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By
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COMPUTER SIMULATION OF MULTI-STOREY STRUCTURES SUBJECTED TO GROUND MOTION

BY H. S. WARD

ABSTRACT

A hybrid computer system has been used to simulate some five-storey structures excited by ground motion. Five types of buildings were investigated to find the effect of some structural parameters on the dynamic response of the buildings. The factors that have been considered are the distribution of stiffness and mass in the structure, the type of foundation condition, and the action of viscous damping.

The results indicate that it may be reasonable to calculate the base shear forces, created by ground motion, as a function of the fundamental period of the building. In the case of lightly damped structures, however, the distribution of forces through the height of the structure is also shown to be dependent on the other dynamic characteristics of the building, as well as the frequency content of the ground motion.

Viscous damping considerably reduces the forces acting on a structure, and also tends to eliminate modes of vibration other than the fundamental. A pinned-end foundation condition is also shown to reduce the forces acting on the lower storeys of a building compared with the fixed-end condition.

INTRODUCTION

The most basic problem involved in earthquake design is the relation between the frequency content of the ground motion and the frequency response of a structure. Housner (1) has studied this problem for single-degree-of-freedom systems, and Clough (2) has considered the extension of this work to the case of multi-degree-of-freedom systems. Clough considered each of the modes of vibration of a structure could be excited independently, and then assumed the response of the structure was equal to the summation of these individual responses.

The work reported here describes an investigation into certain basic factors that influence the response of multi-degree-of-freedom structures that are subjected to ground motion excitation. The structures investigated consist of five-storey buildings, one bay wide, founded on a rigid base. The factors investigated are the distribution of stiffness and mass through the structure, the action of viscous damping, the effect of the support condition at the foundation, and two types of ground motion.

Both the structures and the ground motion were simulated on an analogue computer. Use was also made of a hybrid analogue-digital system to facilitate data reduction. The hybrid computer system provides a method of obtaining useful statistical information about the forces acting on structures during any type of ground acceleration excitation. The structures that have been analysed here are comparatively simple, but the technique can be applied to larger and more complex structures.

Power spectrum analyses of the building responses were made, and this information gives an insight into the behaviour of rigid-framed structures under the dynamic action of ground motion. It appears that the fundamental mode of vibration predominates in the upper storeys and higher modes of vibration appear in the lower storeys.

THE EQUATIONS OF MOTION

The general configuration of the framed structure that was investigated is shown in Fig. 1a. Throughout this work it is assumed that there is no coupling among

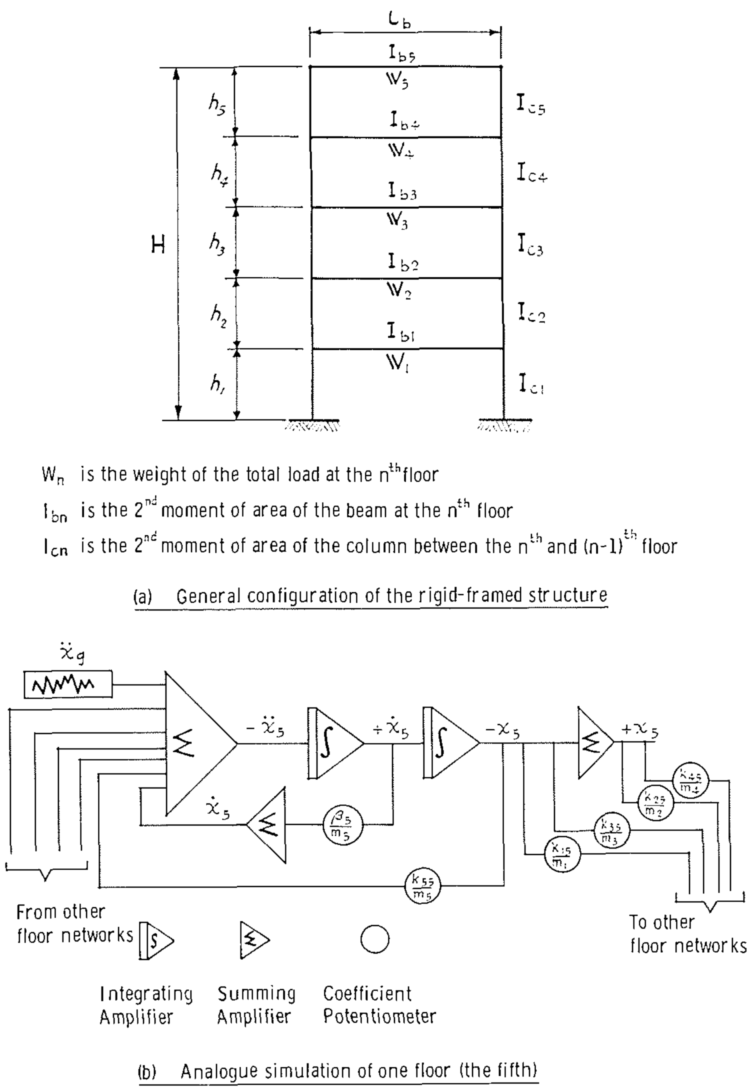

FIG. 1. The structure and the analogue simulation of one floor (note $W_n/g = m_n$).

