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Computing First General Zagreb Index of Operations on Graphs

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ABSTRACT The numerical coding of the molecular structures on the bases of topological indices plays an important role in the subject of Cheminformatics which is a combination of Computer, Chemistry, and Information Science. In 1972, it was shown that the total π-electron energy of a molecular graph depends upon its structure and it can be obtained by the sum of the square of degrees of the vertices of a molecular graph. Later on, this sum was named as the first Zagreb index. In 2005, for $\gamma \epsilon R - \{0, 1\}$, the first general Zagreb index of a graph G is defined as $M^{\gamma}(G) = \sum_{v \epsilon V(G)}[d_G(v)]^{\gamma}$, where $d_G(v)$ is degree of the vertex v in G. In this paper, for each $\gamma \epsilon R - \{0, 1\}$, we study the first general Zagreb index of the cartesian product of two graphs such that one of the graphs is D-sum graph and the other is any connected graph, where D-sum graph is obtained by using certain D operations on a connected graph. The obtained results are general extensions of the results of Deng *et al.* [Applied Mathematics and Computation 275(2016): 422-431] and Akhter *et al.* [AKCE Int. J. Graphs Combin. 14(2017): 70-79] who proved only for $\gamma = 2$ and $\gamma = 3$, respectively.

INDEX TERMS Molecular graphs, topological indices, Cartesian product, sum graphs.

I. INTRODUCTION

For undefined terms, see next section. From the advent of this century, the role of graph theory has increased in various disciplines of science, especially in Computer and Chemistry as well. Topological indices (TI's) associate a numerical number to a molecular graph that provide the useful informations about the involved organic compounds. Every year a number of nanomaterials, drugs and crystalline materials come into view in the various industries including of pharmaceutical. The medical behaviors of the different drugs and compounds have studied with the help of various TI's, see [9], [21]. Moreover, the quantitative structures property relationships (QSPR) and quantitative structure activity relationships (QSAR) are studied with the help of TI's, see [7], [11], [12].

First TI is introduced by famous chemist Harold Wiener in 1947 when he was working on the boiling point of paraffin [2]. This index is known as Wiener index now a day. Gutman and Trinajstić (1972) [14] showed that the total π-electron energy of a molecular graph depends upon its structure and it can be obtained by the sum of square of degrees of the vertices of a molecular graph. Later on, this sum is called by the name of first Zagreb index and an extensive research work has been done on it.

After this, many different TI's came into existence and revolutionized the world of chemistry, see [13], [15]. But, the degree-based topological indices are those one which are most studied. This fact can be emphasized in a recent survey [16] which provides a uniform approach to the degree-based topological indices. Several physicochemical properties such as molecular weight, volume, pressure, density & refraction; boiling, freezing & vaporization point; isomer & edge shift; infrared group frequency; quadruple splitting and polarizability of organic compounds can be obtained using TI's, see [3]–[5], [7], [9], [10].

There are various operations on graphs such as complement, product, switching, addition, deletion and subdivision etc. With the passage of time, the study of

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operations on graphs with respect to the reference of new ones or the use of the old ones paying a key role in the theory of graphs. By the use of various operations, a number of graphs are constructed from the simpler graphs that serve as their basic building blocks. Motivated by this, many researchers studied the well-known families of graphs under the operation of subdivision of graphs.

For a connected graph G, line graph $L(G)$, subdivided graph $S(G)$ and total graph $T(G)$ are defined under the operations L, S and T respectively, see [6]. Moreover, they defined two extra subdivision-related graphs $R(G)$ and $Q(G)$ without any particular name. Yan *et al.* [7] listed these five operations at the same time with respect to their vertex and edge sets. They also studied the behavior of Wiener index and polynomial of graphs under these five decorations. Later on, Eliasi and Taeri [8] assumed $D \in \{S, R, Q, T\}$ and defined the D-sum graph $G_{1+D}G_2$ with the help of cartesian product of the graph $D(G_1)$ (obtained after applying operation D on G_1) and G_2. In addition, they studied the Wiener indices of the new resulting D-sums graphs $G_{1+S}G_2, G_{1+R}G_2, G_{1+Q}G_2$ and $G_{1+T}G_2$. Deng *et al.* [18] studied the first and second Zagreb indices and Akhtar and Imran [19] computed the forgotten index of the aforesaid D-sum graphs.

In this paper, we extend this study and compute the first general Zagreb index of the D-sums graphs $(G_{1+D}G_2)$ for $D \in \{S, R, Q, T\}$. The reset of the paper is settled as; Section 2 includes the basic notions, Section 3 consists on the main results and Section 4 presents some particular results as the consequence of the main results, concludes the work and states the future direction.

II. PRELIMINARIES

Let $G = (V(G), E(G))$ be a simple and connected molecular graph with order $|V(G)| = n$ and size $|E(G)| = m$, where $V(G)$ is a vertex set and $E(G) \subseteq V(G) \times V(G)$ is an edge set. Each vertex of a molecular graph is called atom and bonding between atoms is presented by edges. The degree of a vertex $v \in V(G)$ $(degG(v))$ is number of incident edges on it. Now, some important degree-based TI's are defined as follows:

Definition 1: Let G be a molecular graph, then the first Zagreb index $(M_1(G))$ and second Zagreb index $(M_2(G))$ are defines as:

$$M_1(G) = \sum_{v \epsilon V}[degG(v)]^2 = \sum_{uv \epsilon V}[degG(v) + degG(u)],$$
$$M_2(G) = \sum_{uv \epsilon V}[degG(v)degG(u)].$$

These are two oldest degree-based descriptors of the molecular-structures defined by Gutman and Trinajsti [14]. These indices have been used to study the quantitative and activity structural properties of the molecular graphs such as complexity, chirality, ZE-isomerism, heterosystems, branching and π-electron energy, see [11], [12].

