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COMPUTATIONAL PROBLEMS IN AUTOREGRESSIVE MOVING AVERAGE (ARMA) MODELS

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Abstract

In developing mathematical models of systems from a given input-output data sequence, the choice of the sampling interval and the selection of the order of the model in time-series analysis pose difficult problems. Band-limited (up to 15 Hz) random torque perturbations were applied to the human ankle joint. The applied torque input, the angular rotation output, and the electromyographic activity using surface electrodes from the extensor and flexor muscles of the ankle joint were recorded. Autoregressive moving average models were developed. A parameter constraining technique is applied to develop more reliable models. It is shown that the asymptotic behavior of the system must be taken into account during parameter optimization to develop predictive models.

INTRODUCTION

In a series of previous papers (Agarwal and Gottlieb, 1977 a,b; Gottlieb and Agarwal, 1978; Gottlieb, Agarwal, and Penn, 1978) we have attemped to describe quantitatively the neuromuscluar system dynamics to applied sinusoidal and band-limited gaussian torque perturbations. In these studies, the compliance of the joint was calculated using Fourier series analysis for sinusoidal and power spectral density methods for random perturbations. Although linear analysis methods were used, the system is known to be nonlinear and the parameter values such as the joint viscous and stiffness coefficients are functions of the level of neuromuscular activity.

The purpose of the present paper is to apply time series analysis methods to study the input-output behavior of the neuromuscular system. The time series method is very parsimonious in the use of parameters to represent the model structure. Normalized residual criterion (NRC) will be used to estimate the model order (For details of this method see Suen and Liu, 1977; Osafo-Charles et. al., 1980).

Our previous analysis was limited to analysis of the angular rotation data and calculation of joint compliance. The electromyographic (EMG) data was not analysed due to inherent difficulties in representing this output by linear transfer functions. The time series approach allows nonlinear representations as long as the model is linear in parameter space.

Dufresne, Soechting and Terzuolo (1978) used pseudo-random torque pulses to study the human forearm response. They developed a model of the EMG in terms of the limb position and its derivatives in the following form:

$$EMG(t) = A\ \theta(t - d) + B\ \dot{\theta}(t - d) + C\ \ddot{\theta}(t - d) \qquad (1)$$

where A, B, and C are constant parameters and d is the time delay. They found that the motor output depends primarily on the angular velocity of the joint. The time delay was found to be about 47 msec.

In a subsquent study, Dufresne, Soechting, and Terzuolo (1979) used different time delay parameters for position and its two derivatives. The best estimates for the time delays were found to be 86 msec for position, 25 msec for velocity, and 45 msec for acceleration. The physiological processes associated with these varying delays are not clear. Soechting and Dufresne (1980) found that the linear model given in equation (1) predicted 80% of the EMG response.

Our analysis of the EMG using time series shows that the autoregressive terms of the EMG are important and cannot be ignored as was done in the Dufresne et. al (1978) model.

METHODS

These experiments were done using normal human subjects. A subject sat in a chair with the right foot strapped to a footplate which could rotate about a horizontal, dorsal-plantar axis through the medial malleolus. The plate could be rotated by a DC torque motor. A band-limited gaussian (0-15 Hz) signal was prerecorded from a noise generator. These time-varying signals were superimposed on a biasing mean motor torque level. The subject was instructed to try to maintain a constant mean force against the bias torque of the motor so that the ankle joint movement was nearly symmetrical with respect to the reference angle. The input was applied for 30 sec or more and the data continuously recorded on a digital tape.

The torque was measured by a strain gauge bridge on the side arms of the footplate. Angular rotation was measured by a continuous capacitive transducer. The EMGs were recorded from disc surface electrodes taped over the bellies of the soleus (SM) and the anterior tibial (TA) muscles. These were amplified full-wave rectified and passed through an averaging filter (10 msec averaging time) before recording. A computer generated the motor drive voltage at a conversion rate of 250/s and digitized data on four input channels. The angle and the torque signals were sampled at a rate of 250/s and filtered EMGs at a rate of 500/s. The data analysis was done off-line using the Minitab 2 statistical software package on an IBM 370 computer.

