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Nielsen, Jannie Sønderkær; Sørensen, John Dalsgaard

*Published in:*  
Structure & Infrastructure Engineering

*DOI (link to publication from Publisher):*  
[10.1080/15732479.2017.1387155](https://doi.org/10.1080/15732479.2017.1387155)

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*Publication date:*  
2018

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Nielsen, J. S., & Sørensen, J. D. (2018). Computational framework for risk-based planning of inspections, maintenance, and condition monitoring using discrete Bayesian networks. *Structure & Infrastructure Engineering*, 14(8), 1082-1094. <https://doi.org/10.1080/15732479.2017.1387155>

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# Computational framework for risk-based planning of inspections, maintenance and condition monitoring using discrete Bayesian networks

Jannie Sønderkær Nielsen  and John Dalsgaard Sørensen

Department of Civil Engineering, Aalborg University, Aalborg, Denmark

## ABSTRACT

This paper presents a computational framework for risk-based planning of inspections and repairs for deteriorating components. Two distinct types of decision rules are used to model decisions: simple decision rules that depend on constants or observed variables (e.g. inspection outcome), and advanced decision rules that depend on variables found using Bayesian updating (e.g. probability of failure). Two decision models are developed, both relying on dynamic Bayesian networks (DBNs) for deterioration modelling. For simple decision rules, DBNs are used directly for exact assessment of total expected life-cycle costs. For advanced decision rules, simulations are performed to estimate the expected costs, and DBNs are used within the simulations for decision-making. Information from inspections and condition monitoring are included if available. An example in the paper demonstrates the framework and the implemented strategies and decision rules, including various types of condition-based maintenance. The strategies using advanced decision rules lead to reduced costs compared to the simple decision rules when condition monitoring is applied, and the value of condition monitoring is estimated by comparing the lowest costs obtained with and without condition monitoring.

## ARTICLE HISTORY

Received 23 March 2017  
Revised 20 July 2017  
Accepted 23 August 2017

## KEYWORDS

Maintenance & inspection;  
risk & probability analysis;  
decision support systems;  
monitoring; reliability;  
Bayesian networks

## Introduction

The risk-based approach to inspection and maintenance planning has been applied successfully for offshore oil and gas structures under the name Risk-Based Inspection (RBI) (Faber, 2002; Straub, 2004). The risk-based approach directly considers the influence of the deterioration model and the inspection reliability on maintenance decisions and component reliability. Decisions on inspection schedule and repair criteria are made in order to minimise the total expected lifetime costs. In the approach typically applied, the computations involved are time-consuming, and the decisions do not include information from condition monitoring (CM) and do not fully exploit information from inspections. This is relevant when applying risk-based decision-making for e.g. offshore wind turbine components, which are often monitored or inspected annually. Due to the damage tolerant design of e.g. wind turbine blades, not all detected defects should necessarily be repaired immediately, but the observations could be considered for future decisions (McGugan, Pereira, Sørensen, Toftegaard, & Branner, 2015).

The risk-based approach to inspection/maintenance planning is based on the Bayesian pre-posterior decision analysis (Raiffa & Schlaifer, 1961). In the classical pre-posterior decision analysis, there are two decisions. First, a decision on whether or not to obtain more knowledge (e.g. make an inspection), and secondly a decision on an action (e.g. make a repair). The decision problem can be solved using either the normal or extensive form, but in both cases the costs associated with each branch in the

decision tree are evaluated. For maintenance planning, the two decisions are repeated many times during the lifetime, and the number of branches increases exponentially with the number of decisions / time steps. The probabilities are typically found using Monte Carlo simulations, which are time-consuming, but even if each computation was very fast, the computation time would still explode, and therefore, approximations are needed to solve the problem.

For RBI, Figure 1 shows a typical decision tree. The decision problem is solved by using decision rules for the decisions as in the extensive form analysis and by applying the same decision rules for all time steps. It is possible to significantly reduce the number of computations by assuming that repaired components act as new components or as components with no detected defects (Straub & Faber, 2006). In this approach, the inspection outcome is only used for updating the model through the action (repair/no repair). For example, if the decision rule for repairs is to repair cracks larger than 5 mm, a decision of no repair propagates the information that there was no detection of cracks larger than 5 mm, but the information of a detected 4 mm crack will be lost for instance.

Dynamic Bayesian networks are useful for computational efficient deterioration modelling, and failure probabilities can be updated very fast based on information from inspections and/or CM when all variables are discrete (Jensen & Nielsen, 2007; Nielsen & Sørensen, 2017). It is straightforward to model deterioration models with discrete states such as Markov processes. Although the time spent in each state is limited to be

**CONTACT** Jannie Sønderkær Nielsen  [jjn@civil.aau.dk](mailto:jjn@civil.aau.dk)

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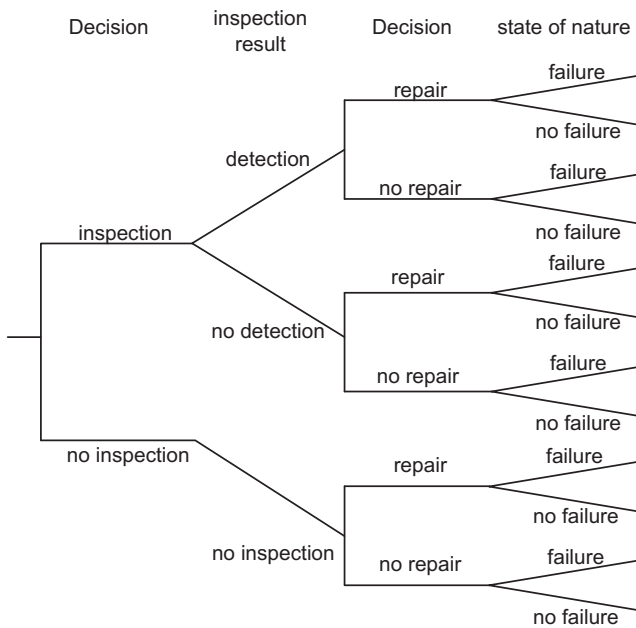


Figure 1. Typical decision tree for RBI.

geometrically distributed for Bayesian network models, it is possible to model semi-Markov models with other sojourn time distributions by adding virtual substates (Welte, 2009) or by using graphical duration models (Donat, Leray, Bouillaut, & Aknin, 2010). Deterioration processes with continuous parameters can be modelled by discretising the damage size and variables (Nielsen, 2013; Straub, 2009). In this way, dBNs can be used to model a variety of deterioration processes including non-Markovian models with time-invariant parameters.

Bayesian networks have previously been applied in relation to maintenance planning with the use of various approaches. Weber, Theilliol, Aubrun, and Evsukoff (2006) used dBNs for model-based fault diagnosis by estimating failure probabilities using several sensors, but did not consider costs in the decision-making. Jung, Véron, Suhner, and Muller (2005) used dBNs to estimate the effect of different time-based maintenance strategies on the performance of a considered sub-process. Bouillaut, Francois, and Dubois (2013) used Bayesian networks to evaluate maintenance strategies for underground rails based on diagnoses from several sources, and included condition-based maintenance where maintenance actions were based on the outcome of ultrasonic inspections. Friis-Hansen (2000) used influence diagrams based on Bayesian networks for inspection planning for fatigue cracks in offshore jacket structures.

Nielsen and Sørensen (2014) summarised several approaches for risk-based inspection and maintenance planning: decision tree, crude Monte Carlo simulations, discrete Bayesian networks, Markov chain Monte Carlo simulation (MCMC), simulation based using Bayesian network for decision support, limited memory influence diagrams (LIMID), and partially observable Markov decision processes (POMDP). Apart from the first two methods, they are all based on Bayesian networks. The methods using discrete Bayesian networks and MCMC can both be used to estimate total maintenance costs for simple stationary strategies, but MCMC does not require a discretisation of the continuous variables as the discrete Bayesian network approach does. Instead, MCMC is

a sampling-based inference method which is much more time-consuming than inference for discrete Bayesian networks, where efficient exact inference methods exist (Nielsen & Sørensen, 2010).