Definition 2: Let R be a set of real numbers, $\gamma \epsilon R - \{0, 1\}$ and G be a molecular graph, then the first general Zagreb

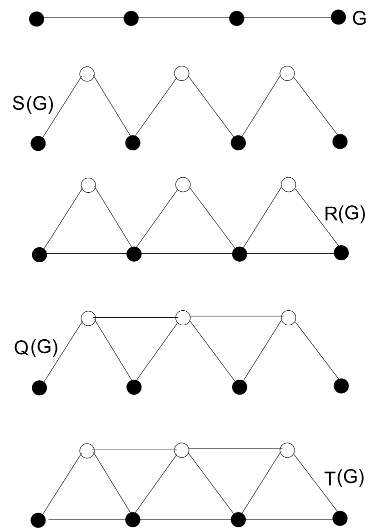

FIGURE 1. ***G*, *S*(*G*), *R*(*G*), *Q*(*G*), and *T*(*G*).**

index is defined as:

$$M^{\gamma} = \sum_{v \epsilon V(G)} degG^{\gamma}(v) = \sum_{uv \epsilon E(G)} [degG^{\gamma-1}(u) + degG^{\gamma-1}(v)].$$

Li and Zheng [20] gave the concept of first general Zagreb index. For $\gamma = 3$, the first general Zagreb index become the forgotten topological index, see [14], [17]. Since, the first general Zagreb index is generalization of the first Zagreb index. Similarly, the general Randić index is generalization of the second Zagreb index which is defined by Randić, see [4].

Definition 3: Let R be a set of real numbers, $\gamma \epsilon R$ and G be a molecular graph, then the general Randić index is defined as

$$R_{\gamma}(G) = \sum_{uv \epsilon E(G)} [degG(u)deg(v)]^{\gamma},$$

where $R_{-\frac{1}{2}}$ is the classical Randić connectivity index.

Now, we present four operations on graph. For the graph G, $S(G)$ is obtained by inserting an additional vertex in each edge of G. $R(G)$ is obtained from $S(G)$ by joining the end (old) vertices of the edges which are incident on each new vertex by an edge. Similarly, $Q(G)$ is obtained from $S(G)$ by joining those pairs of new vertices by edges which have common adjacent (old) vertex. Finally, $T(G)$ is obtained from $S(G)$ after applying both $R(G)$ and $Q(G)$ at the same line. For more explanation see Figure 1.

Definition 4: Let G_1 and G_2 be two connected graphs, D be one of the operations S, R, Q, or T and $D(G_1)$ be a graph (obtained by using the operation D on G_1) with vertex set $V(D(G_1))$ and edge set $E(D(G_1))$. Then, the D-sum graph $G_{1+D}G_2$ is a graph with the set of vertices

$$V(G_{1+D}G_2) = V(D(G_1)) \times (V_2) = (V_1 \cup E_1) \times (V_2)$$

such that two vertices (u_1, u_2) and (v_1, v_2) of $V(G_{1+D}G_2)$ are adjacent iff $[u_1 = v_1 \epsilon V(G_1)$ and $(u_2, v_2) \epsilon E(G_2)]$ or $[u_2 = v_2 \epsilon V(G_2)$ and $(u_1, v_1) \epsilon E(D(G_1))]$.

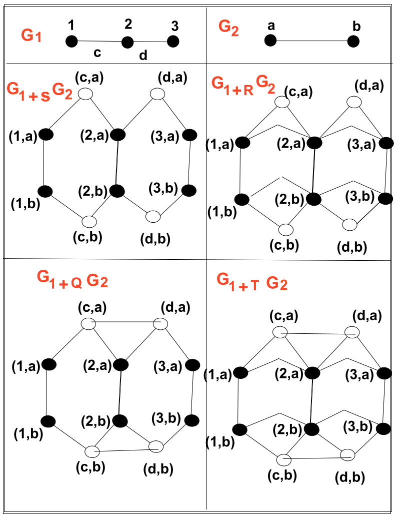

FIGURE 2. $G_{1+D}G_2$ for $G_1 \cong P_3$, $G_2 \cong P_2$ and $D \in \{S, R, Q, T\}$.

We observe that $G_{1+D}G_2$ has $|V(G_2)|$ copies of the graph $D(G1)$ such that these copies are labeled by the vertices of G_2. Moreover, the vertices of V_1 and E_1 are refered as black vertices and white vertices in $G_{1+D}G_2$ respectively and join only black vertices with the same name in $D(G_1)$ in which their corresponding labels are adjacent in G_2. For more explanation, see Figure 2.

III. RESULTS AND DISCUSSION

This section contains the main results of the first general Zagreb index on the D-sum graphs $A_{+S}B$, $A_{+R}B$, $A_{+Q}B$ and $A_{+T}B$, where A and B are assumed two connected graphs.

Theorem 5: Let A and B be two connected graphs and $\theta \in N - \{0, 1\}$. Then, the first general Zagreb index of S-sum graph is

$$M^{\gamma}(A_{+s}B) = \sum_{i=0}^{\theta}\binom{\theta}{i}(M_A^{\theta-i})(M_B^{i+1}) + n_B M_{S(A)}^{\gamma} + \sum_{i=1}^{\theta}\binom{\theta}{i}M_B^{i}M_A^{\gamma-i},$$

where $\theta = \gamma - 1$ and N is set of natural numbers.

Proof: By definition, we have

$$M^{\gamma}(A_{+s}B) = \sum_{(l,m)\epsilon V(A_{+s}B)} d_{A_{+s}B}^{\gamma}(l,m)$$

For $\theta = \gamma - 1$, the above equality in terms of edges becomes,

$$\begin{aligned}M^{\gamma}(A_{+s}B) &= \sum_{(l,m)(n,o)\epsilon E(A_{+s}B)} [d_{A_{+s}B}^{\theta}(l,m) + d_{A_{+s}B}^{\theta}(n,o)]\\ &= \sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)} [d_{A_{+s}B}^{\theta}(l,m) + d_{A_{+s}B}^{\theta}(l,o)]\\ &\quad + \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(S(A))} [d_{A_{+s}B}^{\theta}(l,m) + d_{A_{+s}B}^{\theta}(n,m)] \qquad (1)\end{aligned}$$

