

# Computer-Aided Eulerian Air Traffic Flow Modeling and Predictive Control

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**Eulerian models are used to represent the air traffic environment as traffic flows between interconnected control volumes representing the airspace system. While these models can be manually derived for simple air traffic patterns, computer-based approaches are essential for modeling realistic airspaces involving multiple traffic streams. A computer-aided methodology for deriving large-dimensional Eulerian models of air traffic flow is described here. Starting from the specification of a few airspace parameters, and traffic data, the modeling technique can automatically construct Eulerian models of the airspace. The synthesis of air traffic flow control algorithms using the model predictive control technique in conjunction with these models is given. It is shown that the flow control logic synthesis can be cast as a linear programming problem. The flow control methodology is illustrated using air traffic data over two regions in U.S. airspace.**

## I. Introduction

The development of an Eulerian<sup>1</sup> approach to modeling air traffic was discussed in recent research efforts<sup>2, 3</sup>. These works were motivated by research initiatives currently underway<sup>4, 5</sup> within the air traffic management (ATM) research community to develop decision support tools for analyzing and controlling air traffic flow, to more efficiently manage operations of the U.S. National Airspace System (NAS). The focus of the present paper is on the development of a computer-aided methodology for deriving Eulerian models of the airspace, and employing it for air traffic flow control. The approach uses the NASA-developed Future ATM Concepts Evaluation Tool<sup>6</sup> (FACET) software as its foundation.

The Eulerian approach models the airspace in terms of line elements approximating airways, together with merge and diverge nodes. Since this modeling technique spatially aggregates the air traffic, the order of the airspace model depends only on the number of line elements used to represent the airways, and not on the number of aircraft operating in the airspace. Eulerian models are in the form of linear, time-varying difference equations.

The one-dimensional modeling methodology is an intuitive approach for deriving models of traffic flow networks formed by jet routes and Victor airways. However, not all aircraft in the airspace strictly follow the jet routes or Victor airways. This situation is likely to continue in the future as more and more aircraft opt to fly wind-optimal routes to their destinations<sup>7</sup>. This introduces the need for a more flexible modeling framework. This framework, first advanced in Reference 2, discretizes the airspace into surface elements (*SELS*), within which the traffic flow is aggregated into eight different directions. This modeling provides adequate fidelity in en route airspace where the traffic flow is largely two dimensional. The traffic at all flight levels in Class A airspace (at or above 18,000 ft) is classified as belonging to any one of these eight directions, with inflows and outflows from airports and other external sources. Each surface element is connected to its eight neighbors, with the connection strengths being determined by the actual traffic flow patterns.

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Eulerian models are then derived by examining traffic flows over a specified sample time interval into and between the surface elements. These models are then used for analysis and flow control system design. Details of the modeling approach will be given in Section II. It has been shown in References 2 and 3 that the Eulerian models can be used to carry out a variety of analyses on the air traffic flow, such as controllability, reachability and model decentralization.

An important application of the Eulerian models is in development of quantitative decision support tools for air traffic flow control. The present research has explored the application of the model-predictive control technique<sup>8</sup> (MPC) to the air traffic flow control problem. This will be discussed in Section III, together with two examples. Conclusions from the present research are given in Section IV.

## II. Computer-Aided Eulerian Traffic Flow Modeling

The Eulerian modeling process begins with the definition of a grid of surface elements covering the region of airspace being modeled. The surface element grid is defined by latitude-longitude tessellation on the surface of the earth in geocentric polar coordinates. Each surface element has equal angular dimensions in longitude and latitude as shown in Figure 1. However, due to the spherical nature of the airspace being modeled, surface elements far north or south of the equator will have smaller physical dimensions than those near the equator. All the results reported in this paper are based on one-degree latitude-longitude increments. The eight different en route traffic flow directions within each surface element are indicated in Figure 2. In addition to these, the surface elements above airports will include one output stream for landing aircraft. The aircraft taking off from airports under a surface element are included in one of the eight en route traffic flow directions. Surface elements lying on the boundary of the airspace being modeled will have additional inputs representing traffic entering the system from un-modeled airspace (e.g., international flights).

Since the present form of the Eulerian model is discrete in space and time, a sample interval  $\tau$  must also be specified. Although the spatial and temporal discretizations are based mainly on the level of detail desired in the model, due to the assumption that each surface element is connected only to eight of its neighbors, the sample time interval must be chosen so that no aircraft in a surface element travels beyond its immediate neighbors in a sample interval. Thus, the dimensions of the smallest surface element and the airspeed of the fastest aircraft in the airspace determine the acceptable sample interval.

As in References 2 and 3, the air traffic flow pattern is modeled within each surface element using two sets of parameters. The first of these are the inertia parameters  $a_{ii}$ , one for each of the eight streams representing the fraction of the aircraft that remained from the previous sample time. By definition, in any stream  $i$ , the fraction of aircraft that left the SEL in the previous sample interval is given by  $(1 - a_{ii})$ .

The second set of parameters are the flow divergence parameters  $\beta_{mn}$  representing the aircraft that switched streams within the SEL. Since the aircraft in a stream may stay in it, or switch to any of the other 7 en route streams, or land at an airport, for a given SEL there is a matrix of  $9 \times 8 = 72$  flow divergence parameters. In order to satisfy the principle of conservation of aircraft in a surface element, for each stream  $n$ , the divergence parameters to all the outputs must add up to unity, i.e.;

$$\sum_{m=1}^9 \beta_{mn} = 1$$

Note that one of the  $\beta_{mn}$  is not independent. By convention, let

$$\beta_{nn} = 1 - \sum_{\substack{m=1 \\ m \neq n}}^9 \beta_{mn}$$

It is assumed that an aircraft will nominally remain in the same stream, so the *default* values of the divergence parameters are:

$$\beta_{mn} = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}$$

Figure 3 illustrates the model of a stream in a surface element. The dynamics of the air traffic flow in a SEL can be described using the inertia parameters and the divergence parameters, through the principle of conservation of aircraft. For instance, the difference equation describing the air traffic flow in the easterly stream in the surface element  $i, j$  can be derived as<sup>2,3</sup>:

$$x_{(i,j,3)}(k+1) = a_{(i,j,3,3)} \sum_{m=1}^8 \beta_{(i,j,3,m)} x_{(i,j,m)}(k) + \tau u_{(i,j,3)}(k) + \tau y_{(i,j-1,3)}(k) + \tau q_{(i,j,3)}^{depart}(k) + \tau q_{(i,j,3)}^{exo}(k)$$

In this equation,  $x(k)$  denotes the number of aircraft in the stream at the sample instant  $k$ ,  $u(k)$  are the aircraft flow rates held back in the stream through flow control actions,  $y(\cdot)$  is the air traffic flow rate from the neighboring SEL,  $q^{depart}$  is the air traffic flow rate joining the stream from airports under the SEL and  $q^{exo}$  is the air traffic flow rate entering the airspace. The control variables in this equation are the air traffic flow rates  $u(k)$  for metering actions, and the departure traffic flow rates  $q^{depart}$  from the airports under the SEL.

