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## CHAPTER II

# CONCENTRATION IN CANADIAN MANUFACTURING INDUSTRIES, 1948

### 1. Concentration in Various Industries

It is often said that Canadian manufacturing industries are "highly" concentrated. The indexes shown in Table A-1 of Appendix A make possible more precise and objective statements about the level of concentration. Table 5 shows the frequency distribution of industries and employment by concentration class (number of leading firms accounting for 80 per cent of employment).

TABLE 5

Distribution of 96 Canadian Manufacturing Industries and  
Their Employees by Level of Concentration, 1948

NUMBER OF LARGEST FIRMS REQUIRED TO ACCOUNT FOR 80% OF EMPL. <sup>a</sup>	PERCENTAGE OF NUMBER OF INDUSTRIES		PERCENTAGE OF EMPLOYMENT	
	<i>In Each Class</i>	<i>Cumulative</i>	<i>In Each Class</i>	<i>Cumulative</i>
Less than 1.70	8.3	8.3	2.4	2.4
1.70 to 2.73	8.3	16.7	5.9	8.3
2.73 to 3.65	8.3	25.0	7.6	15.9
3.65 to 4.40	8.3	33.3	1.3	17.2
4.40 to 6.30	8.3	41.7	7.2	24.4
6.30 to 9.10	8.3	50.0	1.8	26.2
9.10 to 12.81	8.3	58.3	6.8	33.0
12.81 to 20.0	8.3	66.7	5.7	38.7
20 to 40	8.3	75.0	11.7	50.3
40 to 90	8.3	83.3	12.5	62.8
90 to 250	8.3	91.7	11.4	74.2
250 and over	8.3	100.0	25.8	100.0
Total	100.0		100.0	

<sup>a</sup> Class intervals include lower limit.

Source: Computed from Appendix A, Table A-1.

On the average (as measured by the median) the largest 9.1 firms in an industry account for 80 per cent of its employment. The 48 industries with lower concentration are, however, considerably larger than the 48 industries with greater concentration. Half of all employment is in industries in which over 35 firms are required to account

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for 80 per cent of employment. Thus the general level of concentration appears lower when size of industry is taken into account than when each industry is given an equal weight. The negative correlation between size of industry and concentration is discussed further below.

Concentration varies greatly about the average level, as Table A-1 indicates. At one end of the range is aluminum production, with a single producer in the industry, and at the other is sawmilling, in which it takes 1,843 firms to account for 80 per cent of employment. The primary metals, cigarettes, cottons, cement, soap are among the industries with very high concentration, while the clothing trades, feed mills, machine shops, and pharmaceuticals are examples of industries with particularly low concentration.

Most of the industries in textiles and apparel, wood products, and paper products have relatively low concentration while most of the industries in the metals, nonmetallic minerals, and chemical groups have high concentration (see Table 6). In the food group the typical level of concentration is close to the average for all industries.<sup>1</sup>

<sup>1</sup> Application of the "Chi square" test to a condensed version of Table 6 indicates that the level of concentration is significantly higher in the foods group than in textiles and significantly higher in metals, minerals, and chemicals (considered as one group) than in foods. The number of industries in the wood products, paper, and miscellaneous groups (taken separately) is too small to permit application of the same test, but the departures from the average pattern are so marked that it is hard to doubt their significance.

The Chi square test was applied to the following tabulation:

Number of Firms Accounting for 80 Per Cent of Employment			Metals, Minerals, Chemicals	Wood, Paper, Misc.	Total
	Foods	Textiles			
Under 9.1	10(11)	5(12)	27(17.5)	6(7.5)	48
9.1 and over	12(11)	19(12)	8(17.5)	9(7.5)	48
Total	<u>22</u>	<u>24</u>	<u>35</u>	<u>15</u>	<u>96</u>

The figures in brackets give the frequencies that would be obtained if the industries in each group were distributed between the two concentration classes in the same proportion as the total of all industries. The value of Chi square is 19.263, with three degrees of freedom. The corresponding probability that the observed tabulation would be obtained if the industry groups represented random groupings of the 96 industries is less than 0.1 per cent.

The contributions to Chi square from the different groups are as follows:

Foods	0.182
Textiles	8.167
Metals, minerals, chemicals	10.314
Wood, paper, miscellaneous	0.600
Total	<u>19.263</u>

If these are regarded as values of Chi square with one degree of freedom,

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### TABLE 6

Distribution of 96 Canadian Manufacturing Industries by  
Concentration Level and Industry Group, 1948

NUMBER OF LARGEST FIRMS REQUIRED TO ACCOUNT FOR 80% OF EMPL. <sup>a</sup>	INDUSTRY GROUP								Total
	<i>Foods, Beverages, Tobacco</i>	<i>Textile Prod- ucts</i>	<i>Wood Prod- ucts</i>	<i>Paper Prod- ucts</i>	<i>Metal Prod- ucts</i>	<i>Non- Metallic Minerals</i>	<i>Chem- icals</i>	<i>Miscel- laneous</i>	
Under 2.73	2	1	0	0	4	4	3	2	16
2.73 to 4.4	3	1	1	0	4	2	4	1	16
4.4 to 9.1	5	3	0	1	1	2	3	1	16
9.1 to 20	3	8	3	0	1	0	1	0	16
20 to 90	3	7	0	2	1	1	2	0	16
90 and over	6	4	4	0	1	1	0	0	16
Total	22	24	8	3	12	10	13	4	96

<sup>a</sup> Class intervals include lower limit.

Source: Appendix A, Table A-1.

The above summary of concentration levels is based on all industries in the sample studied. The sample includes (1) industries with high imports or exports; (2) industries with regionally segregated markets; and (3) industries which correspond more closely to a single national market (see Chapter I). In industries with a single national market, differences in concentration might be expected to have the strongest influence on business policy. The concentration pattern of the 58 industries with "national" markets, as shown in Table 7, is not substantially different from that of the broader group shown in Table 5. The median number of firms required to account for 80 per cent of employment is 8.9 for the industries with national markets and 9.1 for the complete sample. Both number of industries and percentages of employment are distributed similarly by concentration classes for the two groups.

The industries with particularly high imports tend to have higher concentration than the average, while those with regionally segre-

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the corresponding probabilities of obtaining values at least as high from random groupings are as follows:

- Foods, between 0.5 and 0.7
- Textiles, less than 0.01
- Metals, minerals, chemicals less than 0.01
- Wood, paper, miscellaneous, between 0.3 and 0.5

These probabilities must be taken as very approximate since the number of industries in each group is not large enough to permit the use of the Chi square tabulation with confidence. But there can be no doubt of the significance of the difference between textiles and foods, and that between foods and the metals, minerals, and chemicals group.

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TABLE 7

Distribution of 58 Canadian Manufacturing Industries with National Markets, and their Employees, by Level of Concentration, 1948

NUMBER OF LARGEST FIRMS REQUIRED TO ACCOUNT FOR 80% OF EMPL. <sup>a</sup>	INDUSTRIES	PERCENTAGE OF NUMBER OF INDUSTRIES		PERCENTAGE OF EMPLOYMENT	
		<i>In Each Class</i>	<i>Cumula- tive</i>	<i>In Each Class</i>	<i>Cumula- tive</i>
Less than 1.7	Matches				
	Hardwood distillation				
	Cement				
	Gypsum products	6.9	6.9	1.0	1.0
1.7 to 2.7	Automobiles				
	Cotton thread				
	Cigarettes, etc.				
	Pipes and smokers' supplies				
2.7 to 3.5	Pig iron	8.6	15.5	10.3	11.3
	Umbrellas				
	Abrasive products				
	Compressed gases				
	Railway rolling stock				
2.7 to 3.5	Steel ingots and castings	8.6	24.1	10.7	22.0
	Malt products				
	Boiler compounds				
	Writing inks				
	Starch and glucose				
3.5 to 4.1	Excelsior	8.6	32.8	0.5	22.5
	Soap				
4.1 to 6.3	Sugar refining				
	Pens and pencils				
	Tobacco processing				
	Roofing paper	8.6	41.4	3.0	25.5
	Printing inks				
6.3 to 9.0	Leather belting				
	Processed cheese				
	Washing compounds				
	Buttons and fasteners	8.6	50.0	1.0	28.5
	Wine				
9.0 to 12.0	Fur dressing and dyeing				
	Narrow fabrics				
	Biscuits and crackers				
	Synthetic textiles and silk	8.6	58.6	6.9	33.4
12.0 to 17.0	Polishes and dressings				
	Hardwood flooring				
	Dyeing and finishing of textiles				
	Corsets and girdles				
12.0 to 17.0	Coffins and caskets	8.6	67.4	2.5	36.0

(cont. on next page)

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TABLE 7 (cont.)

NUMBER OF LARGEST FIRMS REQUIRED TO ACCOUNT FOR 80% OF EMPL. <sup>a</sup>	INDUSTRIES	PERCENTAGE OF NUMBER OF INDUSTRIES		PERCENTAGE OF EMPLOYMENT	
		<i>In Each Class</i>	<i>Cumula- tive</i>	<i>In Each Class</i>	<i>Cumula- tive</i>
17.0 to 41.0	Leather tanning Paints and varnishes Confectionery, cocoa, etc. Leather gloves Canvas goods	8.6	75.9	6.4	42.4
41.0 to 75.0	Women's clothing contractors Medicinal and pharmaceutical products Hosiery and knit goods Paper boxes and bags Fruit and vegetable preparations	8.6	84.5	17.5	59.9
75.0 to 200	Men's clothing contractors Miscellaneous leather products Boat building Boots and shoes Men's clothing factories	8.6	93.1	16.8	76.7
200 and over	Furniture Fur goods Butter and cheese factories Women's clothing factories	6.9	100.0	23.3	100.0
Total		100.0		100.0	

<sup>a</sup> Class intervals include lower limit.

