Computing Minimum Spanning Tree in Interval Valued Bipolar Neutrosophic Environment

Said Broumi, Assia Bakali, Mohamed Talea, Florentin Smarandache, and Rajkumar Verma

Abstract—**Interval valued bipolar neutrosophic sets is a new generalization of fuzzy set, bipolar fuzzy set, neutrosophic set and bipolar neutrosophic set so that it can handle uncertain information more flexibly in the process of decision making. In this paper, an algorithm for finding minimum spanning tree (MST) of an undirected neutrosophic weighted connected graph (UNWCG) in which the edge weights is represented by a an interval valued bipolar neutrosophic number is presented. The proposed algorithm is based on matrix approach to design the MST of UNWCG. A numerical example is provided to show the effectiveness of the proposed algorithm. Lastly, a comparative study with other existing methods is proposed.**

Index Terms—**Score function, interval valued bipolar neutrosophic sets, Neutrosophic sets, Spanning tree problem.**

I. Introduction

In 1998, Smarandache [1] explored the concept of neutrosophic set (NS) from the philosophical point of view, to represent uncertain, imprecise, incomplete, inconsistent, and indeterminate information that are exist in the real world. The concept of neutrosophic set is a generalization of the concept of the classic set, fuzzy set, intuitionistic fuzzy set (IFS). The neutrosophic sets are characterized by a truth-membership function (t), an indeterminate-membership function (i) and a false-membership function (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. To apply the concept of neutrosophic sets (NS) in science and engineering applications, Smarandache [6] introduced for the first time, the single valued neutrosophic set (SVNS). Later on, Wang *et al.* [2] studied some properties related to single valued neutrosophic sets. The neutrosophic set model is an important tool for dealing with real scientific and engineering applications because it can handle not only incomplete information but also the inconsistent information and indeterminate information. Some more literature about the extension of neutrosophic sets and their applications in various fields can be found in the literature [17].

In classical graph theory, there are common algorithms for solving the minimum spanning tree including Prim and kruskal algorithm. By applying the concept of single valued neutrosophic sets on graph theory, a new theory is developed and called single valued neutrosophic graph theory (SVNGT). The concept of SVNGT and their extensions finds its applications in diverse fields [6]-[16]. Very recently few researchers have used neutrosophic methods to find minimum spanning tree in neutrosophic environment. Ye [4] proposed a method to find minimum spanning tree of a graph where nodes (samples) are represented in the form of SVNS and distance between two nodes which represents the dissimilarity between the corresponding samples has been derived. Kandasamy [3] proposed a double-valued Neutrosophic Minimum Spanning Tree (DVN-MST) clustering algorithm, to cluster the data represented by double-valued neutrosophic information.Mandal and Basu [5] presented a solution approach of the optimum spanning tree problems considering the inconsistency, incompleteness and indeterminacy of the information. The authors consider a network problem with multiple criteria which are represented by weight of each edge in neutrosophic setsThe approach proposed by the authors is based on similarity measure. Recently Mullai [18] solved the minimum spanning tree problem on a graph in which a bipolar neutrosophic number is associated to each edge as its edge length, and illustrated it by a numerical example.

The principal objective of this paper is to propose a new version of Prim‘s algorithm based on matrix approach for finding the cost minimum spanning tree of an undirected graph in which an interval valued bipolar neutrosophic number [19] is associated to each edge as its edge length.

The rest of the paper is organized as follows. Section 2 briefly introduces the concepts of neutrosophic sets, single valued neutrosophic sets and the score function of interval valued bipolar neutrosophic number. Section 3 proposes a novel approach for finding the minimum spanning tree of interval valued bipolar neutrosophic undirected graph. In Section 4, an illustrative example is presented to illustrate the proposed method. In section 5, a comparative study with other existing methods is provided. Finally, Section 6 concludes the paper.

II. Preliminaries

Some of the important background knowledge for the materials that are presented in this paper is presented in this section. These results can be found in [1], [2], [19].

Definition 2.1 [1] Le ξ be an universal set. The neutrosophic set A on the universal set ξ categorized in to three membership functions called the true T_A (x),

Manuscript received October 1, 2017; revised October 30, 2017.

Said Broumi is with the University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco (e-mail: broumisaid78@gmail.com).

Assia Bakali is with Ecole Royale Navale, Morocco (e-mail: assiabakali@yahoo.fr).