In the simulation-based approach using Bayesian network for decision support, it is possible to use more advanced stationary decision rules which depend on e.g. the probability of failure, but the method is much more time-consuming than the simple discrete Bayesian network approach (Nielsen & Sørensen, 2011). Time-variant decision policies can be found using the LIMID and POMDP approaches. In the LIMID approach, the optimal decision policies are found with a single policy updating algorithm, and only one type of decision (inspection or repair) can be optimised, as the algorithm will easily get stuck at a local minimum otherwise (Nielsen & Sørensen, 2011). Additionally, when decision policies and expected costs are found by the LIMID, not all previously obtained information is assumed to be available when decisions are made; therefore, decision policies should preferably be updated each time new information is obtained. When using POMDP, time-invariant decision policies can be found for Markov deterioration models. Nielsen and Sørensen (2015) used a grid-based approach, but more efficient point-based approaches have also been developed (Pineau, Gordon, & Thrun, 2003).

For the framework presented in this paper, the intent was to make a robust model framework, able of including different strategies for a variety of deterioration models (not only Markov models). Two approaches have been selected as the basis for the framework presented in this paper: discrete Bayesian networks and simulation based using Bayesian networks for decision support. This paper presents a computational framework for risk-based maintenance optimisation using Bayesian networks. In the framework, deterioration is modelled using dBNs, and two types of decision models are included for the computation of probabilities of inspections, repairs and failures in each time step:

- Decision model type 1: Bayesian network based.
- Decision model type 2: Simulation based (using Bayesian networks for decision support).

Decision model type 1 uses Bayesian networks directly to estimate probabilities. These computations are fast and exact given the input. This model can be applied for simple decision rules that depend on constants or observed variables, e.g. equidistant inspections, or repair when the inspection outcome exceeds a given threshold. Decision model type 2 uses simulations to estimate probabilities and is, therefore, more time-consuming. The advantage of decision model type 2 is the ability to include advanced decision rules that depend on variables that are updated using inspection and CM information, e.g. the probability of failure within the next time step. For this decision model, dBNs are used within the simulations for updating of the model using observations. Various types of strategies including time-based and condition-based maintenance based on various types of information are implemented in the framework. A numerical example illustrates the model framework and the implemented strategies.

## Computational framework

This section presents the computational framework for risk-based maintenance planning. The framework has been implemented in Matlab (The MathWorks, 2006). Figure 2 illustrates the structure

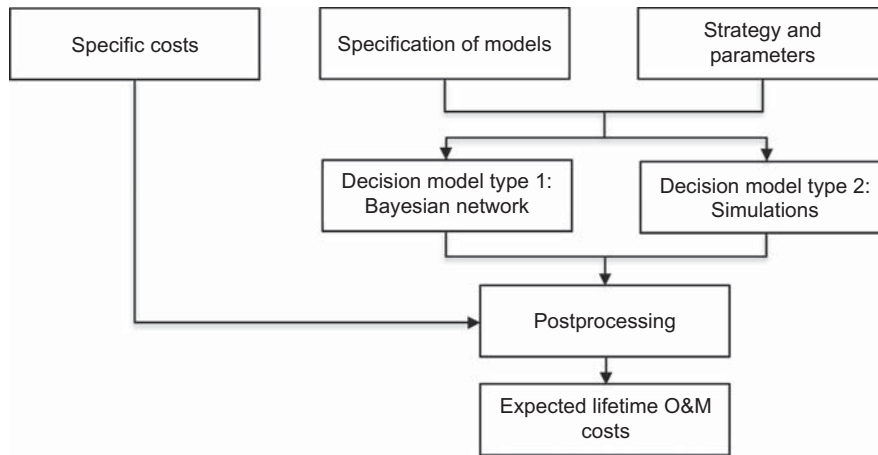


Figure 2. Structure of computational framework.

of the framework. The three blocks in the top row represent the inputs: ‘specific costs’, ‘specification of models’ and ‘strategy and parameters’. The ‘specific costs’ are the expected costs of an inspection, a preventive repair, and a corrective repair following failure, including all contributors such as spare parts, personnel, vessel, fuel and lost revenue. In the block ‘specification of models’, the probabilistic models for deterioration, repair, inspection and CM are defined in terms of conditional probability distributions, as explained in the next section. In the last input block ‘strategy and parameters’, the strategies and associated parameters are chosen. A total of 13 strategies are implemented in the model framework and will be presented in the section ‘Strategies’.

Once the input is defined, the probabilities of inspections, repairs and failures for each time step can be estimated. This is done using one of the two decision models shown in the second row in Figure 2. These decision models are the core of the framework. They estimate the probabilities of inspections, repairs and failures for each time step based on the specified models, strategies and decision parameters. Subsequently, the obtained probabilities are combined with the specific costs of inspection, repair and failure in order to obtain the total expected costs during the planned lifetime (lifetime costs). This is done for all candidate strategies and associated decision parameters, and the one yielding the lowest lifetime costs is the optimal strategy in the risk-based approach.

**Specification of models**

As dBNs are used for calculating probabilities, the input models need to be given as conditional probability distributions for each node, conditioned on the parent nodes, i.e. the nodes that point to a given node. Figure 3 shows the basic dBN including all nodes used in the framework. This dBN does not model any decision rules directly and is the dBN used for decision model type 2. For decision model type 1, decision rules are directly modelled in the network, and the dBN will differ for each strategy.

The nodes included in the model are as follows:

- $D_i$ : damage size
- $D_{R,i}$ : damage size after any repairs
- $M_i$ : model parameter

- $R_i$ : repair decision.
- $I_i$ : inspection outcome
- $I_{M,i}$ : CM outcome
- $I_{D,i}$ : inspection decision

The first four nodes are included for all strategies, and the presence of the last three nodes depends on the strategy applied. Two nodes ( $D_i$  and  $D_{R,i}$ ) are included for the damage size in each time step ( $i$ ) so as to include repairs in the model. Repairs are assumed to be performed in the beginning of a time step if a decision is made on preventive repair or if failure occurred during the last time step. Both repair types are applied between nodes  $D_i$  and  $D_{R,i}$  and deterioration is applied between  $D_{R,i}$  and  $D_{i+1}$ .

**Deterioration model**

The deterioration model is specified in terms of four probability distributions:

- $P(M_0)$ : prior distribution for initial value of model parameter.
- $P(D_0)$ : prior distribution for initial damage size.
- $P(M_i|M_{i-1})$ : conditional probability distribution for model parameter given the value of the model parameter in the previous time step.
- $P(D_i|D_{R,i-1}, M_i)$ : conditional probability distribution for damage size given the damage size after repairs in the previous time step, and given the value of the model parameter.

If nodes  $D_i$  and  $D_{R,i}$  each has  $n_D$  states, and node  $M_i$  has  $n_M$  states,  $P(M_0)$  and  $P(D_0)$  are vectors of size  $n_M$  and  $n_D$ , respectively.  $P(M_i|M_{i-1})$  is a matrix of size  $n_M \times n_M$  and  $P(D_i|D_{R,i-1}, M_i)$  is a three-dimensional matrix of size  $n_D \times n_D \times n_M$ . If the model parameter  $M_i$  is time-invariant, the distribution  $P(M_i|M_{i-1})$  is the identity matrix of size  $n_M$ . If there are no stochastic model parameters in the deterioration model (i.e. for a Markov model), the node  $M_i$  could be omitted, but for simplicity it is included as a scalar value.

