

Computerized Wrist Pulse Signal Diagnosis Using Modified Auto-Regressive Models

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Abstract The wrist pulse signals can be used to analyze a person's health status in that they reflect the pathologic changes of the person's body condition. This paper aims to present a novel time series analysis approach to analyze wrist pulse signals. First, a data normalization procedure is proposed. This procedure selects a reference signal that is 'closest' to a newly obtained signal from an ensemble of signals recorded from the healthy persons. Second, an auto-regressive (AR) model is constructed from the selected reference signal. Then, the residual error, which is the difference between the actual measurement for the new signal and the prediction obtained from the AR model established by reference signal, is defined as the disease-sensitive feature. This approach is based on the premise that if the signal is from a patient, the prediction model previously identified using the healthy persons would not be able to reproduce the time series measured from the patients. The applicability of this approach is demonstrated using a wrist pulse signal database collected using a Doppler Ultrasound device. The classification accuracy is over 82% in distinguishing healthy persons from patients with acute appendicitis, and over 90% for other diseases. These results indicate a great promise of the proposed method in telling healthy subjects from patients of specific diseases.

Keywords Wrist pulse signal · Auto-regressive model · Time series analysis

Introduction

Wrist pulse signals contain vital information of health activities and can reflect the pathologic changes of a person's body condition. Therefore, the practitioners can tell the health conditions of a patient by feeling his wrist pulses, and this method has been used in traditional Chinese medicine for thousands of years. Modern clinical studies demonstrate that there is premature loss of arterial elasticity and endothelial function for patients with certain diseases, such as hypertension, hypercholesterolemia and diabetes [1]. Such loss will eventually decrease the flexibility of vasculature, whilst increase the stress to the circulatory system. As a result, the shape, amplitude and rhythm of patient wrist pulses will also alter in correspondence with the hemodynamic characteristics of blood flow [1].

Although traditional Chinese pulse diagnosis has been attracting more attention in recent years, the wrist pulse assessment is a matter of technical skill and subjective experience [2]. The intuitional accuracy mainly depends upon the individual's persistent practice and quality of sensitive awareness. Different practitioners may not give identical diagnosis results for the same patient. Therefore, it is necessary to develop computerized pulse signal analysis techniques to standardize and objectify the pulse diagnosis method. A couple of methods have been proposed to analyze the digitized pulse signals [3–7]. For example, Leonard et al. [3] revealed that it is possible to distinguish healthy and unwell children by using wavelet power features and wavelet entropy of the pulse signal. Zhang et al. [4] proposed a wavelet transform based method to extract features from carotid blood flow signals, and used a back-propagation (BP) neural network to make the classification among 30 samples. Some other researchers [5–6] also proved that it is possible to identify human sub-health

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status based on pulse signals by using linear discriminant classifier. Moreover, Zhang et al. [7] used the wavelet method to extract different pulse features, including wavelet powers, wavelet packet powers and Doppler ultrasonic diagnostic parameters. Although some of the above methods have achieved encouraging results, their effectiveness are still subject to further assessment due to the limited number of samples and types of diseases. For example, in Leonard's research [3], only 20 samples are used to distinguish well and unwell children, while in Zhang's research [7], two kinds of diseases are investigated.

In this paper, an auto-regressive (AR) based method is proposed to extract the pulse signal features. Since a wrist pulse signal is in essence a time series, using AR model can help to describe the characteristic of this signal and therefore to capture its important features. This AR model is first trained based on the healthy samples and then it is used to predict the input signal. The mean and variance of the prediction error are calculated and selected as features. Except for the AR features, some Doppler ultrasonic diagnosis parameters are also investigated in order to see if they can be helpful to improve the classification accuracy. The selected features are then taken as inputs to a support vector machine (SVM) for pattern classification because the SVM performs well on problems with low training set sizes. The applicability of the proposed method is tested on the established dataset including 100 healthy persons and 148 patients of diseases. There are four kinds of diseases investigated in this paper, i.e. 46 patients with pancreatitis (P), 42 with Duodenal Bulb Ulcer (DBU), 22 with appendicitis (A) and 38 with acute appendicitis (AA). It can be seen that both the sample numbers and disease types are much larger than those of previous researches mentioned above.

The rest of the paper is organized as follows. "The proposed method" section presents the proposed method. The "Experimental results" section performs experiments to validate the developed technique. The "Conclusions and future work" section concludes the paper and makes some discussion.