The Normalized Residual Criterion

Time-series analysis can be extended to obtain discrete linear transfer functions of systems having an input x(t) and output y(t). By x(t) and y(t) we mean pairs of observations that are available at equispaced intervals of time. The behavior of the dynamic system can be adequately represented by the present and past responses and the current and past inputs of the systems. We denote this process as transfer function (TF) models (n,m) and write its equation as

$$y(t) = \alpha_0 + \alpha_1 y(t-1) + \cdots + \alpha_n y(t-n) + \beta_0 x(t) + \ldots + \beta_n x(t-n) + \nu(t) \quad (2)$$

In (2) the parameters to be estimated are $\alpha_0, \ldots, \alpha_n$, $\beta_0, \ldots, \beta_m$, n, and m. The time series $\nu(t)$ is a random term measuring the difference between the response y(t) and the variables used to expalin the time-series data. The parameter α_0 measures the mean output.

Equation (2) reduces to an autoregressive model (AR(n)) if x(t) is omitted from the model, and reduces to a moving average model (MA(m)) if lags of y are omitted. The following assumptions will be made concerning $\nu(t)$ for a given ouput time sequence y(t), t = [0,T],

1) $E[\nu(t)] = 0$

2) $E[\nu(i)\nu(j)] = \sigma_\nu^2 \delta_{ij}$ (3)

where

$$\delta_{ij} = \begin{cases} 1 \text{ for } i=j \\ 0 \text{ for } i\neq j \end{cases}$$

3) $T \gg n$.

From (2),

$$\nu(t) = y(t) - \alpha_0 - \sum_{i=1}^{n} \alpha_i y(t-i) - \sum_{j=0}^{m} \beta_j x(t-j),$$

$$t = 1, 2, \cdots, T-n. \quad (4)$$

Define

$$\sum_{t=1}^{T-n} \nu^2(t) = ||V||^2 \text{ and } \sum_{t=1}^{T-n} y^2(t) = ||Y||^2. \quad (5)$$

Note that in the discussion below V and Y are vectors such that

$$V = \begin{bmatrix} \nu(1) \\ \nu(2) \\ \cdot \\ \cdot \\ \cdot \\ \nu(T-n) \end{bmatrix} \qquad Y = \begin{bmatrix} y(1) \\ y(2) \\ \cdot \\ \cdot \\ \cdot \\ y(T-n) \end{bmatrix} \tag{6}$$

Squaring (4) and normalizing by the total sum of squares, we have

$$\frac{||V||^2}{||Y||^2} = \frac{||y-\alpha_0-\Sigma\alpha_i y - \Sigma\beta_j x||^2}{||Y||^2} = \varepsilon(n, m, T) \tag{7}$$

and therefore

$$E\left[\,||V||^2\,\right] = E\left[\,||Y||^2 \varepsilon(n,m,T)\right] . \tag{8}$$

Since y(t), the data series, is deterministic, (8) can be rewritten as

$$E\left[\,||V||^2\,\right] = ||Y||^2 E\left[\,\varepsilon(n, m, T)\,\right]. \tag{9}$$

From (6) we have

$$E\left[\,||V||^2\,\right] = E\left[\nu^2(1) + \nu^2(2) + \cdots + \nu^2(T-n)\right] , \tag{10}$$

which by assumption 2) in (3) reduces to

$$E\left[||V||^2\right] = \left[T-n\right]\sigma_\nu^2 . \tag{11}$$

Substitution of (11) in (9) we have

$$E\left[\varepsilon(n,m,T)\right] = \frac{(T-n)\sigma_\nu^2}{||Y||^2} \tag{12}$$

and by assumption 3), (12) becomes

$$E\left[\varepsilon(n, m, T)\right] \approx \frac{T\sigma_\nu^2}{||Y||^2} \tag{13}$$

The quantity $\varepsilon(n,m,T)$ depends on n, m, and T and is proportional to the normalized variance of the regression for a given n and m. If this ratio is minimized over n and m, then the data fit as measured by the correlation coefficient ρ will be maximized. Note that

$$\rho = \left[1 - \frac{||V||^2}{||Y||^2} \right]^{1/2} \qquad (14)$$

or

$$\hat{\rho} = \left[1 - \hat{\varepsilon}(n,m) \right]^{1/2} \qquad (15)$$

where T, being a constant for the data, is omitted in the optimization procedure, and $\hat{\varepsilon}(n,m)$ is the minimum value for $\varepsilon(n,m)$. This optimization technique is the so called the Normalized Residual Criterion.

