NASA TECHNICAL NOTE

NASA TN D-8164

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COMPUTER PROGRAM FOR DESIGN ANALYSIS OF RADIAL-INFLOW TURBINES

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1976

1. Report No. NASA TN D-8164	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle COMPUTER PROGRAM FOR DESIGN ANALYSIS OF RADIAL-INFLOW TURBINES		5. Report Date February 1976
		6. Performing Organization Code
7. Author(s) Arthur J. Glassman		8. Performing Organization Report No. E-8394
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		10. Work Unit No. 505-04
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

Input design requirements are power, mass flow rate, inlet temperature and pressure, and rotative speed. The design variables include stator-exit angle, rotor radius ratios, and rotor-exit tangential velocity distribution. Losses are determined by an internal loss model. The program output includes diameters, efficiencies, temperatures, pressures, velocities, and flow angles. Presented are the loss model, the analysis equations, a description of input and output, the FORTRAN program listing and variable list, and sample cases.

17. Key Words (Suggested by Author(s)) Radial-inflow turbine Turbine design	18. Distribution Statement Unclassified - unlimited STAR Category 07 (rev.)

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 64	22. Price* $6.25

* For sale by the National Technical Information Service, Springfield, Virginia 22161

COMPUTER PROGRAM FOR DESIGN ANALYSIS OF RADIAL-INFLOW TURBINES

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SUMMARY

This report presents a computer program for the design analysis of radial-inflow turbines. Input design requirements are power, mass flow rate, inlet temperature and pressure, and rotative speed. The design variables include stator-exit angle, rotor-exit-tip to rotor-inlet radius ratio, rotor-exit-hub to tip radius ratio, and the magnitude and radial distribution of rotor-exit tangential velocity. The turbine losses, which are determined by an internal loss model, include those due to stator and rotor boundary layers, tip clearance, disk friction, and exit velocity. The program output includes diameters, total and static efficiencies, and all absolute and relative temperatures, pressures, velocities, and flow angles at stator inlet, stator exit, rotor inlet, and rotor exit. At the rotor exit, these values are presented at any number of radial positions up to a maximum of 17.

Presented in this report are the loss model, the analysis equations, an explanation of input and output, and the FORTRAN program listing and variable list. Sample cases are included to illustrate use of the program.

INTRODUCTION

The analysis of a power or propulsion system involves many repetitive calculations of component performance and geometry over a range of conditions. Such calculations are most easily and quickly done by a computer. One component of interest for small gas turbine systems is the radial-inflow turbine. Radial-inflow turbine geometries for achieving maximum static efficiency are presented as a function of specific speed in reference 1. However, there appeared to be no readily available computer program for performing the velocity-diagram analysis required for determining geometry and estimating performance.

A computer program for the design analysis of radial-inflow turbines was, therefore, developed. Input design requirements are power, mass flow rate, inlet temperature and pressure, and rotative speed. The design variables include stator-exit angle, rotor-exit-tip to rotor-inlet radius ratio, rotor-exit-hub to tip radius ratio, and the magnitude and radial distribution of rotor-exit tangential velocity. The turbine losses include those due to stator and rotor boundary layers, tip clearance, disk friction, and exit velocity. The program output includes diameters, total and static efficiencies, and all absolute and relative temperatures, pressures, velocities, and flow angles at stator inlet, stator exit, rotor inlet, and rotor exit. At the rotor exit, these values can be presented at any number of radial positions up to a maximum of 17.

This radial-inflow turbine design analysis computer program is described herein. Presented in this report are the loss model, the analysis method, an explanation of input and output, and the FORTRAN program listing and variable list. Sample cases are included to illustrate use of the program.

LOSS MODEL

An important part of any turbine design problem is the estimation of losses. The loss model used for this analysis is a modification and extension of the model used in reference 1. Accounted for by this model are the three-dimensional (profile plus end wall) viscous losses in the stator and the rotor, the disk-friction loss on the back side of the rotor, the loss due to the clearance between the rotor tip and the outer casing, and the exit velocity loss.

Viscous Loss

The stator and rotor viscous losses are each expressed in terms of a kinetic-energy loss coefficient, which is defined as the loss in kinetic energy as a fraction of the ideal kinetic energy of the blade row actual flow. In terms of boundary layer parameters (see ref. 2), the two-dimensional kinetic-energy loss coefficient is expressed as

$$\bar{e}_{2D} = \frac{\psi_{tot}}{s \cos \Phi - \delta_{tot} - t} \tag{1}$$

where the flow angle Φ is equal to α_1 for the stator and β_2 for the rotor. The symbols are defined in appendix A. The station designations are indicated on the schematic cross section of a radial-inflow turbine shown in figure 1. An example velocity diagram indicating angle designations is shown in figure 2. Substituting

$$\psi = E\theta \tag{2}$$

and

$$\delta = H\theta \tag{3}$$

into equation (1), introducing the surface length l, and dividing numerator and denominator by blade-row exit spacing s yield

$$\bar{e}_{2D} = \frac{E\left(\frac{\theta_{tot}}{l}\right)\left(\frac{l}{s}\right)}{\cos \Phi - \frac{t}{s} - H\left(\frac{\theta_{tot}}{l}\right)\left(\frac{l}{s}\right)} \tag{4}$$

The momentum thickness per unit of surface length is assumed to be expressed as a function of Reynolds number as

$$\frac{\theta_{tot}}{l} = C\left(\frac{\theta_{tot}}{l}\right)_{ref}\left(\frac{Re}{Re_{ref}}\right)^{-0.2} \tag{5}$$

The factor C is introduced as a convenience for modifying the loss level if desired. It is further assumed that the ratio of three-dimensional loss to two-dimensional loss is equal to the ratio of three-dimensional (blade surface plus end wall) surface area to two-dimensional (blade wall) surface area (see ref. 2); that is,

$$\bar{e}_{3D} = \bar{e}_{2D}\left(\frac{A_{3D}}{A_{2D}}\right) \tag{6}$$

Combining equations (4), (5), and (6) yields

$$\bar{e}_{3D} = \frac{EC\left(\frac{\theta_{tot}}{l}\right)_{ref}\left(\frac{Re}{Re_{ref}}\right)^{-0.2}\left(\frac{l}{s}\right)\left(\frac{A_{3D}}{A_{2D}}\right)}{\cos\Phi - \frac{t}{s} - HC\left(\frac{\theta_{tot}}{l}\right)_{ref}\left(\frac{Re}{Re_{ref}}\right)^{-0.2}\left(\frac{l}{s}\right)} \tag{7}$$

Equation (7) is used to determine the three-dimensional viscous losses in the stator and the rotor. Evaluation of the various terms in equation (7) is as follows.

Flow angle. - As mentioned previously,

$$\Phi_s = \alpha_1 \tag{8}$$

and

$$\Phi_r = \beta_2 \tag{9}$$

Energy and form factors. - The energy and form factors are obtained from equations (7-14) and (7-13), respectively, of reference 2 using a velocity profile exponent of 0.2. Sufficient numbers of terms are used for each series such that additional terms affect the series sum by less than 0.1 percent. The resultant equations are

$$E = \frac{2\left(\frac{1}{1.92} + \frac{Q}{3.2} + \frac{Q^2}{4.8} + \frac{Q^3}{6.72}\right)}{\frac{1}{1.68} + \frac{Q}{2.88} + \frac{Q^2}{4.4} + \frac{Q^3}{6.24}} \tag{10}$$

and

$$H = \frac{\frac{1}{1.2} + \frac{3Q}{1.6} + \frac{5Q^2}{2.0} + \frac{7Q^3}{2.4} + \frac{9Q^4}{2.8}}{\frac{1}{1.68} + \frac{Q}{2.88} + \frac{Q^2}{4.4} + \frac{Q^3}{6.24}} \tag{11}$$

For the stator,

$$Q_s = \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_1^2 \qquad (12)$$

and for the rotor,

$$Q_r = \frac{\gamma - 1}{\gamma + 1}\left(\frac{W}{W_{cr}}\right)_2^2 \qquad (13)$$

Reference loss coefficients. - The reference values of momentum thickness per unit length and Reynolds number were combined into reference loss coefficients, which were evaluated by matching the program results with the experimental values of stator loss coefficient ($\bar{e}_{3D,s} = 0.064$, which was obtained from unpublished data) and total efficiency ($\eta' = 0.88$) for the turbine of reference 3. This is considered to be representative state-of-the-art performance for a carefully designed turbine. With the coefficients C_s and C_r equal to unity, the reference loss coefficients are

$$\left(\frac{\theta_{tot}}{l\ Re^{-0.2}}\right)_{ref,s} = 0.03734 \qquad (14)$$

and

$$\left(\frac{\theta_{tot}}{l\ Re^{-0.2}}\right)_{ref,r} = 0.11595 \qquad (15)$$

Reynolds number. - Blade-row chord is used as the characteristic length. For the stator,

$$Re_s = \frac{\rho_1 V_1 c_s}{\mu} \qquad (16)$$

and for the rotor

$$Re_r = \frac{\rho_2 W_2 c_r}{\mu} \qquad (17)$$

Expressing velocity in terms of its throughflow component, multiplying both numerator and denominator by annulus area, and applying the continuity equation yield

$$Re_s = \frac{w}{\pi\left(\frac{h}{c}\right)_s \mu D_1 \cos \alpha_1} \tag{18}$$

and

$$Re_r = \frac{w}{\pi\left(\frac{h}{c}\right)_r \mu D_{2,m} \cos \beta_2} \tag{19}$$

Length to spacing ratio. - For the stator, the surface length to exit spacing ratio is expressed as

$$\left(\frac{l}{s}\right)_s = \left(\frac{l}{c}\right)_s \left(\frac{c}{s}\right)_s \tag{20}$$

A stator solidity $(c/s)_s$ is input to the program and internally adjusted to correspond to an integral number of vanes. The stator surface length to chord ratio $(l/c)_s$ is unity for an uncambered vane and obtained from equation (B10) of appendix B for a cambered vane.

For the rotor, the surface length is obtained from equation (C2) of appendix C. The mean spacing at the rotor exit is

$$s_{2,m} = \frac{2\pi r_{2,m}}{n_r} = \frac{\pi(r_{2,h} + r_{2,t})}{n_r} \tag{21}$$

Trailing-edge thickness to spacing ratio. - Trailing-edge thickness is specified as a fraction of blade height using existing turbines as a guide. For the stator, the trailing-edge thickness is 5 percent of the stator blade height; that is,

$$\left(\frac{t}{s}\right)_s = 0.05\left(\frac{h}{s}\right)_s = 0.05\left(\frac{h}{c}\right)_s \left(\frac{c}{s}\right)_s \tag{22}$$

The solidity c/s is an input value, while the aspect ratio h/c is either input or computed from the stator geometry model depending on the case.

For the rotor, the trailing-edge thickness is 4 percent of the rotor blade exit height; that is,

$$t = 0.04(r_{2,t} - r_{2,h}) \tag{23}$$

Dividing equation (23) by spacing as expressed by equation (21) yields

$$\left(\frac{t}{s}\right)_r = \frac{0.04\, n_r\left(1 - \frac{r_{2,h}}{r_{2,t}}\right)}{\pi\left(1 + \frac{r_{2,h}}{r_{2,t}}\right)} \tag{24}$$

Three-dimensional- to two-dimensional-area ratio. - Stator vane surface area for one passage is

$$A_{b,s} = 2l_s h_s = 2\left(\frac{l}{c}\right)_s c_s h_s \tag{25}$$

Stator end-wall surface area for one passage is

$$A_{w,s} = \frac{2}{n_s}\pi\left(r_0^2 - r_1^2\right) \tag{26}$$

Expressing number of stator vanes as

$$n_s = \frac{2\pi r_1}{s_s} \tag{27}$$

and substituting equation (27) into equation (26) yield

$$A_{w,s} = \frac{s_s}{r_1}\left(r_0^2 - r_1^2\right) \tag{28}$$

Since

$$\frac{A_{3D}}{A_{2D}} = \frac{A_b + A_w}{A_b} \tag{29}$$

substitution of equations (25) and (28) into equation (29) yields

$$\left(\frac{A_{3D}}{A_{2D}}\right)_s = 1 + \frac{r_0^2 - r_1^2}{2r_1\left(\frac{l}{c}\right)_s\left(\frac{c}{s}\right)_s h_s} \tag{30}$$

For the rotor,

$$\left(\frac{A_{3D}}{A_{2D}}\right)_r = 1 + \frac{A_{w,t,r} + A_{w,h,r}}{A_{b,r}} \tag{31}$$

where $A_{b,r}$, $A_{w,t,r}$, and $A_{w,h,r}$ are obtained from equations (C5), (C13), and (C17), respectively, of appendix C.

Disk-Friction Loss

The disk-friction loss is calculated using equations (8-7), (8-10), and (8-15) of reference 4 with the coefficient $C_{IV} = 0.085$. For one side of a disk, the friction loss is, therefrom, expressed as

$$L_{df} = \frac{0.02125\,\rho_{1a} U_{1a}^3 r_{1a}^2}{gJ\left(\frac{\rho U r}{\mu}\right)_{1a}^{0.2} w} \tag{32}$$

Tip Clearance Loss

In accordance with the tip clearance discussion in reference 5, it is assumed that the fractional loss due to clearance is equal to the ratio of clearance to passage height at the rotor exit. Clearance is input to the program as a fraction of diameter. The loss due to clearance is, therefore, computed from

$$\frac{L_c}{\Delta h'_{VD,av}} = \frac{h_c}{r_{2,t} - r_{2,h}} = \left(\frac{h_c}{D_t}\right)_2 \frac{2r_{2,t}}{r_{2,t} - r_{2,h}} = \left(\frac{h_c}{D_t}\right)_2 \left(\frac{2}{1 - \frac{r_{2,h}}{r_{2,t}}}\right) \tag{33}$$

where $\left(h_c/D_t\right)_2$ is the input parameter.

Exit Velocity Loss

The kinetic-energy loss associated with the turbine exit velocity is

$$L_{ex} = \frac{V_2^2}{2gJ} \tag{34}$$

ANALYSIS METHOD

The flow analysis is one dimensional at the stator inlet, stator exit, and rotor inlet, each of these calculation stations being at a constant radius. At the rotor exit, where there is a variation in flow-field radius, an axisymmetric two-dimensional analysis is made using constant height sectors. Simple radial equilibrium is used to establish the static pressure gradient at the rotor exit.

The fluid energy corresponding to the shaft power requirement is

$$\Delta h'_{shft} = \frac{C_P P}{Jw} \tag{35}$$

Disk friction and clearance losses, which are expressed by equations (32) and (33), respectively, are added to the shaft work to yield the average total fluid energy extraction; that is,

$$\Delta h'_{VD,av} = \Delta h'_{shft} + L_{df} + L_c \tag{36a}$$

which can also be expressed as

$$\Delta h'_{VD,av} = \frac{\Delta h'_{shft} + L_{df}}{1 - \dfrac{L_c}{\Delta h'_{VD,av}}} \tag{36b}$$

The numerical values needed for evaluating L_{df} are not immediately known, and an iteration is used starting with $L_{df} = 0$.