The proposed method

Feature extraction via AR modelling

AR models [8] are widely used in time series analysis, control and signal prediction. Considering the fact that wrist pulse signals are naturally a time series, the AR model can be used to analyze the time series and then extract the disease-sensitive features. Then, one branch of statistical hypothesis tests called support vector machine (SVM) is applied to the aforementioned features to classify the current subject to either a patient or a healthy person.

When one attempts to apply the time series analysis to the real-world data, it is important to normalize these data in an effort to account for operational and environmental variability. For the wrist pulse signals collected by using the Doppler ultrasound device, the ability to normalize the measured data with respect to varying operational and environmental conditions is essential if one is to avoid false-positive classification. Therefore, each wrist pulse signal $f(t)$ is normalized prior to fitting an AR model:

$$\hat{f}(t) = \frac{f(t) - m_f}{\delta_f} \quad (1)$$

where m_f and δ_f are the mean and standard deviation of $f(t)$, respectively. The reference signal, denoted as $\bar{f}(t)$, is obtained by averaging the normalized pulse signals $\hat{f}(t)$ from all the available training samples from healthy persons. The reference AR model with n terms is then constructed as:

$$\bar{f}(t) = \sum_{i=1}^n a_i \bar{f}(t-i) + \varepsilon_f(t) \quad (2)$$

where $a_i (i=1,2,\dots,n)$ is the i^{th} AR coefficient and $\varepsilon_f(t)$ is a term representing the modelling error. The order of this AR model can be determined by the Akaike information criteria (AIC) [9] and the AR coefficients are calculated using the least square method [10].

After the reference AR model is identified, it is used to fit the input normalized pulse signals. For a given wrist pulse signal $g(t)$, which is obtained from a person with unknown healthy status, it is fitted by the reference AR model as follows:

$$\varepsilon_g(t) = g(t) - \sum_{i=1}^n a_i g(t-i) \quad (3)$$

where $\varepsilon_g(t)$ is the prediction error, representing the discrepancy between the input pulse signal and the reference AR model. The mean and standard deviation of $\varepsilon_g(t)$, denoted by $mean_ \varepsilon_g$ and $std_ \varepsilon_g$, can then be calculated.

Factors like age, gender and the environment of collecting the data, may also affect the sampled wrist pulse waveforms. However, it has been validated in traditional Chinese medicine that these factors mainly affect the amplitude and rhythm while the waveform shapes, which are used in this paper, are less affected [11]. Moreover, the shape of a wrist pulse waveform is mainly dependent on the type of the disease. It can be expected that when a pulse signal is from a healthy person, the reference model which is trained from healthy persons will accurately predict the signal. As a result, the mean and the standard deviation of the prediction error are relatively small. Otherwise, when a pulse signal is from a patient, the reference AR model will not be able to well predict the signal and the mean and the standard deviation of the pre-

Table 1 Sample distribution of the testing database

Diseases	Age				Total
	0–20	20–40	40–60	60 and older	
Healthy	4	23	15	8	50
DBU	2	13	3	3	21
Pancreatitis	8	13	2	0	23
Appendicitis	0	11	0	0	11
Acute appendicitis	10	4	5	0	19

diction error are expected to increase. Therefore, for a given wrist pulse signal $g(t)$, the associate $mean_{\varepsilon_g}$ and std_{ε_g} values are significant features for the classification of $g(t)$.

SVM classification

After the pulse signal features have been extracted, an SVM [12] is employed to classify this signal as being from either healthy persons or patients. Particularly, a soft-margin SVM is adopted in this study. SVM is a supervised learning method for classification. Given a set of points of the form:

$$D = \{(\mathbf{x}_i, y_i) | \mathbf{x}_i \in R^p, y_i \in \{-1, 1\}\}_{i=1}^n \tag{4}$$

where y_i is either 1 or -1, indicating the class to which the point \mathbf{x}_i belongs. Each \mathbf{x}_i is a p -dimensional real vector. The aim of the SVM is to find a separating hyperplane which maximizes the margin between the points having $y_i=1$ and those having $y_i=-1$. This hyperplane can be expressed as:

$$\mathbf{w} \cdot \mathbf{x} - b = 0 \tag{5}$$

where \mathbf{w} is a vector and perpendicular to the hyperplane and b is the offset. It can be found that the width of the margin is

$2/\|\mathbf{w}\|$, where $\|\bullet\|$ represents the Euclidean norm. In case there is no hyperplane that can split the two data sets, the soft-margin SVM will choose a hyperplane that splits the two data sets as cleanly as possible while maximizing the distance to the nearest cleanly split examples [12]. The soft-margin SVM introduces slack variables ξ_i which measure the degree of misclassification of the datum \mathbf{x}_i :

$$y_i(\mathbf{w} \cdot \mathbf{x} - b) \geq 1 - \xi_i, \quad 1 \leq i \leq n \tag{6}$$

The objective function then becomes:

$$\min_{\mathbf{w}, b, \xi} \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^n \xi_i \tag{7}$$

subject to $y_i(\mathbf{w}^T \mathbf{x}_i - b) \geq 1 - \xi_i$ and $\xi_i \geq 0$, where C is the trade-off parameter. Standard quadratic programming technique is used to solve this constrained optimization problem [12].