the lateral, vertical and torsional modes of vibration. The equations of motion for the five-storey structure subjected to a ground acceleration, $\ddot{x}_g$, are

$$(W_n/g)\,(\ddot{x}_g + \ddot{x}_n) = k_{n5}x_5 + k_{n4}x_4 + k_{n3}x_3 + k_{n2}x_2 + k_{n1}x_1 - \beta_n(4\pi f)\dot{x}_n\,, \quad (1)$$

where $n = 1, 2, 3, 4$ and 5. In (1), g is the acceleration due to gravity; β represents the amount of viscous damping referred to the fundamental frequency, f; the k_{ij}'s are elements of the stiffness matrix of the structure; x is the deformation of the structure with respect to the foundation.

Altogether five different structures, designated A, B, C, D and E, were investigated. In structure A all column and beam stiffnesses were equal. For structure B all column stiffnesses were equal and finite, and the beam stiffnesses were infinite (shear-type structure). In practice column stiffnesses will vary through the height of the structure and structure C represents this characteristic. If the value of the stiffness for the top storey column is taken to be one, then the stiffnesses of the remaining members in structure C are as follows: $I_{c4}/h_4 = 1.00$, $I_{c3}/h_3 = 1.34$, $I_{c2}/h_2 = 1.807$, $I_{c1}/h_1 = 2.00$, $I_{b5}/1_b = 1.506$, $I_{b4}/1_b = 1.265$, $I_{b3}/1_b = 1.265$, $I_{b2}/1_b = 1.265$, $I_{b1}/1_b = 1.807$.

Structures A, B and C were all assumed to have a fixed-ended foundation condition. The members for structure D have the same stiffness as structure C but the foundation condition was pin-ended. Finally, structure E had the same column stiffnesses as structure C but all beams were of infinite stiffness; the foundation condition was fixed-ended.

Calculations of the Natural Frequencies and Modes of Vibration

The natural frequencies and modes of undamped free vibrations are obtained from (1) when β_n and $\ddot{x}_g$ are zero, and the displacements, x_n, are defined by

$$x_n = A_n \sin wt. \tag{2}$$

If it is assumed that the structures are constructed in steel and a working stress of 10 tons/sq in. is chosen, then the order of W_n/pg is 0.01, where p is equal to $6EI_{c5}/(h_5)^3$ (E is Young's modulus). With $W_n/pg = 0.01$ and all the W_n equal, the modes and frequency of vibration of structures A, B, C, D and E were calculated; the fundamental frequencies for structures A to E were 0.575 cps, 0.910 cps, 0.703 cps, 0.545 cps and 1.11 cps, respectively. If any other value of W_n/pg is to be used the frequencies are multiplied by $\frac{1}{10}\sqrt{pg/W_n}$, and the mode shapes are unaltered.

Analogue Simulation of the Structures

The basic circuit diagram used for the simulation of the structures is shown in Fig. 1b. The simulation of each floor requires five operational amplifiers. If the velocities and displacements only are of interest it is possible to reduce this requirement to four amplifiers.

For this circuit to be stable the potentiometer settings representing the stiffness coefficients must be set up accurately. The critical settings are the coefficients k_{ij} and k_{ji}. If $k_{ij} \neq k_{ji}$ then the structure does not obey Maxwell's law of reciprocal deflections. In this case any small amount of noise in the system produces large oscillations in the amplifiers. Adequate accuracy can be obtained from the components of the analogue computer in which the potentiometers could be set to four significant digits.

It was possible to change the time scale of the simulation by altering the gains on all integrating amplifiers by a constant ratio. This detail is mentioned since most of the analogue simulation was in real time, but the hybrid computer work was carried out five times slower than real time.

Experimental Work

The experimental work can be divided into two major parts. The first set of experimental results concerned the average maximum displacements that occur at each floor of the structures, due to ground motions that have some of the characteristics of earthquakes.

The results were obtained by subjecting the structures to 25-sec bursts of the simulated ground motion and recording the resulting displacements. For each burst the maximum displacement of each floor was taken to represent the response of the structure. A three-channel frequency-modulated tape recorder was used to record some of the output voltages from the analogue computer, so that power spectrum density analyses of these outputs could be performed. Analyses were made of the displacements at the fifth, third, and first floors to give a representation of what happens through the height of the structure.

The second part of the experimental work was concerned with determining the way in which forces were distributed through the height of a structure at particular instants of time. This phase of the work was performed through the use of a hybrid system consisting of a Pace analogue computer and a Bendix G15 digital computer, coupled together by means of a digital-analogue converter. The converter changes analogue voltages to digital information or vice versa under control of the G15 or an accurate clock. Eight tracks of information can be sampled simultaneously.