For each vertex $l\epsilon V(A)$ and edge $mo \epsilon E(B)$, the first term of (1) will be

$$\begin{aligned}&\sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)} [d_{A_{+s}B}^{\theta}(l,m) + d_{A_{+s}B}^{\theta}(l,o)]\\ &= \sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)} [\sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(l)d_B^{i}(m)\\ &\quad + \sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(l)d_B^{i}(o)]\\ &= \sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)} [\sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(l)[d_B^{i}(m) + d_B^{i}(o)]]\\ &= \sum_{l\epsilon V(A)} [\sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(l)] \sum_{mo\epsilon E(B)} [d_B^{i}(m) + d_B^{i}(o)]\\ &= \sum_{l\epsilon V(A)} [\sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(l)](M_B^{i+1})\\ &= \sum_{i=0}^{\theta}\binom{\theta}{i}(M_A^{\theta-k})(M_B^{i+1}) \qquad (2)\end{aligned}$$

Since $\mid E(S(A)) \mid = 2 \mid E(A) \mid$. Therefore, for each $m\epsilon V(B)$ and $ln\epsilon E(S(A))$ with $l\epsilon V(A)$, $n\epsilon V(S(A)) - V(A)$ the second term of (1) will be

$$\begin{aligned}&\sum_{m\epsilon V(B)}\sum_{ln\epsilon E(S(A))} [d_{A_{+s}B}^{\theta}(l,m) + d_{A_{+s}B}^{\theta}(n,m)]\\ &= \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(S(A))} [\sum_{i=0}^{\theta}\binom{\theta}{i}d_{S(A)}^{\theta-i}(l)d_B^{i}(m) + d_{S(A)}^{\theta}(n)]\\ &= \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(S(A))} [d_{S(A)}^{\theta}(l) + \sum_{i=0}^{\theta}\binom{\theta}{i}d_{S(A)}^{\theta-i}(l).d_B^{i}(m)\\ &\quad + d_{S(A)}^{\theta}(n)]\\ &= \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(S(A))} [d_{S(A)}^{\theta}(l) + d_{S(A)}^{\theta}(n)\\ &\quad + \sum_{i=1}^{\theta}\binom{\theta}{i}d_{S(A)}^{\theta-i}(l)d_B^{i}(m)]\\ &= \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(S(A))} [d_{S(A)}^{\theta}(l) + d_{S(A)}^{\theta}(n)]\\ &\quad + \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(S(A))}\sum_{i=1}^{\theta}\binom{\theta}{i}d_{S(A)}^{\theta-i}(l)d_B^{i}(m)]\\ &= \sum_{m\epsilon V(B)} [M_{S(A)}^{\theta+1}] + \sum_{i=1}^{\theta}\binom{\theta}{i}[M_B^{i}][M_A^{\gamma-i}]\\ &= n_B[M_{S(A)}^{\gamma}] + \sum_{i=1}^{\theta}\binom{\theta}{i}[M_B^{i}][M_A^{\gamma-i}] \qquad (3)\end{aligned}$$

Using (2) and (3) in (1), we get the required result

$$M^{\gamma}(A_{+s}B)=\sum_{i=0}^{\theta}\binom{\theta}{i}(M_A^{\theta-i})(M_B^{i+1})+n_BM_{S(A)}^{\beta}$$
$$+\sum_{i=1}^{\theta}\binom{\theta}{i}M_B^iM_A^{\gamma-i}.$$

Theorem 6: Let A and B be two connected graphs and $\theta\in N-\{0,1\}$. Then, the first general Zagreb index of R-sum graph is

$$M^{\gamma}(A_{+R}B)=\sum_{i=0}^{\theta}\binom{\theta}{i}2^{\theta-i}M_A^{\theta-i}M_B^{i+1}$$
$$+2\sum_{i=0}^{\theta}\binom{\theta}{i}2^{\theta-i}M_A^{\gamma-i}M_B^{i}+2^{\theta}e_An_B,$$

where $\theta=\gamma-1$ and N is set of natural numbers.

Proof: By definition

$$M^{\gamma}(A_{+R}B)=\sum_{(l,m)\epsilon(V(A_{+R}B))}d_{A_{+R}B}^{\gamma}(l,m)$$

For $\theta=\gamma-1$ the above equality in term of edges

$$\begin{aligned}
&M^{\gamma}(A_{+R}B)\\
&=\sum_{(l,m)(n,o)\epsilon(E(A_{+R}B))}[d_{A_{+R}B}^{\theta}(l,m)\\
&\quad+d_{A_{+R}B}^{\theta}(n,o)]\\
&=\sum_{l\epsilon V(A)}\sum_{mo\epsilon(E(B))}[d_{A_{+R}B}^{\theta}(l,m)+d_{A_{+R}B}^{\theta}(l,o)]\\
&\quad+\sum_{m\epsilon V(B)}\sum_{ln\epsilon(E(R(A)))}[d_{A_{+R}B}^{\theta}(l,m)+d_{A_{+R}B}^{\theta}(n,m)]\\
&=\sum_{l\epsilon V(A)}\sum_{mo\epsilon(E(B))}[d_{A_{+R}B}^{\theta}(l,m)+d_{A_{+R}B}^{\theta}(l,o)]\\
&\quad+\sum_{m\epsilon V(B)}\sum_{ln\epsilon(E(R(A)))l,n\epsilon V(A)}[d_{A_{+R}B}^{\theta}(l,m)+d_{A_{+R}B}^{\theta}(n,m)]\\
&\quad+\sum_{m\epsilon V(B)}\sum_{ln\epsilon(E(R(A)))l\epsilon V(A),n\epsilon V(R(A))-V(A)}\\
&\quad\times[d_{A_{+R}B}^{\theta}(l,m)+d_{A_{+R}B}^{\theta}(n,m)] \qquad (4)
\end{aligned}$$