The selection of Doppler ultrasonic diagnostic parameters

In this research, the wrist pulse signals are collected by using a Doppler ultrasonic device. Compared with detecting pulse signal by using the pressure sensor, which is heavily interfered by the artery blood flowing in the wrist [13], capturing pulse signal through ultrasound scanning is more accurate by locating the probe directly on the Styloid processes. In addition, ultrasound scanning can provide new information, which is not available by using the pressure sensor, because it can reflect the deep radial artery changes beneath the skin. Therefore, it would be interesting to see if the Doppler ultrasonic diagnosis parameters can be helpful to improve the classification accuracy. Some previous researchers have found that there are relationships

Fig. 1 The Doppler spectrogram of a wrist pulse signal (left) and its maximum velocity envelope after denoising (right)

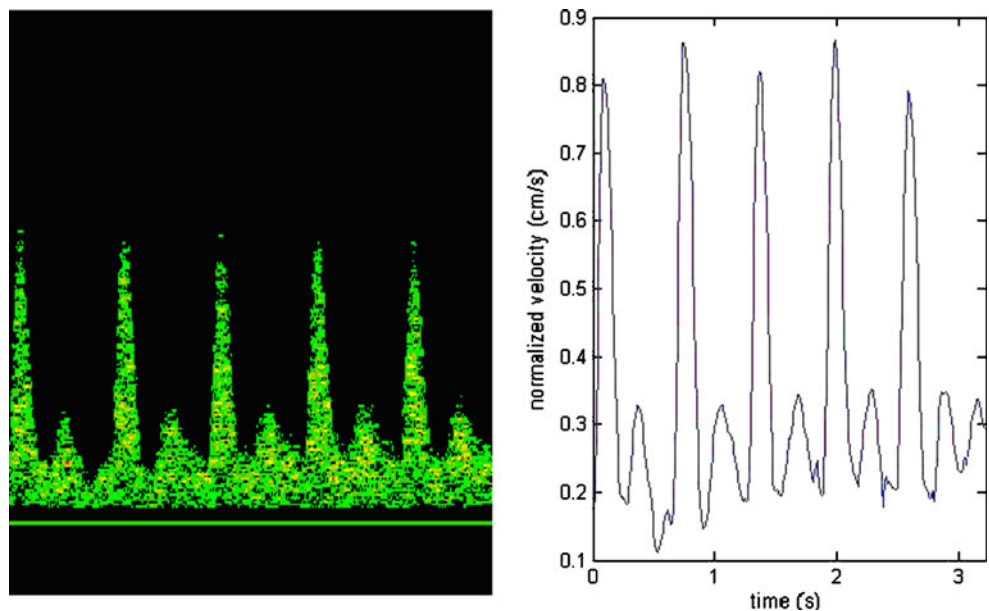
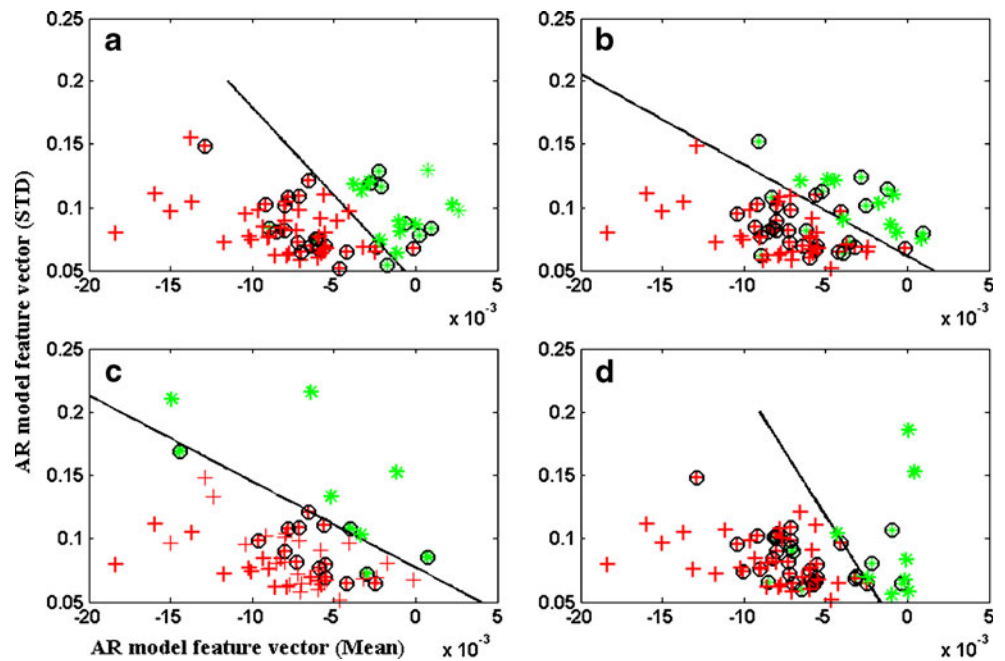


