

Although the function is no longer convex, it seems to find  $\mu(M_a)$  reliably enough to greatly offset this disadvantage. In cases where local instead of global maxima occur, the algorithm simply reverts to (1).

The new method can easily be extended to the formulation of [8] by using the dependencies previously shown. Let  $S \in \mathcal{S}$  be the scaling matrix obtained at a stationary point of the scaled singular value in (1) and denote by  $Y_1$  the corresponding right singular vector. Then the optimal solution  $x^*$  to (16) is

$$x^* = \frac{S^{-1}Y_1}{\|S^{-1}Y_1\|}$$

Replacing  $S$  and  $Y_1$  with the relationships developed earlier and carrying out the multiplication, it is easily seen that

$$\begin{bmatrix} 1 \\ y_{21} \\ \vdots \\ y_{n1} \\ y_{n+1} \\ y_{21}y_{n+1} \\ \vdots \\ y_{n1}y_{n+1} \\ y_{2n+1} \\ y_{21}y_{2n+1} \\ \vdots \\ y_{n1}y_{(n-1)n+1} \end{bmatrix} \quad (17)$$

Here,  $Y_1$  has simply been scaled so that  $y_{11} = 1$ . As before, only  $2(n-1)$  free variables remain making the optimization process much more efficient. Using the system of Example 1, the optimal  $S^{-1}Y_1$  becomes

$$S^{-1}Y_1 = \begin{bmatrix} 1 \\ y_{21} \\ y_{31} \\ y_{41} \\ y_{21}y_{41} \\ y_{31}y_{41} \\ y_{71} \\ y_{21}y_{71} \\ y_{31}y_{71} \end{bmatrix} = \begin{bmatrix} 1 \\ 1.236 \\ 1.094 \\ 0.855 + 0.519i \\ 1.056 + 0.641i \\ 0.935 + 0.567i \\ 0.355 + 0.935i \\ 0.438 + 1.155i \\ 0.388 + 1.023i \end{bmatrix}$$

with corresponding

$$\|M_a x^*\| = 8.25 = \bar{\sigma}(DM_a D^{-1}).$$

## VI. CONCLUSION

A new method that significantly reduces the number of optimization variables required in the calculation of the structured singular value for scalar or block structured uncertainties is presented. This approach takes advantage of the structure of the uncertainty matrix  $\Delta$  to provide a reduction in the number of free optimization variables. For the case of an uncertainty matrix with  $n^2$  nonzero  $1 \times 1$  blocks, we have shown that the structured singular value may be computed with  $2(n-1)$  rather than  $n^2 - 1$  free variables. It should be noted, however, that for this class of uncertainties the nonsimilarity scaling method of [2] also requires only  $2(n-1)$  optimization variables over matrices of dimension  $n \times n$  (not  $n^2 \times n^2$ ). Hence, nonsimilarity scaling still offers the most efficient method of calculating  $\mu$  for this uncertainty class.

Examples 2 and 3 illustrate how the new scaling technique offers significant computational advantages for block structured

uncertainties over previous methods by again reducing the number of optimization parameters. These reductions are also shown to hold for the vector optimization approach developed in [8].

Recent work [11] has revealed that the results presented in this note may also be used to derive explicit expressions for the elements of the nonsimilarity scaling matrices in terms of the similarity scaling variables and further research is proceeding to remove the stationarity assumptions and to extend the results to more general classes of uncertainties.

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## Computing an Optimum Direction in Control Space to Avoid Saddle Node Bifurcation and Voltage Collapse in Electric Power Systems

Ian Dobson and Liming Lu

**Abstract**—This note computes an optimum direction of controls to avoid power system voltage collapse. That is, given a load power forecast, we compute the sensitivity with respect to controls of a load power margin measuring proximity to voltage collapse. The computation is simple enough to contribute to the practical planning and control of a power system to avoid voltage collapse blackouts. The computation applies to avoiding saddle node bifurcation instability of a general dynamical system.