For each vertex $l\epsilon V(A)$ and edge $mo\ \epsilon\ E(B)$, the first term of (4) will be

$$\begin{aligned}
&\sum_{l\epsilon V(A)}\sum_{mo\epsilon(E(B))}[d_{A_{+R}B}^{\theta}(l,m)+d_{A_{+R}B}^{\theta}(l,o)]\\
&=\sum_{l\epsilon V(A)}\sum_{mo\epsilon(E(B))}[[\sum_{i=0}^{\theta}\binom{\theta}{i}d_{R(A)}^{\theta-i}(l).d_B^i(m)]\\
&\quad+[\sum_{i=0}^{\theta}\binom{\theta}{i}d_{R(A)}^{\theta-i}(l).d_B^i(o)]]\\
&=\sum_{i=0}^{\theta}\binom{\theta}{i}\sum_{l\epsilon V(A)}d_{R(A)}^{\theta-i}(l)\sum_{mo\epsilon(E(B))}d_B^i(m)\\
&\quad+\sum_{i=0}^{\theta}\binom{\theta}{i}\sum_{l\epsilon V(A)}d_{R(A)}^{\theta-i}(l).\sum_{mo\epsilon(E(B))}d_B^i(o)\\
&=\sum_{i=0}^{\theta}\binom{\theta}{i}\sum_{l\epsilon V(A)}(2d_A(l))^{\theta-i}\sum_{bo\epsilon(E(B))}d_B^i(m)\\
&\quad+\sum_{i=0}^{\theta}\binom{\theta}{i}\sum_{l\epsilon V(A)}(2d_A(l))^{\theta-i}.\sum_{mo\epsilon(E(B))}d_H^i(o)\\
&=\sum_{i=0}^{\theta}\binom{\theta}{i}(2)^{\theta-i}M_A^{\theta-i}\sum_{mo\epsilon(E(B))}[d_B^i(m)+d_B^i(o)]\\
&=\sum_{i=0}^{\theta}\binom{\theta}{i}(2)^{\theta-i}M_A^{\theta-i}M_B^{i+1} \qquad (5)
\end{aligned}$$

For each vertex $m\epsilon V(B)$ and edge $ln\ \epsilon\ E(R(A))$ $l,n\epsilon V(A)$, the second term of (4) will be

$$\begin{aligned}
&\sum_{m\epsilon V(B)}\sum_{ln\epsilon(E(R(A)B))}[d_{A_{+R}B}^{\theta}(l,m)+d_{A_{+R}B}^{\theta}(n,m)]\\
&=\sum_{m\epsilon V(B)}\sum_{ln\epsilon(E(R(A)))}[[\sum_{i=0}^{\theta}\binom{\theta}{i}d_{R(A)}^{\theta-i}(l).d_B^i(m)]\\
&\quad+[\sum_{i=0}^{\theta}\binom{\theta}{i}d_{R(A)}^{\theta-i}(n).d_B^i(m)]]\\
&=\sum_{m\epsilon V(B)}\sum_{ln\epsilon(E(R(A)))}[[\sum_{i=0}^{\theta}\binom{\theta}{i}d_B^i(m)[d_{R(A)}^{\theta-i}(l)+d_{R(A)}^{\theta-i}(n)]\\
&=\sum_{ln\epsilon(E(R(A)))l,n\epsilon V(A)}[[\sum_{i=0}^{\theta}\binom{\theta}{i}\sum_{m\epsilon V(B)}d_B^i(m)[d_{R(A)}^{\theta-i}(l)\\
&\quad+d_{R(A)}^{\theta-i}(n)]\\
&=\sum_{ln\epsilon(E(R(A)))l,n\epsilon V(A)}[[\sum_{i=0}^{\theta}\binom{\theta}{i}M_B^i[(2d_A(l))^{\theta-i}+(2d_A(n))^{\theta-i}]\\
&=\sum_{ln\epsilon(E(R(A)))l,n\epsilon V(A)}[[\sum_{i=0}^{\theta}\binom{\theta}{i}M_B^i[(2)^{\theta-i}(d_A^{\theta-i}(l))\\
&\quad+(2)^{\theta-i}(d_A^{\theta-i}(n))]\\
&=\sum_{i=0}^{\theta}\binom{\theta}{i}M_B^i\sum_{ln\epsilon(E(R(A)))l,n\epsilon V(A)}[(2)^{\theta-i}(d_A^{\theta-i}(l))\\
&\quad+(2)^{\theta-i}(d_A^{\theta-i}(n))]\\
&=\sum_{i=0}^{\theta}\binom{\theta}{i}M_B^i\sum_{ln\epsilon(E(R(A)))l,n\epsilon V(A)}[(2)^{\theta-i}(d_A^{\theta-i}(l))\\
&\quad+(2)^{\theta-i}(d_A^{\theta-i}(n))]\\
&=\sum_{i=0}^{\theta}\binom{\theta}{i}M_B^i(2)^{\theta-i}\sum_{ln\epsilon(E(R(A)))l,n\epsilon V(A)}[d_A^{\theta-i}(l)+d_A^{\theta-i}(n)]\\
&=\sum_{i=0}^{\theta}\binom{\theta}{i}M_B^i(2)^{\theta-1}[M_A^{\gamma-i}]
\end{aligned}$$

$$= \sum_{i=0}^{\theta}\binom{\theta}{i}M_B^i(2)^{\theta-1}M_A^{\gamma-i} \tag{6}$$

For each vertex $m\epsilon V(B)$ and edge $ln \ \epsilon \ E(R(A))$ $l\epsilon V(A)$, $n\epsilon v(R(A)) - V(A)$. Since we have $d_{R(A)}(l) = 2d_A(l)\ \forall\ l\epsilon V(A)$ also $d_{R(A)}(n) = 2\ \forall\ n\epsilon V(R(A)) - V(A)$. The third term of equation 4 will be,

$$= \sum_{m\epsilon V(B)}\ \sum_{ln\epsilon(E(R(A)B)), l\epsilon V(A), n\epsilon V(R(A))-V(A)} \times [d^{\theta}_{A+_RB}(l,m) + d^{\theta}_{A+_RB}(n,m)]$$

$$= \sum_{m\epsilon V(B)}\ \sum_{ln\epsilon(E(R(A)B)), l\epsilon V(A), n\epsilon V(R(A))-V(A)} \times [\sum_{i=0}^{\theta}\binom{\theta}{i}d^{\theta-i}_{R(A)}(l).d^i_B(m) + d^{\theta}_{R(A)}(n)]$$

