Concepts and Terminology for Sea Level: Mean, Variability and Change, Both Local and Global

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Abstract

Changes in sea level lead to some of the most severe impacts of anthropogenic climate change. Consequently, they are a subject of great interest in both scientific research and public policy. This paper defines concepts and terminology associated with sea level and sea-level changes in order to facilitate progress in sea-level science, in which communication is sometimes hindered by inconsistent and unclear language. We identify key terms and clarify their physical and mathematical meanings, make links between concepts and across disciplines, draw distinctions where there is ambiguity, and propose new terminology where it is lacking or where existing terminology is confusing. We include formulae and diagrams to support the definitions.

1 Introduction and Motivation

Changes in sea level lead to some of the most severe impacts of anthropogenic climate change (IPCC 2014). Consequently, they are a subject of great interest in both scientific research and public policy. Since changes in sea level are the result of diverse physical phenomena, there are many authors from a variety of disciplines working on questions of sea-level science. It is not surprising that sea-level terminology is inconsistent across disciplines (for example, "dynamic sea level" has different meanings in oceanography and geodynamics), as well as unclear or ambiguous even within a single discipline. (For instance, "eustatic" is ambiguous in the climate science literature.) We sometimes experience difficulty in finding correct and precise terms to use when writing about sea-level topics, or in understanding what others have written. Such communication problems hinder progress in research and may even confuse discussions about coastal planning and policy.

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This situation prompted us to revisit the meaning of key sea-level terms, and to recommend definitions along with their rationale. In so doing, we aim to clarify meanings, make links between concepts and across disciplines and draw distinctions where there is ambiguity. We propose new terminology where it is lacking and recommend replacing certain terms that we argue are unclear or confusing. Our goal is to facilitate communication and support progress within the broad realm of sea-level science and related engineering applications.

In the next section, we outline the conventions and assumptions we use in our definitions and mathematical derivations. The following three sections (Sects. 3–5) contain the definitions, with a subsection for each major term defined, labelled with "N" and numbered consecutively throughout. In Sect. 3 we define five key surfaces: reference ellipsoid, sea surface, mean sea level, sea floor and geoid. We consider the variability and differences in these surfaces in Sect. 4, and quantities describing changes in sea level in Sect. 5. In Sect. 6, we show how **relative sea-level change** is related to other quantities in various ways. In Sect. 7 we describe how observational data are interpreted using the concepts we have defined. To facilitate sequential reading of this paper, the material of Sects. 3–7 is arranged to minimize forward references, though we were unable to avoid all.

We give a list of deprecated terms with recommended replacements in Sect. 8, and a list in Sect. 9 of all terms defined, referring to the subsections where they are defined, thus providing an index that also includes our notation. The appendices contain further discussion of some aspects at greater length.

The complexity of sea-level science is evident in the detail of the definitions and discussions in this paper. It may therefore be helpful to keep in mind that from the point of view of coastal planning and climate policy there are three quantities of particular interest. **Extreme sea level** along coasts (i.e., extreme coastal water level) or around offshore marine infrastructure (such as drilling platforms) is of great practical importance because of the enormous damage it can cause to human populations and their built environment and to ecosystems. The expected occurrence of extreme sea level under future climates is therefore relevant to decision-making on a range of time-horizons.

The dominant factor in changes of future local extremes is **relative sea-level change** (RSLC), i.e., the change in sea level with respect to the land (Lowe et al. 2010; Church et al. 2013). Where there is relative sea-level rise, coastal defences have to be raised to afford a constant level of protection against extremes, and low-lying areas are threatened with permanent inundation. Although RSLC depends on many local and regional influences, the majority of coastlines are expected to experience RSLC within a few tens of per cent of **global-mean sea-level rise** (GMSLR) (Church et al. 2013). Projections of GMSLR are therefore of interest to global climate policy, with both adaptation and mitigation in mind.

In general, greater GMSLR is projected for scenarios with higher rates of carbon dioxide emission and on longer timescales. Particular attention is paid, especially by risk-intolerant users, to the probability or possibility of future large changes in sea level (Hinkel et al. 2019). We recommend referring to these as *high-end* scenarios or projections of RSLC or GMSLR (rather than "extreme" scenarios), to avoid confusion with projections of extreme sea level.

2 Conventions and Assumptions

We here summarize the conventions and assumptions employed in the text and formulae of our definitions.

2.1 First Appearance of Terms

Within each definition subsection of Sects. 3–5, we use **bold** font for the first appearance of a term whose definition is the subject of another subsection. When we first define a term that does not have its own subsection, it appears with a *slanted* font. The reader can locate the definitions of terms marked in these ways by looking them up in Sect. 9. In the PDF of this paper, each bold term in Sects. 3–8 is a hyperlink to the relevant definition subsection.

2.2 Time-Mean and Changes

The sea surface varies on all the timescales of the Earth system, associated with sea-surface waves and tides, meteorological variability (from gustiness of winds to synoptic phenomena such as mid-latitude depressions and tropical cyclones), seasonal, interannual and longer-term internally generated climate variability (e.g., El Niño and the Interdecadal Pacific Oscillation), anthropogenic climate change and naturally forced changes (e.g., by volcanic eruptions, glacial cycles and tectonics). For various purposes of understanding and planning for sea-level variations it is helpful to draw a distinction between a time-mean *state* and fluctuations within that state. The definition of a "state" depends on the scientific interest or application. For sea level, the state might be defined by a time-mean long enough to remove tidal influence (about 19 years), or which characterizes a climatological state (conventionally 30 years), but it could be shorter, for example, if interannual variability were regarded as altering the state.

Thus, the time-mean state cannot be absolutely defined, but the concept is necessary. In this paper, **mean sea level** refers to a time-mean state whose precise definition should be specified when the term is used, and which is understood to be long enough to eliminate the effect of meteorological variations at least. We use symbols with a tilde and time-dependence, e.g., $\tilde{X}(t)$ for time-varying quantities, and symbols without any distinguishing mark and no time-dependence, e.g., X for time-mean quantities that characterize the state of the system. On longer timescales, the state itself may change, for example, due to anthropogenic influence. We use the symbol Δ and the word "change" to refer to the difference between any two states; thus, ΔX is "change in X", e.g., change in relative sea level between the time-mean of 1986–2005 and the time-mean of 2081–2100. Anthropogenic sea-level change comes mostly through climate change, but there are other influences too, such as impoundment of water on land in reservoirs.

2.3 Local and Regional

By a *local* quantity, we mean one which is a function of two-dimensional geographical location **r**, specified by latitude and longitude. For some applications, it is important to consider variations of local quantities over distance scales of kilometres or less. Other quantities do not have such pronounced local variation and are typically considered as properties of regions, with distance scales of tens to hundreds of kilometres.

2.4 Global Mean Over the Ocean Surface Area

By *global mean*, we mean the area-weighted mean over the entire connected surface area of the ocean, i.e., excluding the land. The *ocean* includes marginal seas connected to the open ocean such as the Mediterranean Sea, Black Sea and Hudson Bay, but excludes inland

seas such as the Caspian Sea, the African Great Lakes and the North American Great Lakes. It includes areas covered by sea ice and ice shelves, where special treatment is needed to define the level of the **sea surface**. We note that observational estimates of the global mean are often made from systems which lack complete coverage.

For centennial timescales, we can assume the ocean surface area A is constant, with $A = 3.625 \times 10^{14}$ m^2 (Cogley 2012). It is altered substantially by global-mean sea-level changes of many metres, such as on glacial–interglacial timescales or possibly over future millennia due to ice-sheet changes, and on geological timescales due to plate tectonics. The formulae we give for some quantities describing global-mean changes are not exactly applicable under those circumstances.

2.5 Sea-Water Density

Many of the formulae in this paper involve sea-water density. Although sea-water density is a local quantity, we treat it in many contexts as a globally uniform constant with a representative value ρ_* (e.g., 1028 kg m^{-3}). For the density of freshwater added at the sea surface we use a constant $\rho_f = 1000$ kg m^{-3} for convenience, neglecting the variation of $<1\%$ in freshwater density due to temperature.

2.6 Vertical Direction and Distance

Before considering the vertical location of surfaces, or the local vertical distance between two surfaces, we need to specify the meaning of *vertical*. Geodesy is concerned with horizontal and vertical distances measured relative to the **reference ellipsoid**, which is a surface fixed with respect to the solid Earth. Geophysical fluid dynamics, including ocean circulation dynamics, is concerned with horizontal distances on surfaces of constant geopotential, and vertical distances measured perpendicular to such surfaces, especially the **geoid**. We discuss the two frames of reference (one relative to the reference ellipsoid and the other to the geoid) in the subsections describing those two surfaces. The distinction between the two frames is relevant only to the real world, because numerical ocean circulation models implicitly assume an idealized effective gravity field in which the geoid and the reference ellipsoid are identical (often spherical rather than ellipsoidal). In reality, the geoid has an irregular shape, whose vertical separation from the reference ellipsoid is ± 100 m and varies over horizontal length scales of 100s km.

Most of our formulae involve the local vertical coordinate of surfaces such as $X(\mathbf{r})$, for which we use the vertical distance above the reference ellipsoid (negative if below). We make this choice in order to give our formulae a well-defined interpretation. The choice of reference frame (with respect either to the reference ellipsoid or to the geoid) does not affect the geophysical definition of a surface, but the numerical value of its vertical coordinate at any given location is not the same in the two frames, because of the substantial difference between their reference surfaces. However, at a given location, there is negligible difference between the two frames regarding the local vertical direction. Hence, we can ignore the difference between the two definitions of "vertical" in evaluating a vertical gradient, the vertical distance $Y(\mathbf{r}) - X(\mathbf{r})$ between two surfaces, or the change in height $\Delta X(\mathbf{r})$ of a surface.

3 Surfaces

We here define five key surfaces used in sea-level studies. The reference ellipsoid and the associated terrestrial reference frame (depicted in Fig. 1) are geometrical constructions, chosen by convention. The other four surfaces (compared with the reference ellipsoid in Fig. 2) are geophysically defined and established with some uncertainty from observational data. These and other surfaces, such as datums defined by **tides** (e.g., mean lower-low water level), are located relative to the reference ellipsoid (Sect. 2.6), by their geodetic height as a function of geodetic location.

> N1 **Reference ellipsoid**: The surface of an ellipsoidal volume of revolution chosen to approximate the **geoid**.

A reference ellipsoid is a conventional geometric construction used to specify locations in a *terrestrial reference frame*, i.e., relative to the solid Earth. Many reference ellipsoids have been defined by geodesists, and some are intended only for use over limited portions of the globe. A given specification of the reference ellipsoid is time-independent.

For purposes relating to global sea level we make the following requirements of the reference ellipsoid.

1. Its centre is the time-mean centre of mass of the Earth.
2. Its semi-major axis lies in the equatorial plane and its semi-minor axis along the rotation (polar) axis of the Earth.
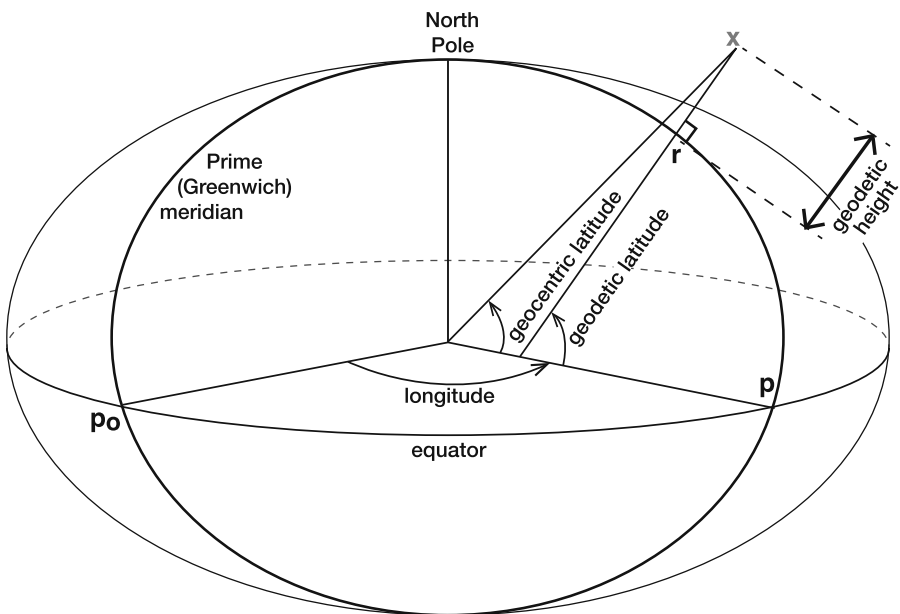3. Its axis of revolution is the rotation axis.