RESULTS

The mathematical modeling problem was considered in two separate parts. For the first model the applied torque is the input and the resulting angular rotation of the joint is the output of the system. For the second model, the angular rotation (and its derivatives) is considered as the input to the system and the resulting stretch reflex electromyographic activity is considered as the output. It should be emphasized that the angular rotation is the net result of two torque inputs applied at the joint; one by the external motor torque and the other muscle forces produced by the stretch reflex mechanism. These mechanisms are also responsible for a significant contribution to the joint viscous and elastic properties. Figure 1 shows a sample of the data at 4 msec sampling interval. The velocity was obtained by digital differentiation.

Angle-Torque Model

Although the data was recorded for 30 seconds at each input (Agarwal and Gottlieb, 1977b), this method does not require such long data records which would also use too much computer time. The time series analysis was done using only two-seconds of the data record. (The first two-seconds of the data were not used to allow the turn-on transients to die out).

The values of $\varepsilon(n,m)$ were computed for a given data record and then plotted against different values of n (see Figure 2). The data sampling interval in this case is 4 msec. This analysis clearly indicates that $n = 2$ and $m = 0$ is adequate to model this data. The same data was analyzed again using the sampling intervals of 12, 20, 40, and 60 msec. Figure 3 shows the $\varepsilon(n,m)$ values for the sampling interval of 20 msec. Note that the minimum value of the normalized residual is about 60 times of that in the first case. For 40 and 60 msec sampling $\varepsilon(n,m)$ did not reach asymptotic values even for model order of (8,8). The normalized residuals at 12 and 20 msec sampling implied a model order of (3,1).

Figure 4 shows the actual angular rotation data (2 to 4 sec interval used in this analysis), the regression fit and the predicted output using 4 msec sampling and model order of (3,1). The regression fit is obtained by using the equation

$$\theta(t) = \alpha_0 + \alpha_1 \theta(t-1) + \alpha_2 \theta(t-2) + \alpha_3 \theta(t-3) + \beta_0 T(t) + \beta_1 T(t-1) \quad (16)$$

The error between the actual data and the regression fit is nearly zero. The correlation coefficient is $\rho = 0.999$. However, when this model is used to predict the output using the first three data output values as the initial conditions, the predicted output is a poor approximation of the actual data (see Figure 4). Figure 5 shows the observed angle and the predicted model values for model orders of (3,1), (7,1), (9,1), and (14,1). Even the fourteenth order model is not able to adequately reproduce the data sequence. These models are not able to capture the steady state (or long term) behavior of the system. Osafo-Charles, et al., (1980) showed that to develop better predictive models, the TF(n,m) models must be constrained to incorporate the steady state response of the system.

Constrained Model

Consider the estimated model given by equation (16). Under conditions of equilibrium

$$\theta(t) = \theta(t-1) = \theta(t-2) = \theta(t-3) = \theta_e$$

and

$$T(t) = T(t-1) = T_e$$

where θ_e and T_e are the steady state response and input respectively. At physical and statistical equilibrium, with $\theta(t) = \theta_e$ and $T(t) = T_e$, equaition (16) became

$$\theta_e = \alpha_1 \theta_e + \alpha_2 \theta_e + \alpha_3 \theta_e + \beta_0 T_e + \beta_1 T_e \quad (17)$$

or

$$\frac{\theta_e}{T_e} = \frac{\beta_0 + \beta_1}{1 - \alpha_1 - \alpha_2 - \alpha_3} = g \quad (18)$$

where (18) expresses the steady state gain in terms of the parameters of the model.