Here $d^{\theta}_{R(A)} = 2^{\theta}$ and $d^{\theta-i}_{R(A)}(l) = (2d_A(l))^{\theta-i}$

$$= \sum_{m\epsilon V(B)}\ \sum_{ln\epsilon(E(R(A)B)), l\epsilon V(A), n\epsilon V(R(A))-V(A)} \times [\sum_{i=0}^{\theta}\binom{\theta}{i}(2d_A(l))^{\theta-i}d^i_B(m) + 2^{\theta}]$$

$$= \sum_{m\epsilon V(B)}\ \sum_{ln\epsilon(E(R(A)B)), l\epsilon V(A), n\epsilon V(R(A))-V(A)} \times [\sum_{i=0}^{\theta}\binom{\theta}{i}(2)^{\theta-i}(d_A(l))^{\theta-i}d^i_B(m) + 2^{\theta}]$$

$$= \sum_{i=0}^{\theta}\binom{\theta}{i}[\sum_{m\epsilon V(B)}\ \sum_{ln\epsilon(E(R(A)B)), l\epsilon V(A), n\epsilon V(R(A))-V(A)} (2)^{\theta-i}(d_A^{\theta-i}(l))d^i_B(m)] + [\sum_{m\epsilon V(B)}\ \sum_{ln\epsilon(E(R(A)B)), l\epsilon V(A), n\epsilon V(R(A))-V(A)} 2^{\theta}]$$

$$= \sum_{i=0}^{\theta}\binom{\theta}{i}(2)^{\theta-i}.M_B^i.M_A^{\gamma-i} + 2^{\theta}e_An_B$$

$$= \sum_{i=0}^{\theta}\binom{\theta}{i}(2)^{\theta-i}.M_B^i.M_A^{\gamma-i} + 2^{\theta}e_An_B \tag{7}$$

Using (5), (6) and (7) in (4), we get

$$M(A+_RB) = \sum_{i=0}^{\theta}\binom{\theta}{i}2^{\theta-i}M_A^{\theta-i}M_B^{i+1} + 2[\sum_{i=0}^{\theta}\binom{\theta}{i}2^{\theta-i}M_A^{\gamma-i}M_B^i] + 2^{\theta}e_An_B.$$

Theorem 7: Let A and B be two connected graphs and $\theta \in N - \{0, 1\}$. Then, the first general Zagreb index of Q-sum graph is

$$M^{\gamma}(A+_QB) = \sum_{i=0}^{\theta}\binom{\theta}{i}(M_A^{\theta-i})(M_B^{i+1}) + \sum_{i=0}^{\theta}\binom{\theta}{i}(M_A^{\gamma-i})(M_B^i) + 2n_B\sum_{i=0}^{\theta}\binom{\theta}{i}\sum_{w_aw_b\epsilon E_A}(d_A^{\theta-i}(w_a).d_A^i(w_b)] + n_B\sum_{w_aw_b\epsilon E(A), w_bw_k\epsilon E(A)}[[\sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(w_a).d_A^i(w_b)] + [\sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(w_b).d_A^i(w_c)]],$$

where $\theta = \gamma - 1$ and N is set of natural numbers.

Proof: By definition

$$M^{\gamma}(A+_QB) = \sum_{(l,m)\epsilon V(A+_QB)} d^{\gamma}_{A+_QB}(l,m)$$
$$= \sum_{(l,m)(n,o)\epsilon E(A+_QB)} [d^{\theta}_{A+_QB}(l,m) + d^{\theta}_{A+_QB}(n,o)]$$
$$= \sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)}[d^{\theta}_{A+_QB}(l,m) + d^{\theta}_{A+_QB}(l,o)] + \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(Q(A))}[d^{\theta}_{A+_QB}(l,m) + d^{\theta}_{A+_QB}(n,m)]$$
$$= \sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)}[d^{\theta}_{A+_QB}(l,m) + d^{\theta}_{A+_QB}(l,o)] + \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(Q(A))l,n\epsilon V(A)}[d^{\theta}_{A+_QB}(l,m) + d^{\theta}_{A+_QB}(n,m)] + \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(Q(A))l\epsilon V(A) and n\epsilon V(Q(A))-V(A)} \times [d^{\theta}_{A+_QB}(l,m) + d^{\theta}_{A+_QB}(n,m)] + \sum_{m\epsilon V(B)}\sum_{ln\epsilon E(Q(A))l,n\epsilon V(Q(A))-V(A)} \times [d^{\theta}_{A+_QB}(l,m) + d^{\theta}_{A+_QB}(n,m)] \tag{8}$$

For each vertex $l\epsilon V(A)$ and edge $mo\ \epsilon\ E(B)$, the first term of (8) will be

$$\sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)}[d^{\theta}_{A+_QB}(l,m) + d^{\theta}_{A+_QB}(l,o)]$$
$$= \sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)}[\sum_{i=0}^{\theta}\binom{\theta}{i}d^{\theta-i}_{Q(A)}(l)d^i_B(m) + \sum_{i=0}^{\theta}\binom{\theta}{i}d^{\theta-i}_{Q(A)}(l)d^i_B(o)]$$
$$= \sum_{l\epsilon V(A)}\sum_{mo\epsilon E(B)}[\sum_{i=0}^{\theta}\binom{\theta}{i}d^{\theta-i}_A(l)[d^i_B(m) + d^i_B(o)]]$$

$$= \sum_{l\epsilon V(A)} [\sum_{i=0}^{\theta} \binom{\theta}{i} d_A^{\theta-i}(l)] \sum_{mo\epsilon E(A)} [d_B^i(m) + d_B^i(o)]$$
$$= \sum_{l\epsilon V(A)} [\sum_{i=0}^{\theta} \binom{\theta}{i} d_A^{\theta-i}(l)](M_B^{i+1})$$
$$= \sum_{i=0}^{\theta} \binom{\theta}{i} (M_A^{\theta-i})(M_B^{i+1}) \tag{9}$$