Fig. 1 The reference ellipsoid, which is used to locate other surfaces in a terrestrial reference frame, whose origin is the centre of the Earth. The figure shows the construction which defines the geodetic coordinates of an arbitrary point **x** in 3D space. The line between **x** and **r** is normal to the reference ellipsoid, on which **r** lies. The equator is intersected at **p** by the meridian through **r**, and at $\mathbf{p}_0$ by the prime meridian, which defines the zero of longitude
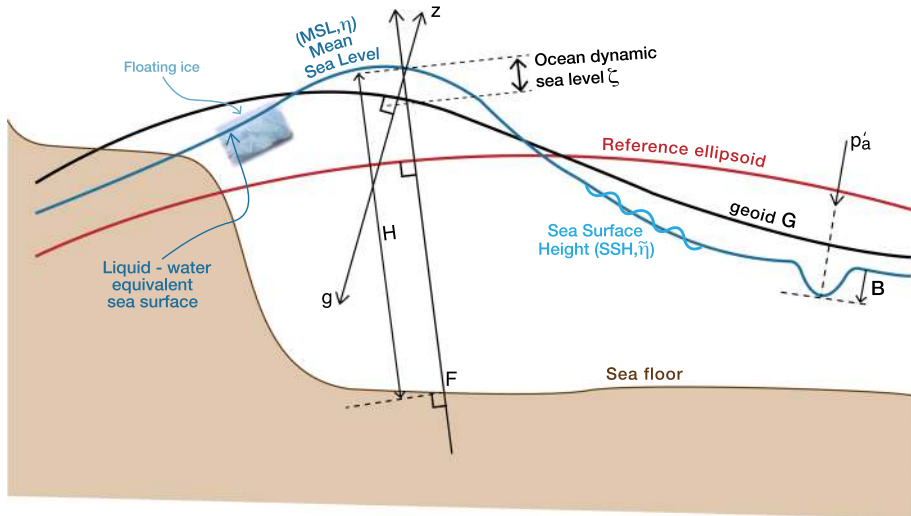
Fig. 2 Relationship between surfaces relating to sea level. The normal to the reference ellipsoid defines the vertical in the terrestrial reference frame. The normal to the geoid is the vertical coordinate (z) for geophysical fluid dynamics, and anti-parallel to the local effective acceleration g due to gravity. The difference between these two definitions of the vertical direction is greatly exaggerated in this diagram; it is negligible in reality. The local vertical coordinates of mean sea level η, the geoid G and the sea floor F are relative to the reference ellipsoid, while dynamic sea level $\tilde{\zeta}$ is relative to the geoid. The local time-mean thickness of the ocean H is the vertical distance between mean sea level and the sea floor. The deviation of atmospheric pressure p'_a from its global mean causes the depression B in sea level by the inverse barometer effect

4. It is fixed with respect to the solid Earth, and it rotates with the Earth.

The International Earth Rotation and Reference Systems Service (www.iers.org) defines the International Terrestrial Reference Frame (ITRF). They recommend the GRS80 ellipsoid.

For more precise geodetic purposes, the ITRF defines the coordinates and their rates of change of a set of stations on the Earth's surface. The coordinates are time-dependent because of tectonic motions and *true polar wander*, i.e., the time-dependence of the Earth's rotation axis with respect to the solid Earth. The latter phenomenon is neglected in the above specification of the ellipsoid. If the rotation axis is invariant, the last point in our specification above is not necessary because, being a volume of revolution, the reference ellipsoid is symmetrical with respect to rotation about the axis.

To locate a point **x** in 3D space in a reference frame based on the reference ellipsoid, we construct a straight line that passes through **x** and is normal to the ellipsoid, which it intersects at **r**. The *geodetic height* of **x** above the ellipsoid is the distance from **r** to **x** along this line, positive outwards. In our formulae, the *vertical* coordinate is the geodetic height, which is sometimes called *ellipsoidal height*. This is not the usual vertical coordinate for models of atmosphere and ocean circulation, which is instead defined relative to the **geoid**.

The *geodetic latitude*, commonly referred to simply as *latitude*, is the angle between the equatorial plane and the normal to the ellipsoid. It is different from the *geocentric latitude*, which is the angle between the equatorial plane and the line from the centre of the Earth to **x**. Geodetic and geocentric latitudes are the same for the poles and the equator, but

elsewhere geodetic latitude is larger (as can be appreciated from Fig. 1), by up to about
0.2°.

To define the *longitude* of **x** ("geodetic" and "geocentric" are the same for longitude),
consider the meridian passing through **r**, which intersects the equator at point **p**, and the
prime meridian (the *Greenwich meridian*), which intersects the equator at $\mathbf{p}_0$. Viewing the
Earth from above, the longitude is the anticlockwise angle between the lines from the
centre of the Earth to $\mathbf{p}_0$ and to **p**.

> N2 **Sea surface** $\tilde{\eta}$: The time-varying upper boundary of the ocean. The sea-surface
> height is the geodetic height of the sea surface above the **reference ellipsoid** (a
> negative value if below).

In ocean areas without floating ice (sea ice, ice shelves or icebergs), the liquid sea surface
is the bottom boundary of the atmosphere. In such areas, the existence of a well-defined
sea-surface height (SSH) $\tilde{\eta}(\mathbf{r}, t)$, that can be represented by a continuous and single-valued
mathematical expression, presupposes a space–time averaging, because the instantaneous
surface is ill-defined in the presence of some short-timescale phenomena that produce foam
and sea spray, such as breaking surface waves and conditions of intense wind. We assume
such averaging when speaking about the sea surface.

In ocean areas with floating ice, the liquid surface boundary is the bottom of the ice. For
those areas, we define the SSH $\tilde{\eta}$ as the *liquid-water equivalent sea surface* $\tilde{\eta}_{\mathrm{LWE}}$ which the
liquid would have if the ice were replaced by an equal mass of sea water of the density ρ_{s}
of the surface water in its vicinity. Following Archimedes' principle,

$$\tilde{\eta}_{\mathrm{LWE}} = \tilde{\eta}_s + \frac{w_i}{g\rho_{\mathrm{s}}}, \tag{1}$$

where $\tilde{\eta}_s$ is the geodetic height of the liquid sea-water surface (beneath the ice) and w_i is
the weight per unit area of floating ice. (The depression of $\tilde{\eta}_s$ relative to $\tilde{\eta}$ is called the
"**inverse barometer** effect of sea ice" by Griffies and Greatbatch 2012, and $\tilde{\eta}_{\mathrm{LWE}}$ is their
"effective sea level".) Although the liquid-water equivalent sea-surface height is not
directly measurable, it is a convenient construct for many practical purposes of sea-level
studies, and dynamically justifiable because the hydrostatic pressure and gravity beneath
the ice are largely unaffected by the replacement of ice with liquid water.

The sea surface varies periodically with various frequencies due to **tides**. It varies also
on all timescales due to **sea-surface waves**, atmospheric pressure, surface flux exchanges
(with the atmosphere), river inflow, variability that is internally generated by ocean
dynamics, motion of the **sea floor** and changes in the mass distribution within the ocean
and solid Earth (discussed in many following entries).

Note that **ocean dynamic sea level** and **ocean dynamic topography** are distinct con-
cepts from sea-surface height and from each other.

> N3 **Mean sea level (MSL)** η: The time-mean of the **sea surface**.

The period for the time-mean must be long enough to eliminate the effects of waves and
other meteorologically induced fluctuations (as discussed in Sect. 2.2). Predicted tidal
variations are subtracted if the period is not long enough to remove time-dependent **tides**,
but permanent tides are included in MSL. For a precise definition of MSL, the period of the
time-mean should be specified, and it could be described either with or without dependence
on the time of year.

MSL is located by its geodetic height $\eta(\mathbf{r})$ above the **reference ellipsoid** (a negative
value if below). In ocean models which regard the **geoid** and the reference ellipsoid as

coincident, η is equally the orthometric height of MSL above the geoid. MSL is sometimes called "mean sea surface". We recommend against using this term, in order to make a clear distinction from "sea-surface height".

N4 **Sea floor** F: The lower boundary of the ocean, its interface with the solid Earth.

The sea floor is the part of the surface of the solid Earth (whether bedrock or consolidated sediment, and lying beneath any unconsolidated sediment, e.g., Webb et al. 2013) that is always or sometimes submerged under sea water. The level of the sea floor varies due to solid-Earth **tides**, accumulation of sediment (with eventual compaction) and **vertical land movement** on a range of timescales.

We specify the instantaneous level of the sea floor by its geodetic height $\tilde{F}(\mathbf{r},t)$ (negative over most of the ocean) relative to the **reference ellipsoid**. The local instantaneous thickness of the ocean (its vertical extent, the depth of the sea floor measured from a ship, a positive quantity sometimes called the *depth of the water column*) is given by

$$\tilde{H}(\mathbf{r},t) = \tilde{\eta}(\mathbf{r},t) - \tilde{F}(\mathbf{r},t) \geq 0, \tag{2}$$

i.e., the vertical distance between the sea surface and the sea floor. The choice of reference surface for vertical coordinates does not affect the value of $\tilde{H}$, because it is the difference between two vertical coordinates; $\tilde{H}$ would be the same if SSH and the sea floor were located by heights relative to the **geoid** rather than the reference ellipsoid. The time-mean *thickness of the ocean H*

$$H(\mathbf{r}) = \eta(\mathbf{r}) - F(\mathbf{r}) \geq 0 \tag{3}$$

is likewise related to MSL η.

The shape of the sea floor is sometimes called the *bottom topography* or the *bathymetry*, for example in describing it as "rough" or "smooth". These two synonymous terms are also used as names for the quantities $-F$, $G - F$ and $\eta - F(= H)$, i.e., the time-mean vertical distance of the sea floor beneath the reference ellipsoid, the geoid or MSL, respectively. In order to be precise, it should be stated which of these alternatives is intended, since G and the reference ellipsoid differ by ±100 m (Sect. 2.6), and G and MSL differ by ±1 m, following time-mean **ocean dynamic sea level**.

N5 **Geoid** G: A surface on which the *geopotential* Φ has a uniform value, chosen so that the volume enclosed between the geoid and the **sea floor** is equal to the time-mean volume of sea water in the ocean (including the liquid-water equivalent of floating ice).

The geopotential is a field of potential energy per unit mass, accounting for the Newtonian gravitational acceleration due to the mass of the Earth plus the centrifugal acceleration arising from the Earth's rotation. We define the sign of the geopotential such that work is required to move a sea-water parcel from a lower geopotential (deeper in the ocean) to a higher geopotential (shallower in the ocean). Note that this sign convention for the geopotential is opposite to that used in geodesy.

The vertical gradient of the geopotential is equal to the local *effective gravitational acceleration*, g, $g(\mathbf{r},t) = \partial\Phi/\partial z$, usually referred to as "*gravitational acceleration*" in geophysical fluid dynamics. Hence, the effective gravitational acceleration is normal to the geoid, because the geoid is an *equipotential surface*, i.e., one on which the geopotential is constant. The Newtonian gravitational acceleration is time-dependent because the distribution of mass in the ocean (liquid and solid), on land (including the land-based

cryosphere) and within the solid Earth is generally changing. The centrifugal acceleration is time-dependent because the Earth's rotation rate and rotation axis are variable.

For models of atmosphere and ocean circulation, the vertical unit vector is directed anti-parallel to the effective gravitational acceleration g (or equivalently it is parallel to the local gradient of the geopotential). The height above the geoid of some point $\mathbf{x}$ is the distance z, measured along the local vertical unit vector, from the geoid to $\mathbf{x}$. The coordinate z (Fig. 2) is also called the *orthometric height* of $\mathbf{x}$. Strictly, the orthometric height is measured along a plumb line, which is always normal to equipotential surfaces, but this distance differs negligibly from that measured along the perpendicular to the **reference ellipsoid**.

