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CONCENTRATION COMPACTNESS PRINCIPLES FOR THE SYSTEMS OF CRITICAL ELLIPTIC EQUATIONS

DONGSHENG KANG

Abstract. In this paper, some important variants of the concentration compactness principle are established. By the variants, some kinds of the elliptic systems can be investigated and the existence of nontrivial solutions to the systems can be verified by the variational methods.

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