Fig. 2 SVM classification results using the normalized AR model: **a** for pancreatitis patients and the healthy persons; **b** for the DBU patients and the healthy persons; **c** for the appendicitis patients and the healthy persons; **d** for the acute appendicitis patients and the healthy persons



between the Doppler ultrasonic parameters (which can be calculated from the Doppler spectrogram) and the status of blood flow, after applying the Doppler ultrasound technique to clinical diagnosis [14]. These ultrasonic parameters have been taken as the evidence of medical diagnosis [7, 15].

Some widely used Doppler ultrasonic diagnostic parameters are defined in appendix. It should be noted that, the sensitivities of the Doppler parameters to different diseases are different. Therefore, in order to increase the accuracy of diagnosis only the Doppler parameters which are sensitive to the diseases are selected as additional features [16]. The procedures of selecting Doppler parameters are described as follows.

Assume that the training database contains a total of m sets of pulse signals from healthy persons and n sets of pulse signals from patients. For each pulse signal, a certain Doppler parameter can be extracted. The Doppler parameters estimated from healthy persons are denoted as $\{DP_1^H, DP_2^H, \dots, DP_m^H\}$

and those estimated from patients are $\{DP_1^P, DP_2^P, \dots, DP_n^P\}$, where $DP_i^H (i = 1, \dots, m)$ and $DP_j^P (j = 1, \dots, n)$ refer to the Doppler parameters estimated from a health person and a patient, respectively. The upper level limit (ULL) and lower level limit (LLL) of $\{DP_1^H, DP_2^H, \dots, DP_m^H\}$ are estimated as:

$$ULL = mean_DP^H + std_DP^H \tag{8}$$

$$LLL = mean_DP^H - std_DP^H \tag{9}$$

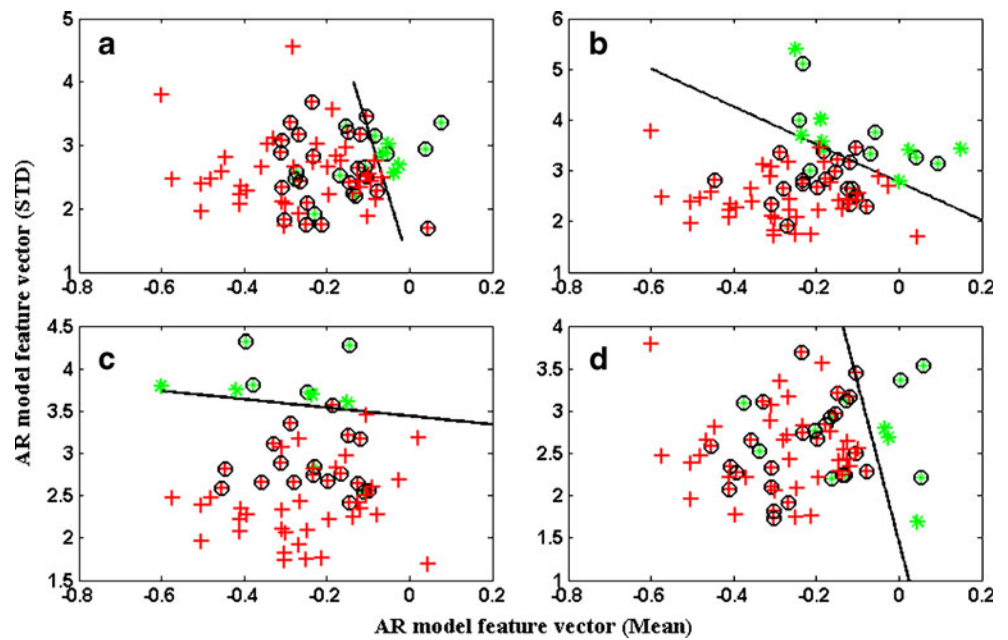
where $mean_DP^H$ and std_DP^H are the mean and standard deviation of $\{DP_1^H, DP_2^H, \dots, DP_m^H\}$.

