Computer modelling of layered conformal contact

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ABSTRACT

This work is concerned with a finite element investigation of conformal contact between cylindrical and spherical layered components to study the effects of material properties and load on the contact. The study stems from the analysis of the contact between the piston barrel and cylinder block and the piston ball and slipper seat of an axial piston pump but could apply equally well to any components with conformal cylindrical or spherical contact such as actuators or hip joints. The purpose of the research is to develop procedures to aid in the selection of lining, sleeve and seat materials which might be chosen for a particular property (eg. chemical resistance, low friction) and then fixed in or onto a standard, and generally less expensive, bulk material. The non-linear nature of the problem means that simple material substitution can lead to significant changes in the size of the contact area with a corresponding variation in the contact pressure.

INTRODUCTION

In this paper the finite element method has been used to study the contact between the piston barrel and cylinder block and piston ball and slipper seat of an axial piston pump. See figure 1. In both of these positions the contact is conformal, the contact stresses becoming part of the general stress distribution in the components. The situation is made more complex by the use of liner and sleeve materials selected for their tribological properties. The components are lubricated by water and as an initial assumption frictional effects have been neglected.

The solution to the problem of the conformal contact of a long elastic cylinder contacting a cylindrical seat was first attempted by Steuermann[1]. He represented the gap between the cylinder and cylindrical seat by a power series and used finite difference methods to solve the integral equation for the contact pressure distribution, though he retained the assumption that both solids can be

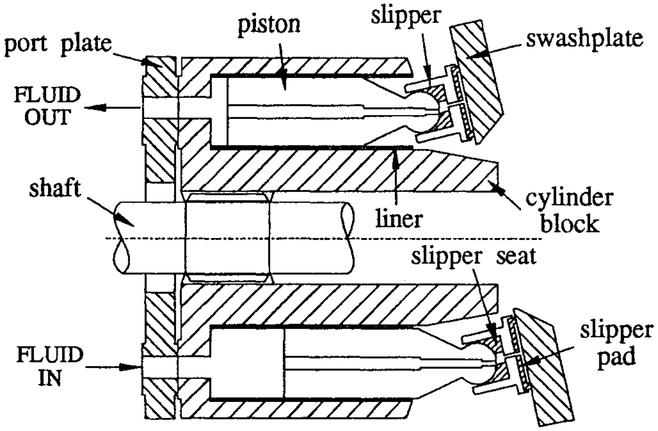

Figure 1 Schematic layout of the main components of an axial piston pump.

regarded as elastic half spaces. The widely quoted standard for the analytical study of conformal contact of concentric cylindrical bodies was conducted by Persson[2] in 1964. He assumed that the contacting surfaces remain cylindrical and determined contact pressure distributions for the plane stress model.

Goodman and Keer[3] presented an extension to the Hertz theory for the solution of an elastic sphere indenting an elastic seat with no friction. Their analysis remained within linear elastic theory and the elastic properties of sphere and cavity were assumed to be identical. However, they assumed that points on both bodies which were initially equidistant from the axis of symmetry came into contact after deformation. This is clearly not true for angles of contact substantially greater than the maximum of 25° which they considered.

With the increased availability of computers many authors began to develop general numerical methods to solve both frictionless and frictional contact problems[4,5]. The use of the finite element method has been a comparitively recent development which has received considerable attention over the past 10-15 years. In early implementations of the method the problems of determining the region of contact between meshes precluded its use for elastic contact problems. However, more recently there have been a large number of numerical approaches to the problem of applying the correct boundary conditions. The majority make use of either Lagrange multipliers or penalty function methods to impose the displacement contstraints at the contact surface between contacting meshes. Francavilla and Zienkiewicz[6] were among the first to provide a contact algorithm that became widely installed in finite element programs. However, this algorithm had the disadvantage of requiring a flexibility, rather than stiffness, approach.

A significant advance was made by Hughes et al[7] who used Lagrange multipliers, a method that could be used in finite element displacement solution routines. However, node-on-node contact was necessary in the contact zone. A general two-dimensional algorithm which did not require node-on-node contact was developed by Bathe and Chaudhary[8] and extended by Pascoe and Mottershead[9,10].

All methods based on so-called gap elements are based on penalty functions where the gap element stiffness is the penalty number. It is this method which is used in some of the popular general purpose finite element packages including the one used for the work described here.

In this application bending loads on the pistons cause them to tilt within the cylinder bores and therefore the problem is essentially three-dimensional. Few authors have considered the effect of three-dimensional conformal cylindrical contact, where the contact area and pressure also vary in the z-direction. Harrison and Harris[11] used the finite element method to study the effect of the diametral gap between a femoral total hip component and a femur. The two components were modelled as conforming concentric cylindrical components with gap elements between and with no friction.