The operations performed by the hybrid system were as follows. First the program was loaded into the digital computer, after which the analogue was automatically put into the "hold" mode and the timing circuit was reset. The computer now waits for the type-in of the mass and height above ground level of each floor. The program was written for a maximum of eight floors, so for the case of the five-storey buildings W_6, W_7, and W_8 were entered as zeros.

After the entry of the last height the analogue computer is put first into the "reset" mode and then the "operate" mode. This switches on the simulated ground motion, and permits the simulated structure to vibrate.

The program now controls an analogue-to-digital conversion. The voltages that were sampled represented the absolute accelerations of each floor. Because of the limited space in the memory of the G15 it was possible to take only 60 samples of each of the eight channel outputs from the analogue computer. A sampling interval of 2 sec was dictated by the execution time required by the G15 to perform the necessary operations, but by running the simulation 5 times slower than real time an effective sampling interval of 0.4 sec was obtained. Thus in real time 60 samples, at 0.4 sec intervals, were obtained for a ground disturbance of 24 sec duration. This is taken to be the length of a typical strong-motion earthquake.

At each sampling time, eight absolute accelerations were sampled and the digital information was stored in memory. After 60 samples the analogue is put into the "hold" mode which locks all the amplifier outputs. The G15 now operates on the stored information in memory. It takes the first sample and multiplies the accelerations of each floor by its mass. The resulting force at each floor is then typed out.

The sum of these forces is formed, to give the base shear, together with the moment of the forces about the base (the overturning moment), and this information is also typed out. The absolute values of each of the forces, base shear, and overturning moment are now stored in memory.

This process is repeated for each of the remaining 59 samples. After the type-out of the last overturning moment the sums of the absolute values of the forces, base shear, and overturning moment are calculated. These sums are divided by 60 and the average values are finally obtained. This ends the program, the total running time of which is 23 minutes.

At this stage the object of the work was to obtain statistical information about the forces acting on a structure. If the object is to describe the motion in detail then the sampling rate is set by the highest mode that is to be detected; the sampling rate must be at least twice the frequency of the highest mode. The sampling rate used during this study will describe the fundamental mode of vibration of most practical structures. It is possible to obtain a higher sampling rate than the one used during the present work by allowing the simulated earthquake to take place over more than one run of the program.

Experimental Results

Displacement Responses to White Noise

The first type of ground motion was simulated by a band-limited white noise generator. The power spectrum of the generator was flat from 0 to 35 cps. Bycroft (3) has shown that bursts of white noise give velocity response results similar to Housner's average earthquake spectra, except for systems with fundamental frequencies greater than the order of 5 cps. Above this frequency the white noise responses were greater than the actual earthquake responses. In the present experiments the generator was adjusted so that the average maximum acceleration of each burst was 7.00 ft/sec^2, and the rms value of the bursts was 2.9 ft/sec^2.

The average maximum displacements for structure A, obtained in each instance from ten experiments, are shown in table I, where the results indicate the consequence of varying W_n/pg and β_n. The standard deviation of the displacements for the case when $W_n/pg = 1.0 \times 10^{-2}$ and $\beta_n = 0$ are also shown. For this particular case the average maximum displacements are compared with the fundamental mode shape, and the two are seen to be in fair agreement. This comparison was obtained by assuming that the fifth floor displacement in the fundamental mode was equal to the experimental displacement.

The displacement results for the other structures subjected to white noise all had the same trend, namely, they increased from the first to the fifth floor. The records of the displacement generally showed that they were in phase, but higher frequency components were apparent especially at the lower floors. A more precise description of the motion can be derived from a power spectrum analysis of the structural vibrations.

The results of a power spectrum analysis for structure A and $W_n/pg = 1.0 \times 10^{-2}$ are shown by the solid line curves in fig. 2. The recording of each floor vibration was made on $\frac{1}{2}$-in. magnetic tape during one of the 25-sec bursts of white noise, and the analysis was performed with a band width of approximately $\frac{2}{3}$ cps. Although the vertical scales in fig. 2 are not absolute values, since the signal on the tape recorder had to be attenuated to avoid overloading the analysis equipment, the same attenuation was applied to each signal so fig. 2 gives a relative indication of the energy in the displacements as a function of the frequency.

It can be seen that there is only one peak in the analysis of the fifth and third floor displacements, but five distinct peaks occur in the first floor analysis. The energy in the second mode of vibration at the first floor is, however, only $\frac{1}{40}$ of that in the first mode. It seemed reasonable to assume that the average maximum displacements due to the white noise excitation occurred simultaneously and in phase.