Source: Appendix A, Tables A-1 and A-4.

gated markets have considerably lower concentration (measured on a national basis) than the average. Industries with very high exports show no particular deviation from the total sample (Table 8).<sup>2</sup> The median values of the concentration index are as follows:

Industries with high imports	4.8
Industries with high exports	7.0
Industries with national markets	8.9
Industries with regionally separated markets	149.2

These difference will be further investigated in a later part of this chapter.

<sup>2</sup> One industry (agricultural machinery) appears on the lists of both those with high imports and those with high exports.

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TABLE 8

Concentration in Selected Groups of Canadian Industries, 1948

<i>Number of Largest Firms Required to Account for 80% of Empl.</i>	<i>Industries with High Imports</i>	<i>Industries with High Exports</i>	<i>Industries with Separate Regional Markets</i>
Less than 4.0	Coal tar distillation Glass Bicycles Petroleum products	Aluminum Nickel Artificial abrasives Distilleries Aircraft Cordage, rope, twine	
4.0 to 9.0	Agricultural imple- ments Asbestos products Carpets, mats, rugs Cotton yarn and cloth Coke products	Agricultural imple- ments Macaroni Vegetable oils	Breweries
9.0 to 40.0	Cotton and jute bags Woolen yarn Plate, cut, and or- namental glass Woolen cloth	Slaughtering and meat packing Condensed milk Ship building Veneer and plywood Flour mills Pulp and paper	
40 and over		Fish curing and packing Saw mills	Iron castings Prepared feeds Cement products Soft drinks Machine shops Planing mills, sash and door factories Feed mills Bread and other bak- ery products

Source: Appendix A, Table A-1.

2. Quality of the Sample

All the above results are based on a sample of industries which is by no means random (see Chapter I, section 3). One may therefore ask how faithfully they reflect the concentration pattern of manufacturing as a whole.

The sample is large, containing over half of all manufacturing industries and nearly three-quarters of total output.<sup>3</sup> The omitted sector

<sup>3</sup> See Appendix C.

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consists of those industries which do not properly correspond with a homogeneous product group (see Chapter I) and the question that should be investigated is whether the results obtained for the sampled *industries* are applicable to the omitted *product groups*.

The omitted products would have to differ greatly from those of the industries covered to produce an average concentration level for manufacturing as a whole (taking industry size into account) very different from that shown in Table 5. Such a radical difference between the sample and the omitted sector is possible but not likely.

There is, however, reason to expect *some* systematic bias toward low concentration in the sample as compared with the omitted sector. The omitted *industries* are smaller than those sampled, as shown by the fact that the sample covers a higher percentage of output than of the number of industries. Moreover, most of the omitted industries produce several heterogeneous products and the number of distinct product groups is larger than the number of industries, so that average output per product group is smaller than the average industry, and a fortiori smaller than the average sampled industry. Table 5 shows that smaller industries tend to have higher concentration, and it is reasonable to suppose that this relation also holds for product output, so that concentration in the markets of the product groups not covered in this study is probably higher than in the markets of the industries covered.

A more conclusive judgment is possible about whether the *variation* of concentration by kinds of manufacturing observed in the sample (Table 6) is representative of manufacturing as a whole. This problem is investigated in Appendix C where it is shown that the differences in concentration among product groups found in the sample are probably applicable to manufacturing as a whole.<sup>4</sup>

### 3. Factors Related to Concentration

How can one account for the observed variation in concentration? This problem will be investigated by seeking factors that can be expected a priori to be determinants of concentration and measuring the strength of their association with concentration in the cross section of industries.

The inquiry has two stages. First, we shall separate the index of concentration into algebraic components that can be said to be determining factors (rather than being determined by concentration) and

<sup>4</sup> With the possible exception of fabricated metal products, where there is insufficient information on concentration by products.

that are reasonably independent of one another. Then we shall, in turn, investigate determinants of these component factors.

Our index of concentration, the number of leading firms accounting for 80 per cent of employment, is equal to the total number of firms multiplied by the *proportion* of firms accounting for 80 per cent of employment. Following common (though not universal) usage we call the latter variable a measure of *inequality* of firm size, to distinguish it from measures of *concentration*. It measures inequality inversely, just as our index of concentration measures concentration inversely.

Concentration therefore increases with inequality of firm size and decreases with an increase in the number of firms. This relation between concentration, inequality of firm size, and the number of firms holds not only for the index used here, but also for other types of measures such as Herfindahl's summary index or the percentage of employment accounted for by given number of the leading firms, all of which measure the extent to which a high percentage of employment is concentrated in a small number of firms.<sup>5</sup>

The number of firms in an industry is equal to the size of the industry divided by the average size of firms (both measured in the same units). These two variables will be regarded as determining the number of firms in our analysis.

It follows that concentration can be regarded as depending on industry size, firm size, and the degree of inequality of firm size. Given a certain value for each of these variables, the concentration index can have only one value. Concentration rises with an increase in average firm size or in inequality of firm size and with a decrease in industry size.

It will be convenient for further discussion to spell out these relations in algebraic terms. Let

- $C$  = concentration index
- $E$  = inequality index
- $N$  = number of firms
- $I$  = industry size
- $F$  = firm size

The corresponding lower case symbols  $c, e, n, i, f$ , will be used for the logarithms of these variables. We then have the following relations:

<sup>5</sup> For a fuller discussion and demonstration of this relation see Gideon Rosenbluth, "Measures of Concentration," in *Business Concentration and Price Policy*, Princeton University for National Bureau of Economic Research, 1955, pp. 61, 62.

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TABLE 9

Frequency Distribution of Number of Firms, 96 Canadian Manufacturing Industries, 1948

CLASS INTERVAL (NUMBER OF FIRMS IN THE INDUSTRY)	NUMBER OF INDUSTRIES							
	Total	Foods, Etc.	Textiles	Wood and Paper Products	Metal Products	Non- metallic Mineral Products	Chemicals	Misc. Industries
1 to 3	3				2		1	
3 to 7	9		1		1	4	2	1
7 to 15	20	6	2	1	3	2	4	2
15 to 35	15	3	4	2	2	2	1	1
35 to 84	19	3	7	3	2	1	3	
84 to 201	9	3	4	1			1	
201 to 485	10	3	3	1	1	1	1	
485 to 1171	7	2	3	1	1			
1171 to 2831	3	2		1				
2831 to 6844	1			1				
Total	96	22	24	11	12	10	13	4
Median Number of Firms	37	48	64	71	15	11	14	12

Source: Appendix A, Table A-7.

TABLE 10

Frequency Distribution of Percentage of Firms Accounting for 80 Per Cent of Employment, 96 Canadian Manufacturing Industries, 1948

CLASS INTERVAL (PERCENTAGE OF FIRMS ACCOUNTING FOR 80% OF EMPL.)	NUMBER OF INDUSTRIES							
	Total	Foods, Etc.	Textiles	Wood and Paper Products	Metal Products	Non- metallic Mineral Products	Chemicals	Misc. Industries
3.91 to 5.28	1	1						
5.28 to 7.01	1				1			
7.01 to 9.67	2	1					1	
9.67 to 13.07	2	1			1			
13.07 to 17.68	6	1			3	1	1	
17.68 to 23.91	13	4	2		1		5	1
23.91 to 32.34	27	3	10	6	2	3	2	1
32.34 to 43.74	28	9	6	4	1	5	2	1
43.74 to 59.15	13	1	6	1	1	1	2	1
59.15 to 80.00	3	1			2			
Total	96	22	24	11	12	10	13	4
Median per- centage	30.38	31.28	33.83	31.69	22.53	36.52	23.43	34.04

Source: Appendix A, Table A-7.

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TABLE 11

Frequency Distribution of Employment, 94 Canadian Manufacturing Industries, 1948

CLASS INTERVAL (TOTAL EMPLOYMENT)	NUMBER OF INDUSTRIES							
	Total	Foods, Etc.	Textiles	Wood and Paper Products	Metal Products	Non- metallic Mineral Products	Chemicals	Misc. Industries
95 to 181	4			1			3	
181 to 342	3		1					2
342 to 648	3						3	
648 to 1,226	14	5	1		1	3	4	
1,226 to 2,323	18	3	7	3		3		2
2,323 to 4,402	13	3	4	1	1	3	1	
4,402 to 8,340	14	3	4	1	3	1	2	
8,340 to 15,805	6	4	1	1				
15,805 to 29,951	13	3	4	2	4			
29,951 to 56,757	6	1	2	2	1			
Total	94	22	24	11	10	10	13	4
Median Em- ployment	3,254	4,825	3,750	6,335	13,224	1,779	723	791

Source: Appendix A, Table A-4, Aluminum and Nickel omitted.