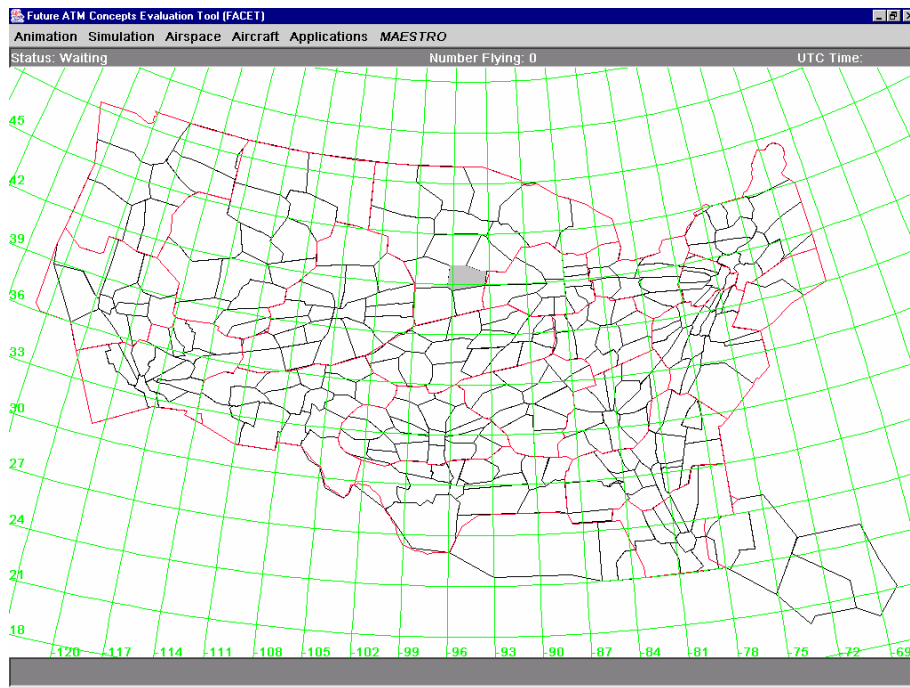


Figure 1. Latitude-Longitude Tessellation Used in Eulerian Flow modeling

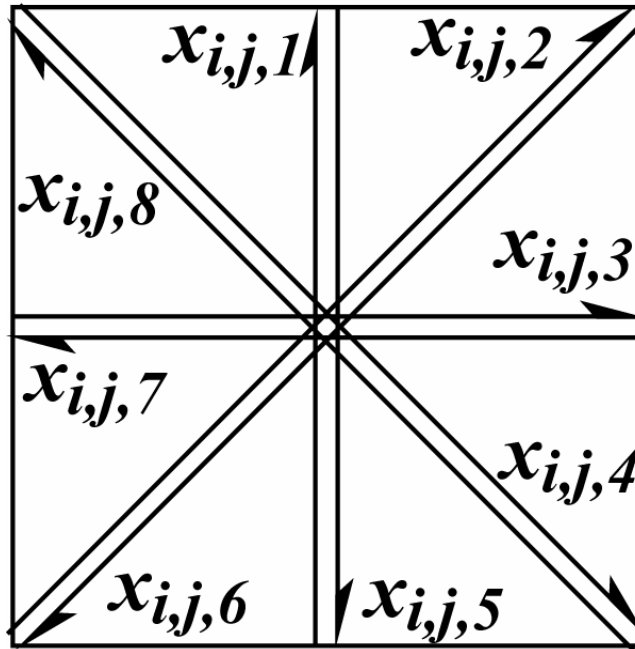


Figure 2. Traffic Flow Directions in a Surface Element

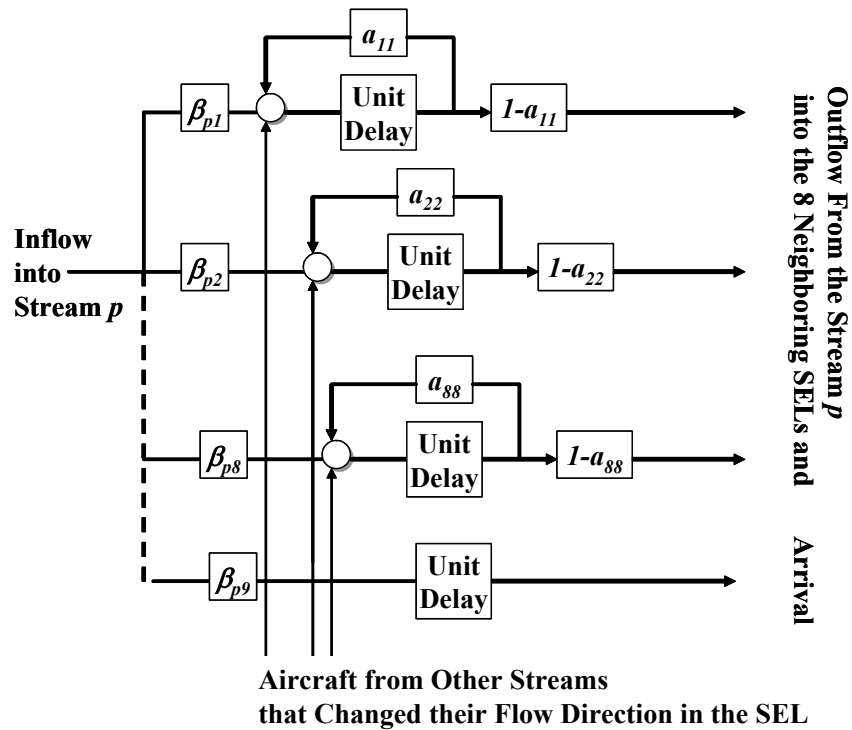


Figure 3. EATF Model of an Air Traffic Stream in a Surface Element

The en route output equations for a surface element can be written as:

$$y_{(i,j,m)}(k) = \left( \frac{1 - \alpha_{(i,j,m,m)}}{\tau} \right) \sum_{n=1}^8 \beta_{(i,j,m,n)} x_{(i,j,n)}(k) - u_{(i,j,m)}(k), \quad m = 1, 2, \dots, 8$$