The obtained ULL and LLL are taken as the thresholds which discriminate patients from healthy persons. If, for example, the DP of an unknown pulse signal is within the range defined by the ULL and LLL, the signal is then classified as from a healthy person. Otherwise, we have some confidence to conclude that the signal is

Table 2 Experimental results to distinguish patients from healthy people

Sample class	Sample number	Accuracy (%) (AR features only)	Accuracy (%) (AR and SW features)	Accuracy (%) (WPT method [7])
Healthy	50	73	88.9	86.3
Pancreatitis	23		80.6	83.3
Healthy	50	71	85.7	82.3
DBU	21		74.3	80.0
Healthy	50	61	90.0	88.2
Appendicitis	11		80.0	81.8
Healthy	50	69	79.4	77.8
Acute appendicitis	19		73.5	76.5
Healthy	50	124	86.0	83.7
All kinds of diseases	74		77.1	80.4
				94.4
				90.9
				86.7
				84.8
				80.5
				88.9
				85.4
				77.1
				76.7
				76.1
				73.3
				80.8
				77.1
				72.4
				60.0
				82.4
				80.0
				72.7

Fig. 3 SVM classification results using the un-moralized AR model: **a** for pancreatitis patients and the healthy persons; **b** for the DBU patients and the healthy persons; **c** for the appendicitis patients and the healthy persons; **d** for the acute appendicitis patients and the healthy persons



from a patient. Based on the above criteria, the percentage of false-positive classification (indication of a disease for a healthy person) can be estimated by counting the number of DP^H which falls outside the range defined by ULL and LLL. Similarly, the percentage of false-negative classification (no indication of disease for a patient) can be calculated by the number of DP^P which falls inside the range. If the percentages of these two false classifications are kept low, the Doppler parameter has a potential to be an effective feature to distinguish healthy persons from patients. After the Doppler parameters have been selected, these parameters are adopted as the features to the pulse signals. These features, combined with those estimated by the AR method, constitute the inputs to the SVM.

Experimental results

Data description

The wrist pulse signals used in this paper were collected by a Doppler ultrasonic blood analyzer module (Edan Instruments, Inc.) from both healthy persons and patients who had been previously diagnosed with certain diseases. There are three steps in each measurement. First is to find the rough position where the fluctuation of pulse is bigger than other positions using the probe; then move the probe slowly and carefully around the rough location and change the angle of the probe against the skin in order to get the most significant signals; finally, these Doppler spectrograms of wrist pulses were recorded and saved. These steps were repeated several times for each measurement to reduce the measurement errors.

By collaborating with the Harbin 211 hospital (Harbin, Heilongjiang Province, China), an experimental database was established, including 248 wrist pulse Doppler ultrasonic blood images for testing. These pulses were collected from people at different ages and with different kinds of diseases, i.e. 100 healthy persons, 46 patients with pancreatitis (P), 42 with Duodenal Bulb Ulcer (DBU), 22 with appendicitis (A) and 38 with acute appendicitis (AA). In this study, the experimental database is split into a training dataset and a testing dataset. For each group (healthy persons and patients), half of the data are randomly selected for the training use and the remaining are for the testing use. Table 1 summarizes the composition of the testing database.

The collected wrist pulse samples are pre-processed before extracting features. First, the maximum velocity

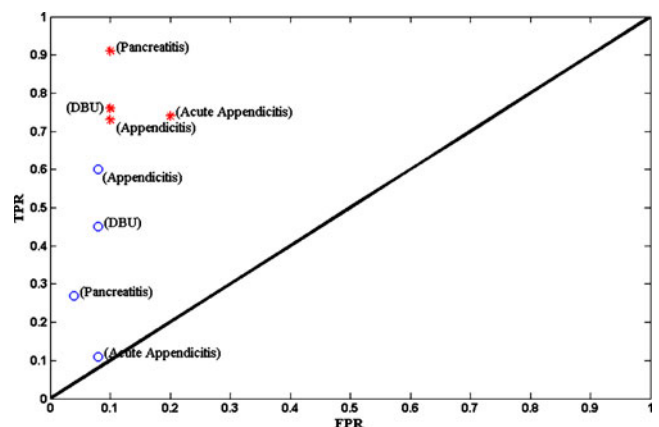


Fig. 4 ROC curve of the four classifiers (*asterisks* classifiers using normalized AR features, *open circles* classifiers using un-normalized AR features)