### I. INTRODUCTION

Voltage collapse is an instability of heavily loaded electric power systems which leads to declining voltages and blackouts. It is associated with bifurcation and reactive power limitations of

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the power system. Power systems are expected to become more heavily loaded in the next decade as the demand for electric power rises while economic and environmental concerns limit the construction of new transmission and generation capacity. Heavily loaded power systems are closer to their stability limits and voltage collapse blackouts will occur if suitable monitoring and control measures are not taken. Progress has been made in understanding voltage collapse and monitoring the proximity to voltage collapse with various indexes (most approaches are covered in [1], [2]). The sensitivity to controls of some of the indexes have been computed so that control action may be taken to optimally improve the index and the voltage stability of the system. For example, Tiranuchit and Thomas [3] computed the sensitivity of the minimum singular value of the system Jacobian and Overbye and DeMarco [4] computed the sensitivity of an energy function index. Another popular index is load power margin which may be computed by direct methods [5]–[7], optimization [8]–[11], continuation [6], [7], [12], and continuation with linear predictor and no corrector [13], [14]. This note computes the first-order sensitivity of a load power margin index to any controls or parameters appearing in the load flow equations.

If system parameters vary slowly and continuously, then voltage collapse can be explained as a dynamic consequence of a saddle node bifurcation instability in which the system operating equilibrium disappears [15]. Discrete contingencies such as loss of a power line can also weaken the system stability and contribute to voltage collapse; if this occurs, or is hypothesized to occur when assessing security, we use the saddle node bifurcation explanation *after* the discrete contingency. This note computes an optimum control direction which steers the system away from a saddle node bifurcation and the associated voltage collapse.

We exploit the geometry of a control space  $\Lambda$ , the vectors of which contain real and reactive powers of the loads and generators and other control parameters such as settings of tap changing transformers and shunt capacitor devices. (The settings of tap changing transformers and shunt capacitor devices are discrete but we compute the control assuming they are continuous and then approximate the control with the nearest discrete setting [16].) Note that the “controls” include load powers which vary freely in normal operation and are only controlled in emergencies by selectively shedding loads. It is also useful to include design parameters in  $\Lambda$  when planning changes to a power system. Define  $\Sigma$  to be the control values in  $\Lambda$  at which the stable operating equilibrium has a saddle node bifurcation.  $\Sigma$  typically consists of curved hypersurfaces and their intersections.<sup>1</sup> We denote the current control parameters by  $\lambda_0$ ; the position of  $\lambda_0$  relative to  $\Sigma$  is the key to monitoring and avoiding voltage collapse. We assume that a load power forecast is given in the form of a linear increase from the current load powers. This forecast is described by a ray in  $\Lambda$  based at  $\lambda_0$ ; controls other than load powers are constant along the ray. The ray will intersect  $\Sigma$  at the point  $\lambda_*$  at which the system

<sup>1</sup> The set of all points of  $\Lambda$  at which there is a bifurcation is called the bifurcation set and it generally contains points other than  $\Sigma$ . These points correspond to saddle node bifurcations not involving the stable equilibrium point, Hopf bifurcations, and other bifurcations not generic in one parameter systems. We do not consider these bifurcations here: Saddle node bifurcations not involving the stable equilibrium do not affect the local stability of the stable operating equilibrium. Hopf bifurcations cause oscillatory instabilities, but are probably not important in voltage collapse because voltage collapses are not observed to be oscillatory. The nongeneric bifurcations are not expected to occur in practice.

bifurcates and loses stability and the proximity to voltage collapse can be measured by the distance  $|\lambda_0 - \lambda_*|$ . Since the controls other than load powers were fixed in the load forecast ray,  $\lambda_0 - \lambda_*$  lies in the load power subspace of  $\Lambda$  and  $|\lambda_0 - \lambda_*|$  is a load power margin. If  $|\lambda_0 - \lambda_*|$  is too small, then control action should be taken to increase  $|\lambda_0 - \lambda_*|$  in order to avoid or reduce the risk of voltage collapse. Control action is represented by a displacement of  $\lambda$  from  $\lambda_0$  in  $\Lambda$ . This note computes  $\nabla_\lambda(|\lambda - \lambda_*|)_{\lambda_0}$ , the gradient with respect to  $\lambda$  of the index  $|\lambda - \lambda_*|$  evaluated at  $\lambda_0$ .  $\nabla_\lambda(|\lambda - \lambda_*|)_{\lambda_0}$  is the sensitivity of the load power margin with respect to control and its direction gives the optimum direction for control. We prove that  $\nabla_\lambda(|\lambda - \lambda_*|)_{\lambda_0}$  is parallel to the normal vector  $n_*$  to  $\Sigma$  at  $\lambda_*$  using generic assumptions and a dynamic power system model and show that it is straightforward to compute  $n_*$  and  $\nabla_\lambda(|\lambda - \lambda_*|)_{\lambda_0}$  from the load flow equations.

