite element. The absolute nodal coordinate formulation has some unique features that distinguish it from other existing large-deformation finite element formulations: it leads to a constant mass matrix; it leads to zero centrifugal and Coriolis forces; it automatically satisfies the principles of mechanics; it correctly describes an arbitrary rigid-body motion including finite rotations; and it can be used to develop several beams, plate, and shell elements that relax many of the assumptions used in classical theorems because this formulation allows for the use of more general constitutive relationships.

Clearly, large-deformation finite element formulations can also be used to solve small deformation problems. However, it is not recommended to use a large-deformation finite element formulation to solve a small-deformation problem. Large-deformation formulations do not exploit some particular features of small-deformation problems, and, therefore, such formulations can be very inefficient in the solution of stiff and moderately stiff systems. It turns out that the development of an efficient small-deformation finite element formulation that correctly describes an arbitrary rigid-body motion requires the use of more elaborate techniques in order to define a local linear problem without compromising the ability of the method to describe large-displacement small-deformation behavior. The finite element floating frame of reference formulation, which is widely used in the analysis of small deformations, is discussed in **Chapter 7** of this book. This formulation allows eliminating high-frequency modes that do not have a significant effect on the solution, thereby leading to a lower-dimension dynamic model that can be efficiently solved using numerical and computer methods.

I would like to thank many students and colleagues with whom I worked for several years on the subject of flexible body dynamics. I was fortunate to collaborate with excellent students and colleagues who educated me in this important field of computational mechanics. In particular, I would like to thank two of my doctorate students, Bassam Hussein and Luis Maqueda, who provided solutions for several of the examples presented in Chapter 4 and Chapter 5. I am grateful for the help I received from Mr. Peter Gordon, the Engineering Editor, and the production staff of Cambridge University Press. It was a pleasant experience working with them on the production of this book. I would also like to thank my family for their help, patience, and understanding during the time of preparing this book.

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Chicago, IL, 2007