TABLE 12

Frequency Distribution of Employment per Firm, 96 Canadian Manufacturing Industries, 1948

CLASS INTERVAL (NUMBER OF EMPLOYEES PER FIRM)	NUMBER OF INDUSTRIES							
	Total	Foods, Etc.	Textiles	Wood and Paper Products	Metal Products	Non- metallic Mineral Products	Chemicals	Misc. Industries
2.40 to 4.96	1	1						
4.96 to 10.22	3			2			1	
10.22 to 21.07	16	4	5	2	1	1	3	
21.07 to 43.43	14	4	3	2		1	2	2
43.43 to 89.54	26	5	9	2	2	2	5	1
89.54 to 184.6	15	5	5	2			2	1
184.6 to 380.6	8	2			3	3		
380.6 to 784.4	6	1	1	1	1	2		
784.4 to 1616.9	3		1		1	1		
1616.9 to 3400	4				4			
Total	96	22	24	11	12	10	13	4
Median Number of Employees per Firm	67.2	50.9	64.8	27.0	508.5	266.2	51.0	45.4

Source: Appendix A, Table A-7.

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$$\begin{aligned} (1) \quad & C = EN \\ (2) \quad & = EI \div F \end{aligned}$$

Or in terms of the logarithms,

$$\begin{aligned} (3) \quad & c = e + n \\ (4) \quad & = e + i - f. \end{aligned}$$

Variation in concentration (differences among industries) is thus related mathematically to variation in one or more of the constituent variables, and we shall assume, for the purpose of this investigation, that it is *due to* variation in the constituent variables.

The frequency distributions of these variables are shown in Tables 9 to 12. The class intervals in these tables increase in proportion to the class limits,<sup>6</sup> hence they represent equal intervals in terms of the logarithms of the variables. The tabulations show that the frequency distributions of the variables are highly skewed to the right, while the distributions of the logarithms are close to symmetrical.

Comparison of Tables 9 to 12 with Table 6 suggests that low concentration in the *textiles* group is associated with large size of industry, small size of firm, and low inequality of firm size, while in *wood products* the even lower general level of concentration is associated with very small firm size as well as large industries. In the *paper* industries firms are large (though this would not be true if the printing and publishing trades had been included) but large industry size more than compensates for the size of firm, and the degree of inequality is low, so that the group has relatively low concentration. In *metals*, on the other hand, concentration is high even though industries are large, because of the large average size of firms and the high degree of inequality.

Most of the sampled industries processing *nonmetallic minerals* are small and have large firms, both characteristics conducive to high concentration. In *chemicals*, however, most of the industries have small firms (although there is a good deal of integration of plants classified in different chemical industries, which is not reflected in these statistics) but the industries are small and inequality of firm size is high, so that concentration is high. The same relation between firm size, industry size, and concentration is found in the four industries drawn from the miscellaneous group.

What is the relative importance of these variables in determining concentration? In one sense they are all of equal importance. The

<sup>6</sup> I.e. the ratio of upper to lower limit is the same for all intervals in a distribution.

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effect on concentration of a given *absolute* change in one of them depends on the level of the others (equation 2) while the effect of a given *proportionate* change (i.e. an absolute change in the logarithms) is the same for all the variables (equations 3 and 4). In language familiar to the economist, the elasticity of concentration is the same in numerical value (namely unity) with respect to firm size, industry size, number of firms, and inequality.<sup>7</sup>

The relative importance of the variables can, however, be discussed in a different sense. To explain the actual variation in concentration one must know how much of it is contributed by variation in each of the determining variables. For this purpose it is convenient to work with the logarithms of the variables since there is a simple linear relation between them (equations 3 and 4).

When concentration is viewed as depending on the number of firms and the inequality of firm size (equation 3 above), a striking result is obtained. The variation in concentration can be ascribed almost entirely to variation in the number of firms, while the influence of variation in inequality is very slight indeed.

This conclusion is based on the analysis shown in the following tabulation:

	96 Industries	94 Industries <sup>8</sup>
Variance of logarithms of concentration index	0.560	0.541
Equals		
Variance of logarithms of inequality index	=0.045	=0.044
Plus		
Variance of logarithms of number of firms	+0.587	+0.551
Plus		
Twice co-variance of logarithms of inequality index and number of firms	-0.072	-0.054

While variation in the number of firms, as measured here, is very nearly the same as variation in concentration, both are nearly twelve times as great as variation in inequality. The negative co-variance between the inequality index and the number of firms is also too small to make a significant contribution.<sup>9</sup>

<sup>7</sup> This proposition is, of course, not true for all conceivable concentration indexes, but it holds for many others beside ours. Cf. Rosenbluth, *op. cit.*, p. 62, note 16.

<sup>8</sup> Excluding aluminum and nickel. These industries are excluded from the subsequent analysis since no accurate figures for firm size and industry size are available.

<sup>9</sup> Since the index measures inequality inversely this negative covariance indicates a slight tendency for inequality to increase with increasing number of firms.

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The same findings can be expressed in the language of the correlation coefficient. Let  $r$  equal the correlation coefficient, and the subscripts  $c, e, n$ , stand for the logarithms of the concentration index, inequality index, and number of firms respectively. The correlation coefficients are as follows:

	<i>96 Industries</i>	<i>94 Industries</i>
$r_{cn} =$	0.96	0.96
$r_{ce} =$	0.06	0.11
$r_{en} =$	-0.22	-0.17

These coefficients indicate that when industries are compared, high concentration is generally associated with a small number of firms, but is no more likely to be associated with a high degree of inequality of firm size than with a low degree. Hence the assumption that one industry is more concentrated than another because it has fewer firms is very likely to be right, but a judgment that one industry is more concentrated than another because its firms differ relatively more in size is no better than a pure guess.<sup>10</sup>

A very important conclusion follows. As a reasonable first approximation, one may regard difference in concentration among industries as reflecting purely differences in the number of firms, and need not inquire into the causes of inequality of firm size.

Variation in the number of firms, according to the scheme outlined above, reflects variation in industry size and in firm size. These two variables are not, however, completely independent<sup>11</sup> and hence there is some difficulty in separating out their respective influence on concentration.

The most reasonable interpretation of the correlation between industry size and firm size is that the former influences the latter, since the size of the market is one of the factors limiting the size of firm.

<sup>10</sup> That these results are not a peculiarity of the particular logarithmic transformation used can be seen by an examination of the corresponding rank correlation coefficients. The values are as follows:

	<i>96 Industries</i>
$R_{cn} =$	0.94
$R_{ce} =$	-0.05
$R_{en} =$	-0.22

where  $R$  is the Spearman rank correlation coefficient. These values of  $R$  are very similar to the corresponding values of  $r$  shown above. While the second correlation is positive in one case and negative in the other, the coefficient is of insignificant magnitude in either case.

<sup>11</sup> The correlation coefficient for the logarithms of industry size and firm size is 0.27. The rank correlation coefficient is 0.21.

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Hence part of the variation in average firm size can be regarded as reflecting variation in industry size while the remaining part is independent. On the basis of this interpretation the variance of logarithms of the number of firms can be redivided into one part reflecting the direct *and indirect* effect of industry size, and another part equal to the *independent* variation in firm size, as follows: <sup>12</sup>

Variance of logarithms of number of firms	0.551
Equals	
Portion attributable directly and indirectly to industry size	=0.229
Plus	
Independent variation in logarithms of average firm size	+0.322

Independent variation in average firm size is thus revealed as the more important factor influencing the number of firms, accounting for 58 per cent of the variation in the latter. While industry size varies somewhat more than average firm size, its effect on the number of firms is reduced by the slight tendency for large industries to have large firms.

The number of firms, which constitutes an "intermediate" variable in our analysis, can now be eliminated in order to examine directly the influence of industry size, firm size, and inequality on concentration. Here again, correlation among the determining variables complicates the analysis. As before, one may assume that industry size influences average firm size. There is also, however, a slight correlation between industry size and the inequality index, and it will be assumed that industry size influences inequality of firm size, larger industries tending to have greater inequality in their firm-size distributions. It is difficult to justify this assumption on theoretical grounds, but it seems more plausible than the alternative assumptions that inequality of firm size influences the size of industry, or that some other factor influences both. It does not appear necessary to assume that average firm size influences inequality of firm size, since the

<sup>12</sup> The part reflecting the direct and indirect influence of industry size is equal to  $r^2_{in} S^2_n$ , where  $r$  is the correlation coefficient,  $S^2$  the variance, and the subscripts  $i$  and  $n$  stand for the logarithms of industry size and firm size respectively.

The part attributable to independent variation in firm size is equal to  $(1-r^2_{if}) S^2_f$  where the subscript  $f$  stands for the logarithm of firm size.

The sum of these two parts is equal to  $S^2_n$ . This equation can be derived from the relation among variances and covariances of  $n$ ,  $i$ , and  $f$ , given the identity  $n = i - f$ .

The analysis is based on 94 industries, with aluminum and nickel excluded (see note 8).