The turbine energy transfer equation is

$$\Delta h'_{VD,av} = \frac{1}{gJ}\left(U_{1a}V_{u,1a} - \frac{1}{w}\sum_{i=1}^{k} w_i U_{2,i} V_{u,2,i}\right) \tag{37}$$

Substituting

$$U_{2,i} = \frac{r_{2,i}}{r_{1a}} U_{1a} \tag{38}$$

into equation (37) and manipulating terms yield

$$\Delta h'_{VD,av} = \frac{U_{1a}V_{u,1a}}{gJ}\left(1 - \frac{1}{w}\sum_{i=1}^{k} w_i \frac{r_{2,i}V_{u,2,i}}{r_{1a}V_{u,1a}}\right) \tag{39}$$

The average change in tangential momentum is

$$\Delta\left(rV_u\right)_{av} = r_{1a}V_{u,1a} - \frac{1}{w}\sum_{i=1}^{k} w_i r_{2,i} V_{u,2,i} \tag{40}$$

As discussed in reference 5, the rotor inlet gas tangential velocity and blade speed can be related as

$$\frac{V_{u,1a}}{U_{1a}} = 1 - \frac{2}{n_r} \tag{41}$$

Substituting equations (40) and (41) into equation (39) and solving for blade speed yield

$$U_{1a} = \left[\frac{gJ\Delta h'_{VD,av}}{1 - \frac{2}{n_r}} \frac{\left(rV_u\right)_{1a}}{\Delta\left(rV_u\right)_{av}} \right]^{1/2} \tag{42}$$

The rV_u ratio is specified as input, and n_r is determined from equation (C19) of appendix C. With U_{1a} known, equation (41) is used to determine $V_{u,1a}$. Rotor inlet diameter is

$$D_{1a} = C_N \frac{U_{1a}}{\pi N} \tag{43}$$

Stator Exit

The conditions at the stator exit, station 1, are computed from the following equations. The radius ratio r_1/r_{1a} and the stator exit angle α_1 are known input values. Stator loss coefficient $\overline{e}_s$ can be either input or computed from the previously presented loss model. If the loss model is used, the stator-exit calculation is iterative between equations (50) to (54).

$$D_1 = \left(\frac{r_1}{r_{1a}}\right) D_{1a} \tag{44}$$

$$V_{u,1} = \left(\frac{r_{1a}}{r_1}\right) V_{u,1a} \tag{45}$$

$$V_1 = \frac{V_{u,1}}{\sin \alpha_1} \tag{46}$$

$$V_{rad,1} = \frac{V_{u,1}}{\tan \alpha_1} \tag{47}$$

$$T_1' = T_0' \tag{48}$$

$$T_1 = T_1' - \frac{V_1^2}{2gJc_p} \tag{49}$$

$$V_{1,id}^2 = \frac{V_1^2}{1 - \bar{e}_s} \tag{50}$$

$$p_1 = \left(1 - \frac{V_{1,id}^2}{2gJc_pT_0'}\right)^{\gamma/(\gamma-1)} p_0' \tag{51}$$

$$p_1' = p_1\left(\frac{T_1'}{T_1}\right)^{\gamma/(\gamma-1)} \tag{52}$$

$$\rho_1 = \frac{p_1}{RT_1} \tag{53}$$

$$h_s = \frac{w}{\rho_1 V_{rad,1}\pi D_1} \tag{54}$$

$$V_{cr,1} = \sqrt{\frac{2\gamma}{\gamma + 1} gRT_1'} \tag{55}$$

Stator Inlet

Stator-inlet diameter D_0 and flow angle α_0 are either input or calculated from the stator geometry model, appendix B. Stator height h_s is assumed constant, and the following four equations are solved simultaneously for the stator-inlet, station 0, conditions V_0, T_0, p_0, and ρ_0:

$$V_0 = \frac{w}{\pi D_0 h_s \rho_0 \cos \alpha_0} \tag{56}$$

$$T_0 = T'_0 - \frac{V_0^2}{2gJc_p} \tag{57}$$

$$p_0 = p'_0 \left(\frac{T_0}{T'_0}\right)^{\gamma/(\gamma-1)} \tag{58}$$

$$\rho_0 = \frac{p_0}{RT_0} \tag{59}$$

Rotor Inlet

Assuming no change in total temperature or total pressure between stator exit and rotor inlet (i.e., $T'_{1a} = T'_1$ and $p'_{1a} = p'_1$), we can solve the following set of six equations for the variables indicated on the left sides:

$$V_{1a} = \frac{V_{u,1a}}{\sin \alpha_{1a}} \tag{60}$$

$$T_{1a} = T'_{1a} - \frac{V_{1a}^2}{2gJc_p} \tag{61}$$

$$p_{1a} = p'_{1a} \left(\frac{T_{1a}}{T'_{1a}}\right)^{\gamma/(\gamma-1)} \tag{62}$$

$$\rho_{1a} = \frac{p_{1a}}{RT_{1a}} \tag{63}$$

$$V_{rad,1a} = V_{rad,1} \frac{\rho_1 D_1}{\rho_{1a} D_{1a}} \tag{64}$$

$$\alpha_{1a} = \tan^{-1} \frac{V_{u,1a}}{V_{rad,1a}} \tag{65}$$

At this point there is sufficient information for evaluation of disk friction loss from equation (32), and the calculation cycles back to equation (36b) until a convergence is obtained.

The rotor-inlet conditions relative to the rotor are

$$W_{u,1a} = V_{u,1a} - U_{1a} \tag{66}$$

$$\beta_{1a} = \tan^{-1} \frac{W_{u,1a}}{V_{rad,1a}} \tag{67}$$

$$W_{1a} = \frac{V_{rad,1a}}{\cos \beta_{1a}} \tag{68}$$

$$T''_{1a} = T_{1a} + \frac{W_{1a}^2}{2gJc_p} \tag{69}$$

$$p''_{1a} = p_{1a}\left(\frac{T''_{1a}}{T_{1a}}\right)^{\gamma/(\gamma-1)} \tag{70}$$

$$W_{cr,1a} = V_{cr,1}\sqrt{\frac{T''_{1a}}{T'_{1a}}} \tag{71}$$

Rotor Exit

At the rotor exit, station 2, the annulus is divided into equal-height sectors, with the sectors being related by simple radial equilibrium. The solution at the rotor exit has to satisfy two input requirements, average specific work and flow. This is done by means of two iteration loops. The outer-loop iteration is started by setting a value for the mean-sector tangential velocity $V_{u,2,m}$. The inner loop is then cycled by varying the mean-sector axial velocity $V_{x,2,m}$ until continuity is satisfied. The average specific work is then checked. If necessary, $V_{u,2,m}$ is adjusted and the iteration process repeated until both loops are simultaneously satisfied.

The rotor-exit hub and tip diameters are known from the calculated rotor-inlet diameter and input diameter ratios. For the k sectors, therefore, the sector mean diameters $D_{2,i}$ are known. Without knowledge of the rotor-exit solution, we can then calculate for all i

$$U_{2,i} = \frac{\pi D_{2,i} N}{C_N} \tag{72}$$

$$T''_{2,i} = T''_{1a} + \frac{U_{2,i}^2 - U_{1a}^2}{2gJc_p} \tag{73}$$

$$W_{cr,2,i} = W_{cr,1a} \sqrt{\frac{T''_{2,i}}{T''_{1a}}} \tag{74}$$

$$p''_{2,id,i} = p''_{1a} \left(\frac{T''_{2,i}}{T''_{1a}}\right)^{\gamma/(\gamma-1)} \tag{75}$$

The rotor loss coefficient $\bar{e}_r$ can be either input or computed from the previously presented loss model using mean-sector conditions. With the rotor-exit mean-sector tangential and axial absolute-velocity component values set by the iteration loops previously discussed, the rotor-exit calculation proceeds as follows, starting with the mean sector:

$$W_{u,2,m} = V_{u,2,m} - U_{2,m} \tag{76}$$

$$\beta_{2,m} = \tan^{-1} \frac{W_{u,2,m}}{V_{x,2,m}} \tag{77}$$

$$W_{2,m} = \frac{V_{x,2,m}}{\cos \beta_{2,m}} \tag{78}$$

$$T_{2,m} = T''_{2,m} - \frac{W_{2,m}^2}{2gJc_p} \tag{79}$$

$$W_{2,id,m}^2 = \frac{W_{2,m}^2}{1 - \bar{e}_r} \tag{80}$$

$$p_{2,m} = \left(1 - \frac{W_{2,id,m}^2}{2gJc_p T''_{2,m}}\right)^{\gamma/(\gamma-1)} p''_{2,id,m} \tag{81}$$

$$\rho_{2,m} = \frac{p_{2,m}}{RT_{2,m}} \tag{82}$$

For the other sectors, the rotor loss coefficient is assumed constant and equal to the mean-sector value. The angular momentum distribution $\left(rV_u\right)_{2,i}/\left(rV_u\right)_{2,m}$ is specified as input. The calculation then proceeds from the mean sector into the hub and from the mean sector out to the tip; thus,

$$V_{u,2,j} = V_{u,2,m} \frac{\left(rV_u\right)_{2,j}}{\left(rV_u\right)_{2,m}} \frac{D_{2,m}}{D_{2,j}} \tag{83}$$

In the following, the subscript j refers to all values of i except i = m:

$$W_{u,2,j} = V_{u,2,j} - U_{2,j} \tag{84}$$

$$p_{2,j} = p_{2,j\pm1} + \frac{\rho_{2,j\pm1}}{2g}\left(\frac{V_{u,2,j}^2}{D_{2,j}} + \frac{V_{u,2,j\pm1}^2}{D_{2,j\pm1}}\right)(D_{2,j} - D_{2,j\pm1}) \tag{85}$$

$$W_{2,id,j}^2 = 2gJc_p T''_{2,j}\left[1 - \left(\frac{p_{2,j}}{p''_{2,id,j}}\right)^{(\gamma-1)/\gamma}\right] \tag{86}$$

$$W_{2,j} = \sqrt{W_{2,id,j}^2 (1 - \bar{e}_r)} \tag{87}$$

$$\beta_{2,j} = \sin^{-1} \frac{W_{u,2,j}}{W_{2,j}} \tag{88}$$

$$V_{x,2,j} = W_{2,j} \cos \beta_{2,j} \tag{89}$$

$$T_{2,j} = T''_{2,j} - \frac{W_{2,j}^2}{2gJc_p} \tag{90}$$

$$\rho_{2,j} = \frac{p_{2,j}}{RT_{2,j}} \tag{91}$$

After equations (83) to (91) are evaluated for all values of j, the mass flow rate at the rotor exit is calculated as

$$w_{calc} = \pi \frac{D_{2,t} - D_{2,h}}{2k} \sum_{i=1}^{k} \rho_{2,i} V_{x,2,i} D_{2,i} \tag{92}$$

If $w_{calc} \neq w$, the value of $V_{x,2,m}$ is adjusted, and the calculation is cycled back to equation (77). When $w_{calc} = w$, the energy transfer determined from equation (37) is compared with the work requirement from equation (36b) and, if necessary, the value of $V_{u,2,m}$ is adjusted and the calculation cycled back to equation (76). When the flow and work requirements are simultaneously satisfied, the rotor-exit solution is established. The following parameters are then calculated:

$$\Delta h'_{VD,i} = \frac{1}{gJ} (U_{1a} V_{u,1a} - U_{2,i} V_{u,2,i}) \tag{93}$$

$$T'_{2,i} = T'_0 - \frac{\Delta h'_{VD,i}}{c_p} \tag{94}$$

$$V_{cr,2,i} = V_{cr,1a} \sqrt{\frac{T'_{2,i}}{T'_{1a}}} \tag{95}$$

$$\alpha_{2,i} = \tan^{-1} \frac{V_{u,2,i}}{V_{x,2,i}} \tag{96}$$

$$p'_{2,i} = p_{2,i}\left(\frac{T'_{2,i}}{T_{2,i}}\right)^{\gamma/(\gamma-1)} \tag{97}$$

$$V_{2,i} = \frac{V_{x,2,i}}{\cos \alpha_{2,i}} \tag{99}$$

Performance

There are five types of losses considered in this analysis. The disk-friction and clearance losses are obtained from equations (32) and (33), respectively. The stator, rotor, and leaving losses are

$$L_s = \frac{V^2_{1,id} - V^2_1}{2gJ} \tag{100}$$

$$L_{r,av} = \sum_{i=1}^{k} \frac{w_i}{w} \frac{W^2_{2,id,i} - W^2_{2,i}}{2gJ} \tag{101}$$

$$L_{ex,av} = \sum_{i=1}^{k} \frac{w_i}{w} \frac{V^2_{2,i}}{2gJ} \tag{102}$$

The sector ideal specific works are

$$\Delta h'_{id,i} = c_p T'_0 \left[1 - \left(\frac{p'_{2,i}}{p'_0}\right)^{(\gamma-1)/\gamma}\right] \tag{103}$$

$$\Delta h_{id,i} = c_p T'_0 \left[1 - \left(\frac{p_{2,i}}{p'_0}\right)^{(\gamma-1)/\gamma}\right] \tag{104}$$

The sector efficiencies based on velocity diagram work are

$$\eta'_{VD,i} = \frac{\Delta h'_{VD,i}}{\Delta h'_{id,i}} \tag{105}$$

$$\eta_{VD,i} = \frac{\Delta h'_{VD,i}}{\Delta h_{id,i}} \tag{106}$$

The overall velocity diagram efficiencies are

$$\eta'_{VD,av} = \frac{\Delta h'_{VD,av}}{\sum_{i=1}^{k} \frac{w_i}{w} \Delta h'_{id,i}} \tag{107}$$

$$\eta_{VD,av} = \frac{\Delta h'_{VD,av}}{\sum_{i=1}^{k} \frac{w_i}{w} \Delta h_{id,i}} \tag{108}$$

while the net overall efficiencies are

$$\eta'_{shft} = \eta'_{VD,av} \frac{\Delta h'_{shft}}{\Delta h'_{VD,av}} \tag{109}$$

$$\eta_{shft} = \eta_{VD,av} \frac{\Delta h'_{shft}}{\Delta h'_{VD,av}} \tag{110}$$

A design parameter that is computed by the program is specific speed

$$N_{sp} = \frac{N\left(w/\rho_2\right)^{1/2}}{\left(J\ \Delta h'_{id,av}\right)^{3/4}} \tag{111}$$

DESCRIPTION OF INPUT AND OUTPUT

This section presents a detailed description of the program input, normal output, and error messages. The input and corresponding printed output for an example problem are included for illustrative purposes.

Input

The program input consists of a title card and a data set in NAMELIST form for each case. The title, which is printed as a heading on the output listing, can be located anywhere in columns 1 to 78 on the title card. A title card, even if it is left blank, must be the first card for each case.

The data are input in data records having the NAMELIST name INPUT. The variables that comprise INPUT along with descriptions, units, and special remarks are presented in the list to follow. Either SI units or U. S. customary units may be used with this program. Values for some of the variables in the input list are internally preset by the program before reading the input. Thus, if a preset value (see input list for values) is appropriate, that particular variable does not have to be specified in the input.