Moreover, the landing air traffic flow rate into the airports under the SEL are given by:

$$y_{(i,j,m)}(k) = \frac{1}{\tau} \sum_{n=1}^8 \beta_{(i,j,m,n)} x_{(i,j,n)}(k), \quad m = 9$$

Several surface elements are required to model realistic airspaces. In the present work, the numbering convention of the surface elements  $(i, j)$  is that the index  $j$  is increasing from left to right, in the easterly direction, and  $i$  is increasing from bottom to top, in the northerly direction. Air traffic flow models of several SELs can be combined to form the overall Eulerian model of the airspace, and can expressed in a compact form as:

$$x(k+1) = A(k)x(k) + Bu(k) + B_d q^{depart}(k) + B_e q^{exo}(k)$$

The departure traffic may be subdivided according to those airports where they will be controlled by a ground delay program, and where they will not. It is assumed that external traffic  $q^{exo}$  cannot be controlled directly. If the controlled inputs are combined into a vector  $v(k)$ , and all other inputs are collected together into a disturbance vector  $w(k)$ , the dynamic equation for the airspace is of the form:

$$x(k+1) = A(k)x(k) + B_1 v(k) + B_2 w(k)$$

The state vector  $x(k)$  can be initialized using traffic data and then propagated forward in time. These equations can be used to facilitate analysis and synthesis of flow control strategies.

Typically, not all states are of interest for analysis or for flow control. An output equation can be formulated to provide the variables of interest as:

$$y(k) = C(k)x(k) + D_1 v(k)$$

The Eulerian air traffic flow model consists of the time-varying difference equation for the state vector, and the time-varying algebraic equation for the output vector. These equations can be formulated for surface elements in any desired region of the NAS, and combined together to form a basis for analysis and flow-control system design.

#### A. Determining the Parameters of the Eulerian Model from Traffic data

While the Eulerian modeling process is intuitively simple to carry out, it is impractical to manually derive these models for airspaces containing more than a few surface elements. During the present research, a computer aided modeling technique has been developed to automatically derive Eulerian models of arbitrary dimension using the FACET software as the traffic propagation engine.

Starting with a specification of the airspace boundaries, surface element size and sample time interval, the first step in the modeling process is that of determining the location of every aircraft in the airspace with respect to the surface element grid. Within each SEL, the heading angle of aircraft is then used to sort them into one of the eight streams. As an additional criterion, this determination may also be based on the surface elements they are likely to occupy at the end of the sample time interval. This provides the initial condition for the Eulerian model.

Next, the aircraft trajectories are propagated using the FACET software for one sample interval. The new locations of the aircraft in the SELs, together with the aircraft location data at the beginning of the sample interval are then used to compute the inertia and divergence parameters for each SEL during the sample time duration. This process is repeated for the time duration of interest.

A flowchart of the automatic modeling methodology is given in Figure 4. The modeling algorithm has been implemented in the form of a software package called *MAESTRO* (Modeling and Analysis Environment for Studying Traffic-flow Requirements and Operations). Starting with the specifications of a few parameters, this software package enables the user to construct models of arbitrary size. The software also incorporates linear algebraic algorithms<sup>9</sup> to help carry out controllability, observability, reachability, order reduction, decentralization and covariance analysis. All the results given in this paper were generated using this software package.

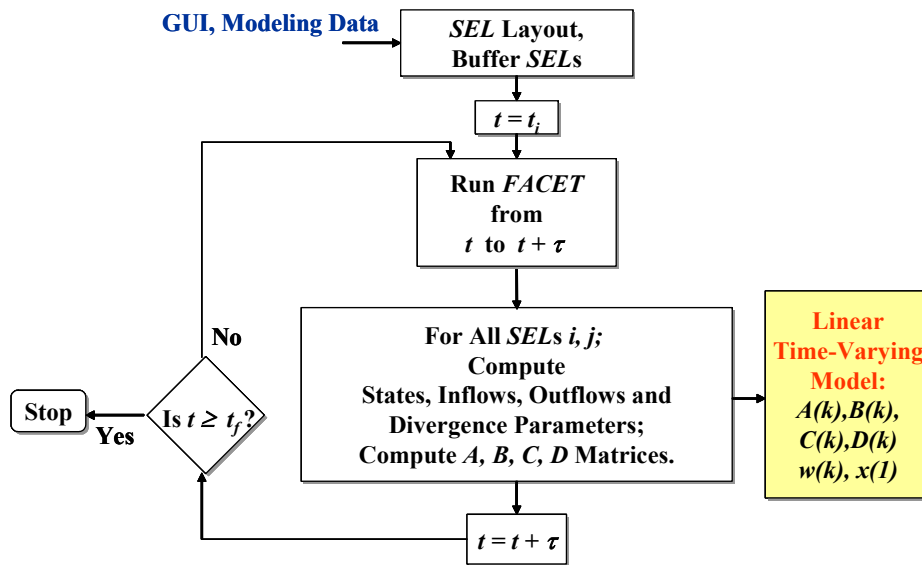


Figure 3. A Flowchart of the Automatic Eulerian Air Traffic Flow Modeling Methodology

### III. Model Predictive Air Traffic Flow Control

One of the objectives of the present research is to demonstrate the application of Eulerian models for the synthesis of closed-loop air traffic flow control algorithms. These algorithms can initially be used as decision support tools, and as other airspace automation initiatives<sup>4,5</sup> mature in the future, they could be used in a more automated mode. A block diagram illustrating the components of the air traffic flow control system is given in Figure 4.