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small co-variance of these two variables can be adequately explained by the correlation between firm size and industry size.<sup>13</sup>

On the basis of these assumptions the variance of logarithms of the concentration index may be analyzed as follows: <sup>14</sup>

Variance of logarithm of concentration index	0.541	100%
Equals		
Direct and indirect influence of industry size	=0.159	29%
Plus		
Effect of independent variation in average firm size	+0.345	64%
Plus		
Independent variation in inequality index	+0.037	7%

Thus "independent" variation in average firm size makes by far the largest contribution to variation in concentration among industries, accounting for nearly two-thirds of the latter. Variation in industry size accounts for less than one-third of the variation in concentration since its effect is reduced by its positive association with firm size. Independent variation in the degree of inequality of firm size accounts for only 7 per cent of the variation in concentration.

In much of the literature on concentration there is a strong tendency to equate concentration and firm size, i.e. to assume, often without discussion, that concentration must be high where firms are large, and low where firms are small. Our findings provide some justification for this practice since the greater part of the variation in concentration among the industries examined can in fact be ascribed to variation in average firm size.

It must be emphasized that these findings apply to a comparison of concentration among industries—to be precise among a selected sample of Canadian manufacturing industries. There is no reason to think that the relative importance of the three variables would be the same in a comparison of concentration in one industry at different

<sup>13</sup> The partial correlation coefficient for firm size and inequality, with industry size held constant, is only  $-0.10$  while the partial correlation coefficient for industry size and inequality, with firm size held constant, is  $-0.35$ .

<sup>14</sup> Data for 94 industries, aluminum and nickel excluded.

The direct and indirect influence of industry size is measured by  $S_c^2 r_{ci}^2$  (using the notation of note 12 and the subscript  $c$  for the logarithm of the concentration index).

The effect of independent variation in average firm size is  $S_c^2 (r_{c,if}^2 - r_{ci}^2)$  where  $r_{c,if}$  is the multiple correlation coefficient for  $c$  as dependent variable and  $i$  and  $f$  as independent variables.

The independent variation in the inequality index is equal to  $S_c^2 (1 - r_{c,if}^2)$ .

points of time, or in comparisons between countries. It seems likely that the results, interpreted as indicating general orders of magnitude, are applicable to manufacturing industries in industrialized countries generally, but this hypothesis must be confirmed by further empirical investigation.

The analysis suggests that for a better understanding of the causes of differences in concentration, the most promising line of investigation concerns the factors (other than industry size) that influence the average size of firms. Some of these will be investigated in the following subsection. We shall also comment on variation in industry size, but for the most part the latter will be treated as an exogenous variable in the analysis that follows.

#### 4. *The Variation in Average Firm Size*

In seeking the causes of variation in firm size it is important to note that the problem is to account for differences in average firm size among industries, and not differences in firm size within an industry.

Many economists believe that firms tend to be large where the technology of an industry requires a high ratio of capital to labor. A very explicit statement of this view can be found in the work of A. S. Dewing: "If the grade of product demands proportionally a small quantity of labor to a given quantity of capital . . . then the most profitable scale of production is relatively large." On the other hand, "the practical businessman, with what he calls 'horse sense' . . . says 'If you have a little capital go into a business with a good deal of handwork.' Sufficient special economies to enable the producer to sell in a free competitive market can be attained in a small-sized shop if labor predominates in the cost of production." These rules, according to Dewing, govern the scale of the multi-plant firm as well as single plants.<sup>15</sup>

Dewing does not give any clear reason for this "law" (though there is some discussion of the economies of scale) but appears to regard it as an empirical fact well known to businessmen. A review of other writings suggests two plausible reasons why size of firm may be related to the ratio of capital to labor.

First, it is argued that where fixed capital is important relative to other factors, a larger number of operations are generally performed mechanically than in industries where fixed capital is less important, so that there will generally be a greater variety of machinery. Lowest-

<sup>15</sup> A. S. Dewing, *The Financial Policy of Corporations*, 4th ed., Ronald, 1941, pp. 869, 871, and 872.

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cost production requires that machines be used in such proportions that each can be operated at full capacity. Hence the minimum efficient scale of plant is the lowest common multiple of machine capacities. This minimum scale will tend to be higher in industries using a greater variety of machinery. A larger minimum scale of plant, in turn, will tend to raise the average size of plants.

P. Sargent Florence found evidence of an association between "horsepower per worker" and representative size of plant in cross sections of American and British manufacturing industries and offered the explanation sketched above for his results.<sup>16</sup>

It may be objected that the average size of *plants* should not be expected to govern the average size of *firms*, since firms in an industry may be considerably larger than the minimum efficient scale of plant. Our data suggest, however, that average firm size and average plant size are closely related, as shown by a rank correlation coefficient of 0.979.<sup>17</sup> Average plant size is, of course, smaller than average firm size, since many firms operate several plants each, but the differences are not sufficiently irregular to produce a low correlation (Table 13).<sup>18</sup> The analysis in Chapter III suggests that while many indus-

TABLE 13  
Average Firm Size and Average Plant Size,  
96 Canadian Manufacturing Industries, 1948

NUMBER OF EMPLOYEES PER FIRM	NUMBER OF INDUSTRIES	UNWEIGHTED AVERAGES	
		<i>Employment Per Firm</i>	<i>Employment Per Plant</i>
Less than 13	12	9.7	9.5
13 to 25	12	18.6	17.7
25 to 47	12	33.9	30.6
47 to 67	12	58.1	55.3
67 to 89	12	79.0	67.7
89 to 172	12	126.2	106.7
172 to 450	12	259.2	210.1
450 and over	12	1,377.6	774.2

Source: Appendix A, Table A-7.

tries contain very large multi-plant firms, a relatively large number of small single-plant firms generally dominate the average firm size.

There is a second, and more important reason for expecting a high

<sup>16</sup> P. Sargent Florence, *Investment, Location and Size of Plant*, Cambridge University Press, 1948, pp. 100-113.

<sup>17</sup> For the 96 industries sampled. Data in Appendix A, Table A-7.

<sup>18</sup> The correlation between plant and firm concentration is only slightly lower, the rank correlation coefficient being 0.947. These relations are discussed further in Chap. III.

capital-labor ratio to be associated with large average firm size. A large firm can generally obtain capital on better terms than a small one, and this advantage is of greater competitive importance where the proportion of capital used to other factors is higher.<sup>19</sup>

The influence of industry size on average firm size has already been mentioned. One factor limiting the size of individual firms in a particular industry is generally the limitation of the market. More can be sold only at lower prices or through higher selling expenses. Greater demand for an industry's product tends to widen the market for individual firms and thus to raise the average size of firms.

When comparing different industries one may construct a common unit of account by expressing demand and cost conditions as functional relations between employment and costs and revenues per unit of employment. Cost conditions and the relation between industry demand and firm demand will, of course, vary considerably among industries, but since there is no reason to expect a systematic association between these conditions and industry size, it follows that larger industries will show some tendency to have larger firms. Industry size is therefore treated as a factor influencing firm size, but its influence is weak, as has been shown.

In addition to industry size and the technical characteristics of the process of production, transportation costs may account for variation in the average size of firms. If the purchasers of a product are geographically dispersed, as is the case in consumer goods industries and many others, greater production in one place will require transportation over greater distances and, therefore, higher transportation costs per unit of product. The importance of this factor in limiting the most profitable size of plant depends, first, on the importance of transportation costs per unit of distance in relation to other costs, second, on the geographical density of the demand, and finally on the importance of economies of scale, that is, on the degree to which unit costs other than transportation costs fall as output in one location is expanded. Plants will tend to be small if transportation costs are relatively high, if demand is "spread thin," and if economies of scale are not sufficiently important to out-weigh these factors. The cost of transporting raw materials to the plant may limit its size in a similar manner.

While high transportation costs limit the size of plants they need

<sup>19</sup> See for example Tibor Scitovsky, *Welfare and Competition*, Irwin, 1951, p. 326; E. A. G. Robinson, *The Structure of Competitive Industry*, London, Nisbet, 1935, p. 62.

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not directly affect the size of firms, which can operate a "chain" of plants. As indicated above, however, average plant size and firm size are closely related, as a result of the relatively large number of single-plant firms in most industries. In the industries with high transport costs of products or materials (Table 14) the leading firms do, on the average, have a very large number of plants (see Chapter III), but there are so many small one-plant firms that the average number of plants per firm is about the same as in other industries (Table 14, last column).

Other industrial characteristics influencing average firm size have been mentioned by various writers. Among them Scitovsky<sup>20</sup> has suggested that firms tend to be small where purchasers are well informed about the significant characteristics of their products, and larger in "uninformed" markets where purchasers rely in part on the "reputation" of the seller, which tends to be correlated with size. Firms tend to be small where purchasers demand variety or craftsmanship in the product and large where they accept standardization and mechanical techniques. Scitovsky believes that there is an association between "informed" markets and demand for variety and craftsmanship, though the reason for this is not clear.

Robinson<sup>21</sup> suggests that the technical unit of production tends to be small where products are small and simple, and large where products are either large or complex, requiring a large number of operations or the assembly of many parts. This classification would appear to be relevant to industries whose products are indivisible units rather than a continuous flow, and suggests a correlation between average plant size and labor requirements per unit of product.

### 5. Variation in Industry Size

Industry size, which has been shown to have a significant influence on concentration, will be taken as "given" in the analysis that follows. The causes of variation in industry size are so numerous and complex that we cannot in this study isolate and measure particular influences.

The problem here is to account for differences in size among industries in the same economy at a given time. For a "closed" economy the theories of general equilibrium direct our attention to the following determinants: the structure of wants;<sup>22</sup> the state of technology; the

<sup>20</sup> *Op. cit.*, pp. 327-335.