The value of g was approximated by the slope of the curve of torque vs. angular rotation in the relaxed ankle during sinusoidal oscillation at 0.1 H_z (Gottlieb and Agarwal, 1978).

For $\theta_e = g\, T_e$ to be true, we must have

$$\beta_0 = g(1 - \alpha_1 - \alpha_2 - \alpha_3) - \beta_1 \tag{19}$$

From equations (19) and (16), we get

$$\begin{aligned}\left[\theta(t) - g\,T(t)\right] = \; & \alpha_1 \left[\theta(t-1) - g\,T(t)\right] \\ & + \alpha_2 \left[\theta(t-2) - g\,T(t)\right] \\ & + \alpha_3 \left[\theta(t-3) - g\,T(t)\right] \\ & + \beta_1 \left[T(t-1) - T(t)\right]\end{aligned} \tag{20}$$

Regression analysis is used again to estimate the parameters α_1, α_2, α_3 and β_1 for a given value of gain g. β_0 is then obtained using (19). Figure 6 shows the output angle and predicted model response for a constrained model with gains of g = 6.5, 7.5, and 8.5. The gain value of 8.5 was considered to provide the best fit in terms of the minimum estimated standard deviation of the regression.

The transfer function for the unconstrained model is:

$$H(z) = \frac{0.00239 - 0.00024\, z^{-1}}{1 - 2.678\, z^{-1} + 2.399\, z^{-2} - 0.7191\, z^{-3}} \tag{21}$$

For the constrained model with a slope of 8.5, the transfer function is:

$$H(z) = \frac{0.00238 + 0.00017\, z^{-1}}{1 - 2.731\, z^{-1} + 2.503\, z^{-2} - 0.7717\, z^{-3}} \tag{22}$$

EMG Model

Our efforts to model EMG as a function of either the angular rotation or the velocity or a combination of both were not successful. As was noted by Dufresne et al. (1978), the velocity of rotation is the most significant input due to spindle properties (Matthews, 1972). However only those components of velocity which stretch the spindle contribute to the EMG of the stretched muscle. (The spindle is silent during shortening). Therefore, a new velocity signal represneting only the stretching velocities was defined as:

$$\dot{\theta}_d(t) = \dot{\theta}(t) \quad \text{if} \quad \dot{\theta} \geq 0$$
$$= 0 \quad \text{if} \quad \dot{\theta} < 0 \qquad (23)$$

The normalized residual analysis indicated a model order of (4,1) using soleus EMG as the ouput and $\dot{\theta}_d$ as the input signal. The predicted output of the unconstrained model and its comparison with the actual EMG signal is shown in Figure 7. Since the EMG signal is a full-wave rectified and filtered (using an averaging filter) signal, it has only negative values (because of negative filter gain). The predicted value of EMG is a poor approximation of the data.

A constrained model was developed using a similar approach as outlined earlier. Figure (8) shows the predicted EMG and the actual data at three values of the gain parameter. The gain of -0.005 was considered to give the most appropriate fit.

For the constrained model with a slope of -0.005, the transfer function is:

$$H(z) = \frac{EMG}{\dot{\theta}_d} = \frac{-.004164 + .00314\, z^{-1}}{1 - 1.19\, z^{-1} + .6685\, z^{-2} - .2947\, z^{-3} + .02104\, z^{-4}} \qquad (24)$$

CONCLUSIONS

The time series approach is a powerful and versatile technique in developing time domain models from a given input-output data sequence. Normalized residual criterion allows effective prediction of the model order. Models developed in this manner may be satisfactory, but may not be good predictive models. It is recommended that constrained parameter modeling which allows incorporating the steady-state behavior be used to obtain better predictive models.