TABLE I

Displacement Results for Structure A and White Noise Excitation

Floor Number	1st	2nd	3rd	4th	5th
Displacement (ft) $W_n/pg = 1.0 \times 10^{-2}$, $\beta_n = 0$	0.105	0.230	0.303	0.395	0.438
Standard Deviation of Displacement (ft)	0.033	0.061	0.063	0.106	0.116
Fundamental Mode Shape	0.084	0.206	0.315	0.394	0.438
Displacement (ft) $W_n/pg = 0.5 \times 10^{-2}$, $\beta_n = 0$	0.074	0.197	0.286	0.360	0.393
Displacement (ft) $W_n/pg = 0.2 \times 10^{-2}$, $\beta_n = 0$	0.044	0.094	0.127	0.145	0.166
Displacement (ft) $W_n/pg = 0.14 \times 10^{-2}$, $\beta_n = 0$	0.047	0.090	0.122	0.146	0.158
Displacement (ft) $W_n/pg = 0.08 \times 10^{-2}$, $\beta_n = 0$	0.037	0.064	0.084	0.104	0.112
Displacement (ft) $W_n/pg = 1.0 \times 10^{-2}$, $\beta_n = 0.20$	0.024	0.055	0.078	0.093	0.100
Displacement (ft) $W_n/pg = 1.0 \times 10^{-2}$, $\beta_n = 0.50$	0.017	0.037	0.048	0.067	0.072
Displacement (ft) $W_n/pg = 1.0 \times 10^{-2}$, $\beta_n = 1.00$	0.012	0.027	0.037	0.043	0.043

The average maximum ground acceleration was 7.00 ft/sec², standard deviation 0.815 ft/sec².

On this assumption the displacement results were used to calculate the lateral forces acting at each floor level assuming that the weight of each floor was W lb. Five different values of W_n/pg for structure A were simulated; fig. 3 shows the results of the lateral forces acting at each floor level for the case of no damping.

The displacement results for structures A, B, and C have been condensed into fig. 4 which presents the base shear as a function of the fundamental period of vibration. The dotted line indicates the base shear that would occur if a rigid structure of weight $5W$ were subjected to a ground acceleration of 7 ft/sec². Because, in many buildings heavy service equipment is placed at the top of the structure, a few experiments were conducted with the mass of the fifth floor twice that of the other floors. Fig. 4 shows that the base shear lies on the curve representing the case for all floors of equal weight. Excessive vibrations of service floors are unlikely to occur in practical structures since they require a combination of large loads and inadequate stiffness of structural members.

The outcome of applying viscous damping to one of the structures is indicated in fig. 5. Power spectrum results showed that viscous damping eliminated modes higher than the fundamental.

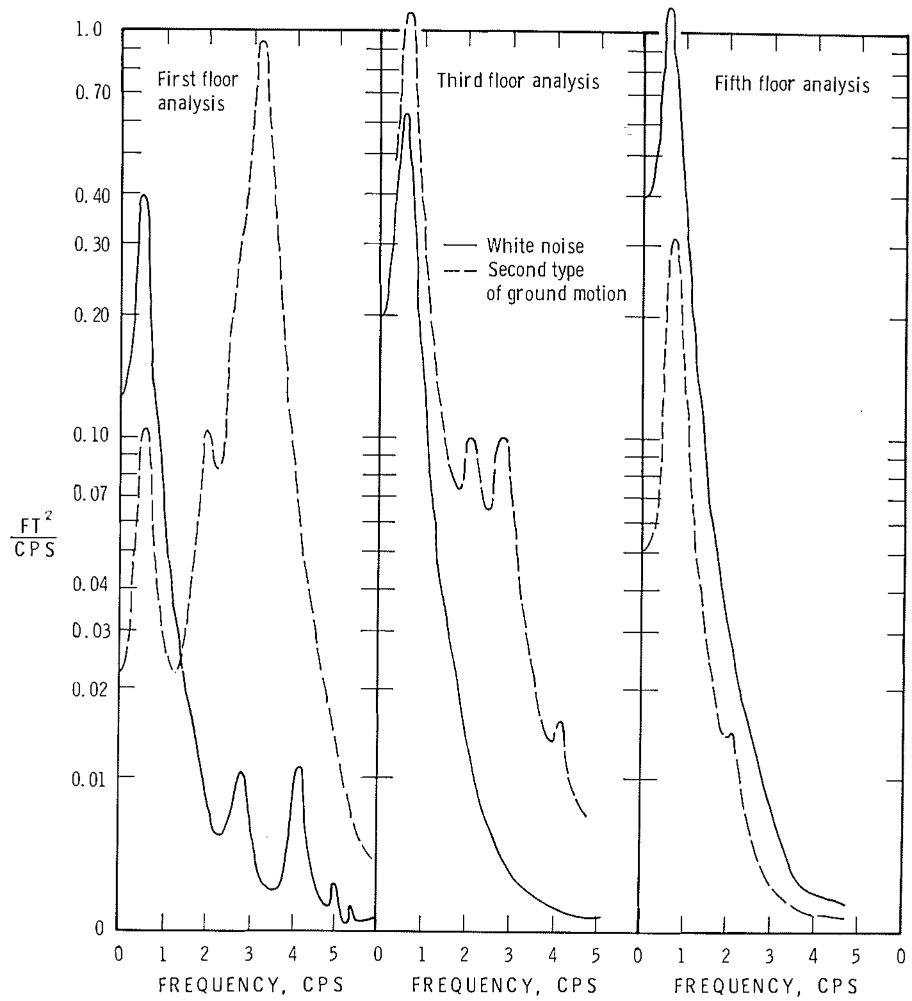

FIG. 2. Power spectra analyses for structure "A" subjected to two ground motions.