<sup>21</sup> *Op. cit.*, pp. 34-35.

<sup>22</sup> For completeness one must include here the state of mind of businessmen

amount, types, and distribution of property (including productive resources) and personal ability; and the structure of markets (degree of competition, etc.). In the simplified model of general equilibrium developed by Leontief,<sup>23</sup> outputs of the various industries are determined by the structure of final demand and by a set of "input-output coefficients" reflecting the state of technology, which are taken as given.

In an open economy like that of Canada the output pattern is modified by international specialization. Where imports are important the domestic industry is smaller, and where exports are important it is larger than domestic demand would require. Moreover, international specialization has indirect effects on the size of industries. For example, those supplying materials chiefly to exporting industries are likely to be larger than they would be in a closed economy, and those supplying materials to importing industries smaller.

The structure of demand, the technical conditions of production, and the distribution of wealth and ability are affected by a multitude of particular influences, reaching far back in the history of the economy, and their analysis would go beyond the scope of the present paper. Industry size will therefore be studied, in conjunction with the other factors influencing firm size, as one of the determinants of concentration, but will not be further analyzed.

It is of interest that, in spite of the importance of international trade in Canada, the pattern of relative industry sizes in Canada is very similar to that of the United States. For a group of forty industries for which classifications can be matched, size of industry is highly correlated in the two countries, as shown by a correlation coefficient of 0.94. It is probable that cultural similarity leads to similar consumption patterns in the two countries, while technological similarity means that the "input-output coefficients" are approximately equal.<sup>24</sup>

### 6. Problems of Measurement

In order to test how adequately the forces discussed in the preceding two sections account for differences in concentration among in-

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investing in plant, equipment, and inventory and the political forces determining government expenditure.

<sup>23</sup> W. W. Leontief, *The Structure of the American Economy, 1919-1939*, 2nd ed., Oxford University Press, 1951, Part II.

<sup>24</sup> See Chap. IV where these relations are discussed more fully. The correlation is based on the logarithms of industry size.

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dustries, the characteristics mentioned must be measured or at least identified in each industry. We used the following methods of measurement and classification.

### CAPITAL PER WORKER

The Canadian Census of Manufactures included data on the "value of capital used in production" up to 1943. This concept included the book value of land, plant, and equipment actually used in production, as well as inventory, cash, and other current assets. On the other hand, such items as idle plant or mineral reserves and investment in other firms were excluded. The series was discontinued after 1943, partly on the ground that many manufacturers had not followed the instructions on Census schedules, so that the figures obtained were heterogeneous in their scope and in the valuation method on which they were based. It is believed that many small manufacturers simply reported the value of their capital *stock* from the liabilities side of their balance sheet.

Nevertheless the figures for individual industries show continuity and what seems to be a reasonable pattern over time. It is worthwhile to use them for a comparison of capital per worker and firm size, on the assumption that if there is an association between these variables, the imperfections in the figures might weaken but not destroy it.

Capital per wage earner and capital per employee were computed for both 1943, the year of wartime peak output, and 1938, a depression year, in order to observe whether the results were affected by the stage of the business cycle. Cyclical variation in employment tends to lower the ratio of capital to labor in prosperity and raise it in depression. On the other hand, cyclical price fluctuations are reflected in the valuation of current additions to real capital, and inventories and cash holdings also vary over the business cycle. It is likely that these cyclical influences affect the ranking of industries by capital-labor ratio.

Both total capital per wage earner and fixed capital per wage earner were examined, since the importance of financing is related to total capital requirements while the reasoning of Florence relates the size of plant to fixed capital requirements.

For 1948, when statistics of capital in use were no longer collected, the horsepower capacity of prime movers per wage earner was used as an index of capital per worker. This measure has obvious limitations that have been repeatedly pointed out.<sup>25</sup> To the extent that the

<sup>25</sup> For example, Harry Jerome, *Mechanization in Industry*, National Bureau of Economic Research, 1934, pp. 209 ff.; Charles A. Bliss, *The Structure of*

manufacturing process involves chemical action and temperature change rather than motion, this ratio must be a poor index. Moreover, where simple equipment is used to work on heavy materials, the ratio of horsepower rating of equipment to its value is likely to be high, so that horsepower per worker will overstate capital intensity as compared with other industries. On the other hand, this measure does not suffer from the inconsistencies in reported capital values that bedevil the capital statistics and is not affected by price fluctuations.

In a comparison of the value of fixed capital per worker and horsepower installed per worker (1943), a rank correlation coefficient of 0.74 was obtained.<sup>26</sup> Such a relatively low coefficient suggests that one measure is only an imperfect index of the other. Since, as has been indicated, each has its own weakness as a measure of real capital per worker, both are examined in relation to concentration and size of firm, in order to test the hypotheses outlined above.

In order to obtain an index of horsepower per worker under normal operating conditions the following correction for cyclical variation in the use of capacity was made. The ratio of horsepower per worker was multiplied by an index of the degree of utilization of capacity, obtained by dividing the consumption of electricity purchased for power and lighting (in kilowatt hours) by the capacity of electric motors operated by purchased power (in horsepower). This adjustment is equivalent to a reduction in those capital-labor ratios that are relatively high because of low utilization of capacity. The resulting index is expressed in kilowatt hours per wage earner.

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*Manufacturing Production*, National Bureau of Economic Research, 1939, p. 12; Florence, *op. cit.*, pp. 9 ff.; W. L. Thorp, "Horsepower Statistics for Manufactures," *Journal of the American Statistical Association*, December 1929, pp. 376-385.

Electric motors operated by purchased electricity are included in the horsepower total, but where power for electric motors is supplied by, e.g. steam engines in the establishment, only the capacity of the steam engines is included, to avoid duplication.

Thorp makes the point that there tends to be more idle capacity in the form of electric motors than other prime movers, so that in industries with a high proportion of electric motors operated by purchased power, mechanization tends to be overstated. However, Bliss found that in the United States in 1929, 12.4 million horsepower of electric motors operated by power generated in the plant were run by 11.7 million horsepower of prime movers, i.e. practically the same rated capacity. He concludes that "no marked discrepancy seems to arise from the different character of power equipment that is primary from the standpoint of the manufacturing establishment" (Bliss, *op. cit.*, p. 12, note 10).

<sup>26</sup> This correlation is based on 66 industries—the 96 listed in Appendix A, Table A-1, with omissions and replacements listed in Table 18 under correlations 5, 6, 7 and the further omission of nonferrous metals, excelsior, and electrical apparatus.

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### INDUSTRY SIZE AND FIRM SIZE

Since the statistics on concentration are in terms of employment, industry size and average firm size are also measured by employment for the purpose of studying their influence on concentration. When the relation between the capital-labor ratio and firm size is investigated, however, the use of employment per firm to measure average firm size tends to understate the degree of correlation. Hence, for this investigation (Second Appendix to Chapter II), horsepower capacity per firm and fixed capital per firm are used as well as employment per firm.

### TRANSPORTATION BARRIERS

In Table 14 the industries that have been classified as selling in regionally segregated markets (Table 8) are supplemented by indus-

TABLE 14

Concentration and Related Variables, Canadian Industries with Segregated Regional Markets for Materials or Products, 1948

<i>Industry</i>	<i>Employment per Firm</i>	<i>Employment</i>	<i>Index of Inequality<sup>a</sup></i>	<i>Index of Concentration<sup>b</sup></i>	<i>Index of Capital per Worker<sup>c</sup></i>	<i>Number of Plants per Firm</i>
Pulp and paper mills	731.3	51,924	31.7	22.5	242,093	1.65
Breweries	221.2	8,407	22.7	8.6	12,181	1.61
Slaughtering and meat packing	182.3	21,879	9.4	11.2	8,247	1.18
Iron castings	88.4	19,354	21.0	45.9	4,848	1.03
Fruit and vegetable canning	44.0	16,644	19.1	72.3	2,546	1.32
Fish curing and packing	23.2	12,243	25.1	132.5	4,054	1.14
Prepared stock and poultry feeds	16.6	4,324	35.5	92.4	15,263	1.22
Soft drinks	16.4	6,683	36.6	149.2	2,630	1.12
Planing mills, etc.	13.0	17,794	27.5	377.0	4,617	1.01
Butter and cheese factories	11.8	21,824	20.0	369.9	4,553	1.06
Bread and bakery products	11.5	31,543	26.7	732.5	1,851	1.04
Machine shops	11.2	5,739	44.7	229.6	3,093	1.00
Cement products	10.4	3,760	33.0	119.2	2,498	1.02
Sawmills	8.3	56,756	26.9	1,843.4	14,325	1.03
Feed mills	2.4	1,799	62.8	469.8	28,502	1.01
Median	16.4	16,644	26.9	132.5	4,617	1.06
Median, all industries	67.2	3,254	30.4	9.1	4,250	1.05

<sup>a</sup> Percentage of firms in an industry required to account for 80 per cent of industry's employment.

<sup>b</sup> Number of largest firms required to account for 80 per cent of industry's employment.

<sup>c</sup> See text, Chap. II, sec. 6. Horsepower capacity of prime movers in use per wage earner, multiplied by electricity purchased for power and lighting (in kilowatt hours) divided by capacity of electric motors operated by purchased power in (in H. P.).