Acknowledgment

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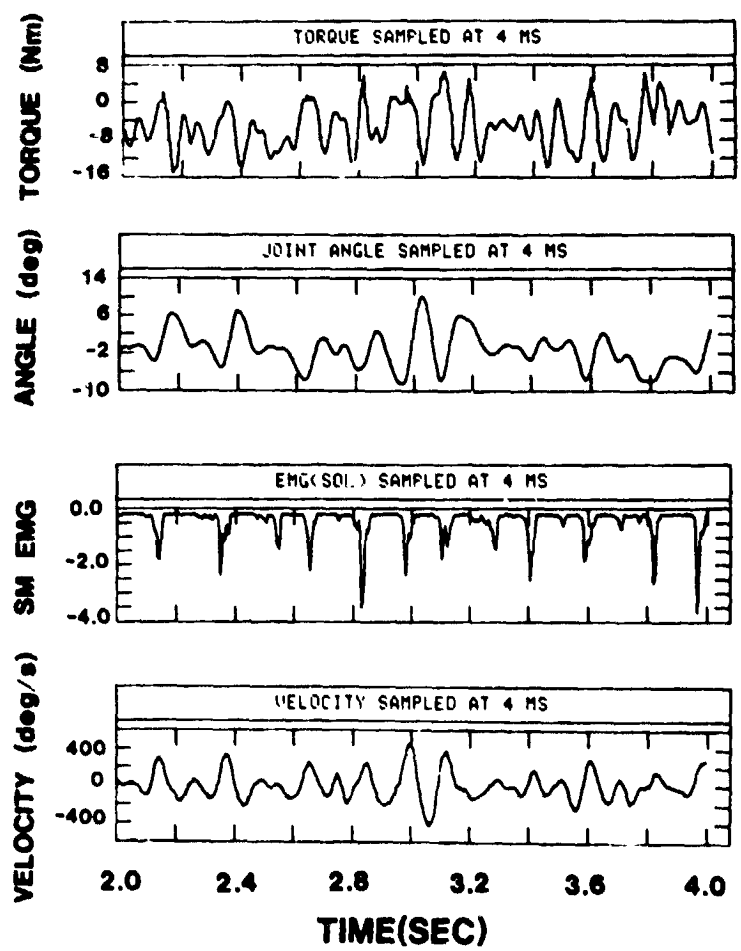
TORQUE SAMPLED AT 4 MS
TORQUE (Nm)
8
0
-8
-16
JOINT ANGLE SAMPLED AT 4 MS
ANGLE (deg)
14
6
-2
-10
EMG(SOL) SAMPLED AT 4 MS
SM EMG
0.0
-2.0
-4.0
VELOCITY SAMPLED AT 4 MS
VELOCITY (deg/s)
400
0
-400
2.0
2.4
2.8
3.2
3.6
4.0
TIME(SEC)