Mohamed Talea is with the University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco (e-mail: taleamohamed@yahoo.fr)

Florentin Smarandache is with the University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA (e-mail: fsmarandache@gmail.com)

Rajkumar Verma is with the Guru Govind Singh Indraprastha University, Delhi, India (e-mail: rkver83@gmail.com)

DOI: 10.7763/IJMO.2017.V7.602

indeterminate $I_A(x)$ and false $F_A(x)$ contained in real standard or non-standard subset of $]^-0, 1^+[$ respectively.

$$^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+ \quad (1)$$

Definition 2.2 [2] Let ξ be a universal set. The single valued neutrosophic sets (SVNs) A on the universal ξ is denoted as following

$$A = \{<x: T_A(x), I_A(x), F_A(x) > x \in\} \quad (2)$$

The functions $T_A(x) \in [0. 1]$, $I_A(x) \in [0. 1]$ and $F_A(x) \in [0. 1]$ are named degree of truth, indeterminacy and falsity membership of x in A, satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \quad (3)$$

Definition 2.3 [4]. An interval valued bipolar neutrosophic set A in X is defined as an object of the form

$$A=\{<x, \begin{array}{l} <\left[T_L^p, T_M^p\right], \left[I_L^p, I_M^p\right], \left[F_L^p, F_M^p\right], \left[T_L^n, T_M^n\right], \\ \left[I_L^n, I_M^n\right], \left[F_L^n, F_M^n\right]> \end{array} >: x \in X\},$$

where T_L^p, T_M^p I_L^p, I_M^p, F_L^p, F_M^p :X $\longrightarrow$ [0, 1] and T_L^n, T_M^n I_L^n, I_M^n, F_L^n, F_M^n : X $\longrightarrow$ [-1, 0] .The positive interval membership degree where T_L^p, T_M^p I_L^p, I_M^p, F_L^p, F_M^p denotes the lower and upper truth membership, lower and upper indeterminate membership and lower and upper false membership of an element $\in$ X corresponding to a bipolar neutrosophic set A and the negative interval membership degree T_L^n, T_M^n I_L^n, I_M^n, F_L^n, F_M^n : denotes the lower and upper truth membership, lower and upper indeterminate membership and lower and upper false membership of an element $\in$ X to some implicit counter-property corresponding to an interval valued bipolar neutrosophic set A.

Deli *et al.* [19], introduced a concept of score function. The score function is applied to compare the grades of IVBNS. This function shows that greater is the value, the greater is the interval valued bipolar neutrosphic sets and by using this concept paths can be ranked

Definition 2.4 [19]. Let $\tilde{A} =<\left[T_L^p, T_M^p\right], \left[I_L^p, I_M^p\right], \left[F_L^p, F_M^p\right], \left[T_L^n, T_M^n\right], \left[I_L^n, I_M^n\right], \left[F_L^n, F_M^n\right]>$ be an interval valued bipolar neutrosophic number, Then, the score function $s(\tilde{A})$, accuracy function $a(\tilde{A})$ and certainty function $c(\tilde{A})$ of an IVBNN are defined as follows:

$$\text{(i) } s(\tilde{A}) = \left(\frac{1}{12}\right) \times \left| \begin{array}{l} T_L^p + T_M^p + 1 - I_L^p + 1 - I_M^p + 1 - \\ F_L^p + 1 - F_M^p + 1 + T_L^n \\ +1 + T_M^n - I_L^n - I_M^n - F_L^n - F_M^n \end{array} \right| \quad (4)$$

$$\text{(ii) } a(\tilde{A}) = T_L^p + T_L^p - F_L^p - F_M^p + T_L^n + T_M^n - F_L^n - F_M^n \quad (5)$$

$$\text{(iii) } c(\tilde{A}) = T_L^p + T_M^p - F_L^n - F_M^n \quad (6)$$

Comparison of interval valued bipolar neutrosophic numbers

Let $\tilde{A}_1 =<\left[T_{L1}^p, T_{M1}^p\right], \left[I_{L1}^p, I_{M1}^p\right], \left[F_{L1}^p, F_{M1}^p\right], \left[T_{L1}^n, T_{M1}^n\right], \left[I_{L1}^n, I_{M1}^n\right], \left[F_{L1}^n, F_{M1}^n\right]>$ and $\tilde{A}_2 =<\left[T_{L2}^p, T_{M2}^p\right], \left[I_{L2}^p, I_{M2}^p\right], \left[F_{L2}^p, F_{M2}^p\right], \left[T_{L2}^n, T_{M2}^n\right], \left[I_{L2}^n, I_{M2}^n\right], \left[F_{L2}^n, F_{M2}^n\right]>$ be two interval valued bipolar neutrosophic numbers then