We define z such that $z = 0$ is the geoid, $z > 0$ is above the geoid, and $z < 0$ is below. By *horizontal* we mean aligned with a surface of constant z. This is not strictly an equipotential surface, but the difference is locally negligible. It is, however, very different from a surface of constant geodetic height $z' = z + G$, where $G(\mathbf{r})$ is the *geoid height* above the reference ellipsoid ($G < 0$ where the geoid is below the ellipsoid).

The **sea surface** would coincide with the geoid if the ocean were in a resting steady state in the rotating frame of the earth. Although defining the geoid in this way is conceptually attractive, it is not realistic or practically useful. (See Appendix 1.)

The sea surface does not really coincide with the geoid because the ocean is not at rest. (See **ocean dynamic sea level**.) For example, **mean sea level** (MSL) north of the Antarctic Circumpolar Current (ACC) is at a higher geopotential than MSL south of the ACC; with respect to the geoid, MSL on the north side is roughly 2 m higher than on the south side.

Referring to its definition, we choose the geoid as the equipotential surface (out of the infinite set of them) which satisfies

$$\int (G - F)\, \mathrm{d}A = V = \int H\, \mathrm{d}A = \int (\eta - F)\, \mathrm{d}A, \tag{4}$$

using Eq. (3) for H, and where V is the volume of the global ocean and $A = \int \mathrm{d}A$ is its surface area. It follows from Eq. (4) that

$$\int \eta\, \mathrm{d}A = \int G\, \mathrm{d}A, \tag{5}$$

i.e., MSL and geoid height above the reference ellipsoid have equal global means.

We define the geoid in terms of MSL η, rather than the sea-surface height $\tilde{\eta}$, in order to restrict changes in G and V to those occurring on the timescales of **global-mean sea-level rise**, rather than on shorter timescales related to meteorological, seasonal and interannual fluctuations. Our definition of the geoid treats it as a geophysical quantity which changes as the Earth system evolves. In some applications, the geoid is defined in a time-independent way as a particular geopotential surface within a particular model of the Earth's gravity field.

We define the geoid to include the permanent ocean tide. With this choice, time-mean **ocean dynamic sea level** ζ is determined solely by ocean dynamics and density. With the zero-tide convention, which is common in gravity-field models, ζ would include the permanent ocean tide, which is almost $+0.1$ m at the equator and -0.2 m at the poles.

We define the geoid as $G(\mathbf{r}) = E(\mathbf{r}, \Phi_G)$, with a choice of Φ_G such that Eq. (4) is satisfied, where $E(\mathbf{r}, \Phi)$ is the geodetic height of the equipotential surface for geopotential Φ. The shapes of the equipotential surfaces, including the geoid, depend on the

geographical distribution of mass over the Earth. According to Eq. (4), the global-mean geoid height must change by

$$\frac{1}{A}\int \Delta G\,\mathrm{d}A = \frac{1}{A}\int \Delta F\,\mathrm{d}A + \frac{\Delta V}{A} \qquad (6)$$

if there is global-mean **vertical land movement** ΔF affecting the sea floor, or a change ΔV in the volume of the global ocean, whether due to change in density or in mass. Consequently, Φ_G must change such that

$$\frac{1}{A}\int \Delta G\,\mathrm{d}A = \frac{\Delta \Phi_G}{A}\int \frac{\partial E(\mathbf{r}, \Phi)}{\partial \Phi}\,\mathrm{d}A = \frac{\Delta \Phi_G}{g}, \qquad (7)$$

if we approximate g as globally uniform.

4 Variations and Differences in Surfaces

In this section we define terms for time-dependent variations in surfaces (on timescales shorter than those of mean sea-level change) and differences between surfaces.

> N6 **Tides**: Periodic motions within the ocean, atmosphere and solid Earth due to the rotation of the Earth and its motion relative to the moon and sun. Ocean tides cause the **sea surface** to rise and fall.

The *astronomical tide* is the dominant constituent of the ocean tides. It is caused by periodic spatial variations in local gravity. Tidal motion of the land surface and sea floor is due to elastic deformation of the solid Earth by gravitational tidal forces. The diurnal and annual cycles of insolation produce periodic variations in atmospheric pressure and winds (sea breezes), which cause the *radiational tide* in the atmosphere and ocean (e.g., Williams et al. 2018). The *predicted tide* is the sum of the astronomical and radiational constituents. Because the ocean and atmosphere are fluids, tidal forces within them cause *tidal currents* as well as displacements.

Sea-surface height (SSH) can be greatly elevated during a storm by a **storm surge**, and the consequent **extreme sea level** is sometimes called a *storm tide*. The *tidal height* is the vertical distance of the SSH due to the predicted tide above a local benchmark or a surface which is fixed with respect to the terrestrial reference frame. Often, this surface is a *tidal datum*, defined by a extremum of the periodic tide, such as mean lower-low water level.

The velocity of the ocean tidal currents depends on water depth. Therefore, **relative sea-level change** (RSLC) affects the tides. In most coastal locations, this interaction alters the tidal variations of the sea surface with respect to **mean sea level** by less than 10% of the RSLC (Pickering et al. 2017).

In most locations, the constituent of the ocean tide with the largest amplitude is the lunar semi-diurnal tide. The orbit of the moon around the Earth modulates the semi-diurnal tide to produce a large amplitude (*spring tide*) at new and full moon, and a small amplitude (*neap tide*) at half-moon. There are many smaller periodic constituents associated with the sun and moon. The precession of the plane of the moon's orbit causes tidal variations with an 18.6-year cycle (the *nodal period*), affecting extreme sea level on this timescale. There are longer tidal periods.

The *pole tide* is caused by variations of the Earth's rotation axis relative to the solid Earth, altering the centrifugal acceleration and local gravity. The two largest components of the pole tide have periods of 1 year and about 433 days. The latter is due to the

Chandler wobble, which is not strictly periodic and arises from the mechanics of the Earth's rotation alone (it is a *free nutation*), rather than being caused by the gravitation of other bodies in the solar system.

The time-means of the tidal forces of the moon and sun are nonzero. Hence, in addition to the periodic constituents, the tides have a constant constituent called the *permanent tide*, which tends to make the Earth and sea surface more oblate. Our definitions of **mean sea level** and the **geoid** use the *mean-tide* convention, including the permanent tide. In gravity models, the *zero-tide* convention is more usual, in which the permanent ocean tide is subtracted, but the permanent elastic tidal deformation of the solid Earth is retained; an estimate of the latter too is subtracted in the *tide-free* convention used by GNSS measurements.

> N7 **Inverse barometer (IB)** B: The time-dependent hydrostatic depression of the sea surface by atmospheric pressure variations, also called *inverted barometer*.

The ocean is almost incompressible. (A uniform change of 1 hPa over the ocean causes a **global-mean sea-level rise** of roughly 0.16 mm.) Therefore changes in atmospheric pressure have a negligible effect on the total volume of the ocean. However, they do move sea water around, and the effect on the **sea surface** depends on the deviation of sea-level pressure $\tilde{p}_a(\mathbf{r})$ from its global (ocean) mean, given by

$$\tilde{p}_a'(\mathbf{r}, t) = \tilde{p}_a(\mathbf{r}, t) - \frac{1}{A} \int \tilde{p}_a(\mathbf{r}, t)\, \mathrm{d}A. \tag{8}$$

For timescales longer than a few days, we can assume the ocean to be in hydrostatic balance. Therefore, the depression of the **sea-surface height** (SSH) $\tilde{\eta}$ by IB is $\tilde{B} = \tilde{p}_a'/(g\,\rho_s)$ where $g(\mathbf{r})$ is the acceleration due to gravity and $\rho_s(\mathbf{r}, \tilde{\eta})$ the surface sea-water density. That is, when $\tilde{p}_a' > 0$ then sea level is depressed locally by $\tilde{B}(\mathbf{r})$, and it is raised when $\tilde{p}_a' < 0$. The latter effect is an important contribution to **storm surge**. In a storm or cyclone, $\tilde{p}_a$ may fall by several 10 hPa, causing SSH to rise by several 100 mm.

The global mean of $g\rho_s$ is approximately 9.9×10^{-5} m Pa^{-1} $\equiv 9.9$ mm hPa^{-1}, with spatial and temporal variations of about 1% around this value. Hence, for most purposes of sea-level studies we can neglect the spatial variations in g and ρ_s, and replace them with constants; thus,

$$\tilde{B} = \frac{\tilde{p}_a'}{g\,\rho_*}. \tag{9}$$

Hence, the global-mean IB correction is zero,

$$\int \tilde{B}(\mathbf{r}, t)\, \mathrm{d}A = 0, \tag{10}$$

which follows by definition of $\tilde{p}_a'$.

The inverse-barometer response of the sea surface compensates for the effect of $\tilde{p}_a'$ on hydrostatic pressure within the ocean, and the subsurface ocean does not feel the fluctuations in atmospheric pressure. Consequently, the ocean behaves dynamically as if the sea-surface height were $\tilde{\eta} + \tilde{B}$, which is called *IB-corrected sea-surface height*. Its time-mean is $\eta + B$, the *IB-corrected mean sea level*. In most climate models, atmospheric pressure variations are not communicated to the ocean. In these models $\tilde{B}$ must be *subtracted* from the simulated SSH to produce a quantity that varies with $\tilde{p}_a$ like the observed $\tilde{\eta}$ does.

N8 **Extreme sea level**: The occurrence or the level of an exceptionally high or low local **sea-surface height**.

Extremely high sea-surface height (SSH) is caused by meteorological conditions as a **storm surge**, by **sea-surface waves** due to various causes and by exceptionally high or low (although predictable) tidal height. When considering coastal impacts, extreme sea level may be called *extreme coastal water level*. For decadal timescales, the main influence on changes in the frequency distribution of extreme sea level is **relative sea-level change** (RSLC), whose effect outweighs that of changes in meteorological forcing (Lowe et al. 2010; Church et al. 2013; Vousdoukas et al. 2018). To avoid confusion, we recommend the phrase *high-end* sea-level change to describe projections of very large RSLC, instead of using the word "extreme" for such projections.

N9 **Storm surge**: The elevation or depression of the **sea surface** with respect to the predicted tide during a storm.

Storm surges are caused during tropical cyclones and deep mid-latitude depressions by low atmospheric pressure, by strong winds pushing water towards the shore (or away from the shore, causing a negative surge) and by **sea-surface waves** breaking at the coast. Wave effects are usually excluded or underestimated by tide-gauges. If the actual **sea-surface height** (SSH) at location $\mathbf{r}$ and time t due to tide and surge combined (sometimes called the *storm tide*) is $\tilde{\eta}(\mathbf{r}, t)$, and the predicted SSH due to the tide alone is $\tilde{\eta}_{\text{tide}}(\mathbf{r}, t)$, the *storm-surge height* σ is

$$\sigma(\mathbf{r}, t) = \tilde{\eta}(\mathbf{r}, t) - \tilde{\eta}_{\text{tide}}(\mathbf{r}, t), \tag{11}$$

also called the *surge residual* or non-tidal residual.

The storm-surge height σ is the sum of three components: the **inverse barometer** (IB) effect of low atmospheric pressure, the *wind setup* caused by the wind-driven current, and the *wave setup*, which is the elevation of the sea surface due to breaking waves. All three effects are normally present, but intensified by storms. IB and wind setup tend to be more important on wide continental shelves, but wave setup can dominate in some cases (Pedreros et al. 2018), especially in areas of steep sea floor slope.

The *swash* is the uprush and backwash of water over the solid surface (e.g., sand or pebbles) generated by each wave. During the uprush, the swash extends above the wave setup. Its maximum height above the predicted tide, called the *wave runup*, gives the highest water level of the storm surge.

Particularly high SSH $\tilde{\eta} = \sigma + \tilde{\eta}_{\text{tide}}$ occurs when the storm surge coincides with high tide. Without the meteorological forcing, storm-surge height σ would be zero, but since the tide level influences the propagation of the storm-forced signal, σ and $\tilde{\eta}_{\text{tide}}$ are not independent (Horsburgh and Wilson 2007).