IU	units indicator (preset value = 2): 1 - SI units 2 - U.S. customary units
NSTAR	stator geometry indicator: 0 - cambered vane with specified diameter ratio 1 - cambered vane with specified aspect ratio 2 - uncambered vane

ALPHA0	stator-inlet flow angle from radial direction, deg (input only for NSTAR = 0 or 1)
ALPHA1	stator-exit flow angle from radial direction, deg
CDT2	ratio of clearance gap to blade tip diameter at rotor exit (preset value = 0.)
CR	rotor loss-coefficient multiplier (preset value = 1.)
CS	stator loss-coefficient multiplier (preset value = 1.)
EBARR	rotor loss coefficient (input only if it is desired that internal loss model not be used for computing $\bar{e}_r$)
EBARS	stator loss coefficient (input only if it is desired that internal loss model not be used for computing $\bar{e}_s$)
GAM	specific heat ratio
K	number of radial sectors at rotor exit
MU	gas viscosity, (N)(sec)/m^2; lb/(sec)(ft)
N	rotative speed, rad/sec; rev/min
POW	shaft power, kW; hp
PTIN	inlet total pressure, N/cm^2; lb/in.^{2}
R	gas constant, J/(kg)(K); (ft)(lbf)/(lbm)($^{\circ}$R)
RH2RT2	rotor-exit-hub - to tip-radius ratio
RT2R1A	rotor-exit-tip - to rotor-inlet-radius ratio
RV1AAV	ratio of rotor-inlet angular momentum to average change in angular momentum (preset value = 1.)
RV2I2M(I), I = 1, K	ratio of i^{th} sector to mean-sector rotor-exit angular momentum (preset value = 1. for all I)
R0R1A	stator-inlet- to rotor-inlet radius ratio (input only for NSTAR = 0)
R1R1A	stator-exit- to rotor-inlet radius ratio
SIGSIN	stator solidity based on exit spacing (preset value = 1.35)
STAR	stator aspect ratio (input only for NSTAR = 1 or 2)
TTIN	inlet total temperature, K; $^{\circ}$R
W	mass flow rate, kg/sec; lb/sec

The program inputs for two sample cases are shown in table I. Both cases are for the same design requirements. The first case is with SI units and the second case is with U. S. customary units. The first card for each case is the previously described title card. After the title card is the NAMELIST data set containing all the variable values. For the second case, only those variables changing in value from the first case need be input. The output corresponding to this sample input is presented and described in the following section.

Output

The program output consists of title headings, the input variable values, and the computed results. This section presents normal output. Error message output is described in the next section.

Tables II and III present the output that corresponds to the sample input shown in table I. Table II is for the case with SI units. The top line of output is a program identification title that is automatically printed with the first case of each data package. The second line is the title card message. The next three lines indicate the units, SI in this case, used for the different variables.

The heading *INPUT* is followed by eight lines showing the input values used for this case. Identification of all the items on the output listing is self-explanatory. The zero value shown for stator aspect ratio indicates that this was not an input for this case. Values of 1.0000 are shown for the stator and rotor loss coefficients to indicate that the actual values are computed using the internal loss model.

The heading *OUTPUT* is followed by the computed results. Absolute temperatures, pressures, flow angles, velocities, and velocity ratios, along with diameter, are shown for each calculation station. At the rotor exit, these values are shown for the mean diameter of each sector as well as for the hub and the tip. Additional output for the rotor-exit sectors include flow rate, specific work, and total and static efficiencies. Also shown are the computed loss coefficients for the stator and rotor, the stator height, number of stator vanes, and number of rotor blades.

Under the heading *OVERALL PERFORMANCE* are the turbine total-to-total and total-to-static pressure ratios, diagram specific work, and both diagram and net total and static efficiencies. Also shown are the individual loss components as fractions of the turbine ideal work and the specific speed.

Table III is similar to table II except that U. S. customary units are used, as indicated by the title message as well as by the specified units themselves. Note that the values for all the dimensionless variables are the same in table III as in table II.

Error Messages

The program contains seven output messages indicating the nonexistence of a solution satisfying the specified input requirements. These messages are presented in this section, and their causes are discussed. In general, when one of these messages appear, the program input should be checked for errors.

(1) NO SOLUTION FOUND AFTER 100 ITERATIONS FOR CONTINUITY AT ROTOR EXIT - This message is caused by the program making 100 iterations through subroutine CONTIN without a solution being found. There is no obvious reason for this situation except possibly for input error in the specification of rotor-exit angular momentum.

(2) ROTOR EXIT CHOKES AT MAXIMUM MASS FLOW RATE = XXXX. XXXX - This message is caused by the choking mass flow rate for the rotor exit being less than the design flow rate specified as program input. If the input design requirements are correct, then possible corrective action includes reducing exit angular momentum, increasing rotor-exit-tip- to rotor-inlet-radius ratio, and decreasing rotor-exit-hub- to tip-radius ratio.

(3) REQUIRED SPECIFIC WORK GREATER THAN ENERGY AVAILABLE IN GAS - See item (4).

(4) SPECIFIC WORK REQUIRED IN SECTOR XX GREATER THAN ENERGY AVAILABLE IN GAS - These last two messages are caused by the turbine-exit total temperature, average value or any sector value, being less than zero. Possible corrective action includes increasing turbine inlet temperature, decreasing turbine power, or increasing mass flow rate.

(5) REQUIRED STATOR IDEAL KINETIC ENERGY GREATER THAN ENERGY AVAILABLE IN GAS - See item (6).

(6) ROTOR IDEAL RELATIVE KINETIC ENERGY REQUIRED IN SECTOR XX GREATER THAN ENERGY AVAILABLE IN GAS - These last two messages are caused by the computed ideal energy required by the stator or the rotor being greater than that available from an infinite expansion of the gas. The probable reason for this condition is an error in the computed or input loss coefficient.

(7) THE PROGRAM CAN NOT FIND A SOLUTION SIMULTANEOUSLY SATISFYING CONTINUITY, RADIAL EQ., AND THE LOSS MODEL AT THE ROTOR EXIT - This message is caused by rotor-exit relative velocity, as determined by radial equilibrium and radial distribution of loss, being less than its tangential component, as determined primarily by blade speed. Corrective action includes decreasing rotor-exit tip radius and increasing rotor-exit-hub to tip radius ratio.

PROGRAM DESCRIPTION

This computer program consists of main program RIFTUD (Radial Inflow Turbine Design), blade loss coefficient subprograms EFFIC, SIMPS1, SHUB, and SHUB2, and rotor-exit continuity subprograms CONTIN and PABC. The entire program is written in FORTRAN IV language and has been run on both an IBM 7094 and a UNIVAC 1110. Running time on the UNIVAC 1110 is about 1 second per design case for a five-sector design. In this section, the functions of the main and subprograms are described, the program variables are defined, and the program listing is presented.

Main Program RIFTUD

Main program RIFTUD performs all input and output operations as well as all of the flow analysis as presented in the section ANALYSIS METHOD.

Program variables. - The variables used in RIFTUD are defined in terms of the following symbols, which are defined in appendix A:

ALPHAA	α_{1a} (degrees)
ALPHA0	α_0 (degrees)
ALPHA1	α_1 (degrees)
ALPHA2(I)	$\alpha_{2,i}$ (degrees)
ALPH0	α_0 (radians)
ALPH1	α_1 (radians)
ALPH1A	α_{1a} (radians)
ALPH2(I)	$\alpha_{2,i}$ (radians)
ALP0	α_0 (degrees)
ALP1A	previous value of α_{1a}
ALSTG	φ
ALUNC	Θ_{cam}
BETA1A	β_{1a} (degrees)
BETA2(I)	$\beta_{2,i}$ (degrees)
BET1A	β_{1a} (radians)
BET2(I)	$\beta_{2,i}$ (radians)
BET2M	$\beta_{2,m}$ (radians)
CDT2	$\left(h_c/D_t\right)_2$
CHRD	c
CN	C_N/π
CP	c_p
CPOW	C_P
CR	C_r
CS	C_s
DELVX	increment in $\left(V_x/V_{cr}\right)_{2,m}$

DHIDS(I)	$\Delta h_{id, i}$
DHIDSA	$\Delta h_{id, av}$
DHIDT(I)	$\Delta h'_{id, i}$
DHIDTA	$\Delta h'_{id, av}$
DHSHFT	$\Delta h'_{shft}$
DHVD(I)	$\Delta h'_{VD, i}$
DHVDAV	$\Delta h'_{VD, av}$
DOR	$180/\pi$
DVUAVC	$\left(DV_u\right)_{2, av, calc}$
DVUTOT	$w\left(DV_u\right)_{2, av, calc}$
DVU2AV	$\left(DV_u\right)_{2, av, required}$
D0	D_0
D1	D_1
D1A	D_{1a}
D2(I)	$D_{2, i}$
D2M	$D_{2, m}$
EBARR	$\bar{e}_r$ input value
EBARS	$\bar{e}_s$ input value
EL	$h_{r, 2}/k$
EN	n_r
ENS	n_s
ENST	truncated integer value of n_s before roundoff
ENT	trancated integer value of n_r before roundoff
ER	$\bar{e}_r$
ES	$\bar{e}_s$
ES1	previous value of $\bar{e}_s$
ETAS	η_{shft}
ETASV	$\eta_{VD, av}$
ETASVD(I)	$\eta_{VD, i}$
ETAT	η'_{shft}
ETATV	$\eta'_{VD, av}$
ETATVD(I)	$\eta'_{VD, i}$
EX	$\gamma/(\gamma - 1)$
F2(I)	$w_{2, i}$
G	g
GAM	γ
HR	h_r
HS	h_s
I	dummy index, usually referring to sector number
ID	sector number increment
II	sector number index for previous sector
IND	solution indicator for rotor-exit continuity calculation
IST	iteration counter for stator calculation

ISTT	iteration counter for stator calculation
ITER	iteration counter for rotor-exit calculation
IU	units indicator - see section Input
J	J
K	number of sectors
KK	K + 2
KP1	K + 1
K1	indicator for print control
LC	L_c
LCDH	$L_c/\Delta h'_{VD,av}$
LCDHIS	$L_c/\Delta h_{id,av}$
LLDHIS	$L_{ex,av}/\Delta h_{id,av}$
LRDHIS	$L_{r,av}/\Delta h_{id,av}$
LSDHIS	$L_s/\Delta h_{id,av}$
LW	L_{df}
LWDHIS	$L_{df}/\Delta h_{id,av}$
LWX	previous value of L_{df}
M	mean sector index
MU	μ
N	N
NSTAR	stator geometry indicator - see section Input
PI	π
POW	P
PS0	p_0
PS1	p_1
PS1A	p_{1a}
PS2(I)	$p_{2,i}$
PTIN	p'_0
PTR1A	p''_{1a}
PTR2(I)	$p''_{2,i}$
PTR2ID(I)	$p''_{2,id,i}$
PT0	p''_0
PT1	p'_1
PT1A	p'_{1a}
PT2(I)	$p'_{2,i}$
Q	$(w/\rho)_{2,av}$
R	R
RHOT0	ρ'_0
RHOX	previous value of ρ_0
RHO0	ρ_0
RHO1	ρ_1
RHO1A	ρ_{1a}
RHO2(I)	$\rho_{2,i}$
RH2RT2	$(r_h/r_t)_2$
RLOSS	$L_{r,av}$
RSTG	r_{cm}
RT2R1A	$r_{2,t}/r_{1a}$

RV1AAV	$\left(rV_u\right)_{1a}/\Delta\left(rV_u\right)_{av}$
RV2IM(I)	$\left(rV_u\right)_{2,i}/\left(rV_u\right)_{2,m}$
RV2I2M(I)	$\left(rV_u\right)_{2,i}/\left(rV_u\right)_{2,m}$
R0R1A	r_0/r_{1a}
R1R1A	r_1/r_{1a}
SIGRV	$\sum_{i=1}^{k}\left[\left(rV_u\right)_{2,i}/\left(rV_u\right)_{2,m}\right]/k$
SIGS	$(c/s)_s$
SIGSIN	input value of $(c/s)_s$
SS	s
SSPD	N_{sp}
STAR	$(h/c)_s$
TGJCP	$2gJc_p$
TITLE(I)	input/output array for title card message
TSPR	$p'_0/p_{2,i}$
TS0	T_0
TS1	T_1
TS1A	T_{1a}
TS2(I)	$T_{2,i}$
TTIN	T'_0
TTPR	$p'_0/p'_{2,i}$
TTR1A	T''_{1a}
TTR2(I)	$T''_{2,i}$
TT0	T'_0
TT1	T'_1
TT1A	T'_{1a}
TT2(I)	$T'_{2,i}$
TT2AV	$T'_{2,av}$
U1A	U_{1a}
U1ASQ	U_{1a}^2
U2(I)	$U_{2,i}$
VCR1	$V_{cr,1}$
VCR1A	$V_{cr,1a}$
VCR2(I)	$V_{cr,2,i}$
VOVCRA	$\left(V/V_{cr}\right)_{1a}$
VOVCR0	$\left(V/V_{cr}\right)_0$
VOVCR1	$\left(V/V_{cr}\right)_1$
VOVCR2(I)	$\left(V/V_{cr}\right)_{2,i}$
VR0	$V_{rad,0}$
VR1	$V_{rad,1}$
VR1A	$V_{rad,1a}$
VUOU1A	$\left(V_u/U\right)_{1a}$
VU0	$V_{u,0}$
VU1	$V_{u,1}$

VU1A	$V_{u,1a}$
VU2O	previous value of $V_{u,2,m}$
VU2(I)	$V_{u,2,i}$
VXVCR	$\left(V_x/V_{cr}\right)_{2,m}$
VXVCRP	previous value of $\left(V_x/V_{cr}\right)_{2,m}$
VX2(I)	$V_{x,2,i}$
V0	V_0
V1	V_1
V1A	V_{1a}
V1IDSQ	$V^2_{1,id}$
V2(I)	$V_{2,i}$
V2LOSS	$L_{ex,av}$
W	w
WCALC	w_{calc}
WCR1A	$W_{cr,1a}$
WCR2(I)	$W_{cr,2,i}$
WGIV	input value of w
WOWCRA	$\left(W/W_{cr}\right)_{1a}$
WOWCRM	$\left(W/W_{cr}\right)_{2,m}$
WOWCR2(I)	$\left(W/W_{cr}\right)_{2,i}$
WU1A	$W_{u,1a}$
WU2(I)	$W_{u,2,i}$
W1A	W_{1a}
W2(I)	$W_{2,i}$
W2IDSQ(I)	$W^2_{2,id,i}$
W2TOT	w_{calc}
Z1	$(T/T')_{1,id}$
Z2	$(T/T'')_{2,id,m}$
Z3	$(T/T'')_{2,id,j}$