If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

If $s(\tilde{A}_1) = s(\tilde{A}_2)$,and $a(\tilde{A}_1) \succ a(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) \succ c(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) = c(\tilde{A}_2)$ then $\tilde{A}_1$ is equal to $\tilde{A}_2$, that is, $\tilde{A}_1$ is indifferent to $\tilde{A}_2$, denoted by $\tilde{A}_1 = \tilde{A}_2$

III. Minimum Spannig Tree Algorithm of IVBN-Undirected Graph

In this section, a neutrosophic version of Prim's algorithm is proposed to handle minimum spanning tree in a neutrosophic environment. In the following, we propose an interval valued bipolar neutrosophic minimum spanning tree algorithm (IVBNMST), whose steps are described below:

Algorithm:

Input: The weight matrix M = $\left\lfloor W_{ij} \right\rfloor_{n\times n}$ for the undirected weighted neutrosophic graph G.

Output: Minimum cost Spanning tree T of G.

Step 1: Input interval valued bipolar neutrosophic adjacency matrix A.

Step 2: Translate the IVBN-matrix into score matrix $\left\lfloor S_{ij} \right\rfloor_{n\times n}$ by using score.

Step 3: Iterate step 4 and step 5 until all (n-1) entries matrix of S are either marked or set to zero or other words all the nonzero elements are marked.

Step 4: Find the score matrix S either columns-wise or row-wise to find the unmarked minimum entries S_{ij} ,which is the weight of the corresponding edge e_{ij} in S.

Step 5: If the corresponding edge e_{ij} of selected S_{ij} produce a cycle with the previous marked entries of the score matrix S then set S_{ij} = 0 else mark S_{ij} .

Step 6: Construct the graph T including only the marked entries from the score matrix S which shall be desired minimum cost spanning tree of G.

Step 7: Stop.

IV. Numerical Example

In this section, a numerical example of IVBNMST is used to demonstrate of the proposed algorithm. Consider the following graph G= (V, E) shown in Figure 2, with fives nodes and fives edges. The different steps involved in the construction of the minimum cost spanning tree are described as follow:

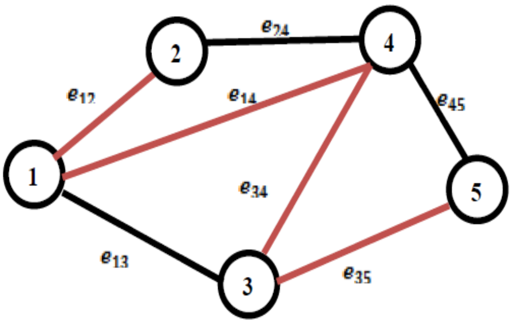

Fig. 2. Undirected IVBN- graphs.

e	Edge length
e_{12}	$<[0.3, 0.4], [0.1, 0.2], [0.2, 0.4], [-0.8, -0.7], [-0.5, -0.3], [-0.1, 0]>$
e_{13}	$<[0.4, 0.5], [0.2, 0.6], [0.4, 0.6], [-0.2, -0.1], [-0.4, -0.2], [-0.5, -0.4]>$
e_{14}	$<[0.6, 0.7], [0.7, 0.8], [0.8, 0.9], [-0.6, -0.5], [-0.4, -0.3], [-0.4, -0.2]>$
e_{24}	$<[0.4, 0.5], [0.8, 0.9], [0.3, 0.4], [-0.2, -0.1], [-0.5, -0.1], [-0.7, -0.5]>$
e_{34}	$<[0.2, 0.4], [0.3, 0.4], [0.7, 0.8], [-0.2, -0.1], [-0.4, -0.1], [-0.4, -0.3]>$
e_{35}	$<[0.4, 0.5], [0.6, 0.7], [0.5, 0.6], [-0.4, -0.3], [-0.4, -0.2], [-0.3, -0.2]>$
e_{45}	$<[0.5, 0.6], [0.4, 0.5], [0.3, 0.4], [-0.4, -0.2], [-0.5, -0.2], [-0.8, -0.6]>$

The IVBN- adjacency matrix A is given below:

$$= \begin{bmatrix} 0 & e_{12} & e_{13} & e_{14} & 0 \\ e_{12} & 0 & 0 & e_{24} & 0 \\ e_{13} & 0 & 0 & e_{34} & e_{35} \\ e_{14} & e_{24} & e_{34} & 0 & e_{45} \\ 0 & 0 & e_{35} & e_{45} & 0 \end{bmatrix}$$

Thus, using the score function, we get the score matrix

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 3. Score matrix.