This note adopts the objectives of Kumano, Yokoyama, and Sekine [17] and we are indebted to these authors for the geometric idea of computing an optimum direction in a load power space which is augmented with other controls. Our computation of the optimum direction is based on the left eigenvector formula in Dobson [18] for the normal vector  $n_*$  to  $\Sigma$ . The formula for the normal vector in [17], which uses a right eigenvector, is only valid for power systems with symmetric Jacobians so that the left and right eigenvectors coincide. The computation in this note is much simpler and more direct than that of [17]. The idea of using a normal vector to determine which loads to shed appeared in [18].

## II. POWER SYSTEM STATIC AND DYNAMIC MODELS AND THE NORMAL VECTOR $n_*$

Consider the load flow equations

$$0 = f(x, \lambda) \quad \text{or} \quad \begin{cases} 0 = f_1(x, \lambda) \\ 0 = f_2(x, \lambda) \end{cases} \quad (1)$$

where  $x \in \mathbf{R}^n$  is a state vector describing the bus voltage phasors and  $\lambda \in \Lambda$  is a control vector.  $f_1$  describes real power balance at the generators and  $f_2$  describes real and reactive power balance at the loads.

We seek to compute the sensitivity  $\nabla_\lambda(|\lambda - \lambda_*|)_{\lambda_0}$  from the static equations (1) but want the results to be consistent with the sensitivity for a general dynamic model extending (1) to which the voltage collapse theory of [15] applies. A dynamic model [18] which extends (1) by including generator swing dynamics and general load dynamics is

$$z = h(z, \lambda) \quad \text{or} \quad \begin{cases} \dot{\omega} = f_1(\delta_G, y, \lambda) - \Delta \omega \\ \dot{\delta}_G = \omega \\ \dot{y} = g(f_2(\delta_G, y, \lambda), \omega). \end{cases} \quad (2)$$

The state vector  $z = (\omega, x) = (\omega, \delta_G, y)$  where  $\delta_G$  are the generator voltage angles in  $x$  and  $\Delta$  is a diagonal matrix of generator dampings.  $g$  defines any smooth dynamic load model which depends on frequency  $\omega$  and the real and reactive power balance at each load (see [15] for an example). Little is known about the large signal behavior of such load dynamics and a convincing function  $g$  is hard to obtain. For our purposes, we require the existence of  $g$  and the assumptions that  $g$  has a unique zero at  $(0, 0)$  and that  $Dg$ , the Jacobian of  $g$  with respect to its first argument, is globally invertible. We also assume the technical condition that the operating region of (2) is contained in a compact positively invariant set; see [15].

We assume a load forecast in which the load powers increase as a linear function of a loading parameter  $t$ :

$$\lambda(t) = \lambda_0 + t\mu \quad (3)$$

where  $\mu$  is a fixed vector of unit length defining the direction of the load forecast ray in  $\Lambda$ . We write  $\lambda_* = \lambda(t_*)$  for the first intersection of the load forecast ray with  $\Sigma$  and  $t_*$  for the loading parameter at the bifurcation. (The disappearance of the operating equilibrium and the existence of  $\lambda_*$  is guaranteed by the absence of any equilibrium for absurdly high loadings.) If the load increase is slow enough to be modeled as a quasistatic variation of (2) and the resulting system  $\dot{z} = h(z, \lambda(t))$  parameterized by  $t$  is assumed to be a generic one parameter system, then the theory of [15], [19] applies and we obtain the following conclusions about the structure of the bifurcation at  $(z_*, \lambda_*)$ : If we write  $h_z|_*$  for the Jacobian of  $h$  with respect to  $z$  evaluated at  $(z_*, \lambda_*)$ , then  $h_z|_*$  has a single, simple zero eigenvalue and corresponding right and left eigenvectors  $v'_*$ ,  $w'_*$  so that  $h_z|_* v'_* = w'_* h_z|_* = 0$ . In addition, the bifurcation is a saddle node bifurcation satisfying the transversality conditions [20], [21], [19]

$$w'_* h_t|_* = w'_* h_\lambda|_* \mu \neq 0 \quad (4.1)$$

$$w'_* h_{zz}|_*(v'_*, v'_*) \neq 0. \quad (4.2)$$

Moreover,  $\Sigma$  is a smooth hypersurface near  $\lambda_*$  and has a well defined normal vector  $n_*$  at  $\lambda_*$ . Note that we write  $w'_*$ ,  $n_*$ , and  $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$  as row vectors.