Program listing. - The FORTRAN listing for main program RIFTUD is as follows:

```
 REAL J,N,MU,LCDH,LW,LWX,LC,LCDHIS,LWDHIS,LLDHIS,LSDHIS,LRDHIS
 DIMENSION D2(17),U2(17),TTR2(17),WCR2(17),PTR2ID(17),VU2(17),
1WU2(17),DHVD(17),TT2(17),VCR2(17),BET2(17),W2(17),TS2(17),PS2(17),
2W2IDSQ(17),RHO2(17),F2(17),RV2IM(17),VX2(17),ALPH2(17),V2(17),
3PT2(17),PTR2(17),VOVCR2(17),WOWCR2(17),DHIDT(17),DHIDS(17),
4ETATVD(17),ETASVD(17),ALPHA2(17),BETA2(17)
 DIMENSION RV2I2M(17),TITLE(13)
 COMMON/EFF/GAM,VOVCR1,W,PI,STAR,MU,D1,ALPH1,NSTAR,ALPH0,ALUNC,SIGS
1,D0,HS,CS,WOWCRM,D1A,D2M,HR,BET2M,EN,RH2RT2,CR
 NAMELIST/INPUT/R,GAM,ALPHA0,ALPHA1,N,TTIN,PTIN,W,POW,RV1AAV,R1R1A,
1R0R1A,RT2R1A,RH2RT2,CDT2,MU,RV2I2M,K,IU,EBARS,EBARR,STAR,NSTAR
2,CS,CR,SIGSIN
 EBARS=1.0
 EBARR=1.0
 CS=1.0
 CR=1.0
 SIGSIN= 1.35
 ES=0.0
 RV1AAV=1.0
 CDT2=0.0
 DO 1 I=1,17
```

```
    1 RV2I2M(I)=1.0
      IU=2
      ITER=0
      PI=3.14159
      DOR=57.2958
      ROR1A=0.0
      ALPHA0=999.99
      WRITE(6,100)
  100 FORMAT(1H1,39X,54HRADIAL INFLOW TURBINE VELOCITY DIAGPAM DESIGN AN
     1ALYSIS)
    2 READ (5,101) TITLE
  101 FORMAT(13A6)
      IF(ITER.GT.0) WRITE(6,1011)
 1011 FORMAT(1H1)
      WRITE(6,102) TITLE
  102 FORMAT(1H ,13A6)
      READ (5,INPUT)
      IF (NSTAR.EQ.0) STAR=0.0
      IST=0
      VXVCRP=0.0
      DELVX=0.01
      ALPH0=ALPHA0/DOR
      ALPH1=ALPHA1/DOR
      IF(NSTAR.NE.2) ALSTG=(ALPH0+ALPH1)/2.
      EN=PI/30.*(110.-ALPHA1)*TAN(ALPH1)
      ENT=AINT(EN)
      IF(EN-ENT.LT..5) EN=ENT
      IF(EN-ENT.GE..5) EN=ENT+1.
      VUOU1A=1.-2./EN
      WGIV=W
      KP1=K+1
      WRITE (6,103)
  103 FORMAT(39H0THIS OUTPUT IS IN THE FOLLOWING UNITS./
     1       129H0TEMPERATURE  PRESSURE    GAS CONST   ROT SPEED   MASS
     2FLOW     POWER      VISCOSITY    VELOCITY     SPFC WORK    DIAMETER
     3   ANGLE)
      GO TO (3,4),IU
    3 CPOW=1000.
      J=1.
      G=1.
      CN=2.
      PT0=PTIN*10000.
      WRITE (6,104)
  104 FORMAT(3X6HKELVIN6X7HN/SQ CM4X8HJLS/KG-K5X7HRAD/SEC5X6HKG/SEC8X2HK
     1W6X10HN-SEC/SQ M5X5HM/SEC6X6HJLS/GM8X2HCM8X7HDEGREES)
      GO TO 5
    4 CPOW=550.
      J=777.649
      G=32.174
      CN=60./PI
      PT0=PTIN*144.
      WRITE (6,105)
  105 FORMAT(12H DEG RANKINE2X8HLB/SQ IN4X8HBTU/LB-R5X7HREV/MIN5X6HLB/SE
     1C8X2HHP7X9HLB/FT-SEC4X6HFT/SEC6X6HBTU/LB8X2HIN8X7HDEGREES)
    5 DHSHFT=CPOW/J*POW/W
      EX=GAM/(GAM-1.)
      CP=EX*R/J
      TGJCP=2.*G*J*CP
C
C     WRITE INPUT VALUES
```

```
C
      WRITE (6,106)TTIN,ALPHA0,EBARS,R,PTIN,ALPHA1,EBARR,GAM,N,STAR,
     1RV1AAV,MU,W,K,POW,R0R1A
  106 FORMAT(8H0*INPUT*/13H INLET TEMP =,F10.4,7X,18HSTATOR IN ANGLE  =,
     1F6.2,6X,19HSTATOR KE LOS COEF=,F6.4,5X,14HGAS CONSTANT =F8.4/13H I
     2NLET PRESS=,F10.4,7X,18HSTATOR EX ANGLE  =,F6.2,6X,19HROTOR KE LOS
     3S COEF=,F6.4,5X,14HSPEC HT RATIO=,F6.4/13H ROTAT SPEED=,F10.3,7X,1
     48HSTATOR ASPECT RAT=,F6.4,
     4                        6X,19HROTOR IN/DEL RVU  =,F6.4,5X,14HVISCOSIT
     5Y    =,E10.4/13H MASS FLOW  =,F10.4,7X,11HDIAM RATIOS,19X,19HROTOR
     6 EX RAD SECTS=,I2/13H SHAFT POWER=,F10.3,8X,17HSTAT IN/ROT IN  =,F
     76.4,6X,19HROT EX SECT/MN RVU=)
      K1=K
      IF(K.GT.10) K1=10
      WRITE (6,107) (RV2I2M(I),I=1,K1)
  107 FORMAT(1H+,78X,F4.2,9F5.2)
      WRITE (6,108) R1R1A
  108 FORMAT(31X,17HSTAT EX/ROT IN  =,F6.4)
      IF(K.GT.10) WRITE (6,109) (RV2I2M(I),I=11,K)
  109 FORMAT(1H+,77X,5F5.2)
      WRITE (6,110)RT2R1A,RH2RT2,CDT2
  110 FORMAT(31X,17HROT EX TP/ROT IN=,F6.4/31X,17HROT EX HUB/TIP  =F6.4,
     16X,19HCL HT/ROT EX TIP D=,F6.4)
      LCDH=CDT2*2./(1.-RH2RT2)
      LW=0.0
    6 DHVDAV=(DHSHFT+LW)/(1.-LCDH)
      ISTT=0
      TT2AV=TTIN-DHVDAV/CP
      IF(TT2AV.LT.0.0) GO TO 203
      U1ASQ=G*J/VUOU1A*DHVDAV*RV1AAV
      U1A=SQRT(U1ASQ)
      VU1A=U1A*VUOU1A
      D1A=CN*U1A/N
C
C     STATION 1 - STATOR EXIT
C
      D1=R1R1A*D1A
      VU1=VU1A/R1R1A
      V1=VU1/SIN(ALPH1)
      VR1=VU1/TAN(ALPH1)
      TT1=TTIN
      TS1=TT1-V1*V1/TGJCP
   25 ES1=ES
      ES=EBARS
      IF(EBARS.EQ.1.0.AND.IST.EQ.0) ES=.05
      IF(EBARS.EQ.1.0.AND.IST.GT.0) CALL EFFIC (ES,ER,1)
      IST=IST+1
      ISTT=ISTT+1
    7 V1IDSQ=V1*V1/(1.-ES)
      Z1=1.-V1IDSQ/TGJCP/TT1
      IF(Z1.LE.0.0) GO TO 204
      PS1=PT0*Z1**EX
      PT1=PS1*(TT1/TS1)**EX
      RHO1=PS1/R/TS1
      HS=W/RHO1/VR1/PI/D1
      VCR1=SQRT(2.*GAM/(GAM+1.)*G*R*TT1)
      VOVCR1=V1/VCR1
C
C     STATION 0 - STATOR INLET
C
```

```
      IF(NSTAR.GT.0) GO TO 71
      D0=D1A*ROR1A
      CHRD=SQRT(D0**2+D1**2-SQRT((D0**2+D1**2)**2-(D0**2-D1**2)**2/
     1COS(ALSTG)**2))/2.
      STAR=HS/CHRD
      ALUNC=ALPH1-ALPH0-A COS((D0**2+D1**2-4.*CHRD**2)/2./D0/D1)
   70 CONTINUE
      IF(IST.EQ.1) SIGS=SIGSIN
      SS=CHRD/SIGS
      ENS= PI*D1/SS
      ENST=AINT(ENS)
      IF(ENS-ENST.LT..5) ENS=ENST
      IF(ENS-ENST.GE..5) ENS=ENST+1.
      SS= PI*D1/ENS
      SIGS=CHRD/SS
      IF(EBARS.NE.1.0) GO TO 26
      IF(ISTT.EQ.1) GO TO 25
      IF(ABS(ES-ES1)/ES.GT..0001) GO TO 25
   26 TT0=TTIN
      RHOT0=PT0/R/TT0
      RHO0=RHOT0
    9 V0=W/PI/D0/HS/COS(ALPH0)/RHO0
      TS0=TT0-V0**2/TGJCP
      PS0=PT0*(TS0/TT0)**EX
      RHOX=RHO0
      RHO0=PS0/R/TS0
      IF(ABS(RHOX-RHO0)/RHO0.GT..0001) GO TO 9
      VU0=V0*SIN(ALPH0)
      VR0=V0*COS(ALPH0)
      VOVCR0=V0/VCR1
C
C     STATION 1A - ROTOR INLET
C
      ALPH1A=ALPH1
    8 VR1A=VU1A/TAN(ALPH1A)
      V1A=VU1A/SIN(ALPH1A)
      TT1A=TT1
      TS1A=TT1A-V1A*V1A/TGJCP
      PS1A=PT1*(TS1A/TT1)**EX
      RHO1A=PS1A/R/TS1A
      VR1A=VR1*RHO1/RHO1A*R1R1A
      ALP1A=ALPH1A
      ALPH1A=ATAN (VU1A/VR1A)
      IF(ABS(ALPH1A-ALP1A).GT..01/DOR) GO TO 8
      LWX=LW
      LW=.0061/G/J*RHO1A*U1A**3*D1A**2/W/(RHO1A*U1A*D1A/MU)**.2
      IF(ABS(LW-LWX)/LW.GT..0001) GO TO 6
      PT1A=PT1
      WU1A=VU1A-U1A
      BET1A=ATAN(WU1A/VR1A)
      W1A=VR1A/COS(BET1A)
      TTR1A=TS1A+W1A**2/TGJCP
      PTR1A=PS1A*(TTR1A/TS1A)**EX
      VCR1A=VCR1
      WCR1A=VCR1A*SQRT(TTR1A/TT1A)
      VOVCRA=V1A/VCR1A
      WOWCRA=W1A/WCR1A
C
C     STATION 2 - ROTOR EXIT
C
```

```
      M=(K+3)/2
      KK=K+2
      D2(KK)=D1A*RT2R1A
      D2(1)=D2(KK)*RH2RT2
      HR=(D2(KK)-D2(1))/2.
      EL=HR/FLOAT(K)
      SIGRV=0.0
      DO 12 I=1,KK
      IF(I.EQ.1.OR.I.EQ.KK) GO TO 11
      D2(I)=D2(1)+(2.*FLOAT(I)-3.)*EL
      RV2IM(I)=RV2I2M(I-1)
      SIGRV=SIGRV+RV2IM(I)/FLOAT(K)
   11 U2(I)=D2(I)*N/CN
      TTR2(I)=TTR1A+(U2(I)**2-U1ASQ)/TGJCP
      WCR2(I)=WCR1A*SQRT(TTR2(I)/TTR1A)
   12 PTR2ID(I)=PTR1A*(TTR2(I)/TTR1A)**EX
      RV2IM(1)=RV2IM(2)-(RV2IM(3)-RV2IM(2))/2.
      RV2IM(KK)=RV2IM(K+1)-(RV2IM(K)-RV2IM(K+1))/2.
      DVU2AV=D1A*VU1A*(1.-1./RV1AAV)
      VU2(M)=DVU2AV/D2(M)*RV2IM(M)/SIGRV
      ITER=1
   13 WU2(M)=VU2(M)-U2(M)
      DHVD(M)=(U1A*VU1A-U2(M)*VU2(M))/G/J
      TT2(M)=TT0-DHVD(M)/CP
      IF(TT2(M).LT.0.0) GO TO 205
      VCR2(M)=VCR1A*SQRT(TT2(M)/TT1A)
      IF(ITER.EQ.1) VXVCR=0.5
      IND=1
   14 VX2(M)=VXVCR*VCR2(M)
      BET2(M)=ATAN(WU2(M)/VX2(M))
      W2(M)=VX2(M)/COS(BET2(M))
      TS2(M)=TTR2(M)-W2(M)**2/TGJCP
      D2M=D2(M)
      BET2M=BET2(M)
      WOWCRM= W2(M)/WCR2(M)
      ER=EBARR
      IF(EBARR.EQ.1.0) CALL EFFIC (ES,ER,2)
   15 W2IDSQ(M)=W2(M)**2/(1.-ER)
      Z2=1.-W2IDSQ(M)/TGJCP/TTR2(M)
      IF(Z2.LE.0.0) GO TO 206
      PS2(M)=PTR2ID(M)*Z2**EX
      RHO2(M)=PS2(M)/R/TS2(M)
      F2(M)=PI*D2(M)*EL*RHO2(M)*VX2(M)
      IF(ITER.EQ.1) WCALC=F2(M)*FLOAT(K)
      IF(ITER.EQ.1) GO TO 51
   67 I=M
      ID=-1
   16 II=I
      I=I+ID
      VU2(I)=VU2(M)*RV2IM(I)*D2(M)/D2(I)
      WU2(I)=VU2(I)-U2(I)
      PS2(I)=PS2(II)+RHO2(II)/G*(VU2(II)**2/D2(II)+VU2(I)**2/D2(I))/2.*(
     1D2(I)-D2(II))
      W2IDSQ(I)=TGJCP*TTR2(I)*(1.-(PS2(I)/PTR2ID(I))**(1./EX))
      Z3=1.-W2IDSQ(I)/TGJCP/TTR2(I)
      IF(Z3.LE.0.0) GO TO 207
      W2(I)=SQRT(W2IDSQ(I)*(1.-ER))
      IF(W2(I).LE.ABS(WU2(I))) GO TO 68
      BET2(I)=A SIN(WU2(I)/W2(I))
      VX2(I)=W2(I)*COS(BET2(I))
```

```
      TS2(I)=TTR2(I)-W2(I)**2/TGJCP
      RHO2(I)=PS2(I)/R/TS2(I)
      IF(I.EQ.1) GO TO 17
      IF(I.EQ.KK) GO TO 18
      F2(I)=PI*D2(I)*EL*RHO2(I)*VX2(I)
      GO TO 16
   17 I=M
      ID=1
      GO TO 16
   18 W2TOT=0.0
      DO 19 I=2,KP1
   19 W2TOT=W2TOT+F2(I)
      WCALC=W2TOT
      IF(K.GT.1) GO TO 51
   69 IF(DVU2AV .EQ. 0.0)GO TO 21
      DVUTOT=0.0
      DO 20 I=2,KP1
   20 DVUTOT=DVUTOT+F2(I)*D2(I)*VU2(I)
      DVUAVC=DVUTOT/W
      IF(ABS(DVUAVC-DVU2AV)/DVU2AV.LE..0001) GO TO 21
      VU20=VU2(M)
      VU2(M)=VU20*DVU2AV/DVUAVC
      GO TO 13
   21 V2LOSS=0.0
      DHIDTA=0.0
      DHIDSA=0.0
      Q=0.0
      RLOSS=0.0
      DO 23 I=1,KK
      DHVD(I)=(U1A*VU1A-U2(I)*VU2(I))/G/J
      TT2(I)=TT0-DHVD(I)/CP
      IF(TT2(I).LT.0.0) GO TO 208
      VCR2(I)=VCR1A*SQRT(TT2(I)/TT1A)
      ALPH2(I)=ATAN(VU2(I)/VX2(I))
      V2(I)=VX2(I)/COS(ALPH2(I))
      PT2(I)=PS2(I)*(TT2(I)/ TS2(I))**EX
      PTR2(I)=PS2(I)*(TTR2(I)/ TS2(I))**EX
      VOVCR2(I)=V2(I)/VCR2(I)
      WOWCR2(I)=W2(I)/WCR2(I)
C
C     SECTOR PERFORMANCE
C
      DHIDT(I)=CP*TT0*(1.-(PT2(I)/PT0)**(1./EX))
      DHIDS(I)=CP*TT0*(1.-(PS2(I)/PT0)**(1./EX))
      IF(I.EQ.1.OR.I.EQ.KK) GO TO 22
      V2LOSS=V2LOSS+F2(I)/W*V2(I)**2/2./G/J
      RLOSS=RLOSS+F2(I)/W*(W2IDSQ(I)-W2(I)**2)/2./G/J
      Q=Q+F2(I)/RHO2(I)
      DHIDTA=DHIDTA+F2(I)/W*DHIDT(I)
      DHIDSA=DHIDSA+F2(I)/W*DHIDS(I)
   22 ETATVD(I)=DHVD(I)/DHIDT(I)
   23 ETASVD(I)=DHVD(I)/DHIDS(I)
C
C     OVERALL PERFORMANCE
C
      ETATV=DHVDAV/DHIDTA
      ETASV=DHVDAV/DHIDSA
      ETAT=DHSHFT/DHIDTA
      ETAS=DHSHFT/DHIDSA
      SSPD=N*SQRT(Q)/(J*DHIDTA)**.75
```

```
      LC=LCDH*DHVDAV
      LCDHIS=LC/DHIDSA
      LWDHIS=LW/DHIDSA
      LLDHIS=V2LOSS/DHIDSA
      LSDHIS=(V1IDSQ-V1*V1)/2./G/J/DHIDSA
      LRDHIS=RLOSS/DHIDSA
      GO TO (30,32),IU
   30 PT0=PT0/10000.
      PS0=PS0/10000.
      PT1=PT1/10000.
      PS1=PS1/10000.
      PT1A=PT1A/10000.
      PS1A=PS1A/10000.
      PTR1A=PTR1A/10000.
      D0=D0*100.
      D1=D1*100.
      D1A=D1A*100.
      HS=HS*100.
      DO 31 I=1,KK
      D2(I)=D2(I)*100.
      PT2(I)=PT2(I)/10000.
      PS2(I)=PS2(I)/10000.
      PTR2(I)=PTR2(I)/10000.
   31 DHVD(I)=DHVD(I)/1000.
      DHVDAV=DHVDAV/1000.
      GO TO 36
   32 PT0=PT0/144.
      PS0=PS0/144.
      PT1=PT1/144.
      PS1=PS1/144.
      PT1A=PT1A/144.
      PS1A=PS1A/144.
      PTR1A=PTR1A/144.
      D0=D0*12.
      D1=D1*12.
      D1A=D1A*12.
      HS=HS*12.
      DO 33 I=1,KK
      D2(I)=D2(I)*12.
      PT2(I)=PT2(I)/144.
      PS2(I)=PS2(I)/144.
   33 PTR2(I)=PTR2(I)/144.
C
C     WRITE CALCULATED VALUES
C
   36 WRITE (6,111)
  111 FORMAT(9H *OUTPUT*/ 26X,2(3HABS,6X),17X,3(3HABS,5X),9X,4HREL ,4(5X
     1,3HREL)/17X,4HDIA-,2(4X,5HTOTAL),4X,2(6HSTATIC,3X),4HFLOW,3X,5HVEL
     2O-,4X,4HCRIT,3X,5HBLADE,2(4X,5HTOTAL),4X,4HFLOW,3X,5HVELO-,4X,4HCR
     3IT/16X,5HMETER,2(5X,4HTEMP,4X,5HPRESS),3X,5HANGLE,4X,4HCITY,9H   VE
     4L RAT,7H  SPEED,5X,4HTEMP,4X,5HPRESS,8H   ANGLE,4X,13HCITY  VEL RA
     5T)
      ALP0=ALPH0*DOR
      ALPHAA=ALPH1A*DOR
      BETA1A=BET1A*DOR
      DO 34 I=1,KK
      ALPHA2(I)=ALPH2(I)*DOR
   34 BETA2(I)=BET2(I)*DOR
      WRITE (6,112)D0,TT0,PT0,TS0,PS0,ALP  0,V0,VOVCR0
```

```
  112 FORMAT(13H STATOR INLET,2(F9.3,F9.2),F9.3,F7.2,F9.2,F7.3)
      WRITE(6,1121) ES,HS,ENS
 1121 FORMAT(11H LOSS COEF=,F6.4,67X,11HSTATOR HGT=,F7.4,17H,NUMBER OF V
     1ANES=,F5.1)
      WRITE (6,113)D1,TT1,PT1,TS1,PS1,ALPHA1,V1,VOVCR1
  113 FORMAT(13H STATOR EXIT ,2(F9.3,F9.2),F9.3,F7.2,F9.2,F7.3)
      WRITE (6,114)D1A,TT1A,PT1A,TS1A,PS1A,ALPHAA,V1A,VOVCRA,U1A,TTR1A,
     1PTR1A,BETA1A,W1A,WOWCRA
  114 FORMAT(13H0ROTOR INLET ,2(F9.3,F9.2)F9.3,F7.2,F9.2,F7.3,2F9.2,
     1F9.3,F8.2,F9.2,F7.3)
      WRITE(6,1141) ER
 1141 FORMAT(11H LOSS COEF=,F6.4)
      WRITE (6,115)(D2(I),TT2(I),PT2(I),TS2(I),PS2(I),ALPHA2(I),V2(I),
     1VOVCR2(I),U2(I),TTR2(I),PTR2(I),BETA2(I),W2(I),WOWCR2(I),I=1,KK)
  115 FORMAT(13H ROTOR EXIT  ,2(F9.3,F9.2)F9.3,F7.2,F9.2,F7.3,2F9.2,
     1F9.3,F8.2,F9.2,F7.3/(13X2(F9.3,F9.2)F9.3,F7.2,F9.2,F7.3,2F9.2,
     2F9.3,F8.2,F9.2,F7.3))
      F2(1)=0.0
      F2(KK)=0.0
      WRITE(6,1161)
 1161 FORMAT(1H )
      WRITE(6,116) EN
  116 FORMAT(26X,4HMASS,5X,4HDIAG/17X,4HDIA-,5X,4HFLOW,5X,4HSPEC,4X,5HTO
     1TAL,3X,6HSTATIC/16X,5HMETER,5X,4HRATE,5X,4HWORK,2(4X,5HEFFIC),3X,
     223HNUMBER OF ROTOR BLADES=,F5.1)
      WRITE (6,117)(D2(I),F2(I),DHVD(I),ETATVD(I),ETASVD(I),I=1,KK)
  117 FORMAT(13X,F9.3,F9.4,F9.3,2F8.3)
      TTPR=PT0/PT2(M)
      TSPR=PT0/PS2(M)
      WRITE (6,118)TTPR,LSDHIS,TSPR,LRDHIS,DHVDAV,LWDHIS,ETATV,LCDHIS,
     1ETASV,LLDHIS,ETAT,ETAS,SSPD
  118 FORMAT(22H0*OVERALL PERFORMANCE*/48X,20HLOSS/IDEAL T-S DEL H/6X,24
     1HTOT-TOT PRESSURE RATIO =,F8.4,13X,10HSTATOR    =,F6.4/6X,24HTOT-ST
     2AT PRESSURE RATIO=,F8.4,13X,10HROTOR     =,F6.4/6X,24HDIAG AVG SPEC
     3IFIC WORK =,F8.4,13X,10HWINDAGE  =,F6.4/6X,24HDIAG TOTAL EFFICIENC
     4Y  =,F8.4,13X,10HCLEARANCE=,F6.4/6X,24HDIAG STATIC EFFICIENCY =,F8
     5.4,13X,10HEXIT KE  =,F6.4/6X,24HNET TOTAL EFFICIENCY   =,F8.4/6X,
     624HNET STATIC EFFICIENCY  =,F8.4,8X,15HSPECIFIC SPEED=,F7.3)
      GO TO 2
   51 IF(IND.GE.6.AND.ABS(WGIV-WCALC)/WGIV.LE..0001) GO TO 65
      CALL CONTIN(VXVCR,WCALC,IND,1,WGIV,.05)
      IF(IND-10)14,61,61
   61 IF(ITER-1)62,62,63
   62 VXVCR=.9
      IF(K.EQ.1) GO TO 63
      IND=1
      ITER=ITER+1
      GO TO 14
   63 IF(IND-10)201,201,202
   65 IF(ITER-1)66,66,69
   66 ITER=ITER+1
      IND=1
      GO TO 67
   68 IF(VXVCR.GE.VXVCRP) GO TO 72
      VXVCR=VXVCRP-DELVX
      DELVX=DELVX/10.
      IF(DELVX.LE..00001) GO TO 209
   72 VXVCR=VXVCR+DELVX
      VXVCRP=VXVCR
```

```
 71 CHRD=HS/STAR
    IF(NSTAR.EQ.2) GO TO 76
    RSTG=(CHRD*COS(ALSTG)+SQRT(CHRD**2*(COS(ALSTG)**2-1.)+D1**2))/2.
    DO=(SQRT(CHRD**2/2.+2.*RSTG**2-D1**2/4.))*2.
    ALUNC=ALPH1-ALPHO-A COS((DO**2+D1**2-4.*CHRD**2)/2./DO/D1)
    GO TO 70
 76 ALPHO=ATAN(SIN(ALPH1)/(CHRD/D1*2.+COS(ALPH1)))
    ALSTG=(ALPHO+ALPH1)/2.
    DO=2.*SQRT(CHRD**2+D1**2/4.+CHRD*D1*COS(ALPH1))
    GO TO 70
201 WRITE(6,120)WCALC
120 FORMAT(48H0 ROTOR EXIT CHOKES AT MAXIMUM MASS FLOW RATE = ,F9.4)
    GO TO 2
202 WRITE(6,121)
121 FORMAT(69H0 NO SOLUTION FOUND AFTER 100 ITERATIONS FOR CONTINUITY
   1AT ROTOR EXIT)
    GO TO 2
203 WRITE(6,122)
122 FORMAT(60H0REQUIRED SPECIFIC WORK GREATER THAN ENERGY AVAILABLE IN
   1 GAS)
    GO TO 2
204 WRITE(6,123)
123 FORMAT(74H0REQUIRED STATOR IDEAL KINETIC ENERGY GREATER THAN ENERG
   1Y AVAILABLE IN GAS)
    GO TO 2
205 WRITE(6,124) M
124 FORMAT(33H0SPECIFIC WORK REQUIRED IN SECTOR,I3,37H GREATER THAN EN
   1ERGY AVAILABLE IN GAS)
    GO TO 2
206 WRITE(6,125) M
125 FORMAT(55H0ROTOR IDEAL RELATIVE KINETIC ENERGY REQUIRED IN SECTOR,
   1I3,37H GREATER THAN ENERGY AVAILABLE IN GAS)
    GO TO 2
207 WRITE(6,125) I
    GO TO 2
208 WRITE(6,124) I
    GO TO 2
209 WRITE (6,128)
128 FORMAT(123H0THE PROGRAM CAN NOT FIND A SOLUTION SIMULTANEOUSLY SAT
   1ISFYING CONTINUITY, RADIAL EQ., AND THE LOSS MODEL AT THE ROTOR EX
   2IT)
    GO TO 2
999 STOP
    END
```