The *skew-surge height* is the elevation of the highest sea surface that occurs within a single tidal cycle above the predicted level of the high tide within that cycle. If the actual SSH is $\tilde{\eta}(\mathbf{r}, t)$ and the predicted SSH due to the tide alone is $\tilde{\eta}_{\text{tide}}(\mathbf{r}, t)$, the skew-surge height is

$$\sigma_k(\mathbf{r}, t) = \max{}_t(\tilde{\eta}(\mathbf{r}, t)) - \max{}_t(\tilde{\eta}_{\text{tide}}(\mathbf{r}, t)), \tag{12}$$

where $\max{}_t(X(\mathbf{r}, t))$ means the maximum value of X that occurs at location $\mathbf{r}$ during the interval of time t from one low tide to the next. For extreme-value analysis, the skew-surge height is preferable to the storm-surge height as a measure of the effect of the meteorological forcing alone in regions where skew-surge height is uncorrelated with tidal height (Williams et al. 2016).

N10 **Sea-surface waves**: Waves on the surface of the ocean, usually surface gravity waves caused by winds.

The amplitude of a *wind wave* depends on the strength of the wind, and the time and the distance of open ocean, called the *fetch*, over which the wind has blown. The sea surface typically exhibits a superposition of many waves of different amplitudes, velocities, frequencies and directions. A *swell wave* is a wind wave of low frequency which was generated far away.

A *tsunami* or *seismic sea wave* is an **extreme sea level** event caused by an earthquake, volcano, landslide or other submarine disturbance that suddenly displaces a volume of water. The displacement propagates as a long-wavelength surface gravity wave, but is not a tidal phenomenon, despite it sometimes being called a "tidal wave".

The *wave height* is the vertical distance from the crest to the trough of a wave, respectively its highest and lowest points. The *wave period* is the interval of time between the passage of repeated features on the waveform such as crests, troughs or upward crossings of the mean level. The *significant wave height* is a statistic computed from wave measurements, defined as either the mean of the largest one-third of the wave heights, or four times the standard deviation of wave heights. (These statistics are approximately equal.) The *significant wave period* is the mean period of the largest one-third of the waves.

N11 **Ocean dynamic sea level** ζ: The local height of the **sea surface** above the **geoid** G, with the **inverse barometer** correction B applied.

Instantaneous ocean dynamic sea level is defined by

$$\tilde{\zeta}(\mathbf{r}, t) = \tilde{\eta}(\mathbf{r}, t) + \tilde{B}(\mathbf{r}, t) - G(\mathbf{r}). \tag{13}$$

It is determined jointly by ocean density and circulation. The time-mean ocean dynamic sea level is

$$\zeta(\mathbf{r}) = \eta(\mathbf{r}) + B(\mathbf{r}) - G(\mathbf{r}), \tag{14}$$

whose global mean

$$\frac{1}{A} \int \zeta(\mathbf{r}) \, \mathrm{d}A = 0, \tag{15}$$

in view of Eqs. (5) and (10).

In the Coupled Model Intercomparison Project (CMIP), $\tilde{\zeta}$ is stored in the diagnostic named `zos`, which is defined to have zero mean (Equation H14 of Griffies et al. 2016). However, some models supply it with a nonzero time-dependent mean. If the global mean of the `zos` diagnostic is found to be nonzero, the global mean should be subtracted uniformly.

N12 Ocean dynamic topography: An estimate of **ocean dynamic sea level** computed from the ocean density structure above a reference level where the velocity is either known or assumed to be zero.

On any horizontal (see **geoid** for definition) level z within the ocean, the hydrostatic pressure is given by

$$\tilde{p}(z) = \tilde{p}_a + g \int_z^{\tilde{\zeta} - \tilde{B}} \tilde{\rho}(z') \, \mathrm{d}z', \tag{16}$$

which is the sum of the atmospheric pressure $\tilde{p}_a$ on the **sea surface** and the weight per unit horizontal area of sea water between z and the sea surface. The coordinate of the sea surface in this case is not $\tilde{\eta}$ but $\tilde{\eta} - G = \tilde{\zeta} - \tilde{B}$ by Eq. (13), the height of the sea surface above the geoid, because we are using the orthometric vertical coordinate z, which is the natural choice for ocean dynamics. Equation (9) leads to the horizontal gradient of the atmospheric pressure $\nabla \tilde{p}_a = \nabla \tilde{p}_a' = g\rho_* \nabla \tilde{B}$. Consequently, the horizontal gradient of pressure within the ocean is given by

$$\nabla \tilde{p} = \nabla \left(\tilde{p}_a + g \int_z^{\tilde{\zeta} - \tilde{B}} \tilde{\rho}(z')\, \mathrm{d}z' \right) \tag{17a}$$

$$= g\rho_* \nabla \tilde{B} + (g\rho_* \nabla \tilde{\zeta} - g\rho_* \nabla \tilde{B}) + g \int_z^{\tilde{\zeta} - \tilde{B}} \nabla \tilde{\rho}(z')\, \mathrm{d}z' \tag{17b}$$

$$= g\rho_* \nabla \tilde{\zeta} + g \int_z^{\tilde{\zeta} - \tilde{B}} \nabla \tilde{\rho}(z')\, \mathrm{d}z'. \tag{17c}$$

In the first step of this derivation, we used Eq. (9) for the **inverse barometer** correction $\tilde{B}$, approximated g and sea-surface $\tilde{\rho} = \rho_*$ as constants, and applied Leibniz's rule to differentiate the integral, which yields the two terms in parentheses in Eq. (17b), but no term for $\tilde{\rho}$ at z because $\nabla z = \mathbf{0}$. From Eq. (17c) we obtain

$$\nabla \tilde{\zeta}(\mathbf{r}) = \frac{1}{g\rho_*} \nabla \tilde{p}(\mathbf{r}, z) - \frac{1}{\rho_*} \int_z^{\tilde{\zeta} - \tilde{B}} \nabla \tilde{\rho}(\mathbf{r}, z')\, \mathrm{d}z', \tag{18}$$

which relates the horizontal gradient of **ocean dynamic sea level** $\tilde{\zeta}$ to the horizontal hydrostatic pressure gradient at a reference level z and the horizontal density gradient above that level. The second term on the right-hand side is the horizontal gradient of the *dynamic height D*

$$D = -\frac{1}{\rho_*} \int_z^{\tilde{\zeta} - \tilde{B}} \tilde{\rho}\, dz' \tag{19}$$

of the sea surface relative to z.

In much of the ocean interior (below the boundary layer and away from coastal and other strong currents), and taking a time-mean sufficient to eliminate tidal currents, *geostrophy* is a reasonable approximation, meaning that there is a balance (no net acceleration) between the pressure gradient and Coriolis forces, and all other forces are negligible. Therefore, $\tilde{\mathbf{v}} \simeq \tilde{\mathbf{v}}_g$, with the *geostrophic velocity* $\tilde{\mathbf{v}}_g$ defined by

$$f\mathbf{k} \times \tilde{\rho} \tilde{\mathbf{v}}_g = -\nabla \tilde{p}, \tag{20}$$

where f is the Coriolis parameter and $\mathbf{k}$ the vertical unit vector. If we can measure $\tilde{\mathbf{v}}$ at some z and assume it is geostrophic, we arrive at

$$\nabla \tilde{\zeta}(\mathbf{r}) = -\frac{f}{g\rho_*}\mathbf{k} \times \tilde{\rho}\tilde{\mathbf{v}}(\mathbf{r},z) - \frac{1}{\rho_*}\int_z^{\tilde{\zeta}-\tilde{B}} \nabla \tilde{\rho}(\mathbf{r},z')\,\mathrm{d}z', \tag{21}$$

from Eq. (18).

Alternatively, if we do not know $\tilde{\mathbf{v}}$ at any z, we assume there exists a *level of no motion* $z = -L$, which is a geopotential (horizontal) surface on which $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}_g = \mathbf{0}$, requiring the horizontal hydrostatic pressure gradient to vanish ($\nabla \tilde{p} = \mathbf{0}$) by Eq. (20). Therefore,

$$\nabla \tilde{\zeta}(\mathbf{r}) = -\frac{1}{\rho_*}\int_{-L}^{\tilde{\zeta}-\tilde{B}} \nabla \tilde{\rho}(\mathbf{r},z)\,\mathrm{d}z, \tag{22}$$

using Eq. (18). There is no motion on $z = -L$ so long as there is a compensation between undulations of dynamic sea level $\tilde{\zeta}$ (on the left-hand side of Eq. 22), and variations of the density structure above $z = -L$ (on the right-hand side). Such exact compensation does not generally occur in the ocean, and the level of no motion does not exist. However, it is a useful approximation in many situations. For example, an anomalous sea-surface high is associated with a depression of the pycnocline in the interior of subtropical gyres (e.g., Figure 3.3 of Tomczak and Godfrey 1994), thus leading to relatively weak flow beneath the pycnocline. In some regions, the approximation is not useful; in particular, sizeable $\tilde{\mathbf{v}}$ occurs at all depths in the Southern Ocean.

The ocean dynamic topography is the estimate of ocean dynamic sea level made using Eqs. (21) or (22). Since Eqs. (21) and (22) are unaffected by adding a constant to $\tilde{\zeta}$, the method provides only the difference in $\tilde{\zeta}$ between any two points (i.e., the gradient); it cannot give $\tilde{\zeta}$ for individual points relative to the geoid. Furthermore, it is not applicable in regions where the reference level for motion is below the **sea floor**, nor for differences between points in basins which are separated by sills that are shallower than the reference level.

5 Changes in Sea Level

The relationships between quantities determining changes in sea level are summarized in Fig. 3. The phrases "sea-level change" (SLC) and "sea-level rise" (SLR) are often used in the literature. These make sense when referring to the phenomenon in general, but more specific terms such as **relative sea-level change** and **global-mean sea-level rise** should be preferred where relevant.

N13 **Ocean dynamic sea-level change** $\Delta \zeta$: The change in time-mean **ocean dynamic sea level**, i.e., the change in IB-corrected **mean sea level** relative to the **geoid**.

For the difference between two time-mean states of the climate, Eq. (13) gives

$$\Delta \zeta(\mathbf{r}) = \Delta \eta(\mathbf{r}) + \Delta B(\mathbf{r}) - \Delta G(\mathbf{r}). \tag{23}$$

Since the time-mean ocean dynamic sea level $\zeta(\mathbf{r})$ always has a zero global mean by Eq. (15), so does $\Delta \zeta$ i.e., **global-mean sea-level rise** is excluded from ocean dynamic sea-level change. This property depends on Eq. (5) and thus requires a different choice of geopotential to define the **geoid** in the two states, if there is any change in global-mean sea level.
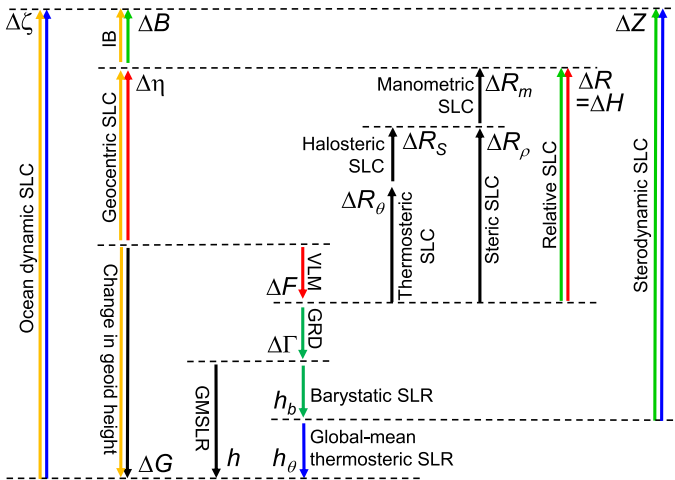
Fig. 3 Relationships between quantities, defined in Sect. 5, that determine changes in sea level. The lengths of the arrows do not have any significance—they are only illustrative—and the dotted horizontal lines serve only to indicate alignment. All of the quantities are differences between two states, and all except h, h_θ and h_b are functions of location **r**. Any closed circuit gives an equality, in which a term has a positive sign when traversed in the direction of its arrow, and a negative sign if in the opposite direction to its arrow. For example, $\Delta R - \Delta \eta + \Delta F = 0$ (Eq. 24) is the circuit marked in red, Eq. (23) in orange, Eq. (38) in blue, and Eq. (54) in green

> **N14 Geocentric sea-level change $\Delta \eta$:** The change in local **mean sea level** with respect to the terrestrial reference frame.