Displacement Responses to Second Type of Ground Motion

The ground motion due to an earthquake is influenced by several parameters. Foremost among these is distance from the epicentre and the type of geological formation through which the waves are transmitted. At points some distance from the epicentre, two distinct phases of ground motion can be distinguished, the P and S waves. This characteristic was approximated in the second type of ground motion, produced by recording on an FM tape recorder the output from a saw-tooth generator. The amplitude and frequency of the generator were varied during the recording to simulate the changes in character that occur in typical earthquakes. Six such simulations were made. The spectrum of one of these is represented by the solid line curve of fig. 6. The broken line represents the power spectrum obtained from an approximation to one of the components of the Olympia, Washingtio, earthquake of 13 April 1949. (Fig. 6 represents the relative values only of the ground accelerations.) The higher frequency peak of the simulated ground motion

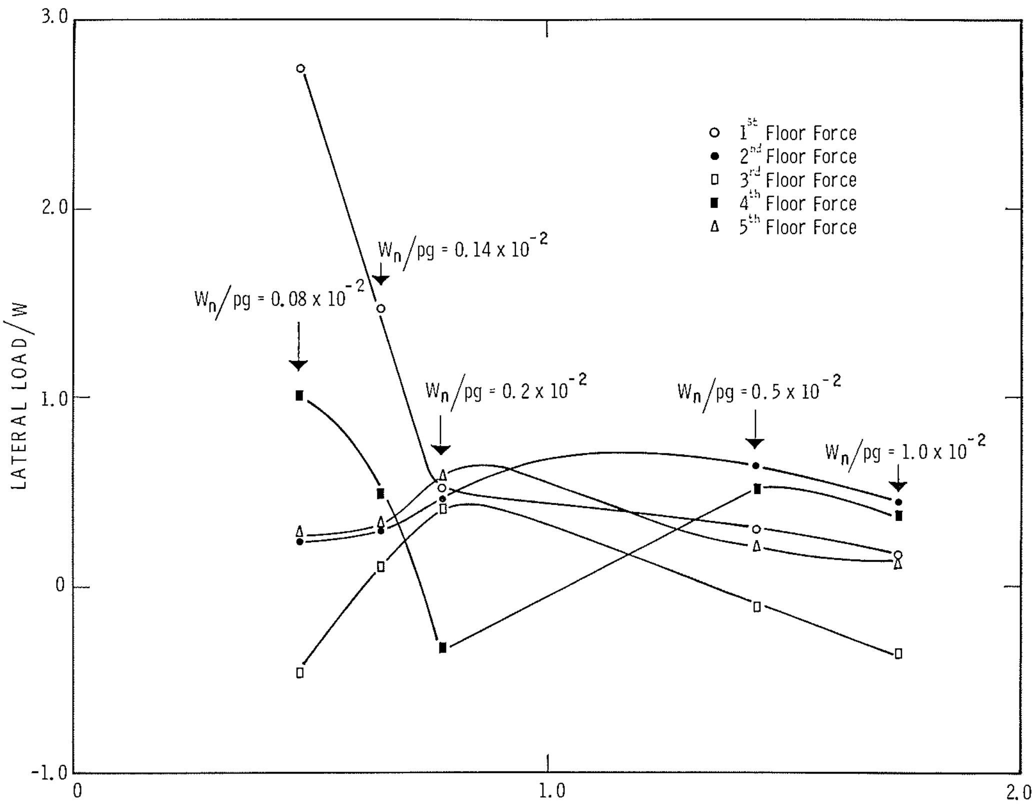

FIG. 3. Distribution of lateral forces for structure "A" excited by white noise.

is larger than in this particular earthquake, but for the structures investigated here ground motion frequencies above 5 cps will not be very significant.

Each of the six recordings was used to investigate structures of type A with five different values of W_n/pg and $\beta_n = 0$. The results of the average maximum deflections for each floor are given in table II. In the second type of ground motion the response was not necessarily in the fundamental mode. This is illustrated by the power spectrum analyses for the case when $W_n/pg = 1.0 \times 10^{-2}$ shown in fig. 2 by the dotted line curves. It can be seen that the third mode predominates on the bottom floor; this occurs since the natural frequency of the third mode is