Blade-Row Loss Coefficient Subprograms

The stator and rotor loss-coefficient calculations, as described in the section LOSS MODEL, are performed by subroutine EFFIC. Function subprograms SIMPS1, SHUB, and SHUB2 are used for the numerical integration required to determine the rotor hub wall area (eq. (C17)). SIMPS1 uses a modification of Simpson's rule wherein more intervals are placed in those regions requiring greater accuracy. SHUB and SHUB2 are the functions being integrated.

Program variables. - Variables transfer between main program RIFTUD and subroutine EFFIC by means of both the EFFIC(ES, ER, J) arguments and labeled common block /EFF/. The arguments ES and ER are the stator and rotor loss coefficients, respectively, and J is the blade-row indicator (J = 1 for stator and J = 2 for rotor). The common block variables, which were defined in the RIFTUD variable list, are ALPH0, ALPH1, ALUNC, BET2M, CS, CR, D0, D1, D1A, D2M, EN, GAM, HR, HS, MU, NSTAR, PI, RH2RT2, SIGS, STAR, VOVCR1, W, and WOWCRM. The remaining variables in subroutine EFFIC are defined as follows:

A(X)	arithmetic statement function for Q where argument $X = \left(V/V_{cr}\right)_1$ or $\left(W/W_{cr}\right)_{2,m}$
AA	a
AR	Q_r
AS	Q_s
A2	a/2
A3A2R	$\left(A_{3D}/A_{2D}\right)_r$
A3A2S	$\left(A_{3D}/A_{2D}\right)_s$
BB	b
BWSR	$A_{b,r}$
CHRDR	c_r
D(Y)	arithmetic statement function for denominator of equation (10) or (11) where argument $Y = Q_s$ or Q_r
E(Y)	arithmetic statement function for E where argument $Y = Q_s$ or Q_r
ELOCS	$(l/c)_s$
ELOSR	$(l/s)_r$
ELOSS	$(l/s)_s$
EWSR	$A_{w,t,r} + A_{w,h,r}$
H(Y)	arithmetic statement function for H where argument $Y = Q_s$ or Q_r
K	error parameter for integration of function SHUB
K2	error parameter for integration of function SHUB2

REFR	$\left(\theta_{tot}/l \ \mathrm{Re}^{-0.2}\right)_{ref, r}$
REFS	$\left(\theta_{tot}/l \ \mathrm{Re}^{-0.2}\right)_{ref, s}$
RER	Re_r
RES	Re_s
SRH	$n_r A_{w, h, r}$
SRH1	part of SRH from first integral in equation (C17)
SRH2	part of SRH from second integral in equation (C17)
SRS	$n_r A_{w, t, r}$
TOSR	$(t/s)_r$
TOSS	$(t/s)_s$
YC	r_{1a}
Y2	$r_{1a} - b\sqrt{3}/2$

Variables transfer among EFFIC, SIMPS1, SHUB, and SHUB2 by means of the function arguments and the labelled common block /SH/. The function arguments are SIMPS1(X1, X2, FUNC, KSIG), SHUB(X), and SHUB2(Y) where

X1	lower limit of integration
X2	upper limit of integration
FUNC	function being integrated, SHUB or SHUB2 in this case
KSIG	error parameter, increases if integration is inaccurate
X	variable of integration
Y	variable of integration

The common block variables, which were defined in the EFFIC variable list, are AA, BB, and YC. These also are the only variables, aside from the arguments, used in SHUB and SHUB2.

A further description of SIMPS1 and definition of its internal variables can be obtained from reference 6. The function described is called SIMPS2, but it becomes exactly SIMPS1 upon deletion of the parameter J from the function arguments.