Geocentric sea-level change is the change in the height $\eta(\mathbf{r})$ of MSL relative to the **reference ellipsoid**. *IB-corrected geocentric sea-level change* is $\Delta \eta + \Delta B$, i.e., the same with the **inverse barometer** correction added. Geocentric sea-level change must be distinguished from **relative sea-level change**.

> **N15 Relative sea-level change (RSLC) ΔR:** The change in local **mean sea level** relative to the local solid surface, i.e., the **sea floor**. Relative sea-level change is also called "relative sea-level rise" (RSLR). (See Sect. 6 for an exposition of the relationship of RSLC to other quantities.)

Both the MSL height η and the sea floor height F may change and thus alter RSL. Hence, RSLC is geodetically expressed as

$$\Delta R(\mathbf{r}) = \Delta \eta(\mathbf{r}) - \Delta F(\mathbf{r}), \tag{24}$$

the difference between **geocentric sea-level change** $\Delta \eta$ and **vertical land movement** ΔF (VLM). *IB-corrected relative sea-level change* is $\Delta R + \Delta B$, i.e., RSLC with the **inverse barometer** correction. Relative sea-level change is the quantity registered by a *tide-gauge*, which measures sea level relative to the solid surface where it is attached.

Since climate models do not include VLM, they do not distinguish between geocentric and relative sea-level change. In climate models where atmospheric pressure changes Δp_a are not applied to the ocean, $-\Delta B$ must be added to include the effect of Δp_a simulated by the atmosphere model. (Note that this adjustment should not be made to **ocean dynamic sea-level change** $\Delta \zeta$, which by definition is IB-corrected; see Eq. 23.)

The term "relative sea level" is not employed in an absolute sense, but only in conjunction with "change", because $\eta - F$ (the analogue of Eq. 24) is simply the depth of the sea floor below MSL, equal to the time-mean thickness of the ocean H (Eq. 3).

In view of Eq. (3), we may also write RSLC as

$$\Delta R(\mathbf{r}) = \Delta H(\mathbf{r}), \tag{25}$$

i.e., the change in local ocean thickness, making it obvious that RSLC is not meaningful at locations which change from land to sea (*transgression*) or vice versa (*regression*), since H is undefined on land.

When considering sea-level change on geological timescales, in the absence of information about **ocean dynamic sea level** $\tilde{\zeta}$ or the inverse barometer effect, we might approximate $\Delta\tilde{\zeta} \simeq 0$ and $\Delta B \simeq 0$, in which case $\Delta\eta \simeq \Delta G$ from Eq. (23), and $\Delta R \simeq \Delta G - \Delta F$ from Eq. (24). This quantity is defined everywhere and thus gives an approximate meaning to RSLC in regions of transgression and regression.

> N16 **Steric sea-level change** ΔR_ρ: The part of **relative sea-level change** which is due to the change $\Delta\rho$ in ocean density, assuming the local mass of the ocean per unit area does not change. It is composed of **thermosteric sea-level change** ΔR_θ, which is the part due solely to the change $\Delta\theta$ in in-situ temperature, and **halosteric sea-level change** ΔR_S, which is the part due solely to the change ΔS in salinity.

The time-mean local mass of the ocean per unit area is

$$m = \int_F^\eta \rho \, dz = H\overline{\rho} \quad \text{with} \quad \overline{\rho} \equiv \frac{1}{H}\int_F^\eta \rho \, dz, \tag{26}$$

where the first factor $H = \eta - F$ is the local time-mean thickness of the ocean (Eq. 3), and the second factor $\overline{\rho}$ is the local vertical-mean time-mean density. If we change the density while keeping m fixed, the thickness of the ocean changes, because

$$0 = \Delta m = \Delta H|_m \overline{\rho} + H\Delta\overline{\rho}|_m \tag{27}$$

(by making a linear approximation). Therefore,

$$\Delta H|_m = -(H/\overline{\rho})\,\Delta\overline{\rho} \quad \text{with} \quad \Delta\overline{\rho} = \frac{1}{H}\int_F^\eta \Delta\rho \, dz. \tag{28}$$

This would exactly define steric sea-level change in a situation where mass did not move horizontally. But in reality there are horizontal transports, making it impossible to separate density changes due to local changes in properties from those due to the movement of mass. For convenience, we approximate $\overline{\rho}$ with the constant ρ_*.

Since the RSLC is given by $\Delta R = \Delta H$ (without **inverse barometer** correction, Eq. 25), steric sea-level change is

$$\Delta R_\rho = -\frac{1}{\rho_*}\int_F^\eta \Delta\rho \, dz = \Delta R_\theta + \Delta R_S \tag{29}$$

with the density increment decomposed into thermal and haline components (by making a linear approximation)

$$\Delta\rho = \frac{\partial\rho}{\partial\theta}\,\Delta\theta + \frac{\partial\rho}{\partial S}\,\Delta S, \tag{30}$$

and with the corresponding thermosteric and halosteric contributions

$$\Delta R_\theta = -\frac{1}{\rho_*} \int_F^\eta \frac{\partial \rho}{\partial \theta} \Delta\theta \, \mathrm{d}z \qquad \Delta R_S = -\frac{1}{\rho_*} \int_F^\eta \frac{\partial \rho}{\partial S} \Delta S \, \mathrm{d}z. \tag{31}$$

Thermosteric sea-level change is often called *thermal expansion*, because $\partial\rho/\partial\theta < 0$, so increasing the temperature gives $\Delta R_\theta > 0$. (See Appendix 2 regarding the dependence of density on salinity.) Relative sea-level change (without inverse barometer correction) is the sum of steric and **manometric sea-level change** (Eq. 35).

N17 **Global-mean thermosteric sea-level rise** h_θ: The part of **global-mean sea-level rise** (GMSLR) which is due to thermal expansion.

This quantity is the global mean of local **thermosteric sea-level change** ΔR_θ (due to temperature change, Eq. 31); thus,

$$h_\theta = \frac{1}{A} \int \Delta R_\theta \, \mathrm{d}A = -\frac{1}{\rho_* A} \int \int_F^\eta \frac{\partial \rho}{\partial \theta} \Delta\theta(\mathbf{r}, z) \, \mathrm{d}z \, \mathrm{d}A. \tag{32}$$

It is the change in global ocean volume due to change in temperature alone, divided by the ocean surface area. The CMIP variable `zostoga` is h_θ calculated with respect to a fixed reference state. (Griffies et al. 2016 define the reference to be the initial state of the experiment for CMIP6.) Hence, differences in `zostoga` between two states give the global-mean thermosteric sea-level rise between those states.

Although **halosteric sea-level change** ΔR_S (due to salinity change, Eq. 31) can be locally of the same order of magnitude as thermosteric, global-mean halosteric sea-level change is practically zero. In Appendix 2 we detail the physical arguments leading to this conclusion. Salinity change should be excluded when calculating h_θ, to avoid including a *spurious* global-mean halosteric sea-level change. (See Appendix 2 here as well as Appendix H9.5 of Griffies et al. 2016.) However, salinity change must of course be included when calculating ΔR_S. It follows that global-mean *steric* sea-level change, which equals h_θ because global-mean halosteric sea-level change is zero, cannot be calculated as the global mean of local *steric* sea-level change. This apparent contradiction is due to the inaccuracy of the approximations made following Eq. (28).

N18 **Manometric sea-level change** ΔR_m: **Definition A:** The part of **relative sea-level change** (RSLC) which is not steric, or alternatively **Definition B:** The part of RSLC which is due to the change $\Delta m(\mathbf{r})$ in the time-mean local mass of the ocean per unit area, assuming the density does not change. In the following, we show that the two definitions are approximately the same.

If we change the local mass m per unit area while keeping density fixed, by Eq. (26) the thickness of the ocean changes by $\Delta H|_\rho = \Delta m / \overline{\rho}$, where $\overline{\rho}$ is the local vertical mean of ρ. (In reality, if the local mass per unit area changes, the density will probably change as well, since mass which is converging horizontally or through the sea surface is unlikely to have $\rho = \overline{\rho}$ exactly.)

Since RSLC $\Delta R = \Delta H$ (without **inverse barometer** correction, Eq. 25), if we approximate $\overline{\rho}$ with the constant ρ_*, we obtain

$$\Delta R_m \simeq \frac{\Delta m}{\rho_*}. \tag{33}$$

This is Definition B of "manometric sea-level change". The local change in mass Δm can be estimated from the gravity field, or from the *bottom pressure* p_b, i.e., the hydrostatic pressure at the **sea floor**, according to $\Delta m = (\Delta p_b - \Delta p_a)/g$. (See Appendix 3.) Because of its relationship to p_b, manometric sea-level change is sometimes referred to as the "bottom pressure term" in sea-level change.

According to Definition B, the global mean of ΔR_m vanishes if the mass of the global ocean is constant, since $1/(A\rho_*) \int \Delta m \, dA = 0$. However, ΔR_m may still be locally non-zero, due to rearrangement of the existing mass of the ocean. If the mass of the global ocean changes, the global mean of ΔR_m is nonzero and equals the **barystatic sea-level rise** (equality is approximate with Definition B of ΔR_m, exact with Definition A), which is part of **global-mean sea-level rise** (GMSLR). Despite the correspondence between (local) manometric and (global) barystatic sea-level rise, we argue that these two concepts are sufficiently different to need distinct terms. (See the subsection for barystatic sea-level rise.)

If mass and density are both allowed to change, Eq. (26) gives

$$\Delta m = \Delta H \, \overline{\rho} + H \, \Delta\overline{\rho} \Rightarrow \Delta H = (1/\overline{\rho})\left(\Delta m - \int_F^\eta \Delta\rho \, dz\right), \tag{34}$$

using the expression for $\Delta\overline{\rho}$ from Eq. (28). Again approximating $\overline{\rho}$ as ρ_* and substituting from Eqs. (25), (29) and (33), we obtain

$$\Delta R = \Delta H \simeq \Delta R_\rho + \Delta R_m, \tag{35}$$

i.e., RSLC (without inverse barometer correction) is the sum of **steric sea-level change** and manometric sea-level change, which are, respectively, the parts due to change in density and change in mass per unit area. Since H is defined only in ocean areas, the formulae are not valid for locations which change from land to sea or vice versa.

With ΔR_m defined by Eq. (33), Eq. (35) is only approximate, because of the replacement of $\overline{\rho}$ with ρ_*. We can make Eq. (35) exact if we retain the definition of Eq. (29) for steric sea-level change involving ρ_* and adopt Definition A of "manometric sea-level change", as

$$\Delta R_m \equiv \Delta R - \Delta R_\rho, \tag{36}$$

i.e., ΔR_m is the part of RSLC that is not steric.

We propose "manometric" as a new term because in the existing literature there is no unambiguous and generally used term for ΔR_m. It may be described as the "mass effect on", "mass contribution to", "mass component of" or "mass term in" sea level or sea-level change, but these descriptions could equally well refer to **GRD**-induced sea-level change (the effects of a change in the geographical distribution of mass) or barystatic sea-level rise, so they can be confusing. ("Manometric" is an existing word, referring to the measurement of hydrostatic pressure using a column of liquid, a concept that is closely related to bottom pressure.)

N19 **Barystatic sea-level rise** h_b: The part of **global-mean sea-level rise** (GMSLR) which is due to the addition to the ocean of water mass that formerly resided within the land area (as land water storage or land ice) or in the atmosphere (which contains a relatively tiny mass of water), or (if negative) the removal of mass from the ocean to be stored elsewhere. It is also called "barystatic sea-level change".

Land water storage, also called *terrestrial water storage*, is water on land that is stored as groundwater, soil moisture, water in reservoirs, lakes and rivers, seasonal snow and permafrost. *Land ice* means ice sheets, glaciers, permanent snow and firn. Barystatic sea-level rise includes contributions from changes in all of these.

It does *not* include changes in the parts of ice shelves and glacier tongues whose weight is supported by the ocean rather than resting on land. (These floating parts constitute the majority of the mass of ice shelves and glacier tongues, but near the grounding line on the seaward side some part of the weight may be supported by the land-based ice.) Where land ice rests on a bed which is below mean sea level, it is already displacing sea water. Therefore, the land ice contribution to barystatic sea-level rise excludes the mass whose liquid-water equivalent volume equals the volume of sea water already displaced. The remainder, which is not currently displacing sea water, is often referred to as the ice mass or volume *above flotation* in glaciology.