**Repair model**

The repair model is specified in terms of the conditional probability distribution  $P(D_{R,i}|D_p, R_i)$  for the damage size after repairs

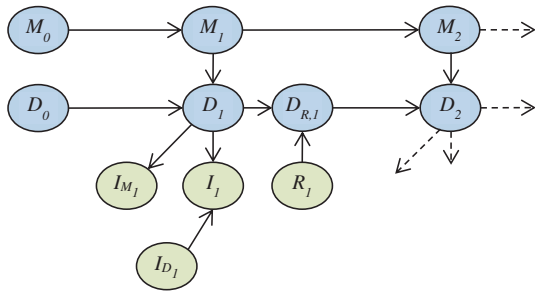


Figure 3. Basic dBN including all nodes used in the framework, and used for decision model type 2.  $D_i$ : damage size,  $D_{R,i}$ : damage size after any repairs,  $M_i$ : model parameter,  $I_i$ : inspection outcome,  $I_{M,i}$ : CM outcome,  $I_{D,i}$ : inspection decision,  $R_i$ : repair decision.

given the damage size before repairs and given the repair decision. The node  $R_i$  for the preventive repair decision has two states: 'no repair' and 'repair', and thereby the distribution  $P(D_{R,i}|D_p, R_i)$  is a matrix of size  $n_D \times n_D \times 2$ . The repair model controls the quality of the repair. If perfect repairs are assumed, the distribution for the damage size would return to the distribution for the initial damage size after a repair. However, the framework also allows for imperfect repairs to be modelled.

### Inspection model

The inspection model defines the reliability of the inspection method. The inspection model is given in terms of the conditional probability distribution  $P(I_i|D_p, I_{D,i})$  for the inspection outcome given the damage size and inspection decision. The node  $I_i$  has  $n_I$  states, and the node  $I_{D,i}$  has two states for 'no inspection' and 'inspection', respectively. The distribution is a matrix of size  $n_I \times n_D \times 2$ . The last state of  $I_i$  is the dummy outcome 'no inspection' if no inspection is made. The other  $n_I - 1$  states are the possible outcomes when an inspection is made.

### Condition monitoring model

The CM outcomes are assumed to be independent given the damage size. The CM model is, therefore, given as the conditional probability distribution  $P(I_{M,i}|D_i)$  for the CM outcome given the damage size. If there are  $n_{I_M}$  possible CM outcomes, the distribution is a matrix of size  $n_{I_M} \times n_D$ .

### Strategies

Each strategy is defined in terms of one decision rule for inspections and one decision rule for preventive repairs. In total, 13 strategies have been implemented. Strategies 0–4 only use simple decision rules, and therefore both decision model types can be used for the computations. Strategies 5–12 also include advanced decision rules where the decision depends on the probability of failure ( $P_f$ ) or the expected damage size ( $E_D$ ). Therefore, decision model type 2 must be used. Strategy 0 only includes corrective maintenance, and the remaining strategies use variants of preventive maintenance:

- (0) Corrective maintenance only.
- (1) No inspections, equidistant repairs.
- (2) No inspections, repairs based on CM outcome.

- (3) Equidistant inspections, repairs based on inspection outcome.
- (4) Inspections based on CM outcome, repairs based on inspection outcome.
- (5) No inspections, repairs based on  $P_f$ .
- (6) Equidistant inspections, repairs based on  $P_f$ .
- (7) Inspections based on  $P_f$ , repairs based on  $P_f$ .
- (8) Inspections based on  $P_f$ , repairs based on inspection outcome.
- (9) No inspections, repairs based on  $E_D$ .
- (10) Equidistant inspections, repairs based on  $E_D$ .
- (11) Inspections based on  $E_D$ , repairs based on  $E_D$ .
- (12) Inspections based on  $E_D$ , repairs based on inspection outcome.

Equidistant inspections (or repairs) can be defined in two ways: as the number of inspections in the lifetime or as the time between inspections. For shorter inspection intervals, e.g. 6 months, one year or two years, the natural choice is to define it in terms of time between inspections. In contrast, for larger inspection intervals where only two or three inspections are performed during the lifetime, it is more natural to let the number of inspections be the decision variable and then schedule the inspections so the time between inspections is the same as the time before the first inspection and after the last inspection.

For decision rules based on CM outcome, inspection outcome,  $P_f$  or  $E_D$ , the decision parameter is the threshold value for when inspections/repairs should be made. For decisions based on  $P_f$  an additional parameter is the reference time for  $P_f$ , for example one month or one year. For decision model type 2 there are two additional parameters: (1) is CM included (yes/no)? and (2) are pre-inspections necessary for repairs (yes/no)? If CM is included, CM outcomes are assumed to be obtained in each time step, and the deterioration model is updated on that basis. If pre-inspections are necessary for repairs, the repair decision will always be 'no repair' if no inspection is performed in that time step.

### Type 1 decision model: Bayesian network

In the type 1 decision model, dBNS are used directly for the computation of probabilities of inspections, repairs and failures in each time step. To do so, the strategies are implemented directly in the dBNS, which is possible for strategies 0–4. This is done in two distinct ways depending on the decision rules used in the strategy. For decision rules based on CM or inspection outcome, the decision rules are modelled using a link from the node for the CM or inspection outcome ( $I_{M,i}$  or  $I_i$ ) to the node for the inspection or repair decision ( $I_{D,i}$  or  $R_i$ ).

The distribution for the decision is then given conditioned on the CM or inspection outcome. Equidistant inspections are modelled by assigning a probability of one for the state 'inspection' in the node  $I_{D,i}$  for time steps with scheduled inspections, and zero in the time steps with no scheduled inspections. Equidistant repairs are modelled in the same way by assigning probabilities for the  $R_i$  nodes. The dBNS for strategies 0–4 are constructed following the description above and shown in Figure 4.

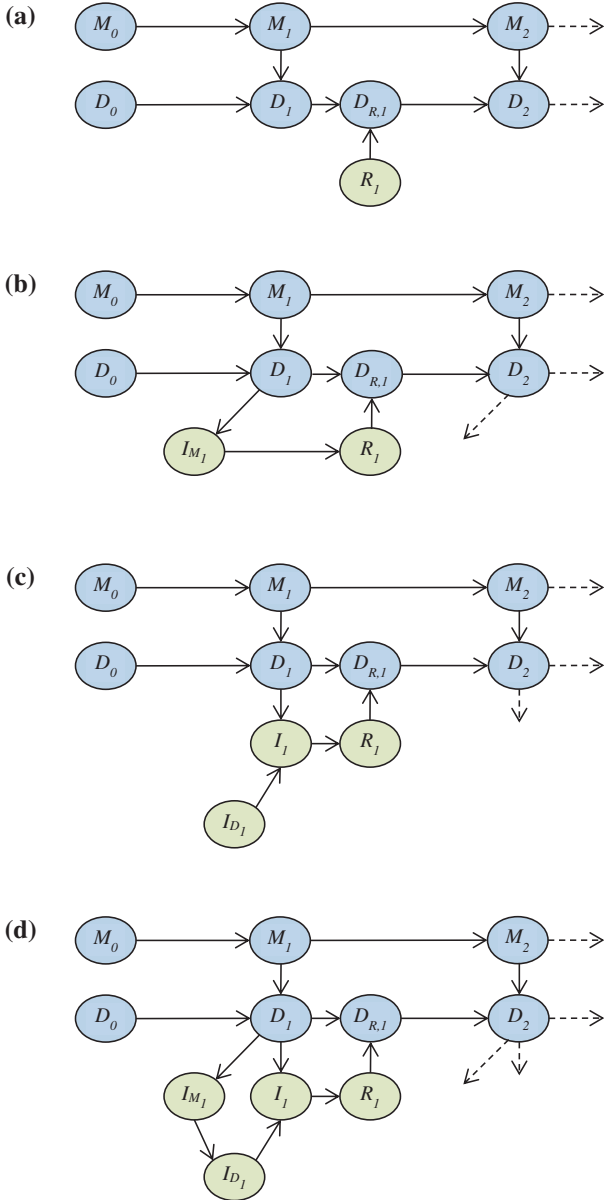


Figure 4. Bayesian networks for decision rules (a) 0–1, (b) 2, (c) 3 and (d) 4 in the type 1 decision model.  $D_i$ : damage size,  $D_{R,i}$ : damage size after any repairs,  $M_i$ : model parameter,  $I_i$ : inspection outcome,  $I_{M,i}$ : CM outcome,  $I_{D,i}$ : inspection decision,  $R_i$ : repair decision.

**Algorithm**

The dBNs in Figure 4 can be used directly for the assessment of probabilities using exact inference algorithms. However, as the network structures are slightly different, the algorithms used to calculate probabilities would be slightly different for each network. To simplify the framework, each network is instead collapsed to the network shown in Figure 5. This way the same algorithm can be used for all strategies in the type 1 decision model. In the collapsed network, the nodes for CM outcome, inspection outcome and inspection decision are omitted and instead a link is added from the node for the damage size ( $D_i$ ) to the node for the repair decision ( $R_i$ ).