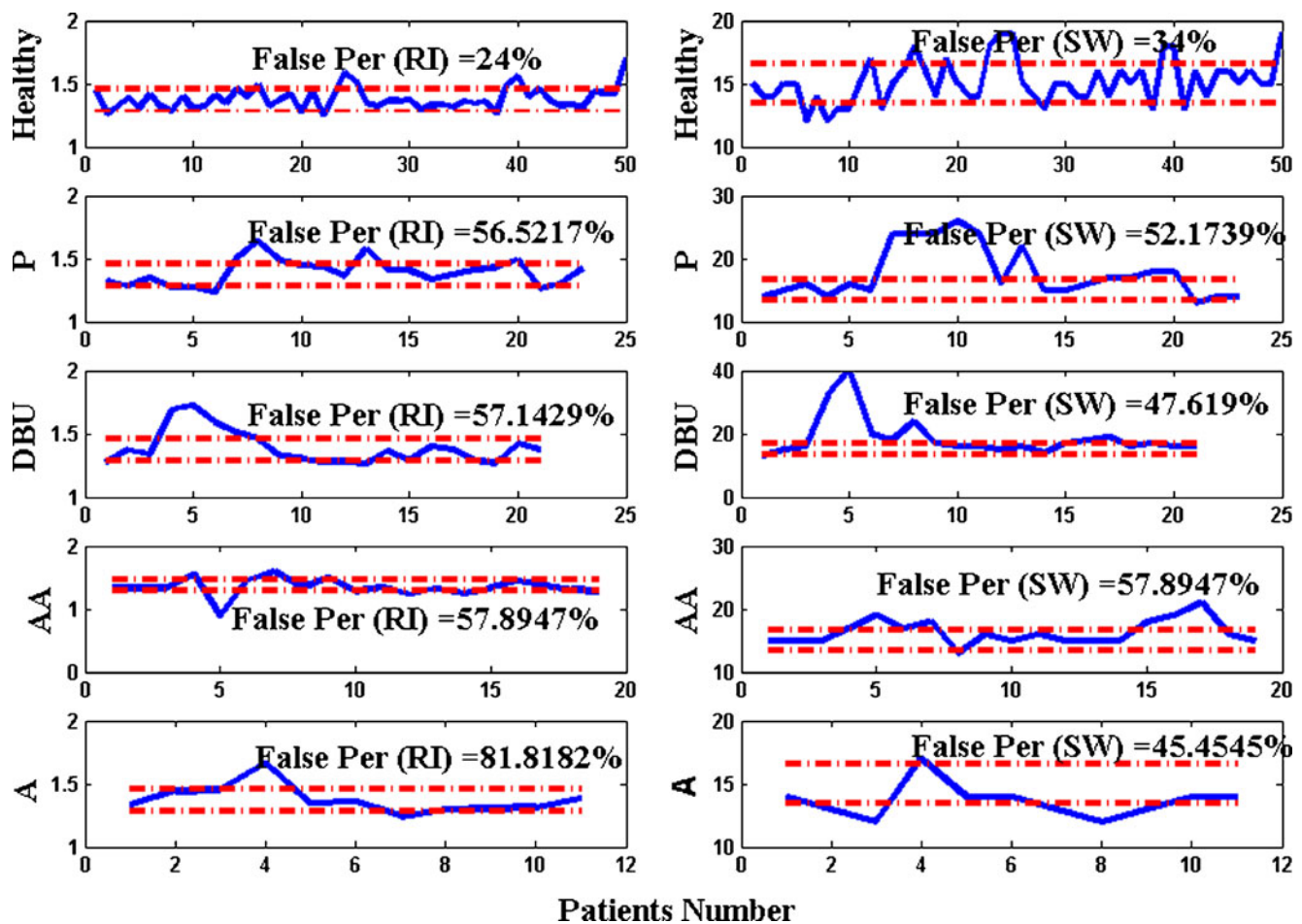


Fig. 5 Illustration of the misclassification percentage of the Doppler parameter RI (*left*) and SW (*right*) for healthy persons and patients with pancreatitis (*P*), duodenal bulb ulcer (*DBU*), appendicitis (*A*) and acute appendicitis (*AA*)

envelope of each pulse waves is extracted and normalized. Then the noise and baseline drift are removed from the normalized signal. In this paper, the wavelet transform is used to remove the noise and baseline drift [17]. Figure 1 shows the Doppler spectrogram of a wrist pulse signal and the extracted velocity envelope.