Closed-loop air traffic flow control logic helps achieve the desired traffic flow rates at arrival airports and keep traffic densities within limits in the national airspace by regulating the departures at airports and by modulating the flow through metering regions in the airspace. If the traffic density is low except at isolated time intervals, and the flow control problems are localized, effective flow control can be achieved using simple strategies. But as the traffic density increases, purely heuristic approaches may result in undesirable flow fluctuations in the airspace, and tools for flow control become important. Note that the traditional approach to air traffic flow control in the NAS is through heuristic means.

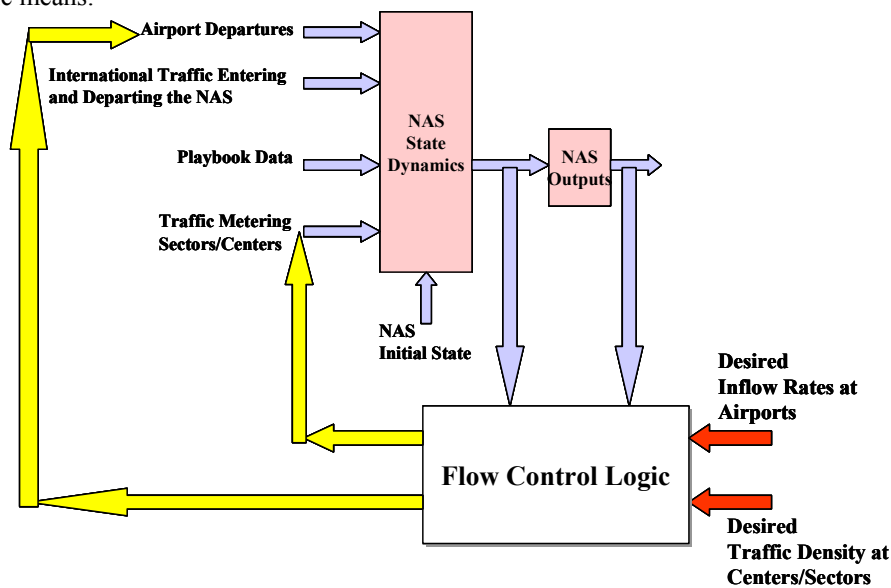


Figure 4. Air Traffic Flow Control System

Although sophisticated decision support tools<sup>10</sup> have been developed to manage aircraft trajectories, computational tools to manage traffic flows have not yet reached comparable levels. This section will demonstrate how the Eulerian models can be used to design en route flow control strategies. Due to the high order of the system dynamics, its time varying nature, and control limits, the model predictive control technique<sup>9-11, 12</sup> is used for the flow control algorithm synthesis.

The basic idea in this control technique is to use a model of the system to predict the outputs up to  $N$  steps ahead (prediction horizon) using a nominal control policy. Nominal control policies are often adopted as either zero or constant values of control, subject to the control constraints. Next, an optimization problem is solved to determine the values of control that will minimize the error between the actual and the desired values of the outputs over the prediction horizon.

The Eulerian air traffic flow model over multiple time steps can be used to readily assemble an output predictor as:

$$\tilde{y} = M_x x(k) + M_u \tilde{u} + M_d \tilde{q}^d$$

where,

$$\tilde{y} = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+N) \end{bmatrix}, \quad \tilde{u} = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+N) \end{bmatrix}, \quad \tilde{q}^d = \begin{bmatrix} q^{depart}(k) + q^{exo}(k) \\ q^{depart}(k+1) + q^{exo}(k+1) \\ \vdots \\ q^{depart}(k+N) + q^{exo}(k+N) \end{bmatrix}$$

$$M_x = \begin{bmatrix} C_0 \\ C_1 A_0 \\ C_2 A_1 A_0 \\ C_3 A_2 A_1 A_0 \\ \vdots \\ C_N A_{N-1} \cdots A_0 \end{bmatrix}, \quad M_u = \begin{bmatrix} D_0 & 0 & 0 & 0 & \cdots \\ C_1 B_0 & D_1 & 0 & 0 & \cdots \\ C_2 A_1 B_0 & C_2 B_1 & D_2 & 0 & \cdots \\ C_3 A_2 A_1 B_0 & C_3 A_2 B_1 & C_3 B_2 & D_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_N A_{N-1} \cdots B_0 & \cdots & \cdots & \cdots & D_N \end{bmatrix}$$

$$M_d = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ \tau C_1 & 0 & 0 & 0 & \cdots \\ \tau C_2 A_1 & \tau C_2 & 0 & 0 & \cdots \\ \tau C_3 A_2 A_1 & \tau C_3 A_2 & \tau C_3 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tau C_N A_{N-1} \cdots A_1 & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

A set of performance variables  $y_{perf}$  is next defined. These performance variables represent the traffic flows that the model predictive control algorithm expects to control, and can be individual air traffic flows in specific surface elements, or linear combinations of traffic flows into airports or regions of interest in the en route airspace.

The essence of model predictive control is that at each time step over the prediction interval, it is desired to minimize the difference between the actual values of the performance variables and the desired or command values  $y_d$ . The variables comprising  $y_{perf}$  and its desired values  $y_d$  are selected based on the specific air traffic flow control objectives.

For the present work, the 1-norm is a suitable choice for the minimization problem in terms of the performance variables and their desired values. As a consequence, the optimization problem can then be cast as a linear programming problem. The objective of the air traffic flow control is to minimize:

$$\|\tilde{y}_d - \tilde{y}_{perf}\|_1 = \sum_{i=1}^{(N+1)p} |\tilde{y}_{d_i} - \tilde{y}_{perf_i}|$$

with  $p$  being the number of performance variables included in the flow control problem. Note that the performance variables can be cast as:  $y_{perf} = Y_p y$  for some matrix  $Y_p$ , and  $\tilde{y}_{perf} = \tilde{Y}_p \tilde{y}$  where  $\tilde{Y}_p$  is a block-diagonal matrix with  $Y_p$  comprising the  $N+1$  blocks. With this definition, the expression for the performance variables can be written as:

$$\tilde{y}_{perf} = \tilde{Y}_p M_x x(k) + \tilde{Y}_p M_u \tilde{u} + \tilde{Y}_p M_d \tilde{q}^d$$