This can be done since the node  $R_i$  is independent of all other nodes, given node  $D_i$ , and it is not necessary to enter evidence into any nodes (including the omitted) to compute the

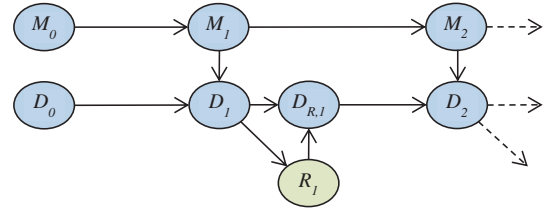


Figure 5. Collapsed Bayesian networks for decision rules 0–4 in the type 1 decision model.  $D_i$ : damage size,  $D_{R,i}$ : damage size after any repairs,  $M_i$ : model parameter,  $R_i$ : repair decision.

probabilities. The distribution  $P(R_i|D_i)$  will depend on the strategy (the distributions for the inspection and repair decision) and on the distribution for the CM model and inspection model. In case equidistant inspections (or repairs) are used, the distribution  $P(R_i|D_i)$  will not be identical for all time steps as repairs will only be possible in time steps with scheduled inspections (or repairs). In this case, the Bayesian network is not formally a dBN as the conditional probability distributions need to be equal for all time steps to be a dBN. However, the same algorithms can be applied.

Each time slice is connected to the past only through the nodes  $D_i$  and  $M_i$ , and the nodes in each time slice are, therefore, independent of all other past nodes given the nodes  $D_i$  and  $M_i$  in their time slice. Therefore, to find the marginal probabilities for each node in the network, the joint distribution  $P(D_i, M_i)$  is found sequentially based on the joint distribution for the previous time step  $P(D_{i-1}, M_{i-1})$  and the conditional probability distributions for the nodes  $D_i, M_i, R_{i-1}$  and  $D_{R,i-1}$ . This can be done by first taking the product of the probability distributions to obtain the joint distribution of all nodes in the previous time step and  $D_i$  and  $M_i$  and then by marginalising out the nodes from the previous time step:

$$P(D_i, M_i) = \sum_{D_{R,i-1}} \sum_{D_{i-1}} \sum_{R_{i-1}} \sum_{M_{i-1}} P(D_i|D_{R,i-1}, M_i) P(D_{R,i-1}|D_{i-1}, R_{i-1}) \times P(R_{i-1}|D_{i-1}) P(M_i|M_{i-1}) P(D_{i-1}, M_{i-1}) \tag{1}$$

This procedure is time-demanding and memory-consuming as a six-dimensional matrix is made. A more effective algorithm can be made using variable elimination where variables are marginalised out as soon as possible:

$$P(D_i, M_i) = \sum_{D_{R,i-1}} P(D_i|D_{R,i-1}, M_i) \sum_{D_{i-1}} \times \sum_{R_{i-1}} P(D_{R,i-1}|D_{i-1}, R_{i-1}) P(R_{i-1}|D_{i-1}) \times \sum_{M_{i-1}} P(M_i|M_{i-1}) P(D_{i-1}, M_{i-1}) \tag{2}$$

In this version of the algorithm, the maximum number of dimensions to handle simultaneously is four, and this version has been implemented in the framework. The algorithm is an exact inference algorithm inspired by the forward operation presented by Straub (2009) and by the forward operation of the frontier algorithm and interface algorithm presented by (Murphy, 2002).

The probability of inspection, repair and failure for each time step is found subsequently based on the distribution  $P(D_i, M_i)$ . First, the marginal distribution  $P(D_i)$  is found by:

$$P(D_i) = \sum_{M_i} P(D_i, M_i) \tag{3}$$

The probability that failure happened during the previous time step is equal to the probability of  $D_i$  being in the state 'fault' (the last state). The marginal probability distribution for the node  $R_i$  is found by:

$$P(R_i) = \sum_{D_i} P(R_i|D_i)P(D_i) \quad (4)$$

The probability of repair for time step  $i$  is equal to the probability of being in the state 'repair' (the second state) for node  $R_i$ . The probability of inspection is always zero for strategies 0–2; for strategy 3 where inspections are equidistant, the probability is either zero or one. For strategy 4, the marginal distribution for the node  $I_{D,i}$  is found by:

$$P(I_{D,i}) = \sum_{I_{M,i}} P(I_{D,i}|I_{M,i}) \sum_{D_i} P(I_{M,i}|D_i)P(D_i) \quad (5)$$

For strategies 3 and 4, the probability of inspection for time step  $i$  is equal to the probability of being in the state 'inspection' (the second state) for node  $I_{D,i}$ .

The distribution  $P(R_i|D_i)$  depends on the strategy. For strategy 0 in all states of  $D_i$ , there are no preventive repairs and  $R_i$  is always in the state 'no repair' (the first state). For strategy 1, equidistant repairs are used and  $R_i$  is in the state 'repair' (the second state) in all states of  $D_i$  in time steps with scheduled repairs, and in the state 'no repair' (the first state) for all other time steps. For strategy 2, repairs are made based on the CM outcome, and the distribution is obtained by:

$$P(R_i|D_i) = \sum_{I_{M,i}} P(R_i|I_{M,i})P(I_{M,i}|D_i) \quad (6)$$

For strategy 3, the distribution is obtained by:

$$P(R_i|D_i) = \sum_{I_i} P(R_i|I_i) \sum_{I_{D,i}} P(I_i|D_i, I_{D,i})P(I_{D,i}) \quad (7)$$

where  $I_{D,i}$  is in the state 'inspection' for time steps with scheduled inspections, and in the state 'no inspection' for other time steps. For strategy 4, the distribution is obtained by:

$$P(R_i|D_i) = \sum_{I_i} P(R_i|I_i) \sum_{I_{D,i}} P(I_i|D_i, I_{D,i}) \sum_{I_{M,i}} P(I_{D,i}|I_{M,i})P(I_{M,i}|D_i) \quad (8)$$

### Type 2 decision model: simulations

In the type 2 decision model, simulations are used to estimate the probabilities of inspections, repairs and failures in each time step. The estimated probabilities are not the exact results as for the type 1 decision model, instead they are the observed frequencies (i.e. the number of simulations with inspections, repairs or failures in each time step) divided by the total number of simulations. The accuracy and computation time increase with the number of simulations. In addition to the overall frequency of inspections, repairs and failures in each time step, the total number of inspections, repairs and failures for each simulation is also saved for the computation of confidence intervals for the lifetime costs.

The type 2 decision model can be used for all strategies and uses the same input models as the type 1 decision model. All decision rules are applied as logic rules within the simulations. For strategies 0–4, only simple decision rules are used and no dBN is needed. For strategies 5–12, the dBN in Figure 3 is used

within each simulation to update the distribution for the damage size, each time CM or inspection outcomes are received. This is necessary to use the advanced decision rules that depend on  $P_f$  or  $E_D$ .

### Algorithm

For strategies 5–12, the dBN is maintained in each time step using information on CM and inspection outcomes and inspection and repair decisions. In the following, capital letters denote stochastic variables and lower-case letters denote realisations of stochastic variables. As for the type 1 decision model, the algorithm is an exact inference algorithm inspired by the forward operation of the frontier and interface algorithm (Murphy, 2002; Straub, 2009). However, for the type 2 decision model, evidence is included using Bayes rule. Evidence is obtained for four nodes in each time step: The nodes for decisions on inspections  $I_{D,i}$  and repairs  $R_i$ , and the nodes for observations from CM  $I_{M,i}$  and inspections  $I_i$ .