We define barystatic sea-level rise as

$$h_b = \frac{\Delta M}{\rho_\mathrm{f} A},\tag{37}$$

i.e., the change in mass ΔM of the global ocean from added freshwater, converted to a change in global ocean volume and divided by the ocean surface area A. Because global-mean halosteric change is negligible, the salinity of the existing sea water does not affect h_b. Any contribution δM to barystatic sea-level rise can be expressed as its *sea-level equivalent (SLE)* $\delta M/(\rho_\mathrm{f} A)$, using the same formula.

The formula provides a convenient method of quantifying the changes in the mass of the ocean if A is constant. However, h_b and SLE may not accurately indicate the contribution of added mass to global-mean ocean thickness if there is a substantial change to A, as for example in the transition from glacial to interglacial.

Calculating the global mean of **manometric sea-level change** ΔR_m from its Definition B (Eq. 33) gives $1/A \int \Delta R_m \, \mathrm{d}A = \Delta M/(\rho_* A) \simeq h_b$, i.e., approximately equal to the barystatic sea-level change, but not exactly since $\rho_* \neq \rho_\mathrm{f}$. With Definition A, the global mean of ΔR_m exactly equals h_b. (See **global-mean sea-level rise**.) Despite this relationship between manometric sea-level change and barystatic sea-level rise, we argue that we need distinct terms for them, rather than referring to the latter as the global mean of the former, for two reasons.

First, barystatic sea-level rise is well defined by conservation of water mass on Earth and can be evaluated from the change in mass of other stores of water, e.g., ice sheets and glaciers, without considering the ocean. This has been the usual approach in observational studies of the budget of global-mean sea-level rise, and is the only possibility for diagnosing h_b from the majority of climate models whose ocean component is Boussinesq or has a linear free surface, and therefore does not conserve water mass. Secondly, the partitioning of RSLC into steric and manometric (Eq. 35) is somewhat arbitrary, because it depends on the choice of ρ_* as a reference density.

Neither reason for the distinction of ΔR_m and h_b applies to thermosteric sea-level change; its contribution to global-mean sea-level rise can only be conceived or evaluated as the global mean of the local ΔR_θ, whose definition by Eq. (31) is well defined.

In recent literature, "eustatic" is often used as a synonym for "barystatic", whereas in geological literature *eustatic sea-level change* means either **global-mean sea-level rise** or **global-mean geocentric sea-level rise**. Because of this confusion of meaning, we deprecate the term "eustatic", following the last three assessment reports of the

Intergovernmental Panel on Climate Change (Church et al. 2001; Meehl et al. 2007; Church et al. 2013).

> N20 **Sterodynamic sea-level change** ΔZ: **Relative sea-level change** due to changes in ocean density and circulation, with **inverse barometer** (IB) correction.

This term is the sum of **ocean dynamic sea-level change** (which includes the IB correction) and **global-mean thermosteric sea-level rise**,

$$\Delta Z(\mathbf{r}) = \Delta \zeta(\mathbf{r}) + h_\theta. \tag{38}$$

It can be diagnosed from ocean models (even those that do not conserve mass as per the commonly used Boussinesq models) as the sum of the changes in zos and zostoga. Sterodynamic sea-level change is the part of relative sea-level change that can be simulated with such models. (As discussed above for ocean dynamic sea-level change $\Delta \zeta$, the change in zos calculated from CMIP data should have zero global mean.)

"Sterodynamic" is a term which is newly introduced in this paper. We propose it because in the existing literature there is no clear, simple or generally used term for ΔZ. It is a concept that appears in the literature, where it is referred to by various cumbersome phrases, such as "the oceanographic part of sea-level change", "steric plus dynamic sea-level change" or "sea-level change due to ocean density and circulation change".

> N21 **Vertical land movement (VLM)** ΔF: The change in the height of the **sea floor** or the land surface.

VLM has several causes, including isostasy, elastic flexure of the lithosphere, earthquakes and volcanoes (due to tectonics). All of these involve a change in height of the existing solid surface. In contrast, landslides and sedimentation alter the solid surface and its height by transport of materials; some authors count them as VLM. Extraction of groundwater and hydrocarbons may cause *subsidence* (sinking of the solid surface) by *compaction* (the reduction in the liquid fraction in the sediment). These anthropogenic effects can be locally large, e.g., in Manila, and can exceed the natural effects by orders of magnitude. Where VLM occurs near the coast, it may cause emergence or submergence of land and thus alter the coastline.

Isostasy or *isostatic adjustment* is the process of adjustment of the *lithosphere* (the crust and the rigid upper part of the mantle) towards a hydrostatic equilibrium in which it is regarded as floating in the *asthenosphere* (the underlying viscous mantle, which is of higher density than the lithosphere), with an equal pressure everywhere at some notional horizontal level beneath the lithosphere. On geological timescales, isostatic adjustment occurs in response to changes in the mass load of the lithosphere upon the mantle beneath (the asthenosphere and lower mantle), due to erosion, sedimentation or emplacement of igneous rocks.

On climate timescales there are large changes in load due to the varying mass of ice on land during glacial–interglacial cycles. (See **glacial isostatic adjustment**.) Isostatic adjustment occurs over multi-millennial timescales determined by the viscous flow of the mantle beneath the lithosphere. An elastic response of the lithosphere, on annual timescales, occurs in response to changes in load. Although it is small compared with the eventual isostatic response, it is much more rapid, and hence responsible for significant VLM due to contemporary and recent historical changes in land ice, for instance in West Antarctica.

> N22 **GRD**: Changes in Earth **G**ravity, Earth **R**otation (and hence centrifugal acceleration) and viscoelastic solid-Earth **D**eformation.

These three effects are all caused by changes in the geographical distribution of ocean and solid mass over the Earth. They are often considered together because they occur simultaneously and may interact. Changes in gravitation and rotation alter the geopotential field and hence the **geoid** $G(\mathbf{r})$, while deformation of the solid Earth changes the **sea floor** topography $F(\mathbf{r})$ through **vertical land movement**. By altering G and F, GRD induces **relative sea-level change** (e.g., Tamisiea and Mitrovica 2011; Kopp et al. 2015), which redistributes but does not change the global ocean volume and thus causes no **global-mean sea-level rise**. *GRD-induced relative sea-level change* $\Delta\Gamma$ is defined as

$$\Delta\Gamma = \Delta G' - \Delta F' \tag{39}$$

(derived in Sect. 6 as Eq. 51) where $\Delta G'$ and $\Delta F'$ are the deviations of the changes in the geoid and in the sea floor from their respective global (ocean) means. By construction, the global (ocean) means of G' and F' are each zero; hence, the global (ocean) mean of $\Delta\Gamma$ is zero.

Whatever the cause, redistribution of the ocean mass itself has GRD effects, and thereby the ocean affects its *own* mass distribution and **mean sea level** (MSL). Thus, MSL, the geoid and the sea floor must all be related in a self-consistent solution, which in the context of **glacial isostatic adjustment** (GIA) is expressed by the *sea-level equation* (Farrell and Clark 1976).

The *ocean GRD* effects are called *self-attraction and loading (SAL)*, where "loading" means the weight on the solid Earth. SAL is caused by climatic change in ocean density and circulation (Gregory et al. 2013), which do not involve any change in the mass of the ocean. SAL is also a component of GRD which is caused by changes in land ice and in the solid Earth; thus, SAL contributes to the sea-level effects of GIA, **contemporary GRD** and **mantle dynamic topography** as well.

We propose the new term "GRD" in the absence of any existing single term to describe this frequently discussed group of effects. GRD-induced relative sea-level change may be described as the "mass effect", "mass contribution", "mass component" or "mass term", but these labels could equally well refer to manometric sea-level change if local, or barystatic sea-level rise if global, so they can be confusing. Moreover, "GRD" is helpful as a label for a concept which unifies SAL, GIA, contemporary GRD and mantle dynamic topography.

> N23 **Glacial isostatic adjustment (GIA)**: **GRD** due to ongoing changes in the solid Earth caused by past changes in land ice.

GIA is caused by the viscous adjustment of the mantle to changes in the load on the lithosphere that occurred when mass was transferred from land ice into the ocean, or the reverse. It is dominated by the ongoing effects of the deglaciation following the Last Glacial Maximum. Due to the reduction in the mass load on land, areas that were beneath former ice sheets are generally rising. This process is sometimes called *post-glacial rebound*, but that term is unsatisfactory because GIA involves remote **vertical land movement** as well, both upward and downward. Areas adjacent to the former ice sheets are subsiding as mantle material moves towards the areas of uplift, while land near to the coast is rising and the **sea floor** is generally subsiding as a result of the increase in the mass of the ocean. The ongoing widespread redistribution of mass also affects the **geoid**. Together, the changes in geoid and sea floor cause *GIA-induced relative sea-level change* $\Delta\Gamma_{\text{GIA}}$.

Previous changes in land ice during the Holocene contribute to GIA as well, but GIA does not include the contributions from any ongoing change in land ice or ocean mass, whose effects we call **contemporary GRD**.

N24 **Contemporary GRD**: **GRD** due to ongoing changes in the mass of water stored on land as ice sheets, glaciers and land water storage.

Such transfers of mass cause instantaneous changes in the **geoid**, and **vertical land movement** (VLM) on annual timescales due to elastic deformation of the solid Earth, which causes further change to the geoid. Together, these effects produce **relative sea-level change** (RSLC). There are also slower responses, both VLM and geoid, due to viscous deformation of the asthenosphere. Note that contemporary GRD excludes GIA; the former arises from *ongoing* change in the mass of water on land, and the latter from *past* change.

The elastic deformation and associated geoid contributions to contemporary GRD-induced relative sea-level change are separately proportional to the mass ΔM which has been added to the ocean. Hence, their sum is

$$\Delta \Gamma_p(\mathbf{r}) = \Delta M \, \gamma_p(\mathbf{r}), \tag{40}$$

where $\gamma_p(\mathbf{r})$ is a geographically dependent constant of proportionality, independent of ΔM. Since the **barystatic sea-level rise** is $\Delta M / \rho_f A$ (Eq. 37), the RSLC due to the combination of these three effects is

$$\Delta R_p(\mathbf{r}) = \Delta M \, \phi(\mathbf{r}) \quad \text{where} \quad \phi(\mathbf{r}) = \gamma_p(\mathbf{r}) + 1/\rho_f A. \tag{41}$$

The addition of freshwater to the ocean will induce sterodynamic sea-level change as well (e.g., Agarwal et al. 2015).

The *barystatic–GRD fingerprint* ϕ is a constant geographical pattern, often called a *sea-level fingerprint*, or sometimes a *static-equilibrium fingerprint* to contrast it with the patterns of **ocean dynamic sea-level change**. "Fingerprint" without qualification can be easily confused with climate detection and attribution studies where the same word refers to the patterns caused by particular climate change forcing agents such as greenhouse gases. "Static-equilibrium" is not informative about the processes concerned. The part of contemporary GRD-induced RSLC due to viscous deformation and associated geoid change cannot be represented by a constant pattern because it depends on convolving the history of mass addition with the time-dependent solid-Earth response.

N25 **Mantle dynamic topography**: **GRD** due to ongoing changes in the solid Earth caused by mantle convection and plate tectonics.

The dynamics of the interior of the Earth cause **vertical land movement**, such as the uplift of mid-ocean ridges by upwelling material and the formation of oceanic trenches due to subduction. At the same time, material with different density is redistributed within the Earth, altering the **geoid**. The consequent GRD-induced **relative sea-level change** can be very large on geological timescales, amounting to hundreds of metres. Mantle dynamic topography does not include **glacial isostatic adjustment** (although that is also due to ongoing changes in the solid Earth).

Mantle dynamic topography is often called "dynamic topography" in the solid-Earth literature and also refers to changes in topography on land. We deprecate "dynamic topography" in a sea-level context because it could be confused with **ocean dynamic topography**.

N26 **Global-mean sea-level rise (GMSLR)** h: The increase ΔV in the volume of the ocean divided by the ocean surface area A, also called "global-mean sea-level change" (GMSLC). Observational estimation of h is described in Sect. 7.