According to the Fig. 3, we observe that the minimum entries 0.358 is selected and the corresponding edge (1, 4) is marked by the brown color. Repeat the procedure until the iteration will exist.

According to the Fig. 4 and Fig. 5, the next non zero minimum entries 0.408 is marked and corresponding edges (3, 5) are also colored

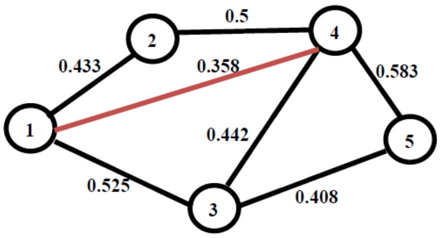

Fig. 4. The marked edge (1,4) of G in next iteration.

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 5. The marked next minimum entries 0.408 of S.

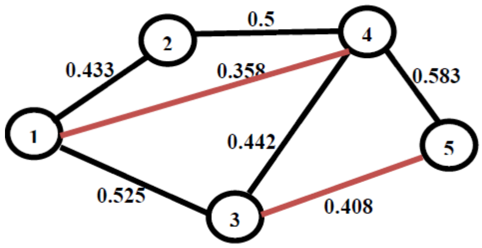

Fig. 6. The marked edge (3,5) of G in next iteration.

$$S= \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 7. The marked next minimum entries 0.433 of S

According to the Fig. 7, the next minimum non zero element 0.433 is marked.

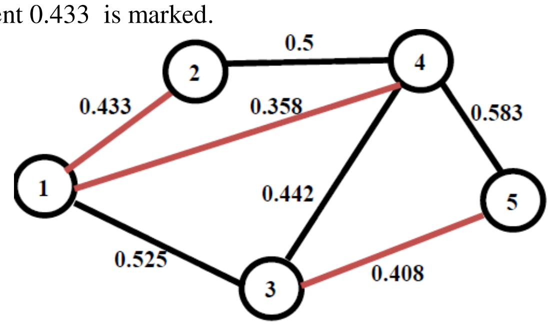

Fig. 8. The marked edge (1, 2) of G in next iteration.

According to the Fig. 9. The next minimum non zero element 0.442 is marked, and corresponding edges (3, 4) are also colored

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & 0.5 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 9. The marked next minimum entries 0.442 of S.

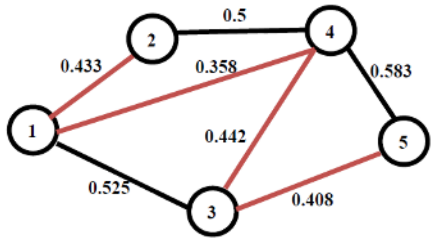

Fig. 10. The marked edge (3, 4) of G in next iteration.

According to the figure 11. The next minimum non zero element 0.5 is marked. But while drawing the edges it produces the cycle. So we delete and mark it as 0 instead of 0.5

$$S = \begin{bmatrix} 0 & 0.433 & 0.525 & 0.358 & 0 \\ 0.433 & 0 & 0 & \cancel{0.5}\ 0 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 11. The marked next minimum entries 0.5 of S.

The next non zero minimum entries 0.525 is marked it is shown in the Fig. 12. But while drawing the edges it produces the cycle. So, we delete and mark it as 0 instead of 0.525

$$S = \begin{bmatrix} 0 & 0.433 & \cancel{0.525}\ 0 & 0.358 & 0 \\ 0.433 & 0 & 0 & \cancel{0.5}\ 0 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 12. The marked next minimum entries 0.525 of S.

According to the Fig. 13. The next minimum non zero element 0.583 is marked. But while drawing the edges it produces the cycle so we delete and mark it as 0 instead of 0.583.

$$S = \begin{bmatrix} 0 & 0.433 & \cancel{0.525}\ 0 & 0.358 & 0 \\ 0.433 & 0 & 0 & \cancel{0.5}\ 0 & 0 \\ 0.525 & 0 & 0 & 0.442 & 0.408 \\ 0.358 & 0.5 & 0.422 & 0 & 0.583\ 0 \\ 0 & 0 & 0.408 & 0.583 & 0 \end{bmatrix}$$

Fig. 13. The marked next minimum entries 0.583 of S.