Source: Appendix A, Tables A-1 and A-7.

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tries which obtain their materials in regionally segregated markets. The industries in this table are regarded as having "high" transport costs in relation to other costs, while the remaining industries in the sample are treated as having "low" transport costs. This simple two-way classification is used to provide a rough test of the influence of transportation barriers on firm size and concentration.

### OTHER INDUSTRY ATTRIBUTES

While the other characteristics discussed in section 4 are difficult to identify in individual industries, their influence should be reflected when industries are classified by the degree of durability of their product (durable, semi-durable, non-durable) and by type of buyer (producers' goods, consumer goods). For example, one would expect buyers of non-durable producers' goods to be "well informed," and buyers of durable consumer goods "uninformed." The influence of these classifications on concentration is therefore investigated. The classification of each industry included in the analysis is shown in Table 15.

### 7. Results of the Statistical Analysis

Regression analysis reveals that differences in industry size, capital-labor ratio, and the importance of transportation costs account for about 62 per cent of variation in concentration among industries.<sup>27</sup> These three variables are not entirely independent, and therefore the influence of each cannot be clearly determined. There is an especially significant correlation between industry size and the importance of transportation costs.<sup>28</sup>

One way to assess the importance of each variable is to consider how much of the variation in concentration could be explained by a regression based on the other two variables alone. Thus elimination of industry size from the analysis would reduce the "explained" proportion of the variation in concentration from 62 per cent to 56 per cent, i.e. by 6 percentage points. Similarly, elimination of the capital-labor ratio would reduce the explained variation by 18 percentage points, and the classification based on the importance of transportation costs accounts for 23 per cent of the variation in concentration.<sup>29</sup>

On the other hand, one can also assess the importance of each variable by considering how much of the variation in concentration

<sup>27</sup> See First Appendix to Chap. II, sec. 1.

<sup>28</sup> See First Appendix to Chap. II, sec. 2.

<sup>29</sup> See First Appendix to Chap. II, sec. 3.

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TABLE 15

Classification of Industries <sup>a</sup>

CONSUMER GOODS

*Durables*

Carpets, mats, rugs  
Furniture  
Boat building  
Coffins and caskets  
Bicycles

Automobiles  
Plate, cut, and ornamental glass  
Umbrellas  
Pipes and smokers' supplies  
Pens and pencils

*Semi-Durables*

Fur goods  
Hosiery and knit goods  
Miscellaneous leather products  
Boots and shoes

Leather gloves  
Corsets and girdles  
Clothing, men's factory  
Clothing, women's factory

*Non-Durables*

Bread and bakery products  
Butter and cheese  
Soft drinks  
Fish curing and packing  
Fruit and vegetable prep.  
Meat packing  
Breweries  
Flour mills  
Sugar refineries  
Macaroni  
Malt and malt products  
Starch and glucose  
Condensed milk

Wine  
Distilleries  
Processed cheese  
Cigarettes, etc.  
Biscuits and crackers  
Cocoa, confectionery, etc.  
Cotton thread  
Polishes and dressings  
Medicinals and pharmaceuticals  
Matches  
Writing inks  
Soap

PRODUCER GOODS

*Durables*

Leather belting  
Canvas goods  
Sawmills  
Planing mills, sash and door  
factories  
Hardwood flooring  
Roofing paper  
Plywood and veneer  
Machine shops  
Iron castings  
Shipbuilding

Agricultural implements  
Pig iron  
Steel ingots and castings  
Aircraft  
Railway rolling stock  
Cement  
Cement products  
Glass  
Artificial abrasives  
Abrasive products

*Semi-Durables*

Cotton and jute bags  
Cordage, rope and twine  
Leather tanning  
Cotton yarn and cloth  
Synthetic textiles and silk  
Dyeing and finishing of textiles

Woolen cloth  
Woolen yarn  
Fur dressing and dyeing  
Clothing contractors, men's  
Clothing contractors, women's  
Asbestos products

(cont. on next page)

MANUFACTURING INDUSTRIES, 1948

TABLE 15 (cont.)

<i>Semi-Durables</i> (cont.)	
Paints and varnishes	Narrow fabrics
Gypsum products	Buttons and fasteners
<i>Non-Durables</i>	
Feed mills	Hardwood distillation
Prepared stock and poultry feeds	Vegetable oils
Tobacco products	Coal tar distillation
Pulp and paper	Boiler compounds
Paper boxes and bags	Printing inks
Petroleum products	Washing compounds
Compressed gases	

<sup>a</sup> The industries included in the statistical analysis are those shown in Appendix A, Table A-1, with the exception of Aluminum, Nickel, Excelsior and Coke products.

Classification by durability follows Charles A. Bliss, *The Structure of Manufacturing Production*, National Bureau of Economic Research, 1939, Appendix I, with one exception. Classification into consumer goods and producer goods is based on the immediate purchaser (omitting wholesale and retail intermediaries) rather than the "ultimate user."

would be explained by a regression based on a single independent variable. On this basis, industry size would account for 30 per cent of the variation in concentration, capital per worker for 14 per cent, and the importance of transportation costs for 33 per cent.

Both methods show the great importance of the transportation cost factor in accounting for differences in concentration. Industries whose product or raw material markets are on a regional basis have very much lower concentration than others, so that the simple two-way classification of industries based on this factor accounts for between 23 and 33 per cent of the variation in concentration.

Variation in capital per worker, while clearly significant, is somewhat less important, accounting for between 14 and 18 per cent of the variation in concentration.

Variation in industry size appears as the least important of the three factors influencing concentration, if the first method of gauging importance is used, and as almost as important as the transportation cost factor if the second method is used. Our data therefore do not yield an unambiguous estimate of the importance of industry size in accounting for variation in concentration. This difficulty is due to the correlation between industry size and the transportation cost factor.

The influence of a given factor, say industry size, depends, of course, both on the "sensitivity" of concentration to variation in industry size and on the degree to which industry size itself varies among the industries examined. The coefficients of the regression

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equation indicate how "sensitive" or "responsive" concentration is to a given variation in each of the independent variables. Since the regression equation is in terms of the logarithms of the variables, the coefficients measure the "elasticity" (in the economist's sense) of concentration with respect to the independent variables, that is to say the *proportionate* variation in concentration per unit of small *proportionate* variation in the independent variable.

The calculations show that the elasticity of concentration with respect to industry size is 0.32. On the average, and assuming the other variables are the same, an industry that is, say, 10 per cent larger than another will have a concentration index about 3 per cent larger (i.e. lower concentration).

The elasticity of concentration with respect to the capital-labor ratio is  $-0.70$ . On the average, and assuming the other variables are the same, an industry having a 10 per cent higher capital-labor ratio than another will have a concentration index about 7 per cent *lower* (i.e. higher concentration).

The influence of the third factor, the importance of transportation costs, cannot be measured in quite this way, since we have treated it as an attribute rather than a variable. The regression equation shows that industries in which there are separate regional markets for raw materials or products have on the average, and assuming other variables are the same, twelve times as high a concentration index as the other industries (i.e. lower concentration).

The reliability of these results can be tested by making the assumption that influences other than those specified lead to random deviations of the actual logarithms of concentration indexes from the values given by the regression equation. The observed values of the (logarithmic) concentration index may then be regarded as one sample from a hypothetical infinite set of observations for the same industries, with the same values of the independent variables (industry size, capital per worker and the transportation variable) but various different combinations of the other random influences. On the assumption that the hypothetical probability distribution of the deviations is normal, the conventional confidence intervals and tests of significance can be employed. The assumption of normality is reasonable, since the actual deviations of observed from estimated values of the (logarithmic) concentration index are about normally distributed.<sup>30</sup>

Computation of "confidence intervals" reveals that it can be assumed with a degree of confidence corresponding to a probability of

<sup>30</sup> See First Appendix to Chap. II, sec. 4.

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95 per cent<sup>31</sup> that the coefficients of the regression equation are within the following limits:

The coefficient for industry size (0.321) is between 0.146 and 0.497.

The coefficient for the index of capital per worker ( $-0.699$ ) is between  $-0.482$  and  $-0.916$ .

The coefficient for the industries with high transportation costs (1.084) is between 0.786 and 1.383. Taking antilogarithms, it is found that the corresponding factor by which the concentration estimate for these industries should be multiplied (12.14) is between 6.109 and 24.13.

These confidence limits indicate that all the regression coefficients are statistically significant. There is therefore no reason to doubt that each of the independent variables is systematically related to concentration. One cannot, however, have great confidence in the precise values of the coefficients, and should therefore regard them as rough indicators.

### OTHER FACTORS INFLUENCING CONCENTRATION

While the three factors discussed so far are clearly of great importance in explaining the variation of concentration among industries, they still leave 38 per cent of the variation unexplained. Does the classification of industries according to durability of goods and type of purchaser (Table 15) contribute further to the explanation of concentration?

The analysis so far has been based on a regression equation which yields "estimates" of the logarithm of the concentration index. The variance of the residual differences between these estimates and the actual values of the logarithm of the concentration index represents the unexplained part of the variation in concentration. We now investigate whether durability of goods and type of purchaser are systematically associated with the size of these residuals.