The formula from [18] for the normal vector  $n_*$  to  $\Sigma$  at  $\lambda_*$  is simply

$$n_* = w'_* h_\lambda|_* \quad (5)$$

where  $h_\lambda|_*$  is the Jacobian of  $h$  with respect to  $\lambda$  evaluated at the bifurcation.

Now we demonstrate that the saddle node bifurcation of the dynamic model (2) at  $(z_*, \lambda_*) = (0, x_*, \lambda_*)$  induces a bifurcation of the solutions of the static equations (1) at  $(x_*, \lambda_*)$  and the Jacobian  $f_x|_*$  has a zero eigenvalue with corresponding left eigenvector  $w_*$ . Moreover the normal vector  $n_*$  of (5) is also given by

$$n_* = w_* f_\lambda|_*. \quad (6)$$

This allows  $n_*$  to be calculated from the static equations (1) assuming only the general form, but not the details, of the dynamical model (2). To derive (6), first note that the unique zero of  $g$  at  $(0, 0)$  implies that (1) has an equilibrium at  $(x_0, \lambda_0)$  if and only if (2) has an equilibrium at  $(0, x_0, \lambda_0)$ . The respective Jacobians of (1) and (2) are

$$f_x = \begin{pmatrix} f_{1x} \\ f_{2x} \end{pmatrix} \quad \text{and} \quad h_z = \begin{pmatrix} -\Delta & f_{1x} \\ I & 0 \\ g_w & Dg f_{2x} \end{pmatrix}.$$

Writing  $w'_* = (w'_{1*}, w'_{2*}, w'_{3*})$ , observe that  $(w'_{1*}, w'_{2*}, w'_{3*}) h_z|_* = 0$  implies that

$$0 = (w'_{1*}, w'_{3*}) \begin{pmatrix} f_{1x} \\ Dg f_{2x} \end{pmatrix} = (w'_{1*}, w'_{3*} Dg) \begin{pmatrix} f_{1x} \\ f_{2x} \end{pmatrix}$$

so that  $w_* = (w'_{1*}, w'_{3*} Dg)$  is the left eigenvector of  $f_x$  corresponding to a zero eigenvalue. ( $w_*$  is unique since if  $w_* f_x = (w'_{1*}, w'_{3*}) f_x = 0$  then  $(w'_{1*}, w'_{1*} \Delta - w'_{3*} (Dg)^{-1} g_w,$

$w'_{3*} (Dg)^{-1} h_z|_* = 0$  and  $w'_{1*} = w'_{1*}$  and  $w'_{3*} = w'_{3*} Dg$  since  $w'_*$  is unique.) Moreover

$$\begin{aligned} n_* &= w'_* h_\lambda|_* = (w'_{1*}, w'_{2*}, w'_{3*}) \begin{pmatrix} f_{1\lambda}|_* \\ 0 \\ Dg f_{2\lambda}|_* \end{pmatrix} \\ &= (w'_{1*}, w'_{3*}) \begin{pmatrix} f_{1\lambda}|_* \\ f_{2\lambda}|_* \end{pmatrix} = w_* f_\lambda|_*. \end{aligned} \quad (7)$$

It is straightforward to check that  $w'_* h_{zz}|_*(v'_*, v'_*) = w_* f_{xx}|_*(v_*, v_*)$  where  $v'_* = (0, v_*)$  and  $v_*$  is the right eigenvector of  $f_x|_*$  corresponding to the zero eigenvalue. This, together with (7), shows that the bifurcation of (1) inherits from (2) the transversality conditions (4).

### III. COMPUTATION OF $n_*$ AND THE LOAD POWER MARGIN SENSITIVITY TO CONTROL

Given a load forecast (3), the system will bifurcate and collapse at  $\lambda_* = \lambda_0 + t_* \mu$ . We compute  $\lambda_*$  by solving a variant of the extended system equations [7], [5], [6]:

$$\left. \begin{aligned} f(x_*, \lambda_*) &= 0 \\ w_* f_x|_* &= 0 \\ w_* u - 1 &= 0 \end{aligned} \right\}. \quad (8)$$

(The last equation of (8) ensures that  $w_*$  is nonzero; in practice  $u$  can be chosen to be any nonzero constant vector.) Equations (8) can be solved for  $(x_*, w_*, t_*)$  using Newton-Raphson iteration. Then  $\lambda_* = \lambda_0 + t_* \mu$ . Note that equations (8) are a variant of the extended system equations in that they use left eigenvectors in place of the usual right eigenvectors. This has the advantage of computing the left eigenvector  $w_*$  required in the formula (6) for  $n_*$  as a by-product of the computation of  $(x_*, \lambda_*)$ . The optimization methods of computing the load power margin also share this advantage since  $w_*$  is the Lagrange multiplier vector [8]. Continuation methods [7], [12], [13], [6] could also be used but would require a further computation of  $w_*$  from  $f_x|_*$ .