FINITE ELEMENT ANALYSES

The analyses consist of several finite element models using gap elements between the components to model the contacting interfaces. They are created from parametric input allowing the key model dimensions, material properties and loading to be changed with the minimum of additional work. The 2D and 3D gap/friction elements in the NISA finite element package were used for the analyses[12].

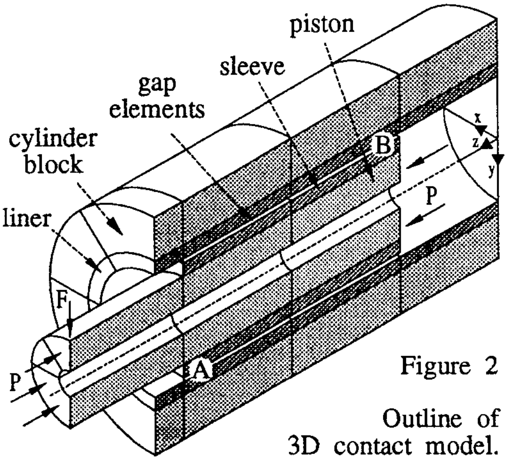

Figure 2

Outline of 3D contact model.

3D Cylindrical Contact

Figure 2 shows the first model which is a three dimensional representation of half a sleeved piston inside a lined cylinder block, taking account of symmetry. The model allows a prediction of the contact forces due to the tilting of the piston inside the bore and consists of four areas, namely the cylinder block, cylinder block liner, piston and piston sleeve. It is meshed using first order 3D solid elements and the interface between the sleeve and liner is modelled with 3D gap elements with no friction, the gap elements connecting nodes across the clearance. The block and liner are constrained from moving in all three coordinate directions at the end furthest removed from the piston. Pressures, P, are applied to each end of the piston along its axis and a force, F, in the negative y-direction causes the piston to tilt within the bore. The values of P and F are calculated in a force analysis program which has been developed as part of this project[13]. 'A' and 'B' show where the components are expected to contact. Several analyses have been run to assess the effects of changing the material properies, sleeve and liner thickness and clearance size on the contact pressure distribution and size.

2D Cylindrical Contact

Figure 3 shows an outline of the second model in the series. It is a two-dimensional representation of a sleeved piston inside a lined cylinder bore. A quarter of the piston/block assembly is modelled, taking account of symmetry and neglecting the non-contacting half. The model is meshed using first order plane strain elements having two degrees of freedom at each node. The assumption is made, therefore, that the same load is applied along the length of the piston. Again gap elements connect nodes across the space between piston sleeve and bore liner. A force is applied to the model at the centre of the piston. Nodes along the symmetry plane are constrained from moving in the y-direction and the nodes along the right edge of the block are constrained from moving in the x-direction.

Several series of finite element analyses were undertaken to establish the effects of load and material properties on the contact size and the forces transmitted between the components.

Axisymmetric Spherical Contact

The next model is an axisymmetric representation of the piston ball in a seated

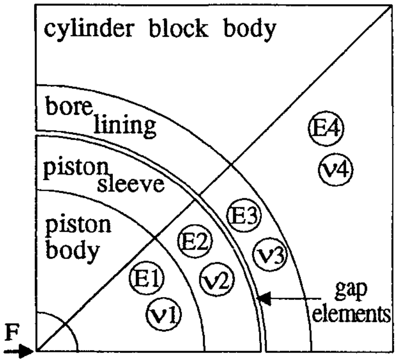

Figure 3 Model representing sleeved piston in lined cylinder bore.

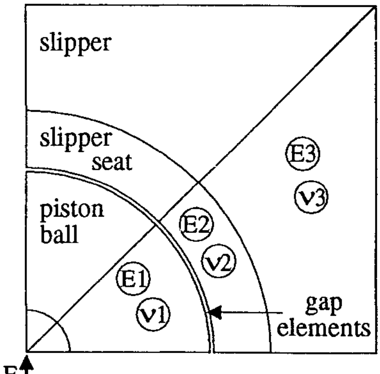

Figure 4 Model representing piston ball in slipper seat.

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slipper, again to assess the effects of material property and clearance changes on the contact size and pressure. See figure 4. The model is meshed using first order axisymmetric elements, the axisymmetric nature of the model implying direct loading between piston and slipper. Again gap elements connect nodes between the components. A force is applied to the model at the centre of the piston ball. Nodes along the axis are constrained from moving in the y-direction and the nodes along the top edge of the slipper are constrained from moving in the x-direction. Several series of analyses were undertaken to establish the effects of load and material properties on the contact size and the forces transmitted between the components.