By definition, GMSLR is

$$h = \frac{\Delta V}{A} = \frac{1}{A} \Delta \left(\int (\eta - F)\, dA \right) = \frac{1}{A} \int \Delta R(\mathbf{r})\, dA = \frac{1}{A} \int \Delta H(\mathbf{r})\, dA, \qquad (42)$$

which follows from Eqs. (4), (24) and (25). Hence, GMSLR is the global mean of **relative sea-level change** ΔR and equals the global mean of the change ΔH in the thickness (or "depth") of the ocean. Note that GMSLR differs from **global-mean geocentric sea-level rise** because GMSLR is unaltered by a global-mean change $(1/A) \int \Delta F\, dA$ in the level of the **sea floor** F, provided the global ocean volume V does not change.

The global ocean volume can change due to changes in ocean density or due to changes in ocean mass. Hence, GMSLR is the sum of **global-mean thermosteric sea-level rise** and **barystatic sea-level rise**,

$$h = h_\theta + h_b. \qquad (43)$$

A satisfactory explanation of historical observed GMSLR in terms of thermosteric and barystatic contributions has been achieved in recent years thanks to improvements in both observations and models (Church et al. 2011; Gregory et al. 2013; Chambers et al. 2017).

Equation (43) is implied by the Definition A of **manometric sea-level change**, as the part of relative sea-level change which is not steric (Eq. 36), whose global mean

$$\frac{1}{A} \int \Delta R_m\, dA = \frac{1}{A} \int \Delta R\, dA - \frac{1}{A} \int \Delta R_\rho\, dA = h - h_\theta, \qquad (44)$$

using Eq. (42) and recalling that global-mean steric sea-level change is purely thermosteric. By definition, the part of h which is not steric is the part which is due to addition of mass, so it must be the case that

$$h_b = \frac{1}{A} \int \Delta R_m\, dA, \qquad (45)$$

i.e., barystatic sea-level rise equals the global mean of manometric sea-level change by Definition A.

The added mass is unlikely to have exactly the temperature of the existing water to which it is added, implying that changes will probably occur to temperature and hence to density. Because of the nonlinearity of the dependence of ρ on θ, there may be a nonzero contribution to h_θ in consequence. Since this is a steric effect, by definition it is not part of h_b.

From Definition B (Eq. 33) we obtain an approximate expression for global-mean manometric sea-level change as $(1/A) \int \Delta m / \rho_*\, dA = \Delta M / (\rho_* A)$, where ΔM is the added mass. This is the same as the expression (Eq. 37) for h_b except that ρ_f is replaced by ρ_*. This difference is the result of the approximation in Eq. (33) that $\rho_* \simeq \overline{\rho}$. Physically, it is because manometric sea-level change ΔR_m (a local quantity) is dominated by redistribution of existing sea water, for which ρ_* is a good choice of representative density, whereas barystatic sea-level rise (a global quantity) is due only to addition or subtraction of freshwater of density ρ_f, since the redistributive effect is zero in the global mean.

On glacial–interglacial and geological timescales, the variation of ocean area cannot be neglected, so GMSLR is ill-defined. However, it is still meaningful to consider *global-mean relative sea-level change* h_R, which is the change in global-mean ocean thickness

$$h_R = \frac{V + \Delta V}{A + \Delta A} - \frac{V}{A} = \frac{V}{A + \Delta A} \left(\frac{\Delta V}{V} - \frac{\Delta A}{A} \right). \qquad (46)$$

If A is constant, $h_R = h$. An increase in A ($\Delta A > 0$) gives a negative contribution to h_R, counteracting the positive contribution from a concomitant increase in V.

In geological literature, global-mean sea-level rise is sometimes called "eustatic sea-level change". Following the last three assessment reports of the Intergovernmental Panel on Climate Change (Church et al. 2001; Meehl et al. 2007; Church et al. 2013), we deprecate "eustatic" because it has become a confusing term, which is also used to mean **global-mean geocentric sea-level rise** or **barystatic sea-level rise**.

> N27 **Global-mean geocentric sea-level rise** h_G: The global-mean change in **mean sea level** with respect to the terrestrial reference frame.

This quantity is the global mean of $\Delta\eta$, the change in MSL relative to the **reference ellipsoid**. From Eqs. (4) and (5) we have

$$
\begin{aligned}
h_G &= \frac{1}{A}\int \Delta\eta\,\mathrm{d}A = \frac{1}{A}\int \Delta G\,\mathrm{d}A = \frac{1}{A}\left(\Delta V + \int \Delta F\,\mathrm{d}A\right) \\
&= h + \frac{1}{A}\int \Delta F\,\mathrm{d}A,
\end{aligned}
\tag{47}
$$

where h is **global-mean sea-level rise** (GMSLR) defined by Eq. (42). Thus, global-mean geocentric sea-level rise h_G differs from GMSLR because the latter is unaltered by a global-mean change $(1/A)\int \Delta F\,\mathrm{d}A$ in the level of the **sea floor** F, provided the volume of the ocean does not change. On geological timescales, when the area of the ocean may change, the global-mean change in level of the sea floor is $\Delta((1/A)\int F\,\mathrm{d}A)$.

6 Relationships Determining Relative Sea-Level Change

Relative sea-level change (RSLC) is $\Delta R = \Delta\eta - \Delta F$ (Eq. 24). By applying the **inverse barometer** correction, we obtain IB-corrected RSLC

$$
\Delta R + \Delta B = \Delta\eta + \Delta B - \Delta F \tag{48a}
$$

$$
= \Delta[\eta - G + B] + \Delta[G - F] \tag{48b}
$$

$$
= \Delta\zeta + \Delta[G - F], \tag{48c}
$$

where in Eq. (48c) we used the definition of **ocean dynamic sea-level change** $\Delta\zeta$ (Eq. 23) to rewrite the first term. From the definition of the geoid (Eq. 4) we obtain

$$
\frac{1}{A}\int \Delta(G - F)\,\mathrm{d}A = \frac{\Delta V}{A} = h, \tag{49}
$$

the **global-mean sea-level rise**. Let us write $\Delta G(\mathbf{r}) = \Delta G'(\mathbf{r}) + (1/A)\int \Delta G\,\mathrm{d}A$ and similarly for ΔF, thus defining $\Delta G', \Delta F'$ as the local deviations of $\Delta G, \Delta F$ from their respective global (ocean) means. Therefore,

$$
\Delta(G - F) = \Delta(G' - F') + \frac{1}{A}\int \Delta(G - F)\,\mathrm{d}A = \Delta(G' - F') + h. \tag{50}
$$

This leads to our expression for **GRD**-induced relative sea-level change (Eq. 39) as

$$\Delta\Gamma(\mathbf{r}) \equiv \Delta[G'(\mathbf{r}) - F'(\mathbf{r})] = \Delta[G(\mathbf{r}) - F(\mathbf{r})] - h. \tag{51}$$

Substituting Eq. (51) in Eq. (48c) gives IB-corrected RSLC as

$$\Delta R(\mathbf{r}) + \Delta B(\mathbf{r}) = \Delta\zeta(\mathbf{r}) + h + \Delta\Gamma(\mathbf{r}), \tag{52}$$

the sum of ocean dynamic sea-level change $\Delta\zeta$, global-mean sea-level rise h and GRD-induced RSLC $\Delta\Gamma$. Using Eqs. (43) and (38) we obtain

$$\Delta\zeta(\mathbf{r}) + h = \Delta\zeta(\mathbf{r}) + h_\theta + h_b = \Delta Z(\mathbf{r}) + h_b, \tag{53}$$

Hence, IB-corrected RSLC is

$$\Delta R(\mathbf{r}) + \Delta B(\mathbf{r}) = \Delta Z(\mathbf{r}) + h_b + \Delta\Gamma(\mathbf{r}), \tag{54}$$

the sum of **sterodynamic sea-level change** $\Delta Z(\mathbf{r})$, **barystatic sea-level rise** h_b and GRD-induced RSLC. The **contemporary GRD**-induced RSLC due to a change δM_i in any of the stores of water on land (as *land water storage* or *land ice*, e.g., in a lake or an ice sheet) has both a barystatic and a GRD-induced effect on sea level, which are related and may interact (e.g., Gomez et al. 2012). Provided they are both proportional to δM_i, we can rewrite Eq. (54) as

$$\Delta R(\mathbf{r}) + \Delta B(\mathbf{r}) = \Delta Z(\mathbf{r}) + \sum_i \delta M_i\, \phi_i + \Delta\Gamma_{\mathrm{GIA}} + \Delta\Gamma_Z, \tag{55}$$

where ϕ_i is the barystatic–GRD fingerprint (Eq. 41) of store i of water, $\Delta\Gamma_{\mathrm{GIA}}$ is GIA-induced RSLC, and $\Delta\Gamma_Z$ is the GRD-induced RSLC of ocean mass redistribution (self-attraction and loading) associated with sterodynamic sea-level change. The last term is typically neglected.

Equation (55) is the means by which MSL projections are derived from coupled atmosphere–ocean general circulation models (AOGCMs). These models do not simulate GRD-induced RSLC (because they have time-independent geoid and sea floor) and are not generally used to compute barystatic sea-level rise (because they do not include adequate representations of land ice or land water storage). RSLR projections are therefore obtained by combining sterodynamic sea-level change simulated by an AOGCM with separately calculated projections of barystatic sea-level rise and GRD-induced RSLC using climate change simulations from the AOGCM applied to models of glaciers, ice sheets and the solid Earth (Church et al. 2013; Kopp et al. 2014; Slangen et al. 2014).

According to Eqs. (35) or (36), $\Delta R = \Delta R_\rho + \Delta R_m$, the sum of **steric sea-level change** ΔR_ρ and **manometric sea-level change** ΔR_m, which are the parts due, respectively, to change in density and change in mass per unit area. In general, $\Delta R_m \neq 0$ even if $h_b = 0$, because ocean mass may be redistributed. In particular, because $\Delta R_m = -1/\rho_s \int_F^\eta \Delta\rho\, \mathrm{d}z$ is small on the continental shelves (where the ocean is shallow), but ocean dynamics will not permit a strong gradient in ζ to develop across the shelf break, global-mean thermosteric sea-level rise demands a redistribution of ocean mass onto the shelves (Landerer et al. 2007; Yin et al. 2010), with consequent ocean GRD (Gregory et al. 2013).

7 Observations of Sea-Level Change

Estimates of **global-mean sea-level rise** (GMSLR) for the last century depend mainly on records from tide-gauges. These instruments register coastal **relative sea-level change** (RSLC) $\Delta R = \Delta\eta - \Delta F$ (Eq. 24), which is affected by local **vertical land movement** (VLM) ΔF. VLM is large in some places, with strong geographical gradients.

GMSLR is calculated as the global mean of RSLC, $h = (1/A) \int \Delta R \, dA$ (Eq. 42). However, tide-gauges measure ΔR only at points on the coast and thus give a sparse, non-uniform and unrepresentative sampling of the global ocean area. The calculation therefore depends on physically based methods for extrapolation. Considering Eq. (52) in the form

$$h = \Delta R + \Delta B - \Delta\zeta - \Delta\Gamma \tag{56}$$

we see that in principle h can be calculated from ΔR from any tide-gauge by applying the **inverse barometer** (IB) correction ΔB, and subtracting local **ocean dynamic sea-level change** $\Delta\zeta$ and local **GRD**-induced RSLC $\Delta\Gamma$. The global mean of each of these three adjustments is zero, so Eq. (42) is satisfied. In practice, using historical records, it is necessary to combine many tide-gauges in order to reduce the influence of unforced variability in $\tilde{\zeta}$.

The IB adjustment is fairly small and can be made accurately from atmospheric pressure records. Various methods are used to allow for the spatial pattern of $\Delta\zeta$, for example by calculating the mean over sets of gauges presumed to be representative of large regions (e.g., Jevrejeva et al. 2008), or by using spatial patterns of $\tilde{\eta}$ variation observed by satellite altimetry during its shorter period of availability (e.g., Church and White 2011). Because **glacial isostatic adjustment** (GIA) is the only part of GRD (including VLM) for which a global field is available, most estimates of GMSLR exclude all tide-gauges where GIA is not the only significant contribution to VLM (those affected by earthquakes, anthropogenic subsidence, sediment compactions, etc.). At the tide-gauges which are retained, we adjust for GIA-induced RSLC $\Delta\Gamma_{\mathrm{GIA}}(\mathbf{r})$ (e.g., Figure 3a of Tamisiea and Mitrovica 2011), estimated by combining solid-Earth models, the sea-level equation and reconstructed histories of deglaciation.