The mean values of residuals obtained when industries are cross-classified by durability of goods and type of purchaser are shown in Table 16. Differences among these mean values are not large, and are quite small when compared to the differences among residual values within each group. Variance analysis indicates that the total variation among the mean values in the table amounts to less than 3 per

<sup>31</sup> For the precise interpretation of confidence limits see, for example, Harold Cramér, *Mathematical Methods of Statistics*, Princeton University Press, 1946, Chap. 34.

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cent of the total variation in the logarithmic concentration index, and that the differences are not statistically significant; differences as large as those shown may well arise in purely random groupings of industries.<sup>32</sup>

TABLE 16

Mean <sup>a</sup> Values of Concentration Residuals <sup>b</sup> by Industry Class

	<i>Producer Goods</i>	<i>Consumer Goods</i>
Durables	-0.0648	-0.1064
Semi-Durables	+0.0432	+0.2794
Non-Durables	+0.1641	-0.1334

<sup>a</sup> Arithmetic mean.

<sup>b</sup> Logarithm of concentration index minus estimated value based on regression equation 1 (see First Appendix to Chap. II).

*Note:* For classification of industries see Table 15.

Our analysis, therefore, does not indicate that factors related to durability of goods and type of purchaser, such as those suggested by Scitovsky, exercise a significant influence on concentration. On the other hand, the possibility that such an influence exists is not ruled out by the data,<sup>33</sup> and it is suggestive that such differences as are found in Table 16, while they may, of course, be mere chance fluctuations, are also consistent with the theories discussed above.

Thus the lowest residuals (i.e. highest concentration, other things being equal) are found in the consumer non-durable group consisting almost entirely of processed food products. Here the importance of advertised brands in determining consumer preferences, which is considered by Scitovsky as one of the characteristics of "uninformed markets,"<sup>34</sup> gives an advantage to large firms and thus makes for high concentration. Cigarettes, beer, meat, distilled liquors, and fruit and vegetable canning are among the industries with the lowest residuals (Table 17).

Next, in order of increasing mean residual values, are the durable goods groups, where both producer and consumer goods have negative mean residuals (i.e. higher concentration than the regression equation would suggest). Here the complexity and size of products makes for larger firms than the capital-labor ratio would suggest, and in the case of consumer goods, consumer ignorance is an additional factor. Agricultural implements, aircraft, railway rolling stock,

<sup>32</sup> See First Appendix to Chap. II, sec. 5.

<sup>33</sup> See, e.g., Lawrence R. Klein, *A Textbook of Econometrics* (Peterson, 1953, pp. 140-141), for a discussion of the danger inherent in uncritical acceptance of the null hypothesis when it cannot be rejected.

<sup>34</sup> Scitovsky, *op. cit.*, p. 403.

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and automobiles are among the industries in this group having very low residuals (Table 17).

TABLE 17

Canadian Manufacturing Industries Grouped by Value of Concentration Index Residual <sup>a</sup>

<i>Residual</i>	<i>Industries (in descending order of residual size)</i>
Above 0.5	Feed mills; Sawmills; Flour; Furniture; Boats; Fur goods; Women's clothing; Vegetable oils; Cut glass; Malt.
0.3 to 0.5	Medicinals; Compressed gases; Canvas goods; Paper boxes and bags; Condensed milk; Flooring; Woolen cloth; Tanning; Paints; Shoes; Miscellaneous leather products.
0.2 to 0.3	Cement; Planing mills; Butter and cheese; Prepared feeds; Synthetic textiles; Bread; Men's clothing; Macaroni; Wine; Hosiery and knit goods.
0 to 0.2	Printing inks; Polishes and dressings; Dyeing and finishing of textiles; Plywood; Women's clothing contractors; Leather belting; Woolen yarn; Pulp and paper; Machine shops; Asbestos products; Washing compounds; Coffins; Men's clothing contractors; Processed cheese; Petroleum products.
-0.2 to 0	Cocoa and confectionery; Roofing paper; Steel ingots; Fur dressing; Cordage and rope; Pig Iron; Coal tar distillation; Starch; Leather gloves; Cotton and jute bags; Shipbuilding; Soap; Sugar; Narrow fabrics; Soft drinks; Fish packing; Cement products; Boiler compounds.
-0.3 to -0.2	Cotton yarn and cloth; Buttons; Carpets; Writing inks; Abrasives; Gypsum products.
-0.5 to -0.3	Biscuits; Bicycles; Abrasive products; Glass; Pens and pencils; Corsets; Tobacco processing.
-0.5 and lower	Distilled liquors; Cotton thread; Hardwood distillation; Fruit and vegetable canning; Iron castings; Agricultural implements; Umbrellas; Aircraft; Railway rolling stock; Pipes and smoker's supplies; Matches; Beer; Automobiles; Meat; Cigarettes.

<sup>a</sup> Logarithm of concentration index minus value estimated by regression equation 1 (see First Appendix to Chap. II).

At the other extreme, the highest mean residual is found in the group of semi-durable consumer goods consisting of apparel and leather products. This group is probably among the "best informed" of consumer markets, and in addition, the relative simplicity of apparel manufacturing operations helps to explain the low concentration (other things being equal) in this group.

It is clear that a great deal of the variation in concentration remains unexplained by our analysis. On inspection of the list of industries with exceptionally high (positive or negative) residuals (Table 17), various suggestions can be made to help explain individual cases. In some industries a measure based on horsepower per worker is a very poor index of capital intensity. For example in 1943 sawmills had 3.9 horsepower per thousand dollars of capital employed while the

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average for manufacturing as a whole was only 1.0.<sup>35</sup> In some cases exceptionally high or low inequality of firm size helps to explain a large residual (e.g. feed mills and fruit and vegetable canning, see Table 14).

In general it is likely that technical factors not directly related to the capital-labor ratio are important determinants of concentration. Dynamic factors, such as the degree of fluctuation of output and the rate of growth of an industry, should also be investigated.

### OTHER MEASURES OF CAPITAL PER WORKER

The statistical analysis has been based on a regression equation in which capital per worker is measured by the index horsepower per worker adjusted for cyclical variation in the use of capacity (see section 6). The use of the other measures of capital per worker discussed in section 6 yields regression equations with slightly lower multiple correlation coefficients.<sup>36</sup> These equations are, of course, different from that used in the above analysis, since capital per worker is measured on different scales and the measures are only imperfectly correlated. Apart from differences in the regression coefficient for capital per worker the results are, however, very similar, so that the detailed analysis based on one equation may be regarded as representative.

### *First Appendix to Chapter II*

#### *Statistical Analysis: Details*

1. Multiple regression analysis of the data for 92 industries yields the equation

$$(1) \quad c' = 0.321i - 0.699m + 1.084t + 2.393$$

(0.088) (0.109) (0.150)

where  $c'$  = the value of the logarithm of the concentration index (number of firms accounting for 80 per cent of employment) estimated by the equation,

$i$  = the logarithm of industry size measured in terms of employment,

$m$  = the logarithm of the index of horsepower per worker adjusted for cyclical variation (see last paragraph of section 6). This index is expressed in terms of kilowatt hours per wage earner.

$t$  = a "dummy variable" taking the value 1 for industries included in Table 14 (having separate regional markets for

<sup>35</sup> *The Manufacturing Industries of Canada, 1943*, Ottawa, Dominion Bureau of Statistics, Table 9.

<sup>36</sup> See First Appendix to Chap. II, sec. 6.

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raw materials or products) and the value 0 for other industries in the sample.<sup>37</sup>

The figures in parentheses are the standard errors of regression coefficients.

The multiple correlation coefficient is 0.79, indicating that 62 per cent of the variation in the concentration variable can be regarded as accounted for by the other three variables.

2. The correlations among the independent variables are given by the following coefficients:

$$r_{im} = 0.12$$

$$r_{it} = 0.41$$

$$r_{mt} = 0.18$$

3. The influence of each of the independent variables is computed from the following tabulation of squared correlation coefficients.

$$\begin{array}{lll} r^2_{ct} = 0.298 & r^2_{c,im} = 0.394 & \\ r^2_{cm} = 0.138 & r^2_{c,it} = 0.442 & r^2_{c,imt} = 0.620 \\ r^2_{ct} = 0.327 & r^2_{c,mt} = 0.560 & \end{array}$$

where  $r_{c,xyz}$  is the multiple correlation coefficient for  $c$  as the dependent variable and  $x, y, z$  as the independent variables.

4. The "Chi square" test of goodness of fit shows that the probability of obtaining, in random samples from a normal population, at least as great a divergence from normality as that actually observed is between 30 and 50 per cent.

The deviations were grouped into ten size classes as follows:

Size of Deviation	Actual Number of Deviations	"Expected" Number of Deviations	$[(2) - (1)]^2 \div (2)$
	(1)	(2)	(3)
above 0.6071	8	9	0.1111
0.4203- 0.6071	5	8	1.1250
0.2802- 0.4203	10	8	0.5000
0.1401- 0.2802	12	10	0.4000
0- 0.1401	11	11	0.0000
-0.1401- 0	14	11	0.8182
-0.2802- -0.1401	8	10	0.4000
-0.4203- -0.2802	6	8	0.5000
-0.6071- -0.4203	7	8	0.1250
-0.6071 or less	11	9	0.4444
Total	92	92	4.4237 = "Chi square"

<sup>a</sup> In a normal distribution with same mean (0) and standard deviation (0.467) as that of the actual deviations.