Figure 1

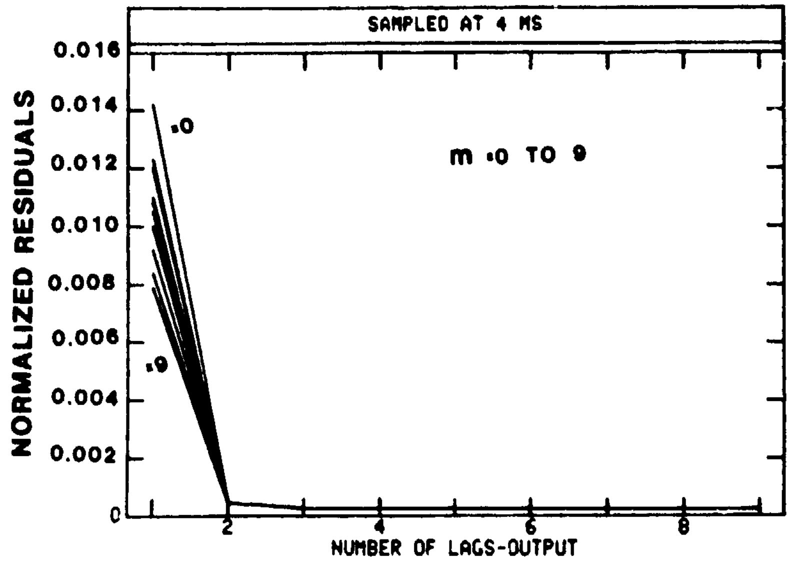

Figure 2

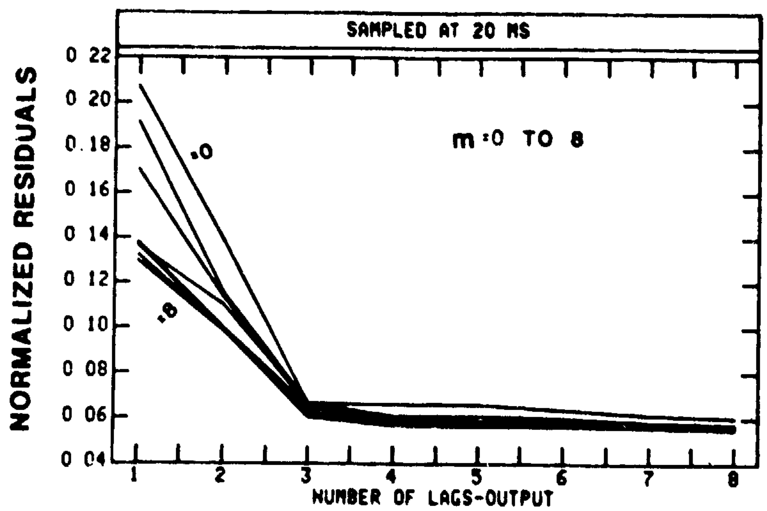

Figure 3

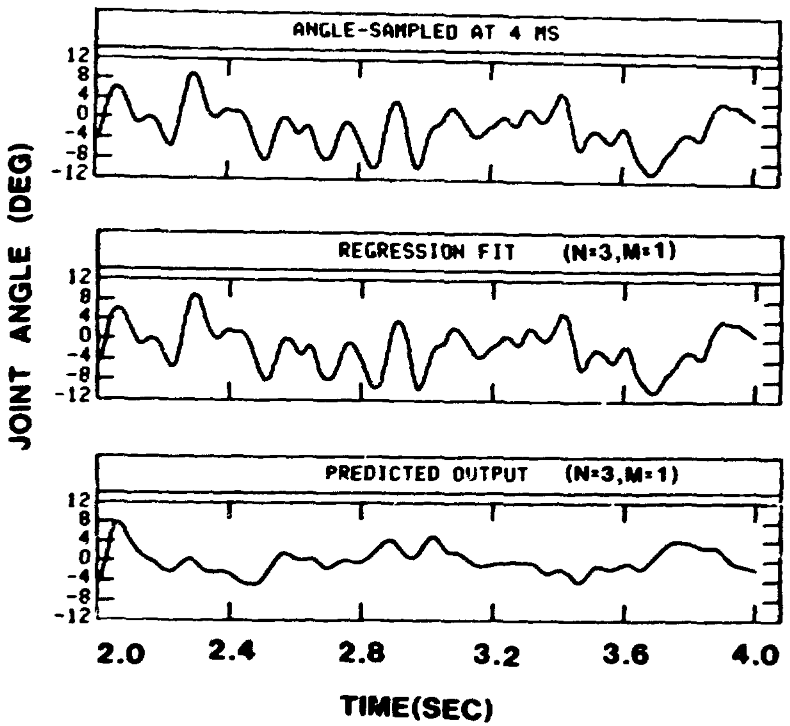

Figure 4

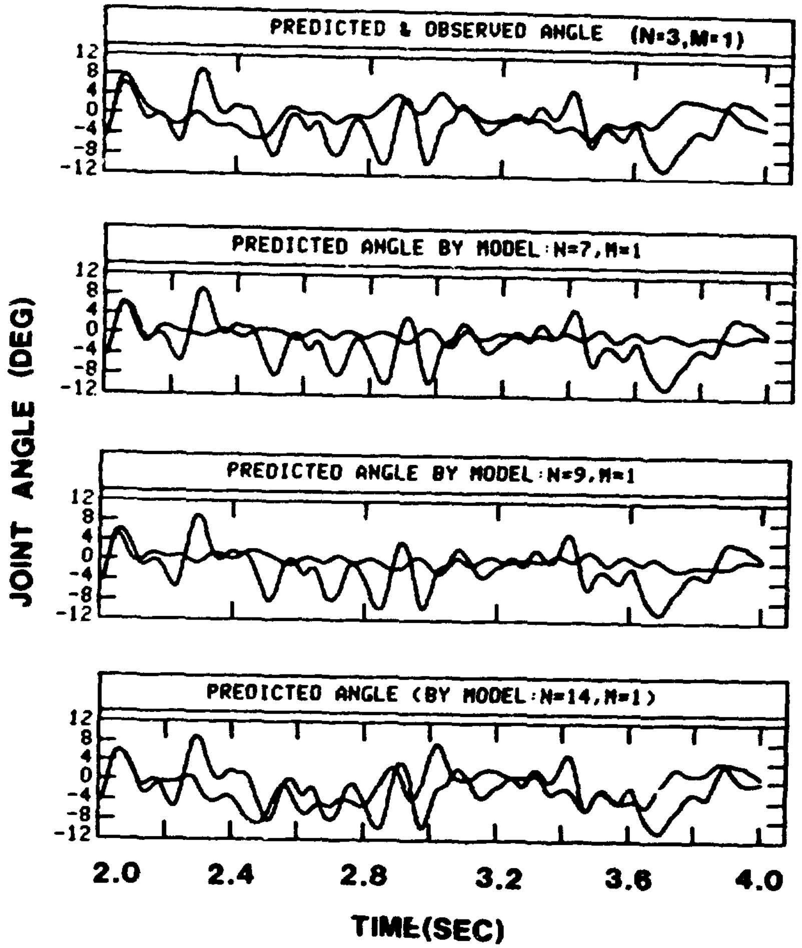