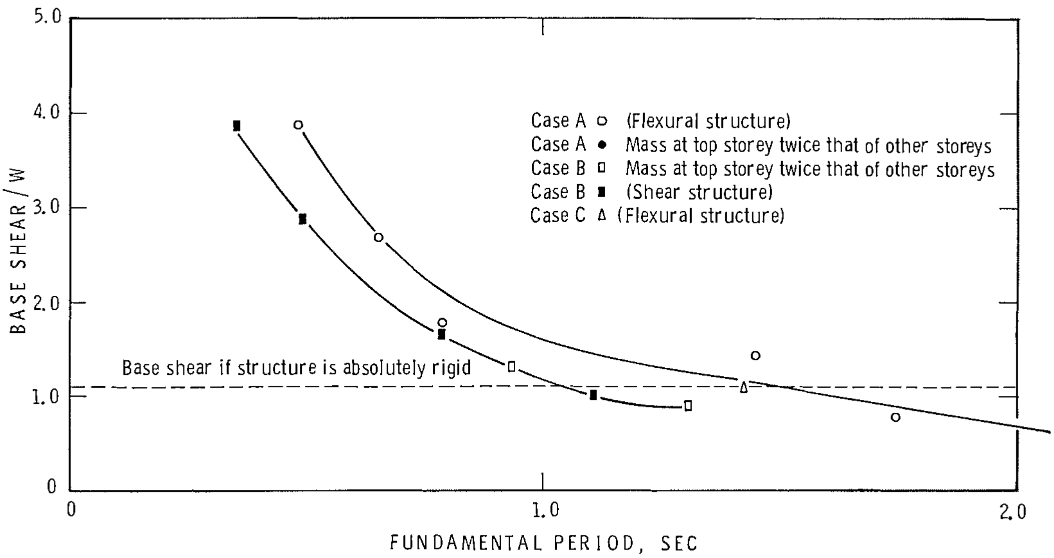

FIG. 4. Base shear forces due to white noise excitation.

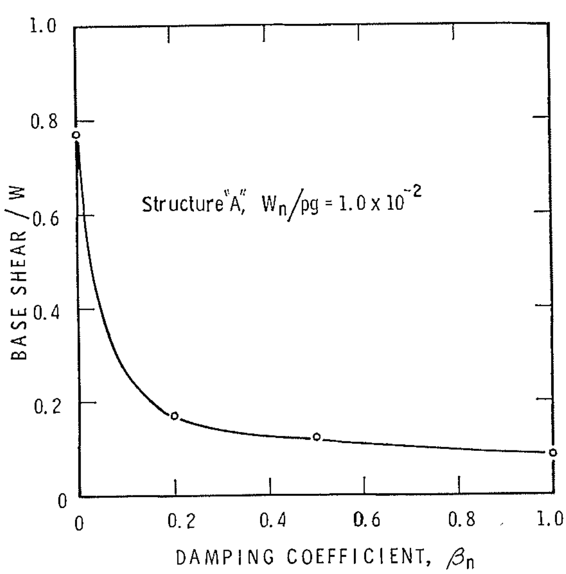

FIG. 5. Reduction in base shear due to viscous damping.

nearest to the peak in the power spectrum for this type of motion (see fig. 6). At the upper floors, however, the fundamental mode becomes dominant again.

Hybrid Computer Results

Use of the hybrid computer meant that the simulation had to be operated at one-fifth of real time. The effective band width of the second type of ground motion was the same for the hybrid computer results as for the displacement results. At the time of the experiments there was no means readily available to limit the band width of the white noise generator. As a consequence the effective band width of the generator was extended to 175 cps. The increase of band width for the white noise generator produced a decrease in the displacement response of the structures by as much as 75 %, for the same rms input.