The evidence on repairs is used when calculating the distribution  $P(D_i, M_i)$  based on the distribution from the previous time step. Otherwise, the algorithm is similar to the algorithm used for the type 1 decision model:

$$P(D_i, M_i) = \sum_{D_{R,i-1}} P(D_i|D_{R,i-1}, M_i) \sum_{D_{i-1}} P(D_{R,i-1}|D_{i-1}, R_{i-1} = r_{i-1}) \times \sum_{M_{i-1}} P(M_i|M_{i-1})P(D_{i-1}, M_{i-1}) \quad (9)$$

The evidence used herein is the repair decision,  $r_{i-1}$ , for time step  $i-1$ . To include evidence from CM and inspections, Bayes rule must be applied to update  $P(D_i, M_i)$ . It is only necessary to update  $P(D_i, M_i)$  as all other nodes are independent on  $I_{M,i}$ ,  $I_i$  and  $I_{D,i}$  given  $D_i$  and  $M_i$ . When CM is available, the distribution  $P(D_i, M_i)$  is updated based on CM outcome  $i_{M,i}$  in the following way:

$$P(D_i, M_i|I_{M,i} = i_{M,i}) \propto P(D_i, M_i, I_{M,i} = i_{M,i}) = P(D_i, M_i)P(I_{M,i} = i_{M,i}|D_i) \quad (10)$$

where ' $\propto$ ' means 'proportional to'. Subsequently, the distribution is scaled to sum to one. In reality, the distribution is not only conditioned on  $I_{M,i} = i_{M,i}$ , but on all past CM and inspection outcomes and decisions, and the same counts for  $P(D_i, M_i)$ . This is omitted from the notation for simplicity and is also omitted from the following equations. Similarly, the distribution is updated based on inspection outcome  $i_i$ :

$$P(D_i, M_i|I_i = i_i) \propto P(D_i, M_i, I_i = i_i) = P(D_i, M_i)P(I_i = i_i|D_i, I_{D,i} = i_{D,i}) \quad (11)$$

where  $i_{D,i}$  is equal to zero when no inspections are made and equal to one when an inspection is made.

For each simulation, the procedure is as follows (where points marked with asterisk (\*) only applies for advanced decision rules):

- Draw value of  $d_0$  from the distribution  $P(D_0)$ .
- Draw value of  $m_0$  from the distribution  $P(M_0)$ .
- Calculate initial distribution:  $P(D_0, M_0) = P(D_0)P(M_0)$ .\*

- For each time step  $i$ :
  - Draw monitoring outcome  $i_{M,i}$  from  $P(I_{M,i}|D_i = d_i)$ .
  - Update distribution  $P(D_i, M_i)$  using Bayes rule.\*
  - Make decision on inspection  $i_{D,i}$  based on the decision rule.
  - Draw inspection outcome  $i_i$  from  $P(I_i|D_i = d_i, I_{D,i} = i_{D,i})$ .
  - Update distribution  $P(D_i, M_i)$  using Bayes rule.\*
  - Make decision on repair  $r_i$  based on the decision rule.
  - Draw  $d_{R,i}$  from  $P(D_{R,i}|D_i = d_i, R_i = r_i)$ .
  - Draw  $d_{i+1}$  from  $P(D_{i+1}|D_{R,i} = d_{R,i}, M_{i+1} = m_0)$ .
  - Update distribution  $P(D_{i+1}, M_{i+1})$  using the algorithm for deterioration and repairs.\*

After each simulation, the contribution to the overall frequencies of inspections, repairs and failures is found as one divided by the number of simulations for time steps with inspections, repairs or failures. These frequencies of inspections, repairs and failures approximate the probability of inspections, repairs and failures. In the same way, the marginal distribution for the damage size can be found for each time step based on the contribution from each of the simulated values  $d_i$ .

**Postprocessing**

When the probability of inspection, repair and failure has been found for each time step using one of the decision models, the total expected lifetime costs can be found using the specific costs of an inspection, a repair and a failure by multiplication. For decision model type 2, confidence intervals are found for the total expected costs using the total number of inspections, repairs and failures for each simulation.

The specific costs should include all direct and indirect costs related to the events such as costs of spare parts, salary to technicians, lease of equipment and lost revenue. The costs could vary with time due to seasons or discounting. When the total expected costs have been found for each candidate strategy and associated parameters, the optimal strategy is identified as the one leading to lowest expected costs.

**Example**

In this section, a numerical example illustrates the model framework, the implemented strategies and how to specify the models. The type of component considered in this example could be an offshore wind turbine component where life safety is assumed not to be an issue in relation to failures, and cost-benefit optimisation can be performed without considering minimum reliability requirements. However, the total costs of a corrective repair following failure including lost revenue are assumed to be significantly larger (€400 k) than the costs of a preventive repair (€30.0 k). The price of an inspection is assumed to be €800 (Salmon, 2015). The costs of online condition monitoring are not included in the estimated costs. Instead, the value of information from condition monitoring is computed, and this value should

be compared to the costs of a condition monitoring system. The planned lifetime of the wind farm is assumed to be 20 years. For simplicity, discounting is not included.

Generally, the models and costs used in this example are fictive, but realistic. For an actual case study, the costs should be estimated based on the weather on the location, the applied vessels, repair methods, inspection methods, and should consider system effects in this relation. The models should be estimated based on available data and knowledge of the applied techniques for e.g. inspections and condition monitoring. Such a case study is beyond the scope of this work, and a comprehensive case study is planned to be published in a separate paper. In this paper, the intention of the example is to provide the reader with a clear idea of the capabilities of the model framework.

**Specification of models**

The node  $D_i$  for the damage size has seven states (0–6) where the last state is ‘fault’. The damage size is measured on a relative scale from zero to one where defects larger than one are in the state ‘fault’. The six non-faulty states are of equal size. Initially, the damage size is assumed to be within the interval covered by the first state. Therefore, the prior distribution for initial damage size is as follows:

$$P(D_0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

The node  $M_i$  for the model parameter has three possible states:  $m_1 = 0.7$ ,  $m_2 = 1.0$  and  $m_3 = 1.3$ . The prior distribution for the initial value of the model parameter is assumed to be uniform:

$$P(M_0) = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} \tag{13}$$

The model parameter is assumed to be time-invariant. Therefore, the conditional probability distribution for the model parameter given the model parameter in the previous time step is the identity matrix of size three:

$$P(M_i|M_{i-1}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

For the deterioration model, it is assumed that it is only possible to transfer one state in each time step (one month), and the transition probability is  $p \cdot M$  for all states. The conditional probability distribution for the damage size given the damage size in the previous time step and given the value of the model parameter is as follows:



$$P(D_i|D_{R,i-1}, M_i = m_k) = \begin{bmatrix} 1 - p \cdot m_k & 0 & 0 & 0 & 0 & 0 & 0 \\ p \cdot m_k & 1 - p \cdot m_k & 0 & 0 & 0 & 0 & 0 \\ 0 & p \cdot m_k & 1 - p \cdot m_k & 0 & 0 & 0 & 0 \\ 0 & 0 & p \cdot m_k & 1 - p \cdot m_k & 0 & 0 & 0 \\ 0 & 0 & 0 & p \cdot m_k & 1 - p \cdot m_k & 0 & 0 \\ 0 & 0 & 0 & 0 & p \cdot m_k & 1 - p \cdot m_k & 0 \\ 0 & 0 & 0 & 0 & 0 & p \cdot m_k & 1 \end{bmatrix} \quad (15)$$

The mean value of the transition probability ( $p$ ) is chosen to result in a mean time to failure of 20 years. As there are six transitions from the initial state to fault, the probability is:  $p = \frac{6}{20 \cdot 12 \text{ months}} = 0.025/\text{month}$ .