Piston Model

Contact pressure information from the previous models can now be applied to detailed models of the relevant components to give a more realistic prediction of the stress distributions than can be made by using point loading. Figure 5 shows a model of a piston as an example, meshed using second order three-dimensional solid elements. Contact forces from the appropriate three-dimensional contact model are used for the upper and lower piston reactions and similarly, contact forces from the appropriate axisymmetric model are used for the slipper reaction force. The model is a half representation of a piston, taking account of symmetry.

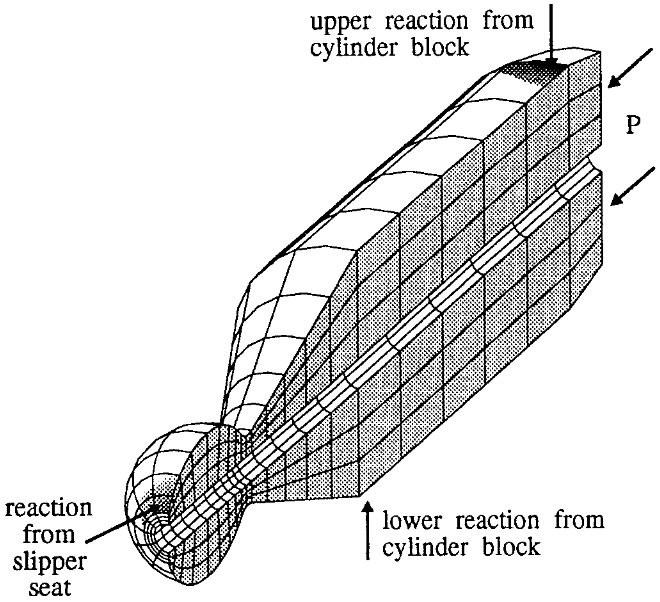

Figure 5 Outline of piston model.

RESULTS

3D Cylindrical Contact

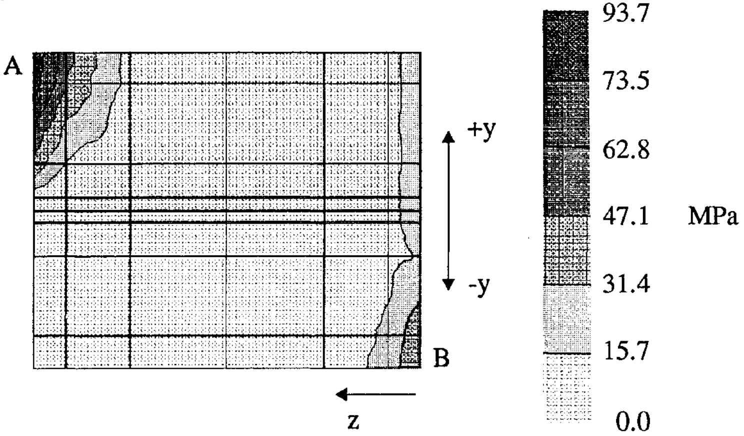

Figure 6 Stress contours on outer surface of piston sleeve.

Figure 6 shows von-Mises stress contours on the surface of the piston sleeve which result from the contact of this component on the cylinder block bore. These results are from a steel piston with a 2.5 mm polymer sleeve contacting a steel cylinder block with no liner. The gap size was 0.025 mm and the pressure 140 bar. The bending or tilting force F was calculated assuming a pump swash angle of 13°. The output information for the gap elements gives the gap status ie. whether or not the gap has closed, and if it has closed, the force transmitted. This has occured in the areas shown by 'A' and 'B' as expected. The resultant contact force in each area is within 16% and 7.5% respectively of the values calculated in the force analysis program, the difference being accounted for by the flexibility of the components.

2D Cylindrical Contact

Figure 7 shows graphs of non-dimensionalised contact pressure against contact angle for a piston in a cylinder block with a 1mm thick liner. The gap size is 0.025mm and the E ratio refers to the Young's Modulus of the liner over the Young's Modulus of the rest of the model. As can be seen, for a stiffer liner material the contact angle is reduced with a consequent increase in contact pressure. Whilst this may be obvious in a qualitative way the quantities involved are more difficult to ascertain.