Program listing. - The FORTRAN listings for subroutine EFFIC and functions SIMPS1, SHUB, and SHUB2 are as follows:

```
      SUBROUTINE EFFIC (ES,ER,J)
      REAL MU
      COMMON/EFF/GAM,VOVCR1,W,PI,STAR,MU,D1,ALPH1,NSTAR,ALPH0,ALUNC,SIGS
     1,D0,HS,CS,WOWCRM,D1A,D2M,HR,BET2M,EN,RH2RT2,CR
      COMMON/SH/AA,BB,YC
      A(X)= (GAM-1.)/(GAM+1.)*X*X
      D(Y)= 1./1.68+Y/2.88+Y*Y/4.4+Y*Y*Y/6.24
      H(Y)= (1./1.2+3.*Y/1.6+5.*Y*Y/2.+7.*Y**3/2.4+9.*Y**4/2.8)/D(Y)
      E(Y)=2.*(1./1.92+Y/3.2+Y*Y/4.8+Y**3/6.72)/D(Y)
      GO TO (1,2),J
    1 AS= A(VOVCR1)
      REFS=.03734
      RES= W/PI/STAR/MU/D1/COS(ALPH1)
      ELOCS=1.
      IF(NSTAR.NE.2.OR.ALUNC.NE.0.0) ELOCS=ALUNC/2./SIN(ALUNC/2.)
      ELOSS= ELOCS*SIGS
      A3A2S= 1.+(D0**2-D1**2)/4./SIGS/HS/D1/ELOCS
      TOSS= .05*SIGS*STAR
      ES=CS*E(AS)*REFS/RES**.2*ELOSS*A3A2S/(COS(ALPH1)-CS*H(AS)*REFS/RES
     1**.2*ELOSS-TOSS)
      GO TO 3
    2 AR= A(WOWCRM)
      REFR=.11595
      CHRDR=0.5*SQRT((D1A-D2M-HR+HS)**2+(D1A-D2M)**2)
      RER=W/PI*CHRDR/HR/MU/D2M/COS(BET2M)
      ELOSR=EN*CHRDR/2./SQRT(2.)/D2M
      BWSR=PI/8.*((D1A-D2M-HR+2.*HS)*(D1A-D2M+HR)-(D1A-D2M-HR)**2)
      SRS=PI/2.*(D1A-D2M-HR)*((PI/2.-1.)*D1A+D2M+HR)
      EXTERNAL SHUB,SHUB2
      K= 0
      K2=0
      AA= (D1A-D2M-HR+2.*HS)/2.
      BB= (D1A-D2M+HR)/2.
      YC=D1A/2.
      A2=AA/2.
      Y2=YC-BB/2.*SQRT(3.)
      SRH1=2.*PI*SIMPS1(0.0,A2,SHUB,K)
      IF(K.GT.0) WRITE(6,10) K
   10 FORMAT(3H K=,I2)
      SRH2=2.*PI*SIMPS1(Y2,YC,SHUB2,K2)
      IF(K2.GT.0) WRITE(6,11) K2
   11 FORMAT(4H K2=,I2)
      SRH=SRH1+SRH2
      EWSR=(SRS+SRH)/EN
      A3A2R=1.+EWSR/BWSR
      TOSR= 0.04/PI*EN*(1.-RH2RT2)/(1.+RH2RT2)
      ER=CR*E(AR)*REFR/RER**.2*ELOSR*A3A2R/(COS(BET2M)-CR*H(AR)*REFR/RER
     1**.2*ELOSR-TOSR)
    3 RETURN
      END
```

```
      FUNCTION SIMPS1 (X1,X2,FUNC,KSIG)
C.....THIS ROUTINE INTEGRATES FUNC(X) FROM X1 TO X2 USING A MULTIPLE
C.....INTERVAL SIMPSON≠S RULE TECHNIQUE.
      LOGICAL SPILL
      DOUBLE PRECISION ANS,Q
      DIMENSION V(200),H(200),A(200),B(200),C(200),P(200),E(200)
      DATA TWO,THREE,FOUR,THIRTY/2.0,3.0,4.0,30.0/
      DATA T,NMAX,NSIG/3.0E-5,200,1/
C.....INITIALIZE FIRST ELEMENTS OF ARRAYS.
      V(1)= X1
      H(1)= (X2 - V(1))/TWO
      A(1)= FUNC ( V )
      B(1)= FUNC ( V(1)+H(1) )
      C(1)= FUNC ( X2 )
      P(1)= H(1)*(A(1) • FOUR*B(1) • C(1))
      E(1)= P(1)
      ANS= P(1)
      N=1
      FRAC=T
      SPILL=.FALSE.
1     TEST=ABS(FRAC*ANS)
      K=N
      DO 3 I=1,K
C.....TEST MAGNITUDE OF 4TH ORDER ERROR IN THIS INTERVAL.
      IF (ABS(E(I)).LE.TEST) GO TO 3
      IF (N.LT.NMAX) GO TO 2
C.....GO TO FINISH IF STORAGE IS FILLED UP.
      SPILL=.TRUE.
      KSIG=KSIG+NSIG
      GO TO 4
C.....SUBDIVIDE INTERVAL AGAIN TO REDUCE 4TH ORDER ERROR.
2     N=N+1
      V(N)=V(I)+H(I)
      H(N)=H(I)/TWO
      A(N)=B(I)
      B(N)=FUNC(V(N)+H(N))
      C(N)=C(I)
      P(N)=H(N)*(A(N)+FOUR*B(N)+C(N))
      H(I)=H(N)
      B(I)=FUNC(V(I)+H(I))
      C(I)=A(N)
      Q=P(I)
      P(I)=H(I)*(A(I)+FOUR*B(I)+C(I))
      Q=P(I)+P(N)-Q
      ANS=ANS+Q
      E(I)=Q
      E(N)=Q
3     CONTINUE
C.....TEST ALL INTERVALS AGAIN IF ANY WERE SUBDIVIDED THE LAST TIME.
      IF (N.GT.K) GO TO 1
4     Q=0.0
      DO 5 I=1,N
5     Q=Q+E(I)
C.....TIGHTEN ERROR LIMIT IF TOTAL ACCUMULATED ERROR TOO LARGE.
      IF (ABS(Q/T).LE.ABS(ANS).OR.SPILL) GO TO 6
      FRAC=FRAC/TWO
      GO TO 1
C.....FINISH OFF CALCULATION.
6     SIMPS1=(ANS+Q/THIRTY)/THREE
      RETURN
C.....THIS ENTRY USED TO GET AT INTERNAL VARIABLES.
      ENTRY SIMPX1 (TT,NN,QQ)
      T=TT
      NN=N
      QQ=Q
      RETURN
      END
```

```
      FUNCTION SHUB(X)
      COMMON/SH/AA,BB,YC
      SHUB=(YC-BB/AA*SORT(AA**2-X*X))*SORT(1.+BB**2/AA**2*X*X/(AA**2-X*X
     1))
      RETURN
      END
```

```
      FUNCTION SHUB2(Y)
      COMMON/SH/AA,BB,YC
      SHUB2=Y*SQRT(1.+AA**2/BB**2*(Y-YC)**2/(BB**2-(Y-YC)**2)'
      RETURN
      END
```

Rotor-Exit Continuity Subprograms

Subroutines CONTIN and PABC provide the means for obtaining the rotor-exit continuity solution; that is, they yield the value of $\left(V_x/V_{cr}\right)_{2,m}$ that satisfies the input mass flow rate. CONTIN provides an estimate for the value of an independent variable X that satisfies a given value of the dependent variable Y by means of curve fitting. The curve is a parabola whose coefficients are calculated by PABC, which is called by CONTIN. The estimation continues until a solution is obtained within the desired tolerance. Subroutines CONTIN and PABC are described in detail in reference 7.

Program variables. - Variables transfer between main program RIFTUD and subroutine CONTIN by means of the CONTIN(XEST, YCALC, IND, JZ, YGIV, XDEL) arguments. Transfer of variables between CONTIN and PABC is by means of the PABC(X, Y, A, B, C) arguments. These agruments are defined as follows:

XEST	on call: value of X used to calculate YCALC on return: value of X to be used to calculate next value of YCALC
YCALC	value of Y corresponding to XEST during call
IND	on call: controls sequence of calculation in CONTIN on return: indicates when a choked solution is found or when no solution can be found
JZ	determines whether subsonic or supersonic solution will be obtained: 1 - subsonic solution 2 - supersonic solution

YGIV	value of Y desired for solution
XDEL	maximum permissable change in XEST between iterations
X	independent variable
Y	dependent variable
A	coefficient A in $y = Ax^2 + Bx + C$
B	coefficient B in $y = Ax^2 + Bx + C$
C	coefficient C in $y = Ax^2 + Bx + C$

The internal variables for CONTIN are defined in reference 7.

Program listing. - The FORTRAN listings for subroutines CONTIN and PABC are as follows:

```
      SUBROUTINE CONTIN(XEST,YCALC,IND,JZ,YGIV,XDEL)
C
C--CONTIN CALCULATES AN ESTIMATE OF THE RELATIVE FLOW VELOCITY
C--FOR USE IN THE VELOCITY GRADIENT EQUATION
C
      DIMENSION X(3),Y(3)
      NCALL = NCALL+1
      IF (IND.NE.1.AND.NCALL.GT.100) GO TO 160
      GO TO (10,30,40,50,60,110,150),IND
C--FIRST CALL
   10 NCALL = 1
      XORIG = XEST
      IF (YCALC.GT.YGIV.AND.JZ.EQ.1) GO TO 20
      IND = 2
      Y(1) = YCALC
      X(1) = 0.
      XEST = XEST+XDEL
      RETURN
   20 IND = 3
      Y(3) = YCALC
      X(3) = 0.
      XEST = XEST-XDEL
      RETURN
C--SECOND CALL
   30 IND = 4
      Y(2) = YCALC
      X(2) = XEST-XORIG
      XEST = XEST+XDEL
      RETURN
   40 IND = 5
      Y(2) = YCALC
      X(2) = XEST-XORIG
      XEST = XEST-XDEL
      RETURN
C--THIRD OR LATER CALL - FIND SUBSONIC OR SUPERSONIC SOLUTION
   50 Y(3) = YCALC
      X(3) = XEST-XORIG
      GO TO 70
```

```
   60 Y(1) = YCALC
      X(1) = XEST-XORIG
   70 IF (YGIV.LT.AMIN1(Y(1),Y(2),Y(3))) GO TO (120,130),JZ
   80 IND = 6
      CALL PABC(X,Y,APA,BPB,CPC)
      DISCR = BPB**2-4.*APA*(CPC-YGIV)
      IF (DISCR.LT.0.) GO TO 140
      IF (ABS(400.*APA*(CPC-YGIV)).LE.BPB**2) GO TO 90
      XEST = -BPB-SIGN(SQRT(DISCR),APA)
      IF (JZ.EQ.1.AND.APA.GT.0..AND.Y(3).GT.Y(1)) XEST = -BPB+
     1SQRT(DISCR)
      IF (JZ.EQ.2.AND.APA.LT.0.) XEST = -BPB-SQRT(DISCR)
      XEST = XEST/2./APA
      GO TO 100
   90 IF (JZ.EQ.2.AND.BPB.GT.0.) GO TO 130
      ACB2 = APA/BPB*(CPC-YGIV)/BPB
      IF (ABS(ACB2).LE.1.E-8) ACB2=0.
      XEST = -(CPC-YGIV)/BPB*(1.+ACB2+2.*ACB2**2)
  100 IF (XEST.GT.X(3)) GO TO 130
      IF (XEST.LT.X(1)) GO TO 120
      XEST = XEST+XORIG
      RETURN
C--FOURTH OR LATER CALL - NOT CHOKED
  110 IF(XEST-XORIG.GT.X(3)) GO TO 130
      IF(XEST-XORIG.LT.X(1)) GO TO 120
      Y(2) = YCALC
      X(2) = XEST-XORIG
      GO TO 70
C--THIRD OR LATER CALL - SOLUTION EXISTS,
C--BUT RIGHT OR LEFT SHIFT REQUIRED
  120 IND = 5
C--LEFT SHIFT
      XEST = X(1)-XDEL+XORIG
      XOSHFT = XEST-XORIG
      XORIG = XEST
      Y(3) = Y(2)
      X(3) = X(2)-XOSHFT
      Y(2) = Y(1)
      X(2) = X(1)-XOSHFT
      RETURN
  130 IND = 4
C--RIGHT SHIFT
      XEST = X(3)+XDEL+XORIG
      XOSHFT = XEST-XORIG
      XORIG = XEST
      Y(1) = Y(2)
      X(1) = X(2)-XOSHFT
      Y(2) = Y(3)
      X(2) = X(3)-XOSHFT
      RETURN
C--THIRD OR LATER CALL - APPEARS TO BE CHOKED
  140 XEST = -BPB/2./APA
      IND = 7
      IF (XEST.LT.X(1)) GO TO 120
      IF(XEST.GT.X(3)) GO TO 130
      XEST = XEST+XORIG
      RETURN
C--FOURTH OR LATER CALL - PPOBABLY CHOKED
  150 IF (YCALC.GE.YGIV) GO TO 110
      IND = 10
      RETURN
```

```
C--NO SOLUTION FOUND IN 100 ITERATIONS
  160 IND = 11
      RETURN
      END
```

```
      SUBROUTINE PABC(X,Y,A,B,C)
C
C--PABC CALCULATES COEFFICIENTS A,B,C OF THE PARABOLA
C--Y=A*X**2+B*X+C, PASSING THROUGH THE GIVEN X,Y POINTS
C
      DIMENSION X(3),Y(3)
      C1 = X(3)-X(1)
      C2 = (Y(2)-Y(1))/(X(2)-X(1))
      A = (C1*C2-Y(3)+Y(1))/C1/(X(2)-X(3))
      B = C2-(X(1)+X(2))*A
      C = Y(1)-X(1)*B-X(1)**2*A
      RETURN
      END
```

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 23, 1975,
505-04.

APPENDIX A

SYMBOLS

A	area, m^2; ft^2
a	semiaxis of ellipse in x-direction, m; ft
b	semiaxis of ellipse in y-direction, m; ft
C	loss coefficient multiplier
C_e	circumference of ellipse, m; ft
C_N	dimensional constant, 2π rad/rev; 60 sec/min
C_P	dimensional constant, 1000 W/kW; 550 (ft)(lb)/(sec)(hp)
c	chord, m; ft
c_p	specific heat at constant pressure, J/(kg)(K); Btu/(lb)($^{\circ}$R)
D	diameter, m; ft
E	energy factor
$\bar{e}$	blade-row loss coefficient
g	dimensional constant, 1; 32.17 (lbm)(ft)/(lbf)(sec^2)
H	form factor
h	blade or clearance height, m; ft
Δh	specific enthalpy difference, J/kg; Btu/lb
J	dimensional constant, 1; 778 (ft)(lb)/Btu
k	number of sectors
L	loss, J/kg; Btu/lb
l	surface length (leading edge to trailing edge), m; ft
N	rotative speed, rad/sec; rev/min
N_{sp}	specific speed, dimensionless; ($ft^{3/4}$)($lbm^{3/4}$)/(min)($sec^{1/2}$)($lbf^{3/4}$)
n	number of blades
P	shaft power, kW; hp
p	absolute pressure, N/m^2; lb/ft^2
Q	parameter defined by equation (12) or (13)

R	gas constant, J/(kg)(K); (ft)(lbf)/(lbm)(°R)
Re	Reynolds number
r	radius, m; ft
s	blade spacing at blade-row exit, m; ft
T	absolute temperature, K; °R
t	trailing-edge thickness, m; ft
U	blade speed, m/sec; ft/sec
V	absolute velocity, m/sec; ft/sec
W	relative velocity, m/sec; ft/sec
w	mass flow rate, kg/sec; lb/sec
x	x coordinate, m; ft
y	y coordinate, m; ft
α	fluid absolute angle measured from radial direction at stations 0, 1, and 1a and from axial direction at station 2, deg
β	fluid relative angle measured from radial direction at stations 0, 1, and 1a and from axial direction at station 2, deg
γ	specific heat ratio
δ	displacement thickness, m; ft
η	efficiency
Θ	sector angle, deg
Θ_{cam}	camber angle, deg
θ	momentum thickness, m; ft
μ	viscosity, (N)(sec)/m^2; lb/(ft)(sec)
ρ	density, kg/m^3; lb/ft^3
Φ	flow angle from throughflow direction, deg
φ	stator vane stagger angle from radial direction, deg
Ψ	angle defined in figure 3, deg
ψ	energy thickness, m; ft

Subscripts:

av	average
b	blade surface
c	clearance
calc	calculated
cm	chord mean
cr	critical condition
df	disk friction
ex	exit
h	hub
i	sector number
id	ideal
j	sector number other than mean sector
m	mean sector
r	rotor
rad	radial direction
ref	reference
s	stator
shft	shaft
sr	surface of revolution
t	tip
tot	total
u	tangential direction
VD	velocity diagram
w	wall
x	axial direction
0	stator inlet
1	stator exit
1a	rotor inlet

2	rotor exit
2D	two dimensional
3D	three dimensional

Superscripts:

'	absolute total value
''	relative total value

APPENDIX B

STATOR GEOMETRY MODEL

The purpose of the stator vane geometry model is to interrelate the stator inlet and exit radii, inlet and exit flow angles, chord, and surface length to chord ratio in a consistent manner. The basic geometry model is shown in figure 3. There are three specific cases considered herein: (1) an uncambered vane with known chord length, (2) a cambered vane with known chord length and inlet flow angle, and (3) a cambered vane with known inlet radius and flow angle. In all cases, the exit radius and flow angle are known.