The repair model is given as the conditional probability for the damage size after any repair, given the damage size before any repairs and given the repair decision. If the node for the repair decision is in the state 'zero' (no preventive repair), the distribution is equal to the identity matrix except for the last column. If the damage size is in the state 'fault' prior to any repairs, a perfect corrective repair is assumed to be made, and the damage size transfers to the initial state:

$$P(D_{R,i}|D_i, R_i = 0) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (16)$$

If the node for the repair decision is in the state one, a perfect preventive repair is assumed to be made, returning the damage size to the initial state:

$$P(D_{R,i}|D_i, R_i = 1) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

The inspection model is the conditional probability distribution for the node for the inspection outcome given the damage size and the inspection decision. The node for the inspection outcome has one more state than the node for the damage size. The first state is '0: no detection', and the last state is '7: no inspection'. Between those, there are six states for detection of defects of sizes 1 to 6. If no inspection is made ( $I_{D,i} = 0$ ), the outcome of the inspection is 'no inspection':

$$P(I_i|D_i, I_{D,i} = 0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (18)$$

If an inspection is made and a defect is detected, it is assumed that the damage size is categorised correctly. Thereby, the only uncertainty is the probability of detection, which depends on the size of the defects. The inspection model when inspections are made ( $I_{D,i} = 1$ ) is assumed to:

$$P(I_i|D_i, I_{D,i} = 1) = \begin{bmatrix} 1 & 0.6 & 0.2 & 0.1 & 0.05 & 0.02 & 0 \\ 0 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.95 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.98 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

For the CM outcome, there are four possible states: 1: no detection, 2: low alarm, 3: high alarm and 4: fault. The fault outcome comes with certainty when the damage size is in the fault state. The overall probability of alarm is assumed to increase with damage sizes 0, 0, 0.1, 0.2, 0.4, 0.8, 1. Unless the damage size is fault, the probability of high alarm given that there is an alarm is assumed to increase with sizes 0, 0, 0.2, 0.4, 0.6, 0.8, 0. These probabilities are assumed not to include false alarms. A false alarm rate of 3% is included independent of damage size (the state fault excluded), divided on 2% chance of false low alarm and 1% chance of false high alarm. The resulting conditional probability distribution for the CM outcome given the damage size is as follows:

$$P(I_{M,i}|D_i) = \begin{bmatrix} 0.97 & 0.97 & 0.87 & 0.77 & 0.57 & 0.17 & 0 \\ 0.02 & 0.02 & 0.10 & 0.14 & 0.18 & 0.18 & 0 \\ 0.01 & 0.01 & 0.03 & 0.09 & 0.25 & 0.65 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (20)$$

## Results

With the models defined, the optimal strategy can be found from a set of candidate strategies and parameters. For strategies 0–4, both decision model types can be used, and for strategies 5–12, only the type 2 model can be used. Initially, the optimal decision parameters are found for strategies 0–4 using the type 1 decision model, and the type 2 decision model is run for the optimal parameters for comparison/validation. Figure 6 shows the total expected lifetime costs for each strategy for both decision model types. For decision model type 2, 10,000 simulations have been used, and the 95% confidence intervals for the total expected costs are shown, assuming normally distributed total costs.

Note that the shown confidence intervals are for the expected value of the total lifetime costs, not for the total lifetime costs.

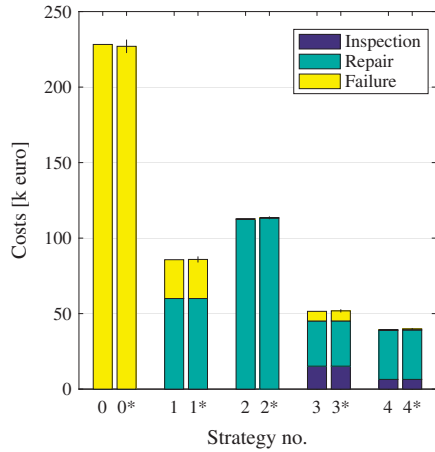


Figure 6. Total expected lifetime costs for strategies 0–4 found using both decision model types. Results found using decision model type 2 are marked with an asterisk (\*). The black vertical lines indicate 95% confidence intervals.

The width of the confidence intervals for the expected value will go towards zero, as the number of simulations goes towards infinite. If too few simulations have been made, this would be evident from the width of the confidence intervals. The total costs obtained using the two methods are generally very similar, and the exact results found using the type 1 decision model are generally within the 95% confidence intervals found using the type 2 decision model. For strategy 0, only corrective maintenance is used, and this results in large costs, €228 k. The lowest costs when using simple decision rules are €39.5 k obtained in strategy 4 where inspections are made based on the CM outcomes, and preventive repairs are made based on the inspection outcomes.

The differences between the models and strategies are clearly seen when considering the direct outcome from the decision models: the probabilities/frequencies of inspections, repairs and failures in each time step. Figures 7–11 illustrate these differences for strategies 0–4. For strategies without inspections or preventive repairs, the frequency for those is always zero; therefore, the figures for these strategies are not shown. The probabilities found using decision model 1 are the exact results as they are found using dBNs. They are shown using a solid line although the results are discrete, for easier distinction between the two decision models.

For strategy 0, Figure 7 shows the probability/frequency of failure for each time step. The probability of failure gradually increases, but stagnates near the end of the lifetime. The results found using simulations are scattered, but generally approximate the exact results well. A more accurate estimate can be obtained by increasing the number of simulations.

Strategy 1 (Figure 8) uses scheduled preventive repairs. The probability of failure is gradually increasing, but drops to the initial value after preventive repairs. The probability of preventive repairs is one in the time steps where repairs are scheduled; otherwise, it is zero.

In strategy 2 (Figure 9), a threshold for the CM outcome determines when it is time for preventive repairs. The probability of preventive repairs first increases and then stagnates at a constant value after approximately 6 years. For this strategy, the probability of failure for each time step is too low to be approximated well by using 10,000 simulations, as only two failures happened

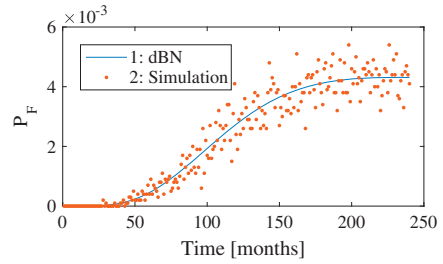


Figure 7. Probability/frequency of failure ( $P_F$ ) for each time step for decision model 1 (1: dBN) and 2 (2: Simulations) for strategy 0.

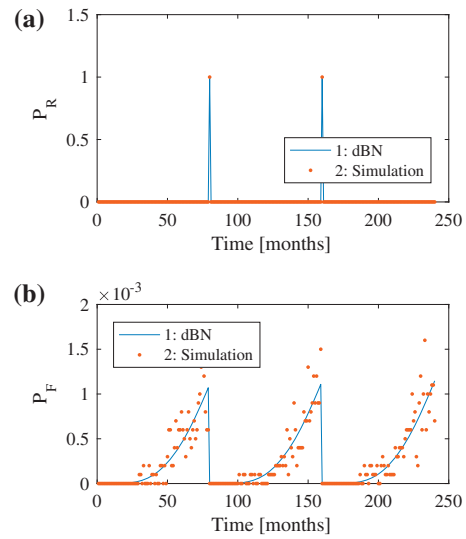


Figure 8. Probability/frequency of (a) repair ( $P_R$ ) and (b) failure ( $P_F$ ) for each time step for decision model 1 (1: dBN) and 2 (2: Simulations) for strategy 1.

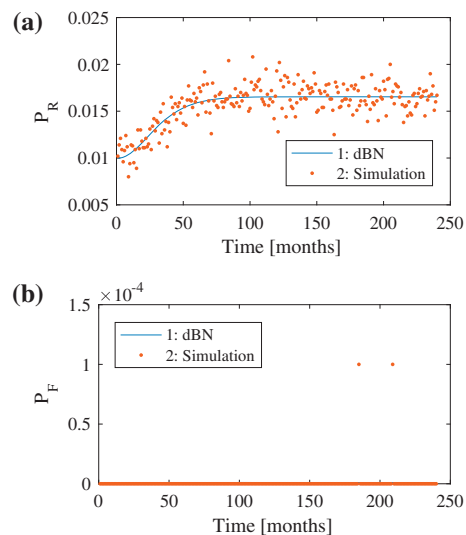


Figure 9. Probability/frequency of (a) repair ( $P_R$ ) and (b) failure ( $P_F$ ) for each time step for decision model 1 (1: dBN) and 2 (2: Simulations) for strategy 2.

during the 10,000 simulations. However, 10,000 simulations are sufficient to estimate the total expected costs, as the contribution from failures is insignificant compared to the contribution from preventive repairs, as is evident from Figure 6.