Axisymmetric Spherical Contact

Figure 8 shows von-Mises stress contours in the contact region for one example of the axisymmetric spherical model, in this case for a steel piston in a steel slipper with no seat. The contact angle in this case is 8.5°. As with the two-dimensional cylindrical model the contact angle is reduced for stiffer

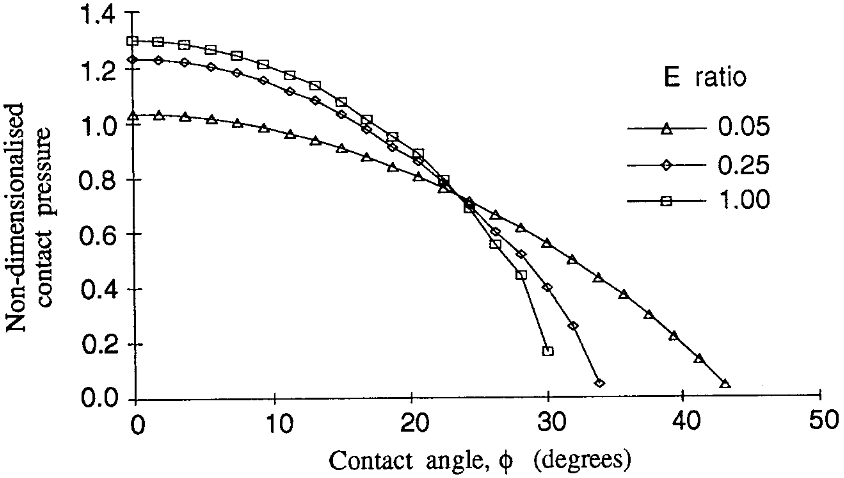

Figure 7 Non-dimensionalised contact pressure against contact angle.

materials with a consequent increase in contact pressure. The non-linear nature of the problem makes this difficult to estimate using analytical methods.

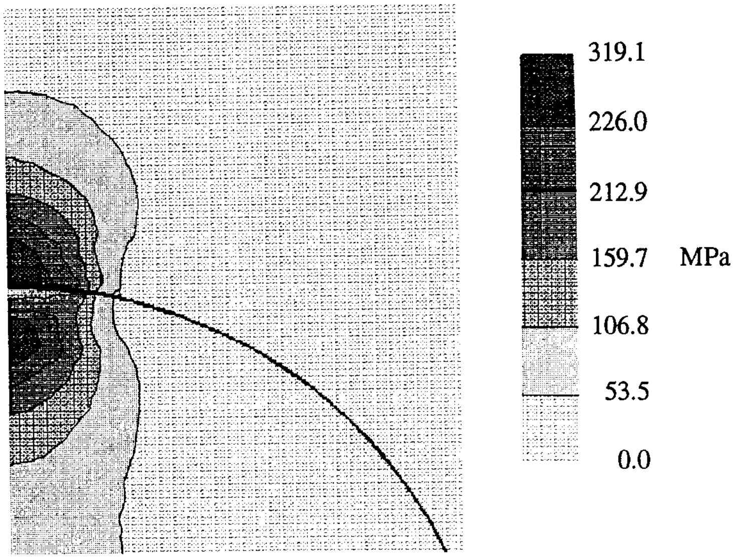

Figure 8 Stress contours in axisymmetric model.

Piston Model

Figure 9 shows the von-Mises stress contours at the neck of a ceramic piston which runs in a cylinder block with a 1 mm polymer liner. By applying the contact loading from the relevant three-dimensional and axisymmetric models the stresses in the bulk of the piston can be estimated with much more confidence than by using point loading.

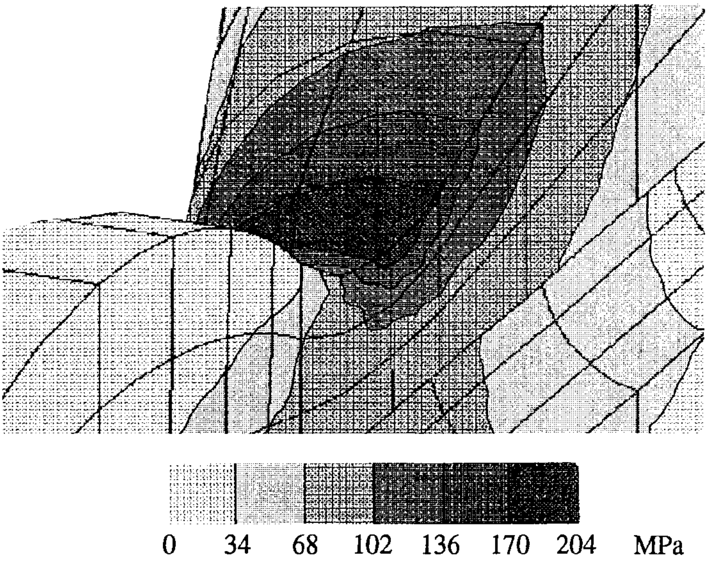

Figure 9 Von-Mises stress contours at the neck of a ceramic piston.

CONCLUSIONS

This paper has described the use of the finite element method to predict the magnitudes of the contact areas and pressures of conformal cylindrical and spherical layered components. Gap elements have been used to represent the clearance between the components. Varying the material properties; clearance and sleeve, liner or seat thicknesses can result in a corresponding change in the contact pressure which, because of its non-linear nature, is difficult to estimate using analytical or other numerical techniques.

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