Experimental results by using the AR features

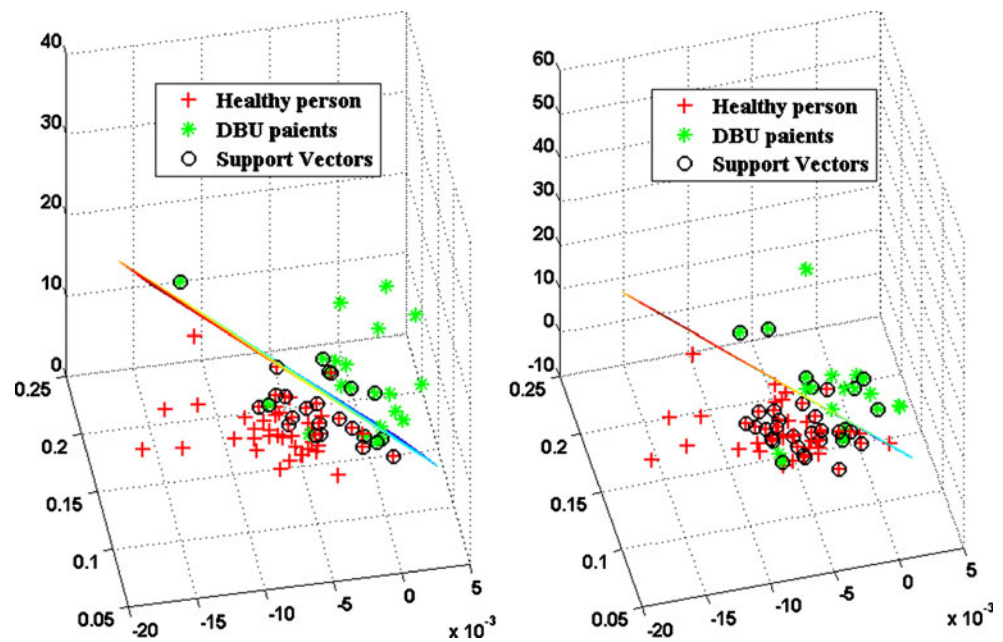
The method described in the “The proposed method” section is applied to the experimental data. As was discussed, the mean and standard deviation of the AR model prediction error $\varepsilon_g(t)$ can be used to distinguish healthy persons from patients. Therefore, these two features are taken as the inputs to the SVM for classification. Figure 2a–d illustrate the classification results for healthy persons versus patients with four kinds of diseases, respectively. In this figure, the estimated support vectors are marked with ‘o’. The classification accuracies using these AR features are listed in Table 2. The classification accuracy corresponding to the healthy

people is defined as the percentage of healthy people who are identified as not having the condition (i.e. specificity). The classification accuracy for the patients is defined as the proportion of actual patients which are correctly identified as such (i.e. sensitivity). Moreover, the average of specificity and sensitivity is also calculated as the total classification rate. It can be seen from Table 2 that the

Table 3 False classification rates (%) of four Doppler parameters for different diseases

	RI	SW	RT	SD
Healthy	24.00	34.00	32.00	32.00
Pancreatitis	56.52	52.17	65.22	52.17
DBU	57.14	47.62	47.62	42.86
Appendicitis	57.89	57.89	52.63	47.37
Acute appendicitis	81.82	45.45	45.45	63.64
Average	55.48	47.43	48.58	47.61

Fig. 6 SVM classification result to distinguish the healthy people from patients with pancreatitis (*left*) and healthy persons from DBU patients (*right*), using the AR model feature vectors (mean and standard deviation) as well as the Doppler parameter (SW)



features extracted by the AR model work well for wrist pulse signal classification.

To demonstrate the effectiveness of normalization procedure when estimating AR features, the classification results using AR features obtained without normalization are shown in Fig. 3a–d. Furthermore, the receiver operating characteristic (ROC) curve of the classifiers using normalized AR features (in Fig. 2) and un-normalized ones (in Fig. 3) is illustrated in Fig. 4. It can be seen from Fig. 4 the classifiers using normalized AR features yield 4 points in the upper left corner of the ROC space, representing high sensitivity (low false negatives) and high specificity (low false positives). On the contrary, classifiers using un-normalized AR features cannot provide comparable classification results.