For each vertex $m\epsilon V(B)$ and edge $ln \epsilon E(Q(A))$ $l, n\epsilon V(A)$, the second term of (8) will be

$$\sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A))} [d_{A+_QB}^{\theta}(l,m) + d_{A+_QB}^{\theta}(n,m)]$$
$$= \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)), l\epsilon V(A), n\epsilon V(Q(A))-V(A)} [d_{A+_QB}^{\theta}(l,m)$$
$$+ d_{A+_QB}^{\theta}(n,m)] + \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)), l, n\epsilon V(Q(A))-V(A)}$$
$$\times [d_{A+_QB}^{\theta}(l,m) + [d_{A+_QB}^{\theta}(n,m)] \tag{10}$$

Now $\forall$ $m\epsilon V(B)$, $ln\epsilon E(Q(A))$ iff $l\epsilon V(A)$ and $n\epsilon V(Q(A)) - V(A)$ the third term of (8) will be

$$\sum_{m^{\epsilon} V(B)} \sum_{ln\epsilon E(Q(A)), l\epsilon V(A), n\epsilon V(Q(A))-V(A)} S$$
$$\times [d_{A+_QB}^{\theta}(l,m)] + [d_{A+_QB}^{\theta}(n,m)]$$
$$= \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)), l\epsilon V(A), n\epsilon V(Q(A))-V(A)}$$
$$\times [d_{Q(A)}(l) + d_B(m)]^{\theta} + [d_{Q(A)}^{\theta}(n)]$$
$$= \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)), l\epsilon V(A), n\epsilon V(Q(A))-V(A)}$$
$$\times [\sum_{i=0}^{\theta} \binom{\theta}{i} d_{Q(A)}^{\theta-i}(l).d_B^i(m) + d_{Q(A)}^{\theta}(n)]$$
$$= \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)), l\epsilon V(A), n\epsilon V(Q(A))-V(A)}$$
$$\times [\sum_{i=0}^{\theta} \binom{\theta}{i} d_{Q(A)}^{\theta-i}(l).d_B^i(m)]$$
$$+ \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)), l\epsilon V(A), n\epsilon V(Q(A))-V(A)} [d_{Q(A)}^{\theta}(n)]$$
$$= \sum_{i=0}^{\theta} \binom{\theta}{i} \sum_{m\epsilon V(B)} d_B^i(m) \sum_{ln\epsilon E(Q(A)), l\epsilon V(A), n\epsilon V(Q(A))-V(A)}$$
$$d_{Q(A)}^{\theta-i}(l) + \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)), l\epsilon V(A), n\epsilon V(Q(A))-V(A)} [d_{Q(A)}^{\theta}(n)]$$
$$= \sum_{i=0}^{\theta} \binom{\theta}{i} M_B^i M_A^{\gamma-i} + n_B. \sum_{ln\epsilon E(Q(A)), l\epsilon V_A, n\epsilon V(Q(A))-V(A)}$$
$$\times [d_{Q(A)}^{\theta}(n)]$$
$$= \sum_{i=0}^{\theta} \binom{\theta}{i} M_B^i M_A^{\gamma-i} + n_B[2 \sum_{w_a w_b\epsilon E(A)} (d_A(w_a) + d_A(w_b))^{\theta}]$$
$$= \sum_{i=0}^{\theta} \binom{\theta}{i} M_B^i M_A^{\gamma-i} + 2n_B \sum_{i=0}^{\theta} \binom{\theta}{i}$$
$$\sum_{w_n w_m\epsilon E_A} (d_A^{\theta-i}(w_a).d_A^i(w_b)] \tag{11}$$

Now $\forall$ $m\epsilon V(B)$ and edge $ln\epsilon E(Q(A))$ iff the vertex $l, n\epsilon V(Q(A)) - V(A)$. Then the fourth term of (8) will be

$$= \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)) l, n, V(Q(A))-V(A)}$$
$$\times [d_{A+_QB}(l,m)^{\theta} + d_{A+_QB}(n,m)^{\theta}]$$
$$= \sum_{m\epsilon V(B)} \sum_{ln\epsilon E(Q(A)) l, n, V(Q(A))-V(A)} [d_{A+_QB}(l)^{\theta} + d_{A+_QB}(n)^{\theta}]$$
$$= \sum_{m\epsilon V(B)} \sum_{w_a w_b\epsilon E(A), w_b w_c\epsilon E(A)} [d_A(w_a) + d_A(w_b)]^{\theta}$$
$$+ [d_A(w_b) + d_A(w_c)]^{\theta}$$
$$= n_B \sum_{w_a w_b\epsilon E(A), w_b w_c\epsilon E(A)} [[\sum_{i=0}^{\theta} \binom{\theta}{i} d_A^{\theta-i}(w_a).d_A^i(w_b)]$$
$$+ [\sum_{i=0}^{\theta} \binom{\theta}{i} d_A^{\theta-i}(w_b).d_A^i(w_c)]] \tag{12}$$

Using (9)-(12) in (8), we get the required result

$$= \sum_{i=0}^{\theta} \binom{\theta}{i} (M_A^{\theta-i})(M_B^{i+1}) + \sum_{i=0}^{\theta} \binom{\theta}{i} M_B^i M_A^{\theta}$$
$$+ 2n_B \sum_{i=0}^{\theta} \binom{\theta}{i} \sum_{w_a w_b\epsilon E'(A)} (d_A^{\theta-i}(w_a).d_A^i(w_b)]$$
$$+ n_B \sum_{w_a w_b\epsilon E'(A), w_b w_c\epsilon E'(A)} [[\sum_{i=0}^{\theta} \binom{\theta}{i} d_A^{\theta-i}(w_a).d_A^i(w_b)]$$
$$+ [\sum_{i=0}^{\theta} \binom{\theta}{i} d_A^{\theta-i}(w_b).d_A^i(w_c)]]$$
$$= \sum_{i=0}^{\theta} \binom{\theta}{i} (M_A^{2\gamma-2-i})(M_B^{2i+1})$$
$$+ 2n_B \sum_{i=0}^{\theta} \binom{\theta}{i} \sum_{w_a w_b\epsilon E(A)} (d_A^{\theta-i}(w_a).d_A^i(w_b)]$$
$$+ n_B \sum_{w_a w_b\epsilon E(A), w_b w_c\epsilon E(A)} [[\sum_{i=0}^{\theta} \binom{\theta}{i} d_A^{\theta-i}(w_a).d_A^i(w_b)]$$
$$+ [\sum_{i=0}^{\theta} \binom{\theta}{i} d_A^{\theta-i}(w_b).d_A^i(w_c)]].$$