There is a sign ambiguity in the formulas (5) and (6) for  $n_*$  since both  $n_*$  and  $-n_*$  are normal vectors to  $\Sigma$ . It is convenient to assume that the normal  $n_*$  is pointing "outward" so that changing  $\lambda$  in the direction  $n_*$  leads to the disappearance of the equilibrium  $x_*$ . Formula (6) becomes

$$n_* = \pm w_* f_\lambda|_*. \quad (9)$$

Strictly speaking, the sign of  $n_*$  should be determined by checking that the equilibrium  $x_*$  disappears when  $\lambda = \lambda_* + \epsilon n_*$  for small positive  $\epsilon$ . In practice, the sign of  $n_*$  should be apparent from the expectation that  $n_*$  should correspond to an increase in power for most loads.  $f_\lambda$  is easy to compute and evaluate at the bifurcation.

Now we prove that the load power margin sensitivity

$$\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0} = -(n_* \mu)^{-1} n_* \quad (10)$$

it follows from (10) that the optimum direction for control is  $-n_*$ .  $\lambda_*$  and  $t_*$  are well defined smooth functions of the current control parameters  $\lambda$  near  $\lambda_0$  because  $\Sigma$  is a smooth hypersurface near  $\lambda_*$  and (4.1) implies that  $n_* \mu \neq 0$  so that the load forecast ray intersects  $\Sigma$  transversally at  $\lambda_*$ .  $\lambda_* = \lambda_0 + t_* \mu$  and  $|\mu| = 1$  imply that  $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0} = \nabla_\lambda t_*|_{\lambda_0}$ . Specify  $\Sigma$  near  $\lambda_*$  as the zero set of a smooth real function  $\alpha$ . Then  $\alpha(\lambda_*) = 0$  and  $t_*$  satisfies  $0 = \alpha(\lambda_0 + t_* \mu)$  and  $0 = (\nabla_\lambda \alpha)|_{\lambda_*} (I + \mu \nabla_\lambda t_*|_{\lambda_0})$ . Hence  $0 = n_* (I + \mu \nabla_\lambda t_*|_{\lambda_0}) = n_* +$



TABLE I

	$\alpha_3$	$V_3$	$\alpha_4$	$V_4$	$\alpha_5$	$V_5$	$\alpha_6$	$V_6$	$\alpha_2$
$x_0$	-0.216	1.106	-0.173	1.035	-0.225	1.101	-0.226	1.170	-0.019
$x_*$	-0.263	0.979	-0.203	0.910	-0.266	0.895	-0.253	0.892	-0.114
$\omega_*$	0.181	0.258	0.145	0.275	0.303	0.473	0.298	0.632	-0.017

TABLE II

	$P_3$	$Q_3$	$P_4$	$Q_4$	$P_5$	$Q_5$	$P_6$	$Q_6$
$n_*$	0.114	0.162	0.091	0.172	0.190	0.296	0.187	0.395
	$P_2$	$V_2$	$C_4$	$C_6$	$n_4$	$n_7$		
	-0.010	-0.616	-0.314	-0.143	0.309	0.117		

tion applies to a dynamic load model with a general form of load dynamics but may be conveniently carried out on the load flow equations. The controls and parameters may appear anywhere in the load flow equations. The computation is simple, with the main computational burden being the computation of a left eigenvector at the bifurcation. However, two of the methods of computing the load power margin produce the required eigenvector as a by product. The computation has several applications to power system planning and security.

The computation applies to control or parameter changes to steer a general parameterized dynamical system away from a saddle node bifurcation if the formula (5) is used to calculate the normal vector  $n_*$ .

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## Robust Discrete Controllers Guaranteeing $l_2$ and $l_\infty$ Performances

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**Abstract**—This note presents an algorithm to design discrete time controllers which are robust with respect to parameter variations and

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