The linear programming problem can then be expressed in the form:

$$\min_{\gamma, \tilde{u}} \begin{bmatrix} \underline{1} & \underline{0} \end{bmatrix} \begin{bmatrix} \gamma \\ \tilde{u} \end{bmatrix}, \text{ such that}$$

$$\begin{bmatrix} I & \tilde{Y}_p M_u \\ I & -\tilde{Y}_p M_u \end{bmatrix} \begin{bmatrix} \gamma \\ \tilde{u} \end{bmatrix} + \begin{bmatrix} \tilde{Y}_p M_x x(k) + \tilde{Y}_p M_d \tilde{q}^d - \tilde{y}_d \\ -\tilde{Y}_p M_x x(k) - \tilde{Y}_p M_d \tilde{q}^d + \tilde{y}_d \end{bmatrix} \geq \underline{0}$$

The vector  $\gamma$  consists of bounding variables, one variable for each term in the 1-norm. Note that the symbol  $\underline{1}$  in the cost function represents a row vector of ones and  $I$  in the inequality is an identity matrix.

Additional constraints in the problem are that the controls must be greater than or equal to zero to be physically meaningful. Since the controls are part of the solution vector, the lower bounds can be handled directly. Another constraint is that the outflows in the streams where metering is taking place must be greater than or equal to zero, which in effect defines the upper bounds on the controls. Since these upper bounds are dependent on the state of the system, they cannot be specified directly, and must be included as constraint equations. Let these constrained outputs be defined as  $y_c = Y_c y$  with  $Y_c$  a matrix of 0's and 1's, and  $\tilde{y}_c = \tilde{Y}_c \tilde{y}$  where  $\tilde{Y}_c$  is a block-diagonal matrix with  $Y_c$  comprising the  $N+1$  blocks. The additional constraint equation is:

$$\begin{bmatrix} \underline{0} & \tilde{Y}_c M_u \end{bmatrix} \begin{bmatrix} \gamma \\ \tilde{u} \end{bmatrix} + \begin{bmatrix} \tilde{Y}_c M_x x(k) + \tilde{Y}_c M_d \tilde{q}^d \end{bmatrix} \geq \underline{0}$$

This constraint equation augments the previous equations. Slack variables are introduced as in standard linear programming problems to transform the inequality constraints into equality constraints. The linear programming problem for the model predictive control problem is then formed as follows. Let the vector of unknown quantities be

$$z^t = \begin{bmatrix} \gamma^t & \tilde{u}^t & r^t & s^t & t^t \end{bmatrix}$$

where  $r$ ,  $s$ , and  $t$  are column vectors of slack variables of dimensions  $(N+1)p$ ,  $(N+1)p$ , and  $(N+1)m$ , respectively, where  $m$  is the number of control inputs,

$$c^t = \begin{bmatrix} \underline{1} & \underline{0} \end{bmatrix}$$

where  $\underline{1}$  is length  $(N+1)p$  and  $\underline{0}$  is length  $(N+1)(2m+2p)$ , and



$$F = \begin{bmatrix} I & \tilde{Y}_p M_u & -I & 0 & 0 \\ I & -\tilde{Y}_p M_u & 0 & -I & 0 \\ 0 & \tilde{Y}_c M_u & 0 & 0 & -I \end{bmatrix}, \quad g = \begin{bmatrix} -\tilde{Y}_p M_x x(k) - \tilde{Y}_p M_d \tilde{q}^d + \tilde{y}_d \\ \tilde{Y}_p M_x x(k) + \tilde{Y}_p M_d \tilde{q}^d - \tilde{y}_d \\ -\tilde{Y}_c M_x x(k) - \tilde{Y}_c M_d \tilde{q}^d \end{bmatrix}$$

For the controls, the lower bounds are zero, and although the upper bounds are determined by constraints, a value of 50 was specified as a practical measure. The lower bounds on the slack variables are set to zero and the upper bounds are set to a large number,  $10^{32}$  in the present work. Likewise, the lower bounds on the bounding variables are set to zero and the upper bounds are set to a large number,  $10^{32}$ .

In the case where departure controls are included, the appropriate columns of  $\tilde{Y}_p M_d$  appear in the  $F$  matrix on the left-hand side of the constraint equation, and an additional constraint is needed. While the requirement that the output must be non-negative could be used, in this case the maximum value of the control is determined by the number of departures in that stream where control is being applied. This gives a simpler expression to implement. Let  $\hat{q}^d$  be the subset of departures where control is to be applied, and let  $\tilde{u}^d$  be the subset of the controls applied to the departures. Then the constraint is:

$$\hat{q}^d - \tilde{u}^d \geq 0.$$

Since this is an inequality, another set of slack variables must be added, so that the vector of unknowns becomes

$$z^t = \begin{bmatrix} y^t & \tilde{u}^t & r^t & s^t & t^t & v^t \end{bmatrix}.$$

For simplicity of notation it will be assumed that the control vector is partitioned so that the metering controls for the prediction horizon  $[k, k+N]$  are first, and the departure controls over the same interval are second; i.e.:

$$\tilde{u} = \begin{bmatrix} \tilde{u}^m \\ \tilde{u}^d \end{bmatrix}$$

Let  $\hat{M}_d$  be the matrix composed of the columns of  $M_d$  corresponding to the departures where control is applied. Then the left and right hand sides of the constraint equations are of the following form.

$$F = \begin{bmatrix} I & \tilde{Y}_p M_u & \tilde{Y}_p \hat{M}_d & -I & 0 & 0 & 0 \\ I & -\tilde{Y}_p M_u & -\tilde{Y}_p \hat{M}_d & 0 & -I & 0 & 0 \\ 0 & \tilde{Y}_c M_u & -\tilde{Y}_c \hat{M}_d & 0 & 0 & -I & 0 \\ 0 & 0 & -I & 0 & 0 & 0 & -I \end{bmatrix} \quad g = \begin{bmatrix} -\tilde{Y}_p M_x x(k) - \tilde{Y}_p M_d \tilde{q}^d + \tilde{y}_d \\ \tilde{Y}_p M_x x(k) + \tilde{Y}_p M_d \tilde{q}^d - \tilde{y}_d \\ -\tilde{Y}_c M_x x(k) - \tilde{Y}_c M_d \tilde{q}^d \\ -\hat{q}^d \end{bmatrix}$$

The linear programming problems for various air traffic flow control situations formulated in this section are solved using a software package called *PCx*<sup>13</sup> from Argonne National Laboratory. This software has been integrated into the Eulerian modeling software (*MAESTRO*) mentioned in the previous section.