Experimental results by using the Doppler parameters as additional features

As described in the “[The selection of Doppler ultrasonic diagnostic parameters](#)” section, the sensitivities of different Doppler parameters may vary for different kinds of diseases. Choosing Doppler parameters which can distinguish healthy persons from patients would help us for further classification. As an example, Fig. 5 illustrates the results of the false classification test for Doppler parameters RI and SW. The ULL and LLL (dashed lines) were determined using the pulse signals from the healthy persons. Table 3 lists the false classification rates of four Doppler parameters for different diseases. It can be seen that the Doppler parameter SW has lower false classifica-

tion rates on average compared with other parameters, and therefore is selected as a feature.

It should be noted that the false classification percentages of SW are still high, which implies that it can not be used alone for classification. Therefore, the Doppler parameter SW should be combined with other features in order to obtain a satisfactory result.

The Doppler parameter SW was selected as the additional feature to the AR features for SVM classification. As an example, Fig. 6 illustrates the classification results for healthy persons versus patients with pancreatitis and DBU, respectively, and the experimental results are listed in Table 2. Compared with the results by using only the AR features, it is clear that the selected Doppler features further improve the classification results. In Table 2, we also listed the classification results in distinguishing between healthy persons and unhealthy persons (i.e. all the patients with four kinds of disease). Moreover, the results by using the wavelet packet transform (WPT) method introduced in [7] are also shown in Table 2 for comparison. It can be seen the proposed method outperforms much the WPT method in most of the cases.

Conclusions and future work

An auto-regressive (AR) modeling method was proposed in this paper to extract features from the wrist pulse signals. The extracted distinctive features were adopted as inputs to a soft margin support vector machine (SVM)

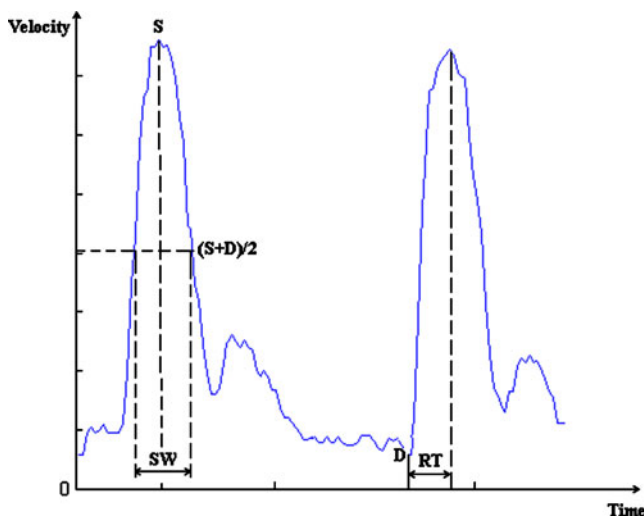


Fig. 7 A typical Doppler signal and some Doppler parameters

for classification. The applicability and performance of this method was evaluated using wrist pulse signals, including both healthy persons and patients. Moreover, some Doppler ultrasonic diagnostic parameters were selected and used as the additional inputs to the SVM. The experimental results showed that, by using the AR method, an accuracy of over 82% in telling the healthy persons from the patients can be reached. A higher accuracy (about 90%) can be achieved by using the combination of the AR method with the Doppler parameters. These results demonstrate the proposed methods have great potentials for computerized pulse diagnosis.

In this research, the AR model was adopted because of its ability to describe time series signals. However, more types of models, such as the autoregressive moving average (ARMA), and the state-space model, can also be used in analyzing time series signals. The investigation of the effectiveness of these models is beyond this paper and will be investigated in the future.

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Appendix: Doppler ultrasonic diagnostic parameters

Figure 7 illustrates a typical Doppler waveform about the wrist artery blood flow, where S and D are the Systolic peak (maximum velocity) and the end of Diastolic velocity, respectively. Two Doppler ultrasonic parameters, RT and

SW, are illustrated in Fig. 7. Other commonly used Doppler parameters are defined as follows [15]:

1. Spectrum Broadening Index (SBI): $SBI = (F_{avpk} - F_{mean}) / F_{avpk}$, where F_{avpk} means frequency excursion of peak systolic velocity and F_{mean} means frequency excursion of mean velocity;
2. Stenosis Index (STI): $STI = 0.9 * (1 - V_m/S)$, where V_m is the mean velocity;
3. Resistance Index (RI): $RI = (S - D)/S$;
4. Ratio of Systolic by Diastolic velocity (S/D).

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