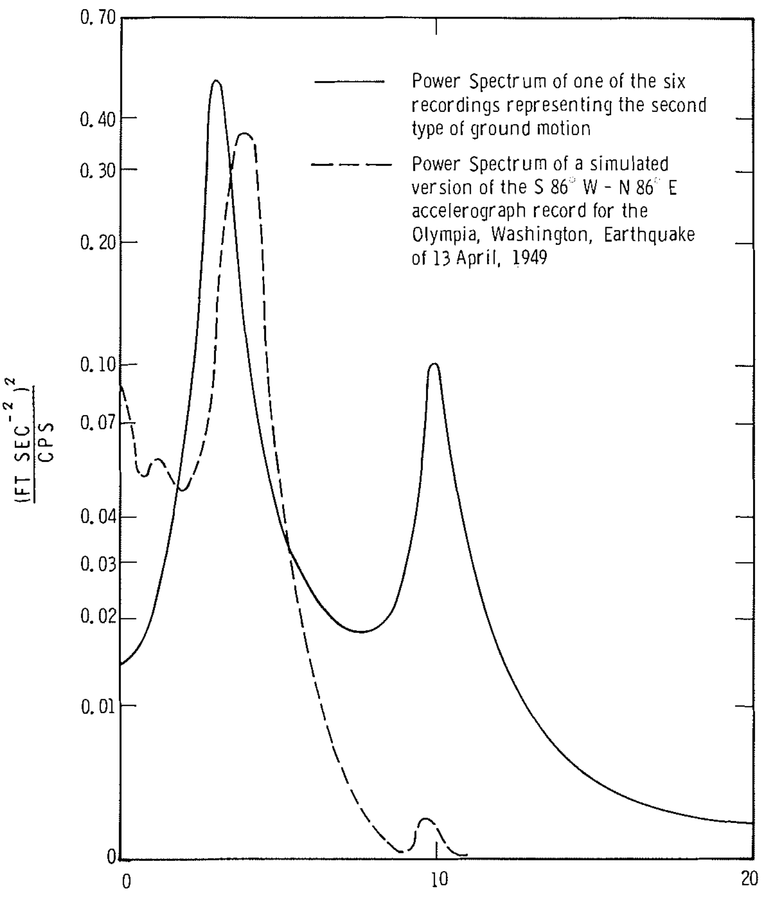

FIG. 6. Power spectrum of second type of ground motion.

The forces calculated from the displacement results were based on the assumption that the average maximum deflections occurred simultaneously and were in phase. In actual fact, higher modes of vibration arise and thus it is possible that a more critical loading might occur. It would be tedious to calculate forces as a function of time from graphical records but the hybrid computer provides 60 such samples with very little labour.

Some of the results for structure C, when $W_n/pg = 1.0 \times 10^{-2}, \beta_n = 0$, and the ground excitation is of the second type, are shown in table III. The results are based on the assumption that each floor has a mass of 1000 lb and each storey height is 10 ft.

The first line of figures in table III represents the forces, base shear, and overturning moments acting on the structure 0.4 sec after the ground motion started. At this time the force at the first floor is equal to 1684 lb-wt, or in other words the absolute acceleration of the first floor is 1.684 ft/sec^2. At 5.2 sec after the motion

TABLE II

DISPLACEMENT RESULTS FOR STRUCTURE A AND THE SECOND TYPE OF GROUND MOTION

Floor Number	1st	2nd	3rd	4th	5th
Displacement (ft) $W_n/pg = 1.0 \times 10^{-2}$, $\beta_n = 0$	0.117	0.102	0.120	0.142	0.143
Displacement (ft) $W_n/pg = 0.5 \times 10^{-2}$, $\beta_n = 0$	0.029	0.053	0.064	0.058	0.064
Displacement (ft) $W_n/pg = 0.2 \times 10^{-2}$, $\beta_n = 0$	0.029	0.049	0.048	0.057	0.083
Displacement (ft) $W_n/pg = 0.14 \times 10^{-2}$, $\beta_n = 0$	0.021	0.042	0.051	0.068	0.085
Displacement (ft) $W_n/pg = 0.08 \times 10^{-2}$, $\beta_n = 0$	0.020	0.035	0.047	0.061	0.069

The average maximum ground acceleration was 8.00 ft/sec^2, standard deviation 0 ft/sec^2.

TABLE III

HYBRID COMPUTER RESULTS FOR STRUCTURE C, $W_n/pg = 1.0 \times 10^{-2}$, $\beta_n = 0$

Time from Start of Earthquake, sec	Force at 1st Floor, lb-wt	Force at 2nd Floor, lb-wt	Force at 3rd Floor, lb-wt	Force at 4th Floor, lb-wt	Force at 5th Floor, lb-wt	Base Shear, lb-wt	Overturning Moment, lb-ft
0.4	−1,684	−1,855	−1,342	−1,489	−1,464	−7,836	−227,050
0.8	−2,050	−1,147	−1,977	−1,245	−1,611	−8,032	−233,160
1.2	−2,392	−756	−1,953	−952	−1,709	−7,763	−221,190
4.0	683	−1,464	5,053	1,489	1,709	7,470	274,170
5.2	1,733	3,125	756	−1,489	−1,196	2,929	−16,845
8.0	−830	−1,806	683	2,807	0	854	88,379
9.6	−415	−1,586	9,155	3,906	−2,832	8,227	253,420
12.0	2,148	1,269	−9,814	−4,443	1,123	−9,716	−369,140
14.8	219	9,057	10,010	6,787	2,124	28,198	861,330
16.0	−1,977	4,833	927	3,247	3,588	10,620	414,060
20.4	−13,501	−7,299	23,315	19,678	−14,722	7,470	469,480
24.0	10,815	10,254	−12,231	−13,623	15,088	10,303	155,760

Average Force at 1st Floor, lb-wt	Average Force at 2nd Floor, lb-wt	Average Force at 3rd Floor, lb-wt	Average Force at 4th Floor, lb-wt	Average Force at 5th Floor, lb-wt	Average Base Shear, lb-wt	Average Overturning Moment, lb-ft
4,901	4,550	6,054	5,706	4,291	10,374	262,890

started the maximum ground acceleration of 8 ft/sec^2 occurred; at 14.8 sec the maximum base shear of all the 60 samples occurred and at 20.4 sec the largest measured lateral force arose at the third floor. The results in table III show that modes of vibration higher than the fundamental can be responsible for large forces at a given floor, e.g., the force at the third floor after 20.4 sec of ground motion.