A flowchart of the model predictive air traffic flow control implementation is given in Figure 4. Since the coefficients of the Eulerian model used for MPC are derived from the traffic data, the application of controls will change the traffic flow. This will in turn cause changes in the model. In order to synthesize correct control decisions for the next sample, the model coefficients must be recomputed using the air traffic resulting from the application of controls in the previous sample. Thus, every control action must be followed by the recomputation of the model coefficients. The new model must then be used in the MPC flow control methodology.

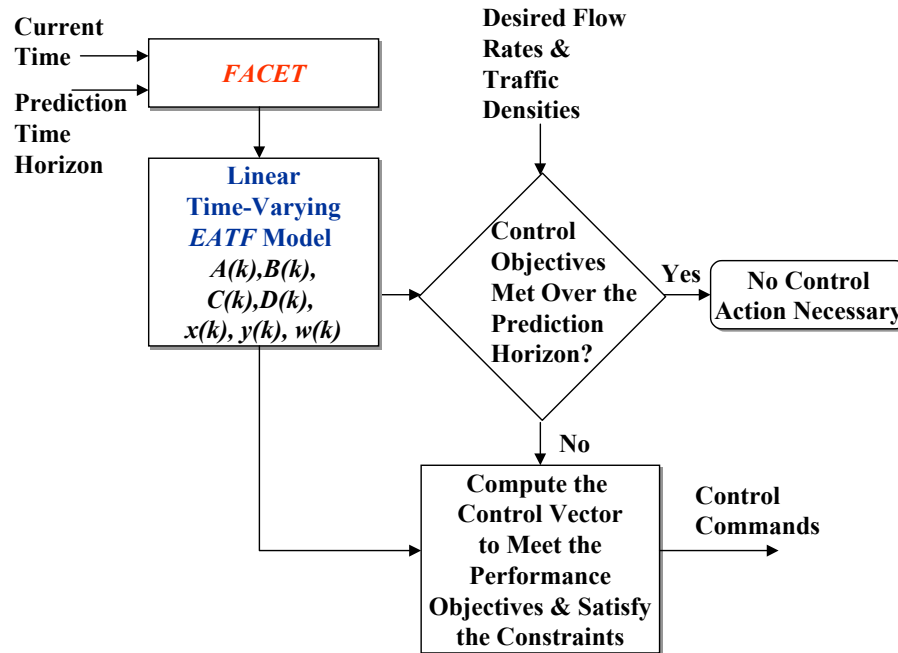


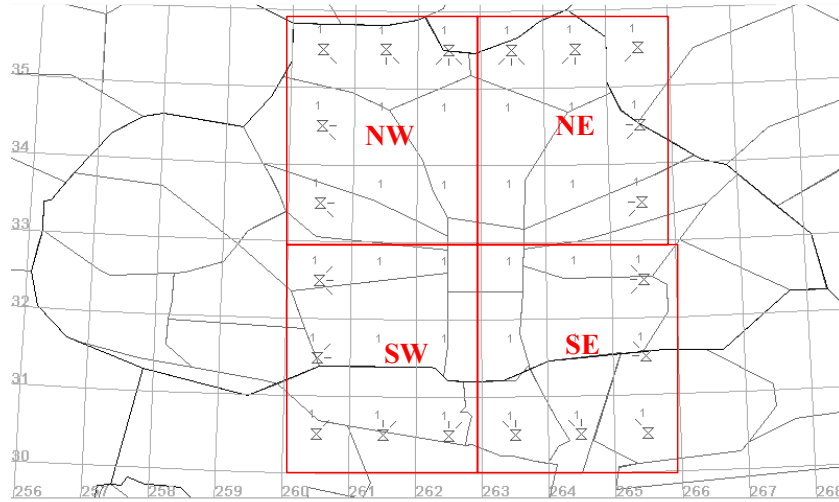
Figure 4. Model Predictive Air Traffic Flow Control Using the EATF Model

The following sections will illustrate two different air traffic flow control examples using the MPC methodology.

#### B. Dallas-Fort Worth Area Metering

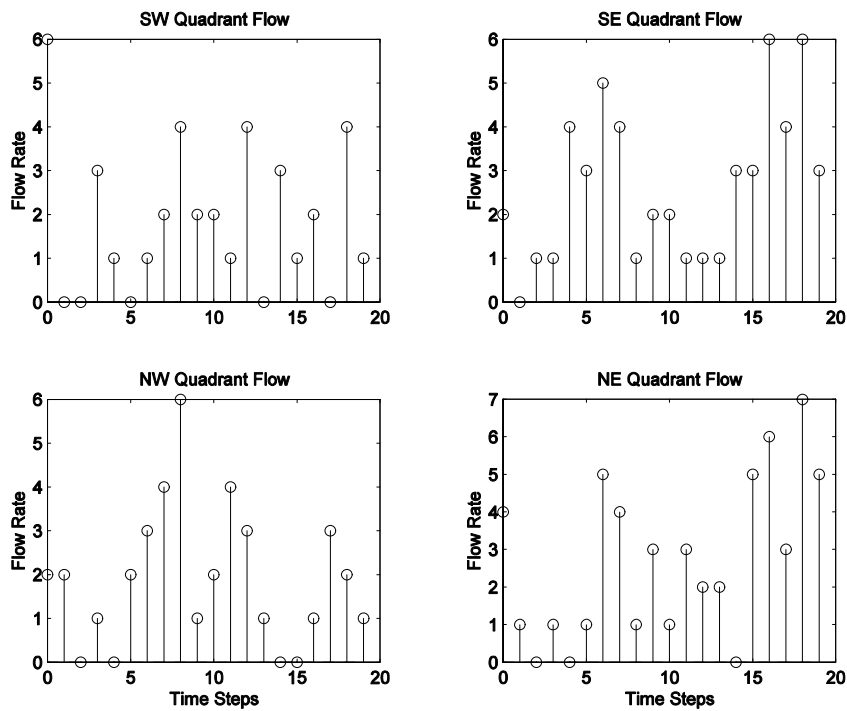
The objective of this example is to regulate the air traffic flow descending into the area surrounding the Dallas-Fort. Worth (DFW) airport for the time period between noon to 2 PM on a typical day. Figure 5 illustrates the region under consideration. Each surface element included in the control problem is denoted with the numeral 1. The airport is near the middle of this figure. The surface elements are 1 degree by 1 degree, or approximately 60 nm north-south by 50 nm east-west. The sample time interval is 6 minutes. The metering locations are indicated by hourglass symbols and the directions are indicated by short line segments in this figure. There are a total of 40 controls, which were selected in part by observing the traffic flow over the specified period. The Eulerian traffic flow model has 512 state variables. For simplicity, the descending air traffic streams are summed by quadrants NW, NE, SE, SW as shown in the figure, resulting in four outputs to be controlled.

