

Computational Methods for Multiphase Flows in Porous Media

**Zhangxin Chen
Guanren Huan
Yuanle Ma**

**Southern Methodist University
Dallas, Texas**



**Society for Industrial and Applied Mathematics
Philadelphia**

Contents

List of Figures	xv
List of Tables	xxiii
Preface	xxvii
1 Introduction	1
1.1 Petroleum Reservoir Simulation	1
1.2 Numerical Methods	2
1.3 Linear System Solvers	3
1.4 Solution Schemes	4
1.5 Numerical Examples	5
1.6 Ground Water Flow Modeling	5
1.7 Basin Modeling	6
1.8 Units	6
2 Flow and Transport Equations	9
2.1 Introduction	9
2.2 Single Phase Flow	10
2.2.1 Single phase flow in a porous medium	10
2.2.2 General equations for single phase flow	13
2.2.3 Equations for slightly compressible flow and rock	15
2.2.4 Equations for gas flow	16
2.2.5 Single phase flow in a deformable medium	17
2.2.6 Single phase flow in a fractured medium	18
2.2.7 Non-Darcy's law	20
2.2.8 Other effects	21
2.2.9 Boundary conditions	21
2.3 Two-Phase Immiscible Flow	22
2.3.1 Basic equations	22
2.3.2 Alternative differential equations	23
2.3.3 Boundary conditions	27
2.4 Transport of a Component in a Fluid Phase	29
2.5 Transport of Multicomponents in a Fluid Phase	30

2.6	The Black Oil Model	31
2.7	A Volatile Oil Model	34
2.8	Compositional Flow	35
2.9	Nonisothermal Flow	37
2.10	Chemical Compositional Flow	40
2.11	Flows in Fractured Porous Media	42
2.11.1	Dual porosity/permeability models	43
2.11.2	Dual porosity models	44
2.12	Concluding Remarks	46
2.13	Bibliographical Information	47
	Exercises	47
3	Rock and Fluid Properties	51
3.1	Rock Properties	51
3.1.1	Capillary pressures	51
3.1.2	Relative permeabilities	53
3.1.3	Rock compressibility	57
3.2	Fluid Properties	57
3.2.1	Water PVT properties	58
3.2.2	Oil PVT properties	60
3.2.3	Gas PVT properties	64
3.2.4	Total compressibility	67
3.2.5	Equations of state	67
3.3	Temperature-Dependent Properties	70
3.3.1	Rock properties	70
3.3.2	Fluid properties	71
3.4	Bibliographical Information	72
	Exercises	72
4	Numerical Methods	75
4.1	Finite Difference Methods	76
4.1.1	First difference quotients	76
4.1.2	Second difference quotients	78
4.1.3	Grid systems	79
4.1.4	Treatment of boundary conditions	80
4.1.5	Finite differences for stationary problems	83
4.1.6	Finite differences for parabolic problems	84
4.1.7	Consistency, stability, and convergence	86
4.1.8	Finite differences for hyperbolic problems	89
4.1.9	Grid orientation effects	93
4.2	Standard Finite Element Methods	94
4.2.1	Finite element methods for stationary problems	94
4.2.2	General domains	117
4.2.3	Quadrature rules	120
4.2.4	Finite element methods for transient problems	121

4.3	Control Volume Finite Element Methods	128
4.3.1	The basic CVFE	128
4.3.2	Positive transmissibilities	131
4.3.3	The CVFE grid construction	132
4.3.4	The upstream weighted CVFE	133
4.3.5	Control volume function approximation methods	136
4.3.6	Reduction of grid orientation effects	141
4.4	Discontinuous Finite Element Methods	142
4.4.1	DG methods	143
4.4.2	Stabilized DG methods	147
4.5	Mixed Finite Element Methods	148
4.5.1	A one-dimensional model problem	149
4.5.2	A two-dimensional model problem	153
4.5.3	Extension to boundary conditions of other kinds	156
4.5.4	Mixed finite element spaces	158
4.5.5	Approximation properties	170
4.6	Characteristic Finite Element Methods	171
4.6.1	The modified method of characteristics	172
4.6.2	The Eulerian–Lagrangian localized adjoint method	178
4.7	Adaptive Finite Element Methods	182
4.7.1	Local grid refinement in space	183
4.7.2	Data structures	187
4.7.3	A posteriori error estimates	187
4.7.4	The eighth SPE project: Gridding techniques	193
4.8	Bibliographical Remarks	198
	Exercises	198
5	Solution of Linear Systems	207
5.1	Tridiagonal Systems	207
5.2	Gaussian Elimination	210
5.3	Ordering of the Nodes	215
5.4	CG	217
5.5	GMRES	220
5.6	ORTHOMIN	223
5.7	BiCGSTAB	224
5.8	Preconditioned Iterations	226
5.8.1	Preconditioned CG	226
5.8.2	Preconditioned GMRES	227
5.9	Preconditioners	230
5.9.1	ILU(0)	232
5.9.2	ILU(I)	232
5.9.3	ILUT	235
5.10	Practical Considerations	236
5.10.1	Decoupling preconditioners	237
5.10.2	COMBINATIVE preconditioners	238