After the above steps, the final path of minimum cost of spanning tree of G is portrayed in Fig. 14.

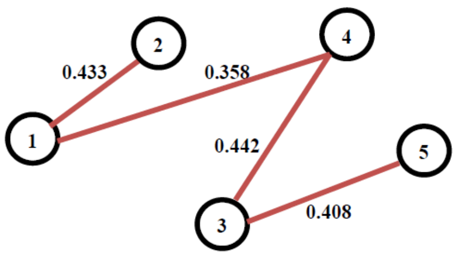

Fig. 14. Final path of minimum cost of spanning tree of G.

Using the above steps described in section 4, hence, the crisp minimum cost spanning tree is 1,641 and the final path of minimum cost of spanning tree is{2,1},{1, 4},{4, 3},{3, 5}.

V. Comparative Study

In order to illustrate the rationality and effectiveness of the proposed method, we apply the algorithm proposed by Mullai *et al.* [18] on our IVBN-graph presented in Section 4. Following the setps of Mullai's algorithm we obtained the results

Iteration 1:

Let C_1={1} and $\overline{C_1}$={2, 3,4 ,5}

Iteration 2:

Let C_2={1,4} and $\overline{C_2}$={2, 3 ,5}

Iteration 3:

Let C_3={1,4, 2} and $\overline{C_3}$={3, 5}

Iteration 4:

Let C_4={1,4, 2, 3} and $\overline{C_4}$={5}

Finally, IVBN minimal spanning tree is

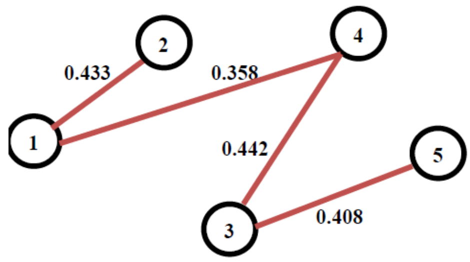

Fig. 15. IVBN minimal spanning tree obtained by Mullai's algorithm.

So, it can be seen that the IVBN minimal spanning tree {2, 1},{1, 4},{4, 3},{3, 5}obtained by Mullai's algorithm, After deneutrosophication of edges'weight using the score function, is the same as the path obtained by proposed algorithm.

The difference between the proposed algorithm and Mullai's algorithm is that our approach is based on Matrix approach, which can be easily implemented in Matlab, whereas the Mullai's algorithm is based on the comparison of edges the in each iteration of the algorithm, which leads to high computation.

VI. Conclusion

This paper deals with minimum spanning tree problem on a network where the edges weights are represented by an interval valued bipolar neutrosophic numbers. This work can be extended to the case of directed neutrosophic graphs and other types of neutrosophic graphs .

Acknowledgment

The authors are very grateful to the chief editor and reviewers for their comments and suggestions, which is helpful in improving the paper

REFERENCES

[1] F. Smarandache, "Neutrosophy, neutrosophic probability, set, and logic," *ProQuest Information & Learning,* Ann Arbor, Michigan, USA, 1998,105 p.

[2] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Multisspace and Multistructure*, vol. 4, 2010, pp. 410-413.

[3] I. Kandasamy, " Double-valued neutrosophic sets, their minimum spanning trees, and clustering algorithm," *Journal of Intelligent System*, 2016, pp.1-17.

[4] J. Ye, "Single valued neutrosophic minimum spanning tree and its clustering method," *Journal of Intelligent System*, vol. 23, 2014, pp. 311-324.

[5] K. Mandal and K. Basu, " Improved similarity measure in neutrosophic environment and its application in finding minimum spanning tree," *Journal of Intelligent & Fuzzy System*, vol. 31, 2016, pp. 1721-1730.

[6] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu, "Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers," in *Proc. the 2016 International Conference on Advanced Mechatronic Systems*, Melbourne, Australia, 2016, pp. 417-422.

[7] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu, " Applying Dijkstra Algorithm for Solving Neutrosophic Shortest Path Problem," in *Proc. the 2016 International Conference on Advanced Mechatronic Systems, Melbourne*, Australia, November 30 - December 3, 2016, pp. 412-416.