The time history of the outputs without flow control inputs are shown in Figure 6. It can be observed that the traffic does not follow any particular pattern. The model predictive control strategy is next implemented, with the requirement that the desired flow rate into each quadrant be less than or equal to 2 aircraft/sample. The prediction time horizon is 4 samples. The time history of aircraft flows into the four quadrants under closed-loop control are given in Figure 7. It can be observed that the flow is much more regular, but the control objectives are not strictly satisfied at all sample instants. Note that it is not possible to achieve exact control due to the fact that the metering controls are constrained, and that the flow is not metered in certain directions.



**Figure 5. Metering Controls in the Vicinity of DFW Airport**

In the foregoing discussions, the aircraft flows in four separate zones were regulated. A more practically useful control objective is the regulation of the total traffic flow into the DFW airport area. In view of this, the MPC problem is next reformulated with one output defined as the sum of all 16 landing outputs, and requiring the desired flow rate to be less than or equal to 8 aircraft/sample. The model predictive flow control then produces the time history shown in Figure 8. This figure also indicates the aircraft flow without control.



**Figure 6. Air Traffic Flows into the DFW Airport Area without Metering**

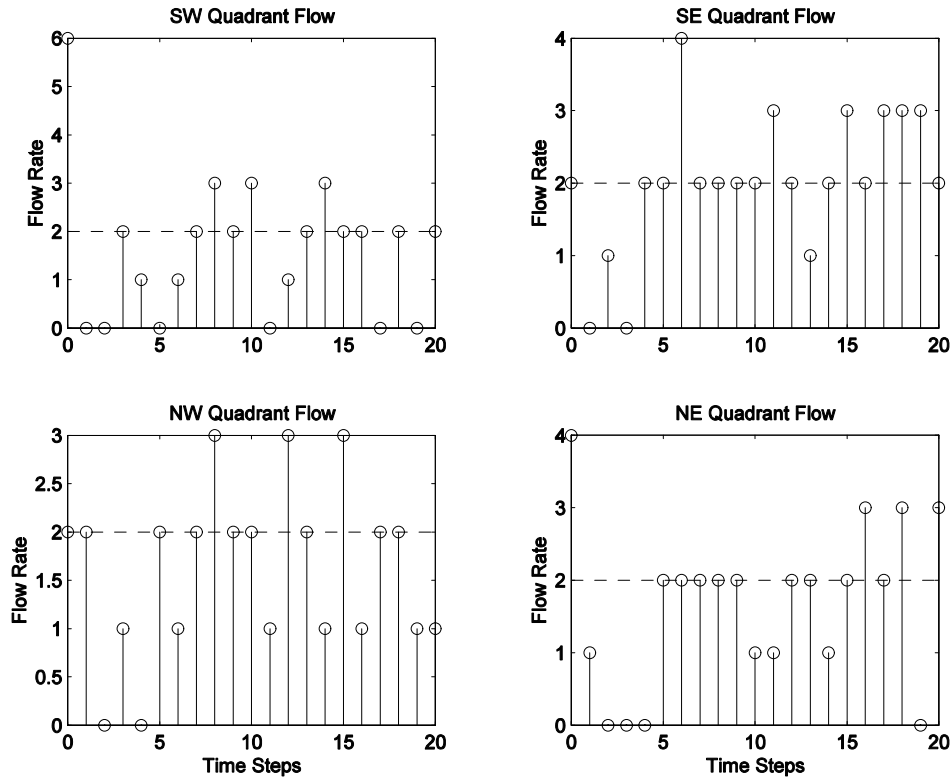


Figure 7. Air Traffic Flows into the DFW Airport Area with Metering

As in the previous case, the present control objective is strictly met only at certain samples. However, the flow rate much more regular under closed-loop control. Achieving a better flow control will require the introduction of additional metering SELs in the modeled region.