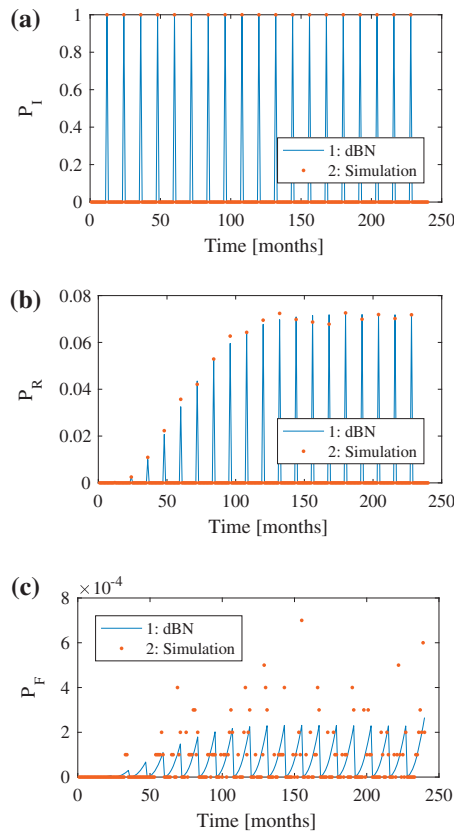


Figure 10. Probability/frequency of (a) inspection ( $P_I$ ), (b) repair ( $P_R$ ) and (c) failure ( $P_F$ ) for each time step for decision model 1 (1: dBN) and 2 (2: Simulations) for strategy 3.

In strategy 3 (Figure 10), inspections are equidistant, and repairs are made following inspections when the inspection outcome exceeds a threshold. The probability of preventive repairs is non-zero in time steps with inspections. It increases the first 10 years, then stagnates. The probability of failure generally increases between inspections and drops immediately following inspections.

In strategy 4 (Figure 11), inspections are made when a threshold for the condition monitoring outcome is exceeded, and preventive repairs are made when a threshold for the inspection outcome is exceeded. All probabilities gradually increase and then stagnate after approximately 10 years. As for strategy 2, the probability of failure per time step is low, but the contribution to the total expected costs is also low; therefore, 10,000 simulations are sufficient to estimate the costs.

Generally, the optimal decision parameters for each strategy are found from a discrete set of candidates as the parameters yielding the lowest expected lifetime costs. It has been examined manually for each strategy that the range of the parameters has been sufficient, i.e. the found optimum should be an internal point, unless the minimum or maximum value used is a fixed boundary, e.g. the largest CM outcome before the outcome corresponding to failure. As an example, Figure 12 shows the total costs for each candidate set of parameters for strategy 3. The minimum expected lifetime costs are obtained when an inspection interval of 12 months is used and when all defects of size 4 or above are repaired preventively.

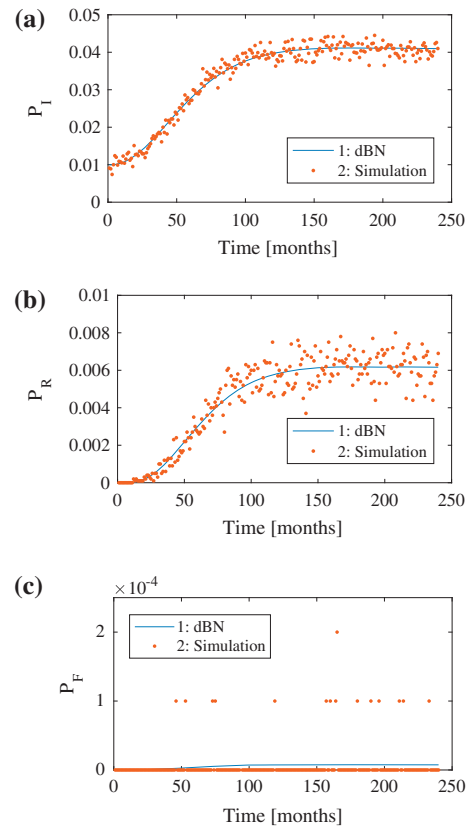


Figure 11. Probability/frequency of (a) inspection ( $P_I$ ), (b) repair ( $P_R$ ) and (c) failure ( $P_F$ ) for each time step for decision model 1 (1: dBN) and 2 (2: Simulations) for strategy 4.

For strategies 5–12, only decision model type 2 can be used as a threshold for either the probability of failure or the expected damage size is used as decision parameter. Strategies 5–8 use the probability of failure, and strategies 9–12 use the expected damage size, but apart from that, strategies 5–8 are equal to strategies 9–12. When a threshold value for the probability of failure is used, the reference time can be chosen. For comparison, the decision model type 2 has been run for strategies 5–8 with a reference time of both 1 and 12 months, and the results are shown in Figure 13. The two reference times generally give similar costs, but a reference time of 12 months gives slightly lower costs; the following will use a reference time of 12 months.

In the costs shown, the cost of CM is not included. Therefore, it would be an additional cost for strategies using information from CM. The value of CM can be found as the difference between the optimal strategy using CM, and the optimal strategy not using CM. This corresponds to the value of information (VoI) (Raiffa & Schlaifer, 1961). Strategies 5–12 can be used both with and without CM. Therefore, the strategies relevant without CM are 0, 1, 3 and 5–12. Of these, 0, 1, 5 and 9 do not include inspections. Strategies with CM are 2, 4 and 5–12. Of these, 2, 5 and 9 do not include inspections.

Table 1 presents the optimal decision parameters and total costs for each strategy without CM, and Figure 14 depicts the total expected costs for preventive strategies without CM, divided on inspections, preventive repairs, and failures. Strategies without inspections are bundled, and strategies with equidistant

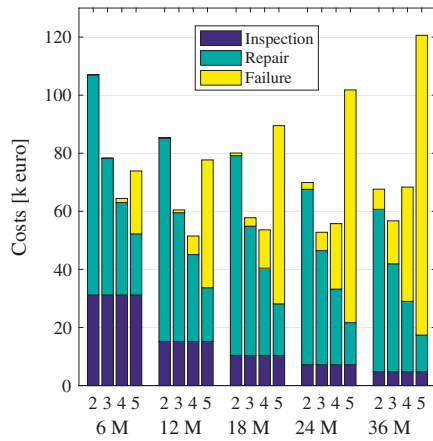


Figure 12. Total expected lifetime costs for strategy 3 for each set of decision parameters: Inspections intervals: 6, 12, 18, 24 and 36 months, repair threshold for inspection outcome: 2, 3, 4 and 5.

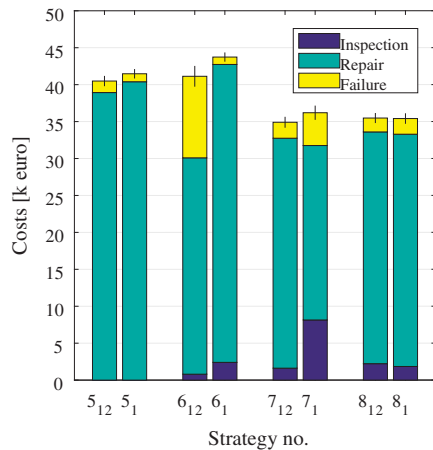


Figure 13. Total expected lifetime costs for strategies 5–8. The subscribed numbers are the reference time for the probability of failure in months.

inspections are bundled. Without CM, the three preventive strategies without inspections (1, 5 and 9) result in similar costs, approximately €85 k. The three strategies with equidistant inspections (3, 6, 10) all use an interval of 12 months and result in similar costs, approximately €52 k. Of the four remaining strategies, a threshold for the probability of failure is used for inspections for strategies 7 and 8, and a threshold for the expected damage size is used for strategies 11 and 12. For strategy 7, the lowest costs are obtained (€50 k), and the costs for strategy 8 are similar.

However, for strategies 11 and 12, the costs are larger (€70 k). The large difference between strategies 7–8 and 11–12 is quite surprising, because the strategies are almost equal, except that different measures/statistics related to the damage size is used. Nonetheless, as CM is not included, the distribution for the damage size is quite uncertain, and the probability of failure is a better measure of the upper tail than the expected damage size and relates closer to the costs. Therefore, decisions made using this variable for optimisation provide lower costs.