[8] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, " Single valued neutrosophic graphs," *Journal of New Theory*, no. 10, 2016, pp. 86-101.

[9] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "On bipolar single valued neutrosophic graphs," *Journal of New Theory*, no. 11, 2016, pp. 84-102.

[10] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "Interval valued neutrosophic graphs," *Critical Review*, XII, 2016. pp.5-33.

[11] S. Broumi, M. Talea, F. Smarandache, and A. Bakali, " Single valued neutrosophic graphs: Degree, order and size," in *Proc. the IEEE International Conference on Fuzzy Systems (FUZZ)*, 2016, pp. 2444-2451.

[12] S. Broumi, F. Smarandache, M. Talea, and A. Bakali, "Decision-making method based on the interval valued neutrosophic graph," *Future Technologie*, 2016, IEEE, pp. 44-50.

[13] S. Broumi, F. Smarandache, M. Talea, and A. Bakali, " Operations on interval valued neutrosophic graphs, chapter in book- new trends in neutrosophic theory and applications- florentin smarandache and surpati pramanik (editors)," 2016, pp. 231-254.

[14] S. Broumi, A. Bakali, T. Mohamed, F. Smarandache, and L. Vladareanu, "Shortest path problem under triangular fuzzy neutrosophic information," in *Proc. 10th International Conference on Software, Knowledge, Information Management & Applications (SKIMA)*, 2016, pp.169-174.

[15] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and M. Ali, "Shortest path problem under bipolar neutrosphic setting," *Applied Mechanics and Materials*, vol. 859, 2016, pp 59-66.

[16] S. Broumi, F. Smarandache, M. Talea, and A. Bakali, "An introduction to bipolar single valued neutrosophic graph theory," *Applied Mechanics and Materials*, vol.841, 2016, pp. 184-191.

[17] Gallup. [Online]. Available: http://fs.gallup.unm.edu/NSS/.

[18] M. Mullai, S. Broumi, and A. Stephen, " Shortest path problem by minimal spanning tree algorithm using bipolar neutrosophic numbers," *International Journal of Mathematic Trends and Technology*, vol. 46, 2017. pp .80-87

[19] I. Deli, Y. Şubaş, F. Smarandache, and M. Ali, "Interval valued bipolar neutrosophic sets and their application in pattern recognition," in *Proc. 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE),* 2016, pp.2460 - 2467

Said Broumi was born in Casablanca, Morocco in 1978, he is a PhD student of University Hassan II - Casablanca. He received his M.Sc in industrial automatic from Hassan II University Ainchok- Casablanca. His search major field is on neutrosophic graph theory, soft set theory, fuzzy theory, intuitionistic fuzzy theory, neutrosophic theory and neutrosophic soft set theory and neutrosophic decision making problem.

Assia Bakali is a professor of Higher Education at the Royal Naval School, Casablanca, Morocco.

Mohamed Talea was born in Casablanca, he is a professor of higher education at the faculty of Sciences Ben M'Sik, University Hassan II Morocco Casablanca. He obtained his PhD in collaboration with the LMP laboratory in Poitiers University, France in 2001. He obtained a doctorate of high graduate studies degree at the University Hassan II-Mohammedia in 1994. Actually he is the director of Information Treatment Laboratory. He has published over twenty papers in conferences and national and international journals. His search major field is on Systems engineering, in security of system information.

Florentin Smarandache is a professor of mathematics at the University of New Mexico in USA. He published over 75 books and 400 articles and notes in mathematics, physics, philosophy, psychology, rebus, literature. In mathematics his research is in number theory, non-Euclidean geometry, synthetic geometry, algebraic structures, statistics, neutrosophic logic and set (generalizations of fuzzy logic and set respectively), neutrosophic probability (generalization of classical and imprecise probability).Also, small contributions to nuclear and particle physics,information fusion, neutrosophy (a generalization of dialectics), law of sensations and stimuli, etc. He got the 2010 Telesio-Galilei Academy of Science Gold Medal Adjunct Professor (equivalent to Doctor HonorisCausa) of Beijing Jiaotong University in 2011, and 2011 Romanian Academy Award for Technical Science (the highest in the country).Dr. W. B. VasanthaKandasamy and Dr. FlorentinSmarandache got the 2012 and 2011 New Mexico-Arizona BookAward for Algebraic Structures.

Rajkumar Verma is with the Guru Govind Singh Indraprastha University, Delhi, India.