Uncambered Vane

For this case, the vane surface length is assumed equal to the chord; that is, $(l/c)_s = 1$. The remaining unknowns are inlet flow angle and inlet radius. From the geometry of figure 3(a),

$$\tan \varphi_0 = \frac{r_1 \sin \varphi_1}{c + r_1 \cos \varphi_1} \tag{B1}$$

Since the vane is uncambered, $\alpha_0 = \varphi_0$ and $\alpha_1 = \varphi_1$. Therefore, equation (B1) becomes

$$\tan \alpha_0 = \frac{\sin \alpha_1}{\frac{c}{r_1} + \cos \alpha_1} \tag{B2}$$

The inlet radius is found by using the law of cosines; that is,

$$r_0^2 = c^2 + r_1^2 - 2cr_1 \cos (180 - \alpha_1) \tag{B3}$$

Since $\cos \alpha_1 = -\cos (180 - \alpha_1)$, we get

$$r_0 = \sqrt{c^2 + r_1^2 + 2cr_1 \cos \alpha_1} \tag{B4}$$

Cambered Vane

It is assumed that the surface length is a circular arc, as shown in figure 3(b). The arc length to chord ratio can be related to the sector angle Θ. Arc length is

$$l = \frac{\pi r \Theta}{180} \tag{B5}$$

Chord is

$$c = 2r \sin\frac{\Theta}{2} \tag{B6}$$

Dividing equation (B5) by equation (B6) then yields

$$\frac{l}{c} = \frac{\pi\Theta}{360 \sin\frac{\Theta}{2}} \tag{B7}$$

The sector angle Θ can be related to the camber angle Θ_{cam}. Remembering that a tangent is perpendicular to the radius at the point of tangency, referring to figure 3(b), we can write

$$(180 - \Theta_{cam}) + \Theta = 180 \tag{B8}$$

Therefore,

$$\Theta = \Theta_{cam} \tag{B9}$$

and equation (B7) becomes

$$\left(\frac{l}{c}\right)_s = \frac{\pi\Theta_{cam}}{360 \sin\frac{\Theta_{cam}}{2}} \tag{B10}$$

Referring to figure 3(a), the camber angle can be expressed as

$$\Theta_{cam} = 180 - \alpha_0 - (180 - \alpha_1) - \Psi = \alpha_1 - \alpha_0 - \Psi \tag{B11}$$

From the law of cosines,

$$\cos \Psi = \frac{r_0^2 + r_1^2 - c^2}{2r_0r_1} \tag{B12}$$

Substituting equation (B12) into equation (B11) yields

$$\Theta_{cam} = \alpha_1 - \alpha_0 - \cos^{-1}\left(\frac{r_0^2 + r_1^2 - c^2}{2r_0r_1}\right) \tag{B13}$$

Equations (B13) and (B10) are used to determine surface length to chord ratio once the inlet radius and chord are known.

To determine inlet radius or chord for the two cambered vane cases being considered, it is assumed that the stagger angle at the chord midpoint is equal to average flow angle; that is,

$$\varphi_{cm} = \frac{\alpha_0 + \alpha_1}{2} \tag{B14}$$

Referring again to figure 3(a), the law of cosines yields

$$r_1^2 = r_{cm}^2 + \left(\frac{c}{2}\right)^2 - 2r_{cm}\left(\frac{c}{2}\right)\cos \varphi_{cm} \tag{B15}$$

and, since $\cos \varphi_{cm} = -\cos (180 - \varphi_{cm})$, also

$$r_0^2 = r_{cm}^2 + \left(\frac{c}{2}\right)^2 + 2r_{cm}\left(\frac{c}{2}\right)\cos \varphi_{cm} \tag{B16}$$

Adding equations (B15) and (B16) gives

$$r_0^2 + r_1^2 = 2r_{cm}^2 + 2\left(\frac{c}{2}\right)^2 \tag{B17}$$

Known chord. - In this case, we are trying to find the inlet radius. With the chord known, equation (B15) is solved for r_{cm} using the quadratic formula

$$r_{cm} = \frac{1}{2}\left[c \cos \varphi_{cm} + \sqrt{\left(c \cos \varphi_{cm}\right)^2 + \left(4r_1^2 - c^2\right)}\right] \tag{B18}$$

With r_{cm} evaluated thusly, equation (B17) yields the inlet radius

$$r_0 = \sqrt{2r_{cm}^2 + \frac{c^2}{2} - r_1^2} \tag{B19}$$

Known inlet radius. - In this case, we are trying to find the chord. Subtracting equation (B15) from equation (B16) and solving for r_{cm} result in

$$r_{cm} = \frac{r_0^2 - r_1^2}{2c \cos \varphi_{cm}} \tag{B20}$$

Substituting equation (B20) into equation (B17) yields

$$c^4 - 2c^2\left(r_0^2 + r_1^2\right) + \left(\frac{r_0^2 - r_1^2}{\cos \varphi_{cm}}\right)^2 = 0 \tag{B21}$$

Using the quadratic formula and taking the positive root finally give

$$c = \sqrt{r_0^2 + r_1^2 - \sqrt{\left(r_0^2 + r_1^2\right)^2 - \left(\frac{r_0^2 - r_1^2}{\cos \varphi_m}\right)^2}} \tag{B22}$$

APPENDIX C

ROTOR GEOMETRY MODEL

The purpose of the rotor geometry model is to provide a consistent basis for evaluating mean surface length, chord length, blade-surface and end-wall surface areas, and number of blades. Shown in figure 4 is the rotor geometry with dimensions expressed in terms of various radii and the stator blade height. The model is based on the meridional-plane projection of the rotor. The tip contour is assumed to be circular and the hub contour is assumed to be elliptical, each being a 90° arc. It is further assumed that the rotor consists of full blades only.

Surface and Chord Lengths

The mean surface length and mean chord are the elliptical arc and straight line, respectively, joining the midpoints (see fig. 4) of the rotor leading and trailing edges. The general formula for the circumference of an ellipse with semiaxes a and b is

$$C_e \approx 2\pi \sqrt{\frac{a^2 + b^2}{2}} \tag{C1}$$

Therefore, the mean surface length of the rotor is

$$l_r = \frac{\pi}{2} \sqrt{\frac{1}{2}\left[\left(r_{1a} - r_{2,t} + \frac{h_s}{2}\right)^2 + \left(r_{1a} - r_{2,m}\right)^2\right]} \tag{C2}$$

From the Pythagorean theorem, the chord length is

$$c_r = \sqrt{\left(r_{1a} - r_{2,t} + \frac{h_s}{2}\right)^2 + \left(r_{1a} - r_{2,m}\right)^2} \tag{C3}$$

Wall Areas

The blade surface area is the area between the elliptical hub and the circular tip. The general formula for the area within an ellipse is

$$A = \pi ab \tag{C4}$$

Therefore, the blade surface area for one passage (i.e., two sides) is

$$A_{b,r} = \frac{\pi}{2}\left[(r_{1a} - r_{2,t} + h_s)(r_{1a} - r_{2,h}) - \left(r_{1a} - r_{2,t}\right)^2\right] \tag{C5}$$

The tip and hub wall areas are determined as the surfaces of revolution of the tip and hub curves around the turbine axis. The general formula for the surface of revolution of a curve $y = f(x)$ between (x_1, y_1) and (x_2, y_2) is

$$A_{sr} = 2\pi \int_{x_1}^{x_2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = 2\pi \int_{y_1}^{y_2} y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \tag{C6}$$

With the turbine axis taken as $y = 0$, the centers of the tip and hub curves are at $x = 0$ and $y = r_{1a}$. Therefore, the equation of the circular tip is

$$x^2 + \left(y - r_{1a}\right)^2 = \left(r_{1a} - r_{2,t}\right)^2 \tag{C7}$$

while that of the elliptical hub is

$$\frac{x^2}{a^2} + \frac{\left(y - r_{1a}\right)^2}{b^2} = 1 \tag{C8}$$

where the semiaxes a and b are

$$a = r_{1a} - r_{2,t} + h_s \tag{C9}$$

and

$$b = r_{1a} - r_{2,h} \tag{C10}$$

For the circular tip, differentiation of equation (C7) yields

$$\frac{dy}{dx} = -\frac{x}{y - r_{1a}} \tag{C11}$$

Substitution of equations (C11) and (C7) into equation (C6) results in

$$A_{sr,t} = 2\pi(r_{1a} - r_{2,t})\int_0^{r_{1a}-r_{2,t}} \left[\frac{r_{1a}}{\sqrt{(r_{1a} - r_{2,t})^2 - x^2}} - 1\right] dx \tag{C12}$$

Integrating equation (C12) and dividing by the number of rotor passages then gives the tip wall area for one passage

$$A_{w,t,r} = \frac{2\pi}{n_r}(r_{1a} - r_{2,t})\left[\left(\frac{\pi}{2} - 1\right) r_{1a} + r_{2,t}\right] \tag{C13}$$

For the elliptical hub, the area of the surface of revolution cannot be analytically expressed because equation (C6) cannot be integrated analytically. Further, neither form of equation (C6) can be integrated numerically over its entire range because of an infinite slope at one limit. However, if we break the curve into two pieces, we can write

$$A_{sr,h} = 2\pi \int_0^{a/2} y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx + 2\pi \int_{r_{1a}-b\frac{\sqrt{3}}{2}}^{r_{1a}} y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \tag{C14}$$

From equation (C8), the two derivatives can be determined as

$$\frac{dy}{dx} = \frac{bx}{a\sqrt{a^2 - x^2}} \tag{C15}$$

and

$$\frac{dx}{dy} = -\frac{a(y - r_{1a})}{b\sqrt{b^2 - \left(y - r_{1a}\right)^2}} \tag{C16}$$

Substituting equations (C8), (C15), and (C16) into equation (C14) and dividing by the number of rotor passages then give the following hub wall area for one passage:

$$A_{w,h,r} = \frac{2\pi}{n_r}\left\{\int_0^{a/2}\left(r_{1a} - \frac{b}{a}\sqrt{a^2 - x^2}\right)\sqrt{1 + \frac{b^2x^2}{a^2(a^2 - x^2)}}\,dx + \int_{r_{1a}-b\frac{\sqrt{3}}{2}}^{r_{1a}} y\sqrt{1 + \frac{a^2\left(y - r_{1a}\right)^2}{b^2\left[b^2 - \left(y - r_{1a}\right)^2\right]}}\,dy\right\} \tag{C17}$$

Equation (C17) can be integrated numerically. In the program, each of the integrals is evaluated by Simpson's rule.

Blade Number

From the approach of reference 8 along with the additional assumption that $V_{u,1a} = U_{1a}$, a number of blades can be computed as

$$n_r = 2\pi \tan \alpha_{1a} \tag{C18}$$

This number of blades is based on not permitting the velocity to fall below zero anywhere within the rotor channel; the limiting condition, therefore, is zero velocity on the pressure surface at the rotor inlet. Equation (C18) yields a rather large number of blades, especially at the higher flow angles. Studies such as the one reported in reference 5 have shown that the number of blades can be reduced significantly from these high values without significant degradation in performance. Therefore, a reduction factor is used in equation (C18). Also, the small difference between α_{1a} and α_1 is neglected. With these changes, the number of rotor blades is computed herein as

$$n_r = \frac{\pi}{30}(110 - \alpha_1)\tan\alpha_1 \qquad \text{(C19)}$$

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TABLE I. - INPUT FORM WITH SAMPLE DATA

TITLE

PROJECT NUMBER

ANALYST

SHEET _____ OF _____

STATEMENT NUMBER | CONT | FORTRAN STATEMENT | IDENTIFICATION

```
      THIS IS A SAMPLE CASE USING SI UNITS
 $INPUT R=208.1,GAM=1.667,ALPHA0=55.6,ALPHA1=72.0,N=4031.7,TTIN=1083.33,
 PTIN=9.101,W=,2771,POW=22.371,RIR1A=1.0216,ROR1A=1.2581,RT2R1A=0.7133,
 RH2RT2=0.3493,CDT2=.0023,MU=.5804E-04,K=5,IU=1,NSTAR=0 $
      THIS IS THE SAME CASE USING U.S. CUSTOMARY UNITS
 $INPUT R=38.68,N=38500.,TTIN=1950.,PTIN=13.2,W=.611,POW=30.,MU=.39E-04,IU=2 $
```