5.10.3	Bordered systems	238
5.10.4	Choice of initial solutions	238
5.11	Concluding Remarks and Comparisons	239
5.12	Bibliographical Remarks	245
	Exercises	245
6	Single Phase Flow	247
6.1	Basic Differential Equations	247
6.2	One-Dimensional Radial Flow	248
6.2.1	An analytic solution	248
6.2.2	Numerical comparisons	251
6.3	Finite Element Methods for Single Phase Flow	252
6.3.1	Linearization approaches	255
6.3.2	Implicit time approximations	255
6.3.3	Explicit time approximations	257
6.4	Bibliographical Remarks	258
	Exercises	258
7	Two-Phase Flow	259
7.1	Basic Differential Equations	259
7.2	One-Dimensional Flow	260
7.2.1	An analytic solution	260
7.2.2	An example	263
7.3	IMPES and Improved IMPES	265
7.3.1	Classical IMPES	265
7.3.2	The seventh SPE project: Horizontal well modeling	267
7.3.3	Improved IMPES	270
7.4	Alternative Differential Formulations	274
7.4.1	Phase formulation	274
7.4.2	Weighted formulation	274
7.4.3	Global formulation	275
7.4.4	Numerical comparisons	275
7.5	Numerical Methods for Two-Phase Flow	277
7.5.1	Mixed finite element methods	277
7.5.2	CVFE methods	278
7.5.3	Characteristic finite element methods	279
7.5.4	Comparison between numerical methods	280
7.6	Miscible Displacement	280
7.7	Bibliographical Remarks	281
	Exercises	281
8	The Black Oil Model	283
8.1	Basic Differential Equations	283
8.1.1	The basic equations	283
8.1.2	Rock properties	286

8.1.3	Fluid properties	286
8.1.4	Phase states	287
8.2	Solution Techniques	288
8.2.1	The Newton–Raphson method	288
8.2.2	The SS technique	289
8.2.3	The sequential technique	299
8.2.4	Iterative IMPES	307
8.2.5	Well coupling	311
8.2.6	The adaptive implicit and other techniques	313
8.3	Comparisons between Solution Techniques	314
8.3.1	An undersaturated reservoir	314
8.3.2	A saturated reservoir	319
8.3.3	The ninth SPE project: Black oil simulation	324
8.3.4	Remarks on numerical experiments	330
8.4	The Second SPE Project: Coning Problems	331
8.5	Bibliographical Remarks	340
	Exercises	340
9	The Compositional Model	347
9.1	Basic Differential Equations	347
9.1.1	The basic equations	347
9.1.2	Equations of state	349
9.2	Solution Techniques	351
9.2.1	Choice of primary variables	351
9.2.2	Iterative IMPES	353
9.3	Solution of Equilibrium Relations	358
9.3.1	Successive substitution method	358
9.3.2	Newton–Raphson’s flash calculation	359
9.3.3	Derivatives of fugacity coefficients	360
9.3.4	Solution of Peng–Robinson’s cubic equation	361
9.3.5	Practical considerations	363
9.4	The Third SPE Project: Compositional Flow	364
9.4.1	PVT phase behavior study	369
9.4.2	Reservoir simulation study	375
9.4.3	Computational remarks	378
9.5	Bibliographical Remarks	379
	Exercises	379
10	Nonisothermal Flow	381
10.1	Basic Differential Equations	381
10.1.1	The basic equations	382
10.1.2	Rock properties	384
10.1.3	Fluid properties	385
10.2	Solution Techniques	386
10.2.1	Choice of primary variables	387
10.2.2	The SS technique	388