Theorem 8: Let A and B be two connected graphs then the first general Zagreb index of their T-sum graph is

$$M^{\gamma}(A_{+T}B)$$
$$= \sum_{i=0}^{\theta} \binom{\theta}{i} (2)^{\theta-i} M_A^{\theta-i} M_H^{k+1}$$

$$+\sum_{i=0}^{\theta}\binom{\theta}{i}M_B^i(H)(2)^{\theta-i}M_A^{\gamma-i}$$
$$+\sum_{i=0}^{\theta}\binom{\theta}{i}(2)^{\theta-i}M_B^iM_A^{\gamma-k}+2n_B\sum_{i=0}^{\theta}\binom{\theta}{i}$$
$$\sum_{w_aw_b\epsilon E(A)}(d_A^{\theta-i}(w_a).d_A^i(w_b)]$$
$$+n_B\sum_{w_aw_b\epsilon E(A),w_bw_c\epsilon E(A)}[[\sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(w_a)+d_A^i(w_b)]$$
$$+[\sum_{i=0}^{\theta}\binom{\theta}{i}d_A^{\theta-i}(w_a)+d_A^i(w_c)]],$$

where $\theta=\gamma-1$ and $\gamma\in N^+-\{0,1\}$.

Proof: Since we have $d_{A_{+T}B}(a,b) = d_{A_{+R}B}(a,b)$ for each vertex $a\epsilon V(A)$ and $b\epsilon V(B)$, also $d_{A_{+T}B}(a,b) = d_{A_{+Q}B}(a,b)$ for each vertex $a\epsilon V(T(A)) - V(A)$ and $b\epsilon V(B)$. Therefore, the result follows by the proof of Theorems 6 and 7.

Theorem 9: Let $\Re$ be a set of real numbers, $\gamma\in\Re-\{0,N^+\}$, $\theta=\gamma-1$ and A & B be two connected graphs. Then, the first general Zagreb index of D-sum graphs ($A_{+S}B$, $A_{+R}B$, $A_{+Q}B$ and $A_{+T}B$) are

(i) $M^{\gamma}(A_{+s}B)$
$$=\sum_{i=0}^{\infty}\binom{\theta}{i}(M_A^i)(M_B^{\theta-i+1})+n_BM_{S(A)}^i$$
$$+\sum_{i=1}^{\infty}\binom{\theta}{i}M_B^{\theta-1}M_A^{i+1}$$

(ii) $M^{\gamma}(A_{+R}B)$
$$=\sum_{i=0}^{\infty}\binom{\theta}{i}2^iM_A^iM_B^{\theta-i+1}$$
$$+2[\sum_{i=0}^{\infty}\binom{\theta}{i}2^iM_A^{i+1}M_B^{\theta-i}]+2^{\gamma}e_An_B$$

(iii) $M^{\gamma}_{A_{+Q}B}$
$$=\sum_{i=0}^{\infty}\binom{\theta}{i}(M_A^i)(M_B^{\theta-i+1})+\sum_{i=0}^{\infty}\binom{\theta}{i}(M_A^{i+1})$$
$$(M_B^{\theta-i})+2n_B\sum_{i=0}^{\infty}\binom{\theta}{i}\sum_{w_aw_b\epsilon E(A)}(d_A^i(w_a).d_A^{\theta-i}(w_b)]$$
$$+n_B\sum_{w_aw_b\epsilon E(A),w_bw_c\epsilon E(A)}[[\sum_{i=0}^{\infty}\binom{\theta}{i}d_A^i(w_a).d_A^{\theta-i}(w_b)]$$
$$+[\sum_{i=0}^{\infty}\binom{\theta}{i}d_A^i(w_b).d_A^{\theta-i}(w_c)]]$$

(iv) $M^{\gamma}(A_{+T}B)$
$$=\sum_{i=0}^{\infty}\binom{\theta}{i}2^iM_A^iM_B^{\theta-i+1}$$
$$+[\sum_{i=0}^{\infty}\binom{\theta}{i}2^iM_A^{i+1}M_B^{\theta-i}]$$
$$+\sum_{i=0}^{\infty}\binom{\theta}{i}(M_A^{i+1})(M_B^{\theta-i})$$
$$+2n_B\sum_{i=0}^{\infty}\binom{\theta}{i}\sum_{w_aw_b\epsilon E(A)}(d_A^i(w_a).d_A^{\theta-i}(w_b)]$$
$$+n_B\sum_{w_aw_b\epsilon E(A),w_bw_c\epsilon E(A)}[[\sum_{i=0}^{\infty}\binom{\theta}{i}d_A^i(w_a).d_A^{\theta-i}(w_b)]$$
$$+[\sum_{i=0}^{\infty}\binom{\theta}{i}d_A^i(w_b).d_A^{\theta-i}(w_c)]]$$

Proof: The proof is similar as of Theorem 5-Theorem 8.

If we assume that γ is a negative integer, then from Theorem 9 the following Corollary is obtained:

Corollary 10: Let γ be a negative integer, $\theta=\gamma-1$ and A & B be two connected graphs. Then, the first general Zagreb index of D-sum graph ($A_{+S}B$, $A_{+R}B$, $A_{+Q}B$ and $A_{+T}B$) are

(i) $M^{\gamma}(A_{+s}B)$
$$=\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}(M_A^i)(M_B^{\theta-i+1})$$
$$+n_BM_{S(A)}^i+\sum_{i=1}^{\infty}(-1)^i\binom{\theta+i-1}{i}M_B^{\theta-1}M_A^{i+1}$$

(ii) $M^{\gamma}(A_{+R}B)$
$$=\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}2^iM_A^iM_B^{\theta-i+1}$$
$$+2[\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}2^iM_A^{i+1}M_B^{\theta-i}]+2^{\gamma}e_An_B$$