With CM included, the results are shown in Table 2 and Figure 15. The three strategies without inspections are bundled, and the strategies with equidistant inspections are bundled. For strategies

Table 1. Optimal decision parameters and lifetime costs for strategies without CM.

Strategy	Optimal inspection decision parameter	Optimal repair decision parameter	Lifetime costs (k€)
0	–	–	228.2
1	–	2 repairs	85.7
5	–	$P_f = 0.02$	87.6
9	–	$E_D = 0.4$	84.4
3	12 months	$l = 4$	51.5
6	12 months	$P_f = 0.03$	52.6
10	12 months	$E_D = 0.7$	52.2
7	$P_f = 0.01$	$P_f = 0.03$	50.0
8	$P_f = 0.01$	$l = 4$	51.6
11	$E_D = 0.35$	$E_D = 0.4$	70.4
12	$E_D = 0.35$	$l = 2$	69.8

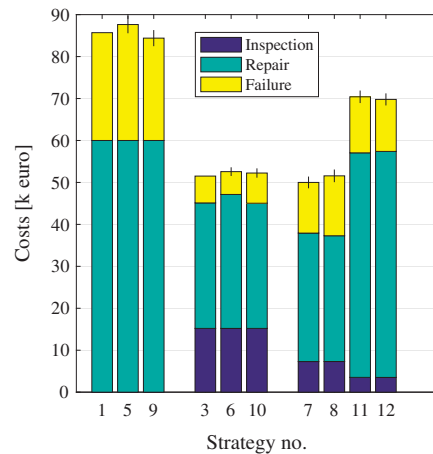


Figure 14. Total expected lifetime costs for preventive strategies without CM.

without inspections, the strategies using advanced decision rules result in much lower costs (€39.0–40.5 k) than strategy 2 using simple decision rules (€112.7 k). For equidistant inspections, preventive repairs made based on the probability of failure (strategy 6) result in larger costs (€41.1 k) than when based on the expected damage size (strategy 10) (€38.8 k). However, the optimal inspection interval is very large (120 months), and similar costs are obtained when not using inspections at all. Therefore, with the models considered in this example, equidistant inspections are not a good option when CM is used. For the remaining strategies, the CM outcome is considered when decisions on inspections are made. For strategy 4, the best of the strategies with only simple decision rules, a threshold value for the CM outcome is used for decisions on inspections, and the costs obtained are €39.5 k. Slightly lower costs are generally obtained when the inspection decision is made based on the probability of failure or expected damage size. The lowest costs (€34.9 k) are obtained for strategy 7.

The value of CM can be found as the difference in costs between the best strategy with and without CM. If only simple strategies are considered, the best without CM is strategy 3 (€51.5 k), and the best strategy with CM is strategy 4 (€39.5 k). Hereby, the value of CM is estimated to €12.0 k. When advanced strategies are also considered, the best overall strategy without CM is strategy 7 (€50.0 k), and with CM strategy 7 (€34.9 k). This results in a CM value of €15.1 k. These values are the expected

Table 2. Optimal decision parameters and lifetime costs for strategies with CM.

Strategy	Optimal inspection decision parameter	Optimal repair decision parameter	Lifetime costs (k€)
2	–	$I_M = 3$	112.7
5	–	$P_f = 0.03$	40.5
9	–	$E_D = 0.7$	39.0
6	120 months	$P_f = 0.1$	41.1
10	120 months	$E_D = 0.7$	38.8
4	$I_M = 3$	$l = 4$	39.5
7	$P_f = 0.03$	$P_f = 0.03$	34.9
8	$P_f = 0.02$	$l = 4$	35.5
11	$E_D = 0.7$	$E_D = 0.7$	36.4
12	$E_D = 0.7$	$l = 4$	35.8

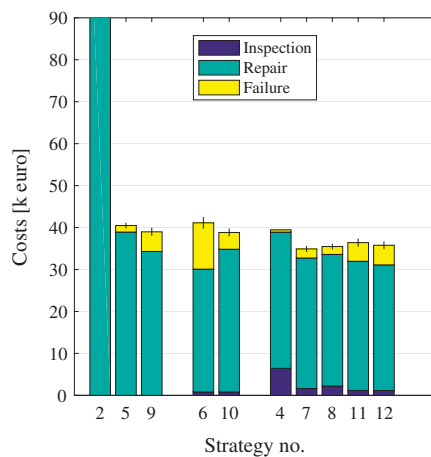


Figure 15. Total expected lifetime costs for preventive strategies with CM.

lifetime benefits of CM. These should be compared to the lifetime costs of CM, which would include initial costs of sensors and data infrastructure, installation of the system and operation and maintenance of the system.

## Conclusions

This paper presented a computational framework for risk-based maintenance planning using Bayesian networks. The framework allows for computation of optimal strategies for inspections and preventive repairs when probabilistic models for deterioration, repairs, inspections and condition monitoring are available in addition to the specific costs of inspections, preventive repairs and failures, which could be time variant.

In the example, lower costs were obtained for all preventive strategies compared to corrective maintenance only, and strategies using inspections or CM performed much better than scheduled repairs. When no condition monitoring was available, the best strategy using only simple decision rules performed similar to the best strategy using advanced decision rules. The best simple strategy (strategy 3) used equidistant inspections and repairs when the inspection outcome exceeded a threshold. The best advanced strategy (strategy 7) used inspections and repairs when the probability of failure exceeded thresholds. It was not important whether a reference period of 1 or 12 months was used, but the strategies using the probability of failure in some cases performed much better than the strategies using the expected damage size.

When condition monitoring was included, the strategies using advanced decision rules for the planning of inspections performed best; strategies using the probability of failure or expected damage size gave similar results, but 12% could be saved compared to the simple strategy where inspection decisions were made based on the most recent condition monitoring outcome alone. The value of condition monitoring was 26% higher when advanced decision rules were used. These results are specific to the costs and models used in the example, and the presented framework can be used for assessment of total costs, optimal strategy and value of condition monitoring for other examples where the required models and costs can be specified.

For industrial implementation of the model framework, a tool with a graphical user interface could be developed for use by the strategic maintenance manager. He would need to provide the input to the model, for example with help from modules developed for this purpose. The tool could then be used for assessment of expected lifetime costs for relevant strategies and could, therefore, support decisions on maintenance strategies. If he decides to use simple strategies, these are straightforward to implement. For advanced decision rules, the daily maintenance manager would need access to a decision tool based on Bayesian networks continuously updated using condition monitoring and inspection outcomes, in order to apply the advanced decision rules.

The current version of the framework only includes the possibility of one preventive repair type, and one inspection type. In both cases, the framework could be extended by allowing the nodes  $R_i$  and  $I_{D,i}$  to have more states than two. For example, three states for the repair node: 'no repair', 'repair' and 'exchange', to model different reliability for repairs and component exchanges. For example, the reliability of a component after an imperfect repair will be lower than for a new component.

The framework also only allows for one stochastic parameter to be included in the deterioration model. However, extension to more parameters is possible, although it requires modification of the algorithms. Another relevant extension could be to include correlation between condition monitoring outcomes. This could be done by adding a link between adjacent  $I_M$  nodes or by adding a time-invariant node for the reliability of the CM method. This implementation would also require a modification of the algorithms as an extra node would be needed to make the future independent from the past.

Another limitation of the framework is that it only considers one failure mode. To obtain an optimal maintenance strategy, system effects should be included. To avoid computational intractability, system effects could be included in an approximate way, for example in relation to costs, by using a modular modelling approach. The costs of inspections, repairs and exchanges could be reduced if operations requiring similar equipment are bundled. The probability of being able to bundle operations could be estimated based on the expected number of operations in each time step.

## Acknowledgements

The research leading to these results has been conducted under the LEANWIND project which has received funding from the European Union Seventh Framework Programme under the agreement SCP2-GA-2013-614020.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## Funding

This work was supported by European Union Seventh Framework Programme [agreement number SCP2-GA-2013-614020].

## ORCID

Jannie Sønderkær Nielsen  <http://orcid.org/0000-0003-2752-7673>

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