10.3	The Fourth SPE Project: Steam Injection	393
10.3.1	The first problem	394
10.3.2	The second problem	395
10.4	Bibliographical Remarks	397
	Exercises	397
11	Chemical Flooding	399
11.1	Basic Differential Equations	400
11.2	Surfactant Flooding	403
11.2.1	Effective salinity	404
11.2.2	Binodal curves	404
11.2.3	Tie lines for two phases	405
11.2.4	Tie lines for three phases	406
11.2.5	Phase saturations	406
11.2.6	Interfacial tension	406
11.2.7	Interfacial tension without mass transfer	407
11.2.8	Trapping numbers	407
11.2.9	Relative permeabilities	408
11.3	Alkaline Flooding	408
11.3.1	Basic assumptions	409
11.3.2	Mathematical formulations of reaction equilibria	409
11.4	Polymer Flooding	411
11.4.1	Viscosity	411
11.4.2	Permeability reduction	412
11.4.3	Inaccessible pore volume	412
11.5	Foam Flooding	413
11.5.1	Critical oil saturation	413
11.5.2	Critical surfactant concentration	413
11.5.3	Critical capillary force	413
11.5.4	Oil relative permeability effects	414
11.5.5	Gas-liquid ratio effects	414
11.5.6	Gas velocity effects	414
11.6	Rock and Fluid Properties	415
11.6.1	Adsorption	415
11.6.2	Phase-specific weights	416
11.6.3	Phase viscosities	416
11.6.4	Cation exchange	417
11.7	Numerical Methods	418
11.8	Numerical Results	418
11.8.1	Example 1	419
11.8.2	Example 2	421
11.8.3	Example 3	422
11.9	Application to a Real Oilfield	426
11.9.1	Background	426
11.9.2	The numerical model	427
11.9.3	Numerical history matching	428

11.9.4	Predictions	431
11.9.5	Assessment of different development methods	431
11.10	Bibliographical Remarks	432
	Exercises	432
12	Flows in Fractured Porous Media	433
12.1	Flow Equations	434
12.1.1	Dual porosity/permeability models	434
12.1.2	Dual porosity models	436
12.2	The Sixth SPE Project: Dual Porosity Simulation	438
12.3	Bibliographical Remarks	443
	Exercises	443
13	Welling Modeling	445
13.1	Analytical Formulas	445
13.2	Finite Difference Methods	447
13.2.1	Square grids	447
13.2.2	Extensions	448
13.3	Standard Finite Element Methods	450
13.3.1	Triangular finite elements	450
13.3.2	Rectangular finite elements	452
13.4	Control Volume Finite Element Methods	453
13.4.1	Well model equations	453
13.4.2	Horizontal wells	453
13.4.3	Treatment of faults	454
13.4.4	Corner point techniques	456
13.5	Mixed Finite Element Methods	457
13.5.1	Rectangular mixed spaces	457
13.5.2	Triangular mixed spaces	457
13.6	Well Constraints	459
13.7	The Seventh SPE Project: Horizontal Well Modeling	460
13.8	Bibliographical Remarks	475
	Exercises	475
14	Special Topics	477
14.1	Upscaling	477
14.1.1	Single phase flow	477
14.1.2	Two-phase flow	478
14.1.3	Limitations in upscaling	478
14.2	History Matching	479
14.3	Parallel Computing	480
14.3.1	Domain decomposition	480
14.3.2	Load balancing	481
14.3.3	Data communication	481
14.3.4	Time step size and communication time control	482

14.4	Oil Recovery Optimization	482
14.5	Surface Network Systems	483
14.5.1	Hydraulic models of flow devices	483
14.5.2	Models of links and nodes	484
14.6	Bibliographical Remarks	485
15	Nomenclature	487
15.1	English Abbreviations	487
15.2	Subscripts	488
15.3	Base Quantities	488
15.4	English Symbols	488
15.5	Greek Symbols	492
15.6	Generic Symbols Used in Chapters 4 and 5	494
16	Units	499
16.1	Unit Abbreviations	499
16.2	Unit Conversions	500
16.3	SI and Other Metric Systems	502
Bibliography		503
Index		523