(iii) $M^{\gamma}_{A_{+Q}B}$
$$=\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}(M_A^i)(M_B^{\theta-i+1})$$
$$+\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}(M_A^{i+1})(M_B^{\theta-i})$$
$$+2n_B\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}\sum_{w_aw_b\epsilon E(A)}(d_A^i(w_a).d_A^{\theta-i}(w_b)]$$
$$+n_B\sum_{w_aw_b\epsilon E(A),w_bw_c\epsilon E(A)}[[\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}$$
$$\times d_A^i(w_a).d_A^{\theta-i}(w_b)]+[\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}$$
$$\times d_A^i(w_b).d_A^{\theta-i}(w_c)]]$$

(iv) $M^{\gamma}(A_{+T}B)$
$$=\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}2^iM_A^iM_B^{\theta-i+1}$$

$$+[\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}2^iM_A^{i+1}M_B^{\theta-i}]$$
$$+\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}(M_A^{i+1})(M_B^{\theta-i})$$
$$+2n_B\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}\sum_{w_aw_b\epsilon E(A)}(d_A^i(w_a).d_A^{\theta-i}(w_b)]$$
$$+n_B\sum_{w_aw_b\epsilon E(A),w_bw_c\epsilon E(A)}[[\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}$$
$$\times d_A^i(w_a).d_A^{\theta-i}(w_b)]+[\sum_{i=0}^{\infty}(-1)^i\binom{\theta+i-1}{i}$$
$$\times d_A^i(w_b).d_A^{\theta-i}(w_c)]]$$

IV. APPLICATIONS AND CONCLUSION

We illustrate the obtained results of Section 3 (Theorem 5-Theorem 8) with the help of the following examples.

Example 1: Let P_n and P_m be two graphs with $n \geq 3$ and $m \geq 2$. Then, we have

1. $M^\gamma(P_{n+S}P_m)$
$$=\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-1-k}(n-2)+2][2^{k+1}(m-2)+2]$$
$$+\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-k}(n-2)+2][2^k(m-2)+2]$$
$$+m(2^\gamma(2n-3)+2)$$
2. $M^\gamma(P_{n+R}P_m)$
$$=\sum_{k=0}^{\gamma}C_K^{\gamma-1}2^{\gamma-1-k}[2^{\gamma-1-k}(n-2)+2]$$
$$\cdot[2^{k+1}(m-2)+2]$$
$$+2\sum_{k=0}^{\gamma}C_K^{\gamma-1}2^{\gamma-1-k}[2^{\gamma-k}(n-2)+2]$$
$$\times[2^k(m-2)+2]+2(n-1)m$$
3. $M^\gamma(P_{n+Q}P_m)$
$$=\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-1-k}(n-2)+2][2^{k+1}(m-2)+2]$$
$$+\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-k}(n-2)+2][2^k(m-2)+2]$$
$$+2m\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-1-k}+(n-2)2^{\gamma-1}+2^k]$$
$$+m\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^k+2(n-1)2^{\gamma-1}+2^{\gamma-1-k}]$$
4. $M^\gamma(P_{n+T}P_m)$
$$=\sum_{k=0}^{\gamma}C_K^{\gamma-1}2^{\gamma-1-k}[2^{\gamma-1-k}(n-2)+2]$$
$$\times[2^{k+1}(m-2)+2]$$
$$+2\sum_{k=0}^{\gamma}C_K^{\gamma-1}2^{\gamma-1-k}[2^{\gamma-k}(n-2)+2]$$
$$\times[2^k(m-2)+2]$$
$$+2m\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-1-k}+(n-2)2^{\gamma-1}+2^k]$$
$$+m\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^k+2(n-1)2^{\gamma-1}+2^{\gamma-1-k}]$$

Example 2: Let C_n and C_m be two graphs for $n \geq 3$ and $m \geq 3$. Then, we have

1. $M^\gamma(C_{n+S}C_m)$
$$=\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-1-k}(n)][2^{k+1}(m)]$$
$$+\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-k}n][2^k)]+m(2^{\gamma+1}n+2^\gamma n)$$
2. $M^\gamma(C_{n+R}C_m)$
$$=\sum_{k=0}^{\gamma}C_K^{\gamma-1}2^{\gamma-1-k}[2^{\gamma-1-k}(n)][2^{k+1}(m)]$$
$$+2\sum_{k=0}^{\gamma}C_K^{\gamma-1}2^{\gamma-1-k}[2^{\gamma-k}n][2^k)]+2^\gamma nm$$
3. $M^\gamma(C_{n+Q}C_m)$
$$=\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-1-k}(n)][2^{k+1}(m)]$$
$$+\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-k}n][2^k)]$$
$$+2m\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-1}n]+2m\sum_{k=0}^{\gamma}C_K^{\gamma-1}[(2n)2^{\gamma-1}]$$
4. $M^\gamma(C_{n+T}C_m)$
$$=\sum_{k=0}^{\gamma}C_K^{\gamma-1}2^{\gamma-1-k}[2^{\gamma-1-k}(n)][2^{k+1}(m)]$$
$$+2\sum_{k=0}^{\gamma}C_K^{\gamma-1}2^{\gamma-1-k}$$
$$\times[2^{\gamma-k}n][2^k)]+2m\sum_{k=0}^{\gamma}C_K^{\gamma-1}[2^{\gamma-1}n]$$
$$+2m\sum_{k=0}^{\gamma}C_K^{\gamma-1}[(2n)2^{\gamma-1}].$$

In this paper, the four D operations on graphs are studies and D-sum graphs $A_{+S}B$, $A_{+R}B$, $A_{+Q}B$ and $A_{+T}B$ are obtained for two connected graphs A and B. Mainly, we have computed the first general Zagreb index of the D-sum graphs in terms of their factor graphs for each $\gamma\epsilon R-\{0,1\}$. The results of [18] and [19] are the special cases of our work for $\gamma=2$ and

$\gamma = 3$ respectively. Moreover, the problem to compute the general Randić index (a generalization of the second Zagreb index) of the D-sum graphs is still open.

CONFLICTS OF INTEREST

There is no conflict of interest.

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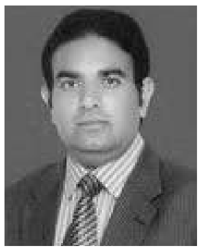

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