Alternatively, tide-gauge records may be corrected for VLM using vertical motion calculated from collocated GNSS (e.g., GPS) receivers. Effectively, this transforms RSLC to **geocentric sea-level change** $\Delta\eta = \Delta R + \Delta F$ (Eq. 24). Geoid adjustments must be applied to GNSS-corrected tide-gauge records just as for satellite altimetry, as described in the next paragraph.

Geocentric sea-level change $\Delta\eta(= \Delta\zeta - \Delta B + \Delta G$, Eq. 23) has been measured over most of the global ocean since the early 1990s by satellite radar *altimetry*, using instruments which are located in a terrestrial reference frame (equivalent to the reference ellipsoid), and measure their vertical distance from the sea surface. To study the contemporary causes of observed geocentric sea-level change we must subtract $\Delta G_{\mathrm{GIA}}(\mathbf{r})$, the effect of GIA on the geoid (e.g., Figure 3b of Tamisiea and Mitrovica 2011), from IB-corrected geocentric sea-level change, thus:

$$\Delta\eta + \Delta B - \Delta G_{\mathrm{GIA}} = \Delta\zeta + \Delta G_{\mathrm{NGIA}}, \tag{57}$$

where $\Delta G_{\mathrm{NGIA}} = \Delta G - \Delta G_{\mathrm{GIA}}$ is due to ongoing redistribution of water mass on the Earth's surface.

To convert **global-mean geocentric sea-level rise** $h_G = (1/A) \int \Delta\eta \, dA$ to GMSLR h requires an adjustment for the global mean of ΔF, according to Eq. (47).

Although several processes can produce large local VLM, the only large global-mean effect is GIA. There is no contemporary GMSLR associated with GIA, so Eq. (49) gives $(1/A) \int \Delta F_{\mathrm{GIA}} \, \mathrm{d}A = (1/A) \int \Delta G_{\mathrm{GIA}} \, \mathrm{d}A \Rightarrow (1/A) \int \Delta G_{\mathrm{NGIA}} \, \mathrm{d}A = h + (1/A) \int \Delta F_{\mathrm{NGIA}} \, \mathrm{d}A$. Hence, the global mean of Eq. (57) becomes

$$h_G = h + \frac{1}{A} \int \Delta F_{\mathrm{GIA}} \, \mathrm{d}A + \frac{1}{A} \int \Delta F_{\mathrm{NGIA}} \, \mathrm{d}A, \tag{58}$$

recalling that the global means of ΔB and $\Delta \zeta$ are zero. In response to the shift of mass from the land (as ice) into the ocean since the Last Glacial Maximum, and the consequent mantle adjustment, the sea floor is subsiding on average, giving a trend in $(1/A) \int \Delta F_{\mathrm{GIA}} \, \mathrm{d}A$ of about -0.3 mm year $^{-1}$ (Tamisiea and Mitrovica 2011). Thus, $h_G < h$ due to GIA. Contemporary changes in land ice cause elastic deformation of the sea floor. This gives a negative $(1/A) \int \Delta F_{\mathrm{NGIA}} \, \mathrm{d}A$ which reduces h_G by about 8% of the **barystatic sea-level rise** (Frederikse et al. 2017).

8 Deprecated Terms and Recommended Replacements

Deprecated term	Recommended replacement
Eustatic sea-level change	**Barystatic sea-level rise** or barystatic sea-level change for global-mean sea-level rise due to change in the mass of the ocean, but not its density
	Global-mean sea-level rise for the global mean of relative sea-level change, due to the change in the volume of the ocean
	Global-mean geocentric sea-level rise for the global mean of change in mean sea level relative to the terrestrial reference frame, due to the combined effects of change in the volume of the ocean and change in the level of the sea floor
Dynamic topography	**Ocean dynamic sea level** for mean sea level above the geoid due to ocean dynamics
	Ocean dynamic topography for ocean dynamic sea level estimated from ocean density
	Sea-level change due to mantle dynamic topography for GRD-induced relative sea-level change due to solid-Earth dynamics
Mean sea surface	**Mean sea level** (MSL)
Mean sea-level change or local sea-level change	**Relative sea-level change** (RSLC) or relative sea-level rise (RSLR) for the change in mean sea level relative to the land
	Geocentric sea-level change for the change in mean sea level relative to the terrestrial reference frame
Global sea-level change (GSLC)	**Gobal-mean sea-level rise** (GMSLR) or global-mean sea-level change (GMSLC) for the global mean of relative sea-level change, due to the change in the volume of the ocean
	Global-mean geocentric sea-level rise for the global-mean change in mean sea level relative to the terrestrial reference frame

Deprecated term	Recommended replacement
Extreme sea level	**Extreme sea level** for the occurrence of exceptionally high local sea surface due to short-term phenomena, or **extreme coastal water level** when considering coastal impacts.
	High-end sea-level change for projections or scenarios of very large RSLR or GMSLR
Sea-level change due to thermal expansion	**Thermosteric sea-level change** for contribution to relative sea-level change
	Global-mean thermosteric sea-level rise for contribution to global-mean sea-level change
Sea-level change due to oceanographic processes or steric plus dynamic sea-level change	**Sterodynamic sea-level change** for the change in relative sea-level due to change in ocean density and circulation
Sea-level fingerprint or static-equilibrium fingerprint	**Barystatic–GRD fingerprint** for the sum of the RSLC from GRD (elastic and geoid) and the barystatic sea-level rise due to the addition of a unit mass of water to the global ocean
Post-glacial rebound (PGR)	**Glacial isostatic adjustment** (GIA)
Mass effect on, mass term in, mass component of, or mass contribution to sea level or to sea-level change	**Barystatic sea-level rise** for the contribution to global-mean sea-level rise from the change of mass of the global ocean (associated with changes in mass of water and ice on land), as opposed to global-mean thermosteric sea-level rise
	Manometric sea-level change for the contribution to relative sea-level change due to change in the local mass of the ocean per unit area, as opposed to steric sea-level change
	GRD-induced relative sea-level change for the effects on relative sea level from geoid change and vertical land movement, as opposed to steric, ocean dynamic and barystatic sea-level change
Bottom pressure term in sea-level change	**Manometric sea-level change**

9 List of Defined Terms and Notations

This table gives the entry number or section (not the page number) in which each term is defined. In the PDF, each term is a hyperlink to the relevant text. The rows for which there is no entry number are included only to define notation.

	Above flotation	N19
	Altimetry	Section 7
A	Area of the global ocean	
	Asthenosphere	N21
	Astronomical tide	N6
p_a	Atmospheric pressure at the sea surface	
h_b	**Barystatic sea-level rise**	N19

ϕ	*Barystatic–GRD fingerprint*	N24
	Bathymetry	N4
	Bottom pressure	N18
	Bottom topography	N4
	Chandler wobble	N6
	Compaction	N21
	Contemporary GRD	N24
ρ	Density of water	
	Depth of the water column	N4
D	*Dynamic height*	N12
g	*Effective gravitational acceleration*	N5
	Ellipsoidal height	N1
	Equipotential surface	N5
	Eustatic sea-level change	N19
	Extreme coastal water level	N8
	Extreme sea level	N8
	Fetch	N10
	Free nutation	N6
	Geocentric latitude	N1
$\Delta\eta$	**Geocentric sea-level change**	N14
	Geodetic height	N1
	Geodetic latitude	N1
G	**Geoid**	N5
	Geoid height	N5
Φ	*Geopotential*	N5
	Geostrophy	N12
$\tilde{\mathbf{v}}_g$	*Geostrophic velocity*	N12
$\Delta\Gamma_{GIA}$	*GIA-induced relative sea-level change*	N23
	Glacial isostatic adjustment (GIA)	N23
	Global mean	Section 2.4
h_G	**Global-mean geocentric sea-level rise**	N27
h_R	*Global-mean relative sea-level change*	N26
h	**Global-mean sea-level rise (GMSLR)**	N26
h_θ	**Global-mean thermosteric sea-level rise**	N17
	Gravitational acceleration	N5
	GRD	N22
$\Delta\Gamma$	*GRD-induced relative sea-level change*	N22
	Greenwich meridian	N1
	Halosteric sea-level change ΔR_S	N16
	High-end	N8
	Horizontal	N5
	IB-corrected geocentric sea-level change	N14
	IB-corrected mean sea level	N7
	IB-corrected relative sea-level change	N15
	IB-corrected sea-surface height	N7
θ	In-situ temperature of ocean water	

	Significant wave height	N10
	Significant wave period	N10
	Skew-surge height	N9
	Spring tide	N6
	State	Section 2.2
	Static-equilibrium fingerprint	N24
ΔR_ρ	**Steric sea-level change**	N16
ΔZ	**Sterodynamic sea-level change**	N20
	Storm surge	N9
	Storm tide	N6
σ	*Storm-surge height*	N9
	Subsidence	N21
	Surge residual	N9
	Swash	N9
	Swell wave	N10
	Terrestrial reference frame	N1
	Terrestrial water storage	N19
	Thermal expansion	N16
	Thermosteric sea-level change ΔR_θ	N16
H	*Thickness of the ocean*	N4
	Tidal currents	N6
	Tidal datum	N6
	Tidal height	N6
	Tide-free	N6
	Tide-gauge	N15
	Tides	N6
	Transgression	N15
	True polar wander	N1
	Tsunami	N10
	Vertical	N1
ΔF	**Vertical land movement (VLM)**	N21
V	Volume of the global ocean	
	Wave height	N10
	Wave period	N10
	Wave runup	N9
	Wave setup	N9
	Wind setup	N9
	Wind wave	N10
	Zero-tide	N6

"Addressing the Grand Challenge of regional sea level change prediction". The Grand Challenge steering committee provided a forum for discussions during the preparation of this paper. We are grateful to Palanimuthu and colleagues for their careful typesetting of this complicated manuscript.

Appendix 1: No Resting Steady State Exists for a Realistic Ocean

A state of rest requires zero acceleration parallel to the surface, which must therefore be an equipotential. This condition is satisfied by the geoid because it is an equipotential surface by definition. However, zero acceleration at the surface is not a sufficient condition for the ocean to remain at rest. If there are any horizontal density gradients within the ocean, there will be pressure gradients beneath the horizontal surface, producing forces that will set the ocean into motion. So another necessary condition for zero ocean circulation is the absence of density gradients along horizontal surfaces, i.e., density is a function of depth only. This configuration is quite unlike the real state of the ocean.

Appendix 2: Why We Can Ignore Global Halosteric Sea-Level Change

When freshwater enters the ocean, such as from melting continental ice sheets, it adds to the ocean mass and in turn increases global-mean sea level (barystatic sea-level rise). Ocean salinity also changes due to the dilution of sea water, thus suggesting a role for a global halosteric sea-level change (Munk 2003; Levitus et al. 2005). However, the net effect on global-mean sea level is almost entirely barystatic since the global halosteric effect is negligible (Lowe and Gregory 2006). We can understand why this is so by recognizing that freshwater entering the ocean sees its salinity increase while the ambient sea water is itself freshened. These compensating salinity changes (which are often ignored, as by Munk 2003 and Levitus et al. 2005) have corresponding compensating sea-level changes, thus bringing the global halosteric effect to near zero. We demonstrate this effect in the following subsections, by considering a two-bucket thought experiment where one bucket holds freshwater (bucket-1) and the other holds sea water (bucket-2). We ask how the total water volume changes upon homogenizing the water in the two buckets, while conserving the masses of freshwater and salt. As we will see, the total volume of homogenized water is very nearly equal to the sum of the volume initially in the two separate buckets (to within 0.1%).

Conservation of Mass for Freshwater and Salt

Let the two buckets contain water of mass M_n, volume V_n, salinity S_n, and density ρ_n, $n = 1, 2$, and assume they have equal Conservative Temperature and equal pressure. Now homogenize the water from the two buckets into a single larger bucket, and assume no change in pressure nor any heat of mixing so that Conservative Temperature also remains unchanged