<sup>37</sup> All data are for 1948. The 92 industries included are those shown in Appendix A, Table A-1, excluding aluminum, nickel, excelsior, and coke products.

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The value of Chi square is 4.424. The number of degrees of freedom is  $10 - 6 = 4$ .

5. The residuals of regression equation (1) are grouped into six classes, based on durability of goods and type of purchaser (Table 16). Variance analysis yields a value of  $F$  that is not statistically significant, as follows:

<i>Source of Variation</i>	<i>Sum of Squares</i>	<i>Degrees of Freedom</i>	<i>Mean Square</i>
Total	22.1680	91	
Between means	1.6467	5	0.3294
Within groups	20.5213	86	0.2386

$$F = 1.38$$

$F_{.05}$  is between 2.33 and 2.30

6. Using the unadjusted index of horsepower per wage earner (see Second Appendix to Chapter II, section 6) the regression equation is

$$(2) \quad c'' = 0.437i - 0.492h + 0.996t - 0.317$$

(0.093) (0.099) (0.159)

with a multiple correlation coefficient of 0.752, where

$c''$  = the value of the logarithm of the concentration index estimated by means of the equation,

$h$  = the logarithm of horsepower capacity of equipment per wage earner (unadjusted),

$i$  and  $t$  have the same meaning as in equation (1).

The figures in brackets are the standard errors of the regression coefficients.

When the value of capital employed per worker is substituted for the measures based on horsepower the following equations are obtained:

$$(3) \quad c_3 = 0.347i - 0.883k + 0.896t + 3.296$$

(0.089) (0.159) (0.150)

with a multiple correlation coefficient of 0.769 for 1943, and

$$(4) \quad c_4 = 0.362i - 0.871k' + 0.884t + 2.928$$

(0.087) (0.148) (0.148)

with a multiple correlation coefficient of 0.775 for 1938. Here

$c_3$  and  $c_4$  represent the estimated values of the logarithm of the concentration index,

$k$  = the logarithm of the dollar value of capital used per employee in 1943 and

$k$  = the logarithm of the value of capital per employee in 1938.<sup>38</sup>

*Second Appendix to Chapter II*

*Determinants of Average Firm Size*

Since the amount of capital per worker presumably influences concentration *via* its relation with firm size (Chapter II, section 4), it is of interest to see how various indexes of capital per worker are related to different measures of average firm size and plant size.

The rank correlations in Table 18 show that much higher correlations are obtained when firm size or plant size are measured in terms of capital than when they are measured in terms of employment (compare correlations 1 and 9, 3 and 10, 4 and 11). This is not surprising since capital per firm and employment per firm are not perfectly correlated. For industries at a given average level of capital per firm, high employment per firm means a low capital-labor ratio. But for any given level of employment per firm, high capital per firm means a high capital-labor ratio.

While the significance of small differences in rank correlation coefficients is doubtful,<sup>39</sup> some of the smaller differences found in Table 18 are suggestive. In all cases in which both plant size and firm size are correlated with the same index of capital per worker, the correlation with firm size is slightly higher (compare correlations 1 and 2, 3 and 4, 6 and 8, 10 and 11). This relation suggests that even where a high capital-labor ratio is associated with small plants, it tends to make for a large size of firm.

The correlations involving the "adjusted" figures of horsepower per wage earner are slightly better than those involving the unadjusted figures, which suggests that cyclical fluctuations in the use of capacity do tend to obscure the normal relation between relative importance of capital and size of firm (compare correlations 1 and 3, 2 and 4, 9 and 10). Comparison of correlations 5 and 7 indicates that the distortion is due to an association between high amplitude of cyclical fluctuations and large plant size.<sup>40</sup>

<sup>38</sup> Equations (3) and (4) are based on 93 industries, consisting of the 96 shown in Table 1 with the exception of aluminum, nickel, and plywood and veneer.

<sup>39</sup> Nor have statistical tests for it been developed.

<sup>40</sup> The difference between correlations 5 and 7 is that industries with relatively large fluctuations in proportion of capacity utilized have relatively higher capital-labor ratios in correlation 7 (1938). Since this improves the correlation, these industries must have relatively large plants.

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TABLE 18

Correlation between Capital per Worker and Plant Size or Firm Size

Variables	Rank Correlation Coefficient	Number of Industries
1. Horsepower per wage earner and average employment per plant, 1948	0.229	92
2. Horsepower per wage earner and average employment per firm, 1948	0.260	92
3. Adjusted horsepower <sup>a</sup> per wage earner and average employment per plant, 1948	0.325	92
4. Adjusted horsepower <sup>a</sup> per wage earner and average employment per firm, 1948	0.367	92
5. Value of fixed capital per wage earner and average fixed capital per plant, 1943	0.505	69
6. Value of total capital per wage earner and average fixed capital per plant, 1943	0.517	69
7. Value of fixed capital per wage earner and average fixed capital per plant, 1938	0.561	69
8. Value of total capital per wage earner and estimated fixed capital per firm, <sup>b</sup> 1943	0.586	66
9. Horsepower per wage earner and average horsepower per plant, 1948	0.687	92
10. Adjusted horsepower <sup>a</sup> per wage earner and average horsepower per plant, 1948	0.737	92
11. Adjusted horsepower <sup>a</sup> per wage earner and average horsepower per firm, 1948	0.763	89

<sup>a</sup> Adjusted horsepower per wage earner refers to horsepower per wage earner multiplied by the ratio of electricity purchased for power and lighting to capacity of electric motors operated by purchased power (see Chap. II, sec. 6).

<sup>b</sup> Obtained by multiplying fixed capital per plant, 1943, by the number of plants per firm, 1948, as shown in Appendix A, Table A-3.

Source: Appendix A, Tables A-1, A-7, and A-8.

The industries included in these correlations are the 96 listed in Appendix A, Table A-1, with the following adjustments, required by the data available:

CORRELATION  
NUMBER

ADJUSTMENTS

1, 3, 9, 10	Omit excelsior, aluminum, nickel, coke products Replace pig iron and steel ingots by primary iron and steel (combined) Replace pulp and paper mills by separate components: Pulp mills, Paper mills, Combined pulp and paper mills Replace soap and washing compounds by combined industry: Soap and washing compounds
2, 4	Omit excelsior, aluminum, nickel, coke products
5, 6, 7	Omit butter and cheese factories, fruit and vegetable canning, biscuits, cocoa and confectionery, flour mills, bread, feed mills, prepared feeds, soft drinks, beer, narrow fabrics, men's clothing contractors, women's clothing contractors, sawmills, plywood and veneer, planing mills

(cont. on next page)

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TABLE 18 (cont.)

CORRELATION NUMBER	ADJUSTMENTS
	Replace pig iron and steel ingots by primary iron and steel
	Omit iron castings, machine shops
	Replace aluminum and nickel by nonferrous metals
	Add electrical apparatus
	Replace abrasive products and artificial abrasives by combined industry: Abrasive products
	Omit coke products, glass, cut glass
	Replace printer's ink and writing ink by combined industry: Inks
	Replace soaps and washing compounds by combined industry
	Omit boiler compounds, matches
8	In addition to adjustments made for correlations 5, 6, 7, omit abrasives (combined), soap and washing compounds (combined), and inks (combined)
11	Omit aluminum, nickel, excelsior, coke products, soap, washing compounds
	Replace pig iron and steel ingots by primary iron and steel (combined)

There are minor errors in some of the basic data for a few of the industries included; correction of these would not alter the correlation coefficient significantly.

The combined influence of capital per worker, industry size, and transportation barriers on firm size is illustrated by the following multiple regression equation <sup>41</sup>

$$(5) \quad f = 0.560i + 0.718m - 1.035t - 2.596$$

$$(0.076) \quad (0.094) \quad (0.128)$$

where  $f$  is the estimated logarithm of average employment per firm and  $i$ ,  $m$ , and  $t$  have the same interpretation as in equation (1), First Appendix to Chapter II. The multiple correlation coefficient is 0.747, and the standard errors of individual regression coefficients are given in parentheses. A considerably higher correlation would, of course, be obtained if firm size were measured in terms of capital.

TABLE 19

Mean <sup>a</sup> Value of Firm-Size Residuals <sup>b</sup>

	<i>Producer Goods</i>	<i>Consumer Goods</i>
Durables	+0.0838	+0.0789
Semi-Durables	+0.0496	-0.1106
Non-Durables	-0.0775	+0.0419

<sup>a</sup> Arithmetic mean.

<sup>b</sup> Logarithm of employment per firm minus value computed from regression equation 5.

<sup>41</sup> This regression is based on the same 92 industries as equation (1), First Appendix to Chap. II.

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The residuals of this regression equation <sup>42</sup> have the mean values shown in Table 19 for different classes of industries. Differences among the means for the various groups are not statistically significant.<sup>43</sup> Generally the groups with high firm-size residuals have low concentration-index residuals (Table 16). The main exception is the fact that non-durable consumer goods (mainly food products), which have the lowest concentration-index residual, follow the two durable goods groups and producers' semi-durables in terms of firm-size residual.

<sup>42</sup> Actual logarithm of employment per firm less value estimated by the equation.

<sup>43</sup> A value of  $F$  of 0.49 is obtained, with 86 and 5 degrees of freedom.