Since the nature of an earthquake cannot be predicted beforehand the most reasonable approach for determining earthquake loads must be based on some statistical process. The complete set of results of which table III are a part can be used to plot statistical information concerning the base shear. Thus, for example, one can determine the frequency of occurrence of base shears within a given interval and the probability of base shear exceeding a given value. In this paper, however, the remaining results are presented in terms of the average forces at each level, the average base shear, and the overturning moment.

TABLE IV

HYBRID COMPUTER RESULTS FOR THE SECOND TYPE OF GROUND MOTION

Floor Number	1st	2nd	3rd	4th	5th	Base Shear, lb-wt	Overturning Moment, lb-ft
	Lateral force, lb-wt	Lateral force, lb-wt	Lateral force, lb-wt	Lateral force, lb-wt	Lateral force, lb-wt		
Structure C, $\beta_n = 0$, $W_n/pg = 0.01$	4,901	4,505	6,054	5,706	4,291	10,374	262,890
Structure C, $\beta_n = 0$, $W_n/pg = 0.005$	26,220	38,392	32,282	7,327	30,125	46,983	286,040
Structure C, $\beta_n = 0$, $W_n/pg = 0.0033$	7,813	3,965	7,633	9,180	10,698	18,701	799,410
Structure C, $\beta_n = 0.284$, $W_n/pg = 0.01$	1,232	1,939	3,694	2,656	1,357	7,888	244,560
Structure C, $\beta_n = 0.20$, $W_n/pg = 0.005$	4,707	8,285	7,499	2,179	5,203	13,975	249,980
Structure C, $\beta_n = 0.164$, $W_n/pg = 0.0033$	1,418	1,443	1,647	3,025	3,419	7,018	287,650
Structure D, $\beta_n = 0$, $W_n/pg = 0.01$	3,905	3,407	5,352	3,866	3,347	10,437	319,590
Structure D, $\beta_n = 0$, $W_n/pg = 0.005$	3,406	4,389	4,670	2,997	3,469	13,032	399,840
Structure D, $\beta_n = 0$, $W_n/pg = 0.0033$	29,782	33,164	15,991	14,026	35,686	31,524	922,050
Structure E, $\beta_n = 0$, $W_n/pg = 0.01$	16,382	28,901	24,474	3,572	26,667	40,757	288,870
Structure E, $\beta_n = 0$, $W_n/pg = 0.005$	18,263	20,282	13,728	8,390	9,875	29,477	949,740
Structure E, $\beta_n = 0$, $W_n/pg = 0.0033$	4,496	2,878	6,210	7,065	7,268	17,717	683,400

The average maximum ground acceleration was 8 ft/sec^2 with a standard deviation of 0 ft/sec^2.

The information for the second type of ground motion is shown in table IV. The results apply for structures C, D, and E, subjected to a ground motion with an average maximum value of 8 ft/sec^2. The most critical condition appears when the frequency of the second mode coincides with the peak frequency of the ground motion. This is illustrated by the results for case C, $\beta_n = 0$, $W_n/pg = 0.005$ and case D, $\beta_n = 0$, $W_n/pg = 0.0033$. The pin-ended condition has the expected consequence of reducing the lateral load at the first floor in comparison to the fixed-ended condition. The results for the shear-type building, structure E, do not show such a sharply resonant condition as the flexural type buildings, C and D. It can be seen that viscous damping significantly reduces the structrual response but it does not prevent the second mode from being excited for structure C when $W_n/pg = 0.005$.

Conclusions

For the structures investigated here, both white noise and the second type of ground motion caused significant excitation of the higher modes of vibration; this is particularly true of the displacements on the lower floors. The white noise excitation probably contains too large a high-frequency content to give a true representation of an earthquake, other than in epicentral regions; the second type of ground motion, even though it is contrived, probably gives a more realistic representation of the frequency content of a distant earthquake. Viscous damping of the order of 10 to 20 per cent of critical damping considerably reduces the structural response and also reduces the effect of higher modes of vibration.

The results shown in fig. 4 indicate that it may be reasonable to calculate the base shear of a multi-degree-of-freedom system as a function of its fundamental period. The results of fig. 3, however, show that for undamped systems this base shear is not necessarily distributed through the structure as a function of its fundamental mode. The manner in which the forces are distributed is dependent mainly on the dynamic characteristics of the structure and the frequency content of the ground motion. It seems probable that the hybrid computer system could be used to investigate these factors for larger and more complex structures than those considered in this paper.

For white noise excitation the average base shear acting on structures with a fundamental period less than a second is greater than that predicted on the assumption that the structure is absolutely rigid. In the case of the second type of ground motion the base shear exceeds this predicted value only when one of the natural frequencies of the structure coincides with the peak frequency of the ground motion.

Structures C and D were investigated to obtain some idea of the effect on the structural response of the support condition at the foundation. The results showed that the over-all forces were not significantly different, but the lateral load at the first floor was smaller for the pin-ended condition.

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