TABLE II. - OUTPUT FOR SAMPLE CASE USING SI UNITS

```
                                       RADIAL INFLOW TURBINE VELOCITY DIAGRAM DESIGN ANALYSIS
      THIS IS A SAMPLE CASE USING SI UNITS

THIS OUTPUT IS IN THE FOLLOWING UNITS.

TEMPERATURE  PRESSURE    GAS CONST   ROT SPEED   MASS FLOW     POWER     VISCOSITY   VELOCITY    SPEC WORK   DIAMETER       ANGLE
  KELVIN      N/SQ CM    JLS/KG-K     RAD/SEC     KG/SEC         KW     N-SEC/SQ M     M/SEC       JLS/GM       CM         DEGREES

*INPUT*
INLET TEMP = 1083.3300       STATOR IN ANGLE  = 55.60      STATOR KE LOS COEF=1.0000     GAS CONSTANT =208.1110
INLET PRESS=    9.1011       STATOR EX ANGLE  = 72.00      ROTOR KE LOSS COEF=1.0000     SPEC HT RATIO=1.6670
ROTAT SPEED=  4031.710       STATOR ASPECT RAT= .0000      ROTOR IN/DEL RVU  =1.0000     VISCOSITY     =  .5804-04
MASS FLOW  =     .2771       DIAM RATIOS                   ROTOR EX RAD SECTS= 5
SHAFT POWER=    22.371        STAT IN/ROT IN  =1.2581      ROT EX SECT/MN RVU=1.00 1.00 1.00 1.00 1.00
                              STAT EX/ROT IN  =1.0216
                              ROT EX TP/ROT IN= .7133
                              ROT EX HUB/TIP  = .3493      CL HT/ROT EX TIP D= .0023
*OUTPUT*
                         ABS      ABS                       ABS     ABS     ABS               REL      REL     REL     REL     REL
                DIA-    TOTAL    TOTAL    STATIC   STATIC   FLOW   VELO-    CRIT   BLADE     TOTAL    TOTAL    FLOW   VELO-    CRIT
               METER     TEMP    PRESS     TEMP    PRESS   ANGLE    CITY  VEL RAT  SPEED      TEMP    PRESS   ANGLE    CITY  VEL RAT
STATOR INLET   19.550  1083.33    9.101  1072.02    8.865  55.60   108.49   .204
LOSS COEF= .0640                                                                   STATOR HGT= 1.8527,NUMBER OF VANES= 16.0
STATOR EXIT    15.875  1083.33    8.995  1013.94    7.623  72.00   268.67   .506

ROTOR INLET    15.539  1083.33    8.995  1010.85    7.565  71.92   274.59   .517   313.24  1020.45    7.746  -31.50    99.93   .194
LOSS COEF= .2820
ROTOR EXIT      3.872   926.12    5.925   908.32    5.644    .00   136.09   .277    78.05   931.98    6.019  -29.83   156.88   .319
                4.593   926.12    5.914   908.99    5.644    .00   133.49   .272    92.59   934.36    6.046  -34.74   162.46   .330
                6.035   926.12    5.886   910.68    5.644    .00   126.74   .258   121.66   940.35    6.115  -43.83   175.69   .355
                7.478   926.12    5.852   912.83    5.644    .00   117.60   .240   150.74   947.96    6.203  -52.04   191.19   .385
                8.920   926.12    5.810   915.43    5.644    .00   105.45   .215   179.82   957.21    6.310  -59.61   208.46   .418
               10.363   926.12    5.762   918.50    5.644    .00    89.06   .181   208.90   968.07    6.436  -66.91   227.09   .453
               11.084   926.12    5.735   920.20    5.644    .00    78.48   .160   223.44   974.11    6.507  -70.65   236.82   .470

                         MASS     DIAG
                DIA-     FLOW     SPEC    TOTAL   STATIC
               METER     RATE    WORK     EFFIC    EFFIC    NUMBER OF ROTOR BLADES= 12.0
                3.872    .0000   81.769    .919    .834
                4.593    .0414   81.769    .916    .834
                6.035    .0516   81.769    .907    .834
                7.478    .0592   81.769    .896    .834
                8.920    .0631   81.769    .883    .834
               10.363    .0617   81.769    .868    .834
               11.084    .0000   81.769    .860    .834

 *OVERALL PERFORMANCE*
                                                LOSS/IDEAL T-S DEL H
      TOT-TOT PRESSURE RATIO =  1.5553             STATOR    = .0252
      TOT-STAT PRESSURE RATIO=  1.6125             ROTOR     = .0779
      DIAG AVG SPECIFIC WORK = 81.7686             WINDAGE   = .0048
      DIAG TOTAL EFFICIENCY  =   .8915             CLEARANCE= .0059
      DIAG STATIC EFFICIENCY =   .8339             EXIT KE   = .0658
      NET TOTAL EFFICIENCY   =   .8800
      NET STATIC EFFICIENCY  =   .8232        SPECIFIC SPEED=   .739
```

TABLE III. - OUTPUT FOR SAMPLE CASE USING U.S. CUSTOMARY UNITS

```
     THIS IS THE SAME CASE USING U.S. CUSTOMARY UNITS

THIS OUTPUT IS IN THE FOLLOWING UNITS.

TEMPERATURE  PRESSURE    GAS CONST   ROT SPEED   MASS FLOW     POWER      VISCOSITY   VELOCITY    SPEC WORK   DIAMETER       ANGLE
DEG RANKINE  LB/SQ IN    BTU/LB-R     REV/MIN     LB/SEC         HP       LB/FT-SEC    FT/SEC      BTU/LB        IN         DEGREES

*INPUT*
INLET TEMP = 1950.0000        STATOR IN ANGLE  = 55.60      STATOR KE LOS COEF=1.0000     GAS CONSTANT = 38.6800
INLET PRESS=   13.2000        STATOR EX ANGLE  = 72.00      ROTOR KE LOSS COEF=1.0000     SPEC HT RATIO=1.6670
ROTAT SPEED= 38500.000        STATOR ASPECT RAT= .0000      ROTOR IN/DEL RVU  =1.0000     VISCOSITY    =  .3900-04
MASS FLOW  =     .6110        DIAM RATIOS                   ROTOR EX RAD SECTS= 5
SHAFT POWER=    30.000          STAT IN/ROT IN   =1.2581    ROT EX SECT/MN RVU=1.00 1.00 1.00 1.00 1.00
                                STAT EX/ROT IN   =1.0216
                                ROT EX TP/ROT IN= .7133
                                ROT EX HUB/TIP   = .3493    CL HT/ROT EX TIP D= .0023
*OUTPUT*
                        ABS      ABS                          ABS     ABS     ABS                REL      REL      REL      REL      REL
               DIA-    TOTAL    TOTAL    STATIC   STATIC    FLOW    VELO-    CRIT   BLADE      TOTAL    TOTAL    FLOW    VELO-    CRIT
               METER    TEMP    PRESS     TEMP     PRESS    ANGLE    CITY  VEL RAT  SPEED       TEMP    PRESS    ANGLE    CITY  VEL RAT
STATOR INLET   7.697  1950.00   13.200  1929.63   12.858   55.60   355.93   .204
LOSS COEF= .0640                                                                STATOR HGT=  .7294,NUMBER OF VANES= 16.0
STATOR EXIT    6.250  1950.00   13.046  1825.10   11.057   72.00   881.46   .506

ROTOR INLET    6.118  1950.00   13.046  1819.53   10.973   71.92   900.88   .517  1027.71  1836.81   11.235  -31.50   327.85    .194
LOSS COEF= .2820
ROTOR EXIT     1.524  1667.02    8.593  1634.98    8.186     .00   446.49   .277   256.06  1677.56    8.729  -29.83   514.70    .319
               1.808  1667.02    8.577  1636.19    8.186     .00   437.98   .272   303.76  1681.86    8.769  -34.74   533.00    .330
               2.376  1667.02    8.537  1639.23    8.186     .00   415.83   .258   399.16  1692.64    8.869  -43.83   576.40    .355
               2.944  1667.02    8.487  1643.09    8.186     .00   385.83   .240   494.56  1706.34    8.996  -52.04   627.26    .385
               3.512  1667.02    8.427  1647.78    8.186     .00   345.96   .215   589.96  1722.97    9.152  -59.61   683.91    .418
               4.080  1667.02    8.357  1653.30    8.186     .00   292.19   .181   685.36  1742.53    9.335  -66.91   745.05    .453
               4.364  1667.02    8.318  1656.37    8.186     .00   257.47   .160   733.06  1753.41    9.438  -70.65   776.96    .470

                        MASS     DIAG
               DIA-     FLOW     SPEC     TOTAL    STATIC
               METER    RATE     WORK     EFFIC    EFFIC    NUMBER OF ROTOR BLADES= 12.0
               1.524   .0000   35.178     .919     .834
               1.808   .0914   35.178     .916     .834
               2.376   .1138   35.178     .907     .834
               2.944   .1305   35.178     .896     .834
               3.512   .1392   35.178     .883     .834
               4.080   .1361   35.178     .868     .834
               4.364   .0000   35.178     .860     .834

*OVERALL PERFORMANCE*
                                                LOSS/IDEAL T-S DEL H
     TOT-TOT PRESSURE RATIO =  1.5553              STATOR    = .0252
     TOT-STAT PRESSURE RATIO=  1.6125              ROTOR     = .0779
     DIAG AVG SPECIFIC WORK = 35.1777              WINDAGE   = .0048
     DIAG TOTAL EFFICIENCY  =   .8915              CLEARANCE= .0059
     DIAG STATIC EFFICIENCY =   .8339              EXIT KE   = .0658
     NET TOTAL EFFICIENCY   =   .8800
     NET STATIC EFFICIENCY  =   .8232           SPECIFIC SPEED= 95.355
```

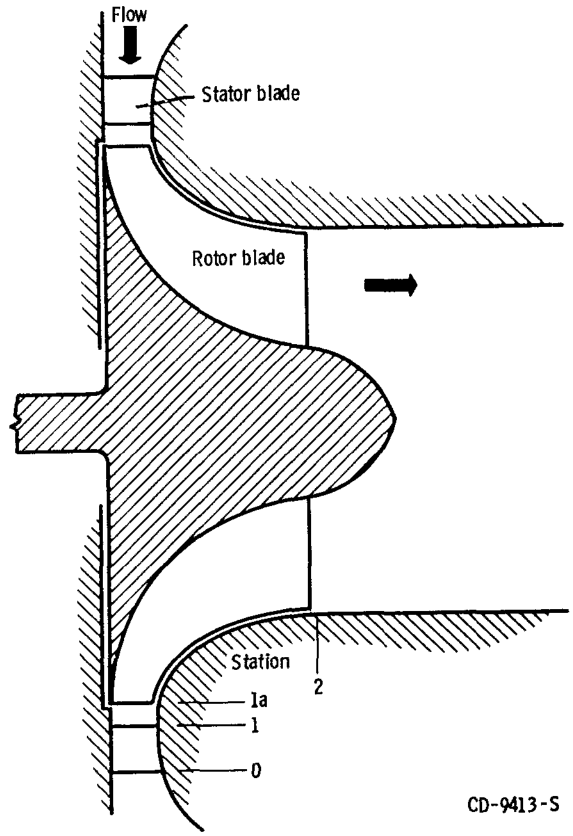


Figure 1. - Schematic cross section of radial-inflow turbine.

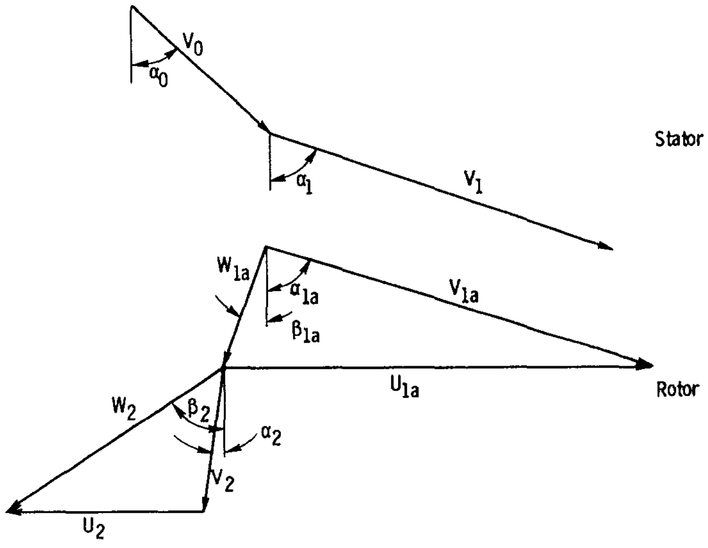
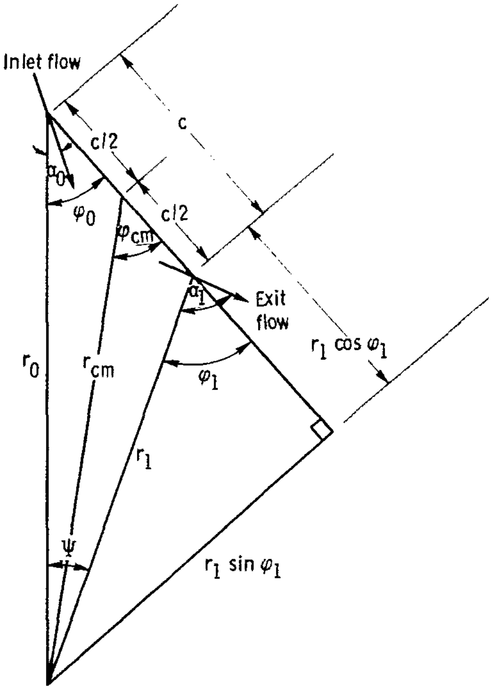
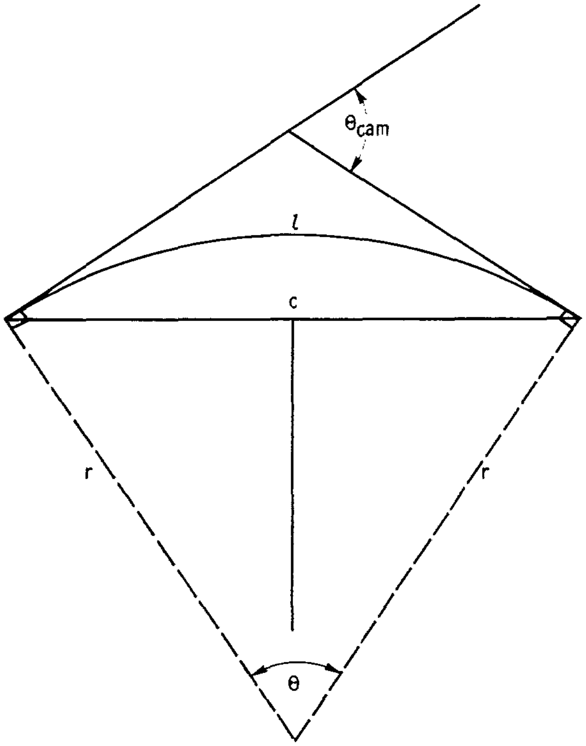


Figure 2. - Velocity diagram.



(a) Chord.



(b) Surface.

Figure 3. - Stator geometry model.

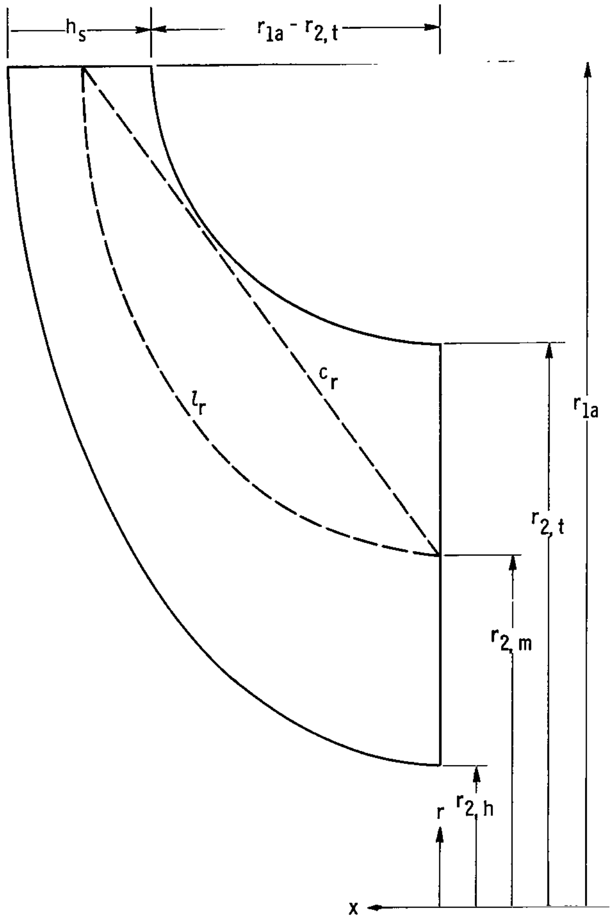


Figure 4. - Rotor geometry model.

NASA-Langley, 1976 E-8394