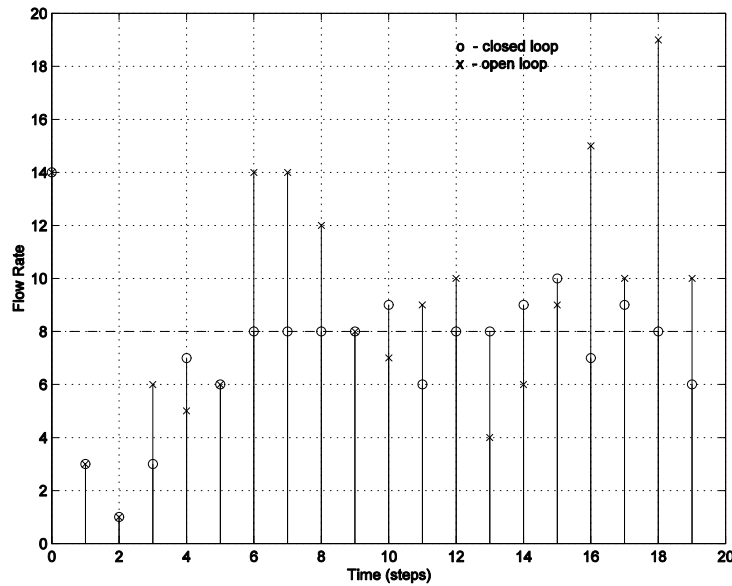


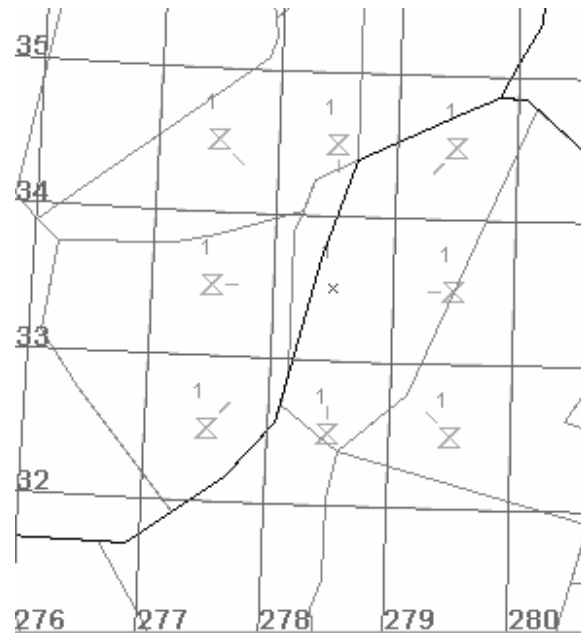
Figure 8. Air Traffic Flow Rate into DFW Airport Area Under Metering

### C. Air Traffic Density Control in a Region

A flow control problem that sometimes arises in the NAS is that of maintaining the density of air traffic in certain regions below a certain specified level to limit the workload on the human air traffic controller. This example illustrates how the MPC methodology can be used for traffic density control. For illustrative purposes, in this example, the aircraft density in a single surface element will be regulated. Extending this approach to multiple surface elements is straightforward.

The traffic density is defined in this paper as the total number of aircraft within a surface element at a sample instant. Since the airspace volume represented by surface element is known, the actual traffic density/unit volume and the total aircraft count are proportional. The density is computed by summing the aircraft in each of the eight streams in the surface element of interest. For the present example, the surface element of interest is between 33 and 34 degrees North latitude and 82 and 83 degrees West longitude, an area east of Atlanta's Hartsfield airport. This region was selected arbitrarily among those that appeared to have a steady flow of traffic. Metering points were chosen at the streams in neighboring SELs that feed into the controlled SEL. The surface element of interest is shown in Figure 9. This example has 72 states and 8 controls.

Note that metering controls have been placed in neighboring surface elements, in directions pointing towards the surface element. The desired density is then chosen to be 4 aircraft or less, and the number of prediction steps is taken as 2. A comparison of the controlled and uncontrolled densities is shown in Figure 10. The MPC-based controller is able to maintain the desired density to within 2 aircraft or less in most cases.



**Figure 9. Surface Elements Considered in the Traffic Density Control Example**

The two examples given in this section illustrate the MPC-based air traffic flow control logic synthesis. Although the performance of the MPC controllers was satisfactory, significant improvements are possible through additional analysis of the Eulerian models.

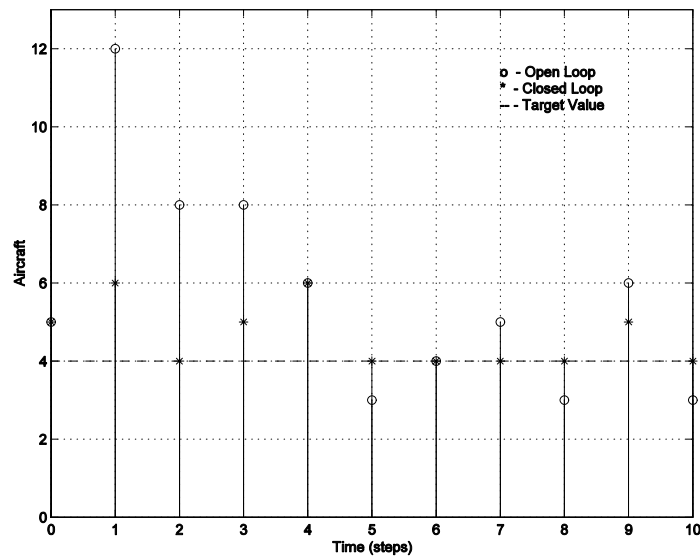


Figure 10. Density Control Results

#### IV. Conclusions

This paper presented the development of a computer-aided Eulerian air traffic flow modeling methodology and its application to deriving quantitative flow control strategies. The flow control algorithms can initially be used as decision support tools, and as other airspace automation initiatives mature in the future, they can be used in a more fully automatic mode.

The Eulerian modeling methodology divides the airspace into interconnected surface elements, and the dynamics of the traffic flow through and between these surface elements is then derived by invoking the principle of conservation. Although the Eulerian approach preserves no information on the motion of individual aircraft, it provides a convenient formalism for aggregating air traffic flow information. An automatic procedure for deriving the Eulerian models from individual aircraft trajectories was developed during the present research. In this approach, the user provides inputs such as the region of the national airspace to be modeled, spatial discretization, sample time, metering locations, airports subject to departure control, the output locations and the time interval of interest. The automatic modeling procedure then uses the FACET software to assemble the Eulerian model. A software package termed as the *MAESTRO* (Modeling and Analysis Environment for Studying Traffic-flow Requirements and Operations) has been developed and integrated into FACET to assist the user in automatically constructing Eulerian air traffic flow models.

Various types of analysis can be conducted using the Eulerian traffic flow models. By aggregating the traffic information in the form of discrete-time, linear time-varying models, the Eulerian model enables several types of useful analyses on the air traffic flow in the airspace.

The model predictive control technique was employed in conjunction with the Eulerian model to synthesize air traffic flow control algorithms. The MPC technique uses the Eulerian model to make predictions over a specified time-horizon about the future values of performance variables under a nominal control policy. An optimization technique is then used to refine the nominal control policy so as to achieve the desired values of states and outputs.

The present research considered the 1-norm of the error between desired and predicted performance variables as the performance index. Inequality constraints were specified on the control variables and output variables. Since Eulerian models are linear, the resulting optimization problem is in the form of a linear program. This linear programming problem was then solved using a well-known software package. Flow control synthesis for two different problems was then demonstrated.

This paper demonstrated that it is feasible to use the Eulerian air traffic flow models for analysis and flow control system synthesis. The methodology given here can be readily tailored to practical traffic flow control problems to improve the efficiency of en route air traffic flow control in the future. Investigation of alternate control techniques will be future interest.

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