Figure 5

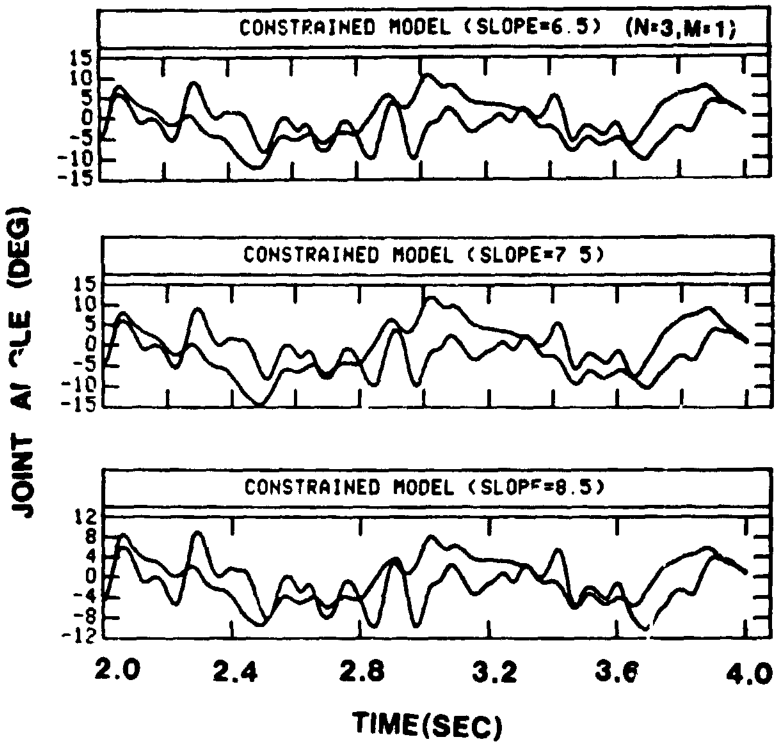

Figure 6

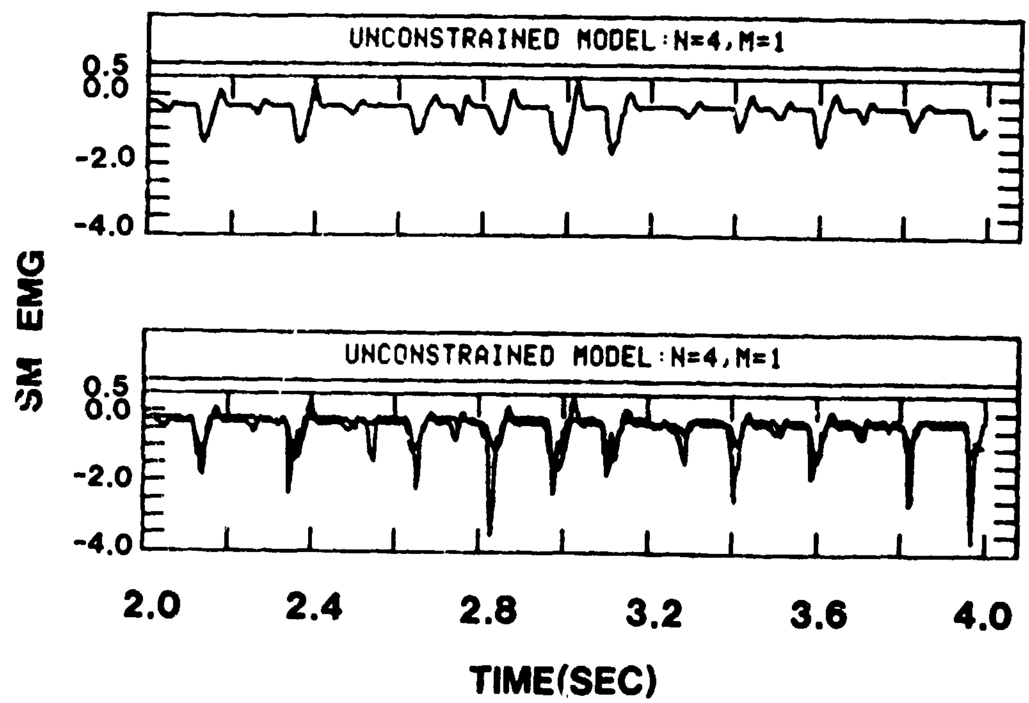

Figure 7

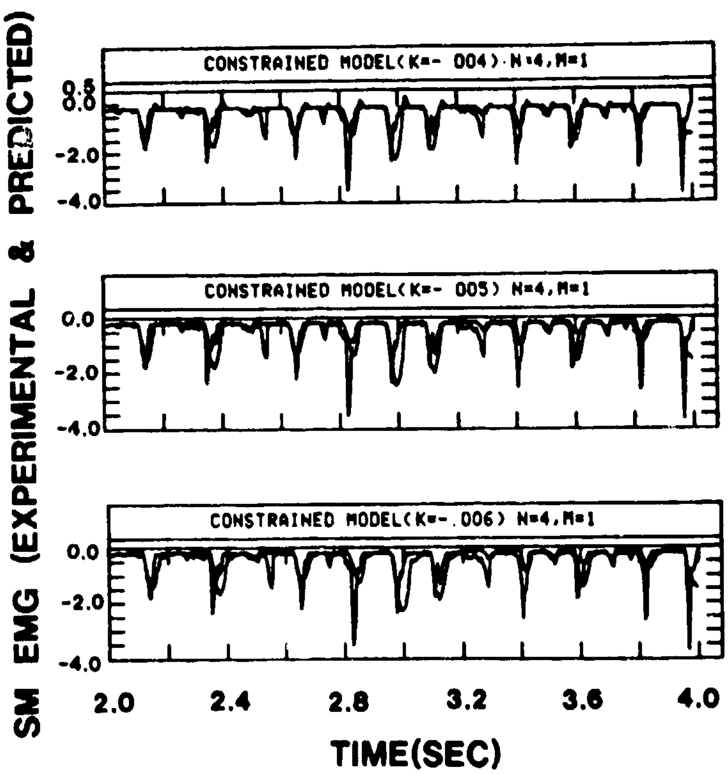
CONSTRAINED MODEL(K=-.004) N=4,M=1
CONSTRAINED MODEL(K=-.005) N=4,M=1
CONSTRAINED MODEL(K=-.006) N=4,M=1
0.5
0.0
-2.0
-4.0
SM EMG (EXPERIMENTAL & PREDICTED)
2.0
2.4
2.8
3.2
3.6
4.0
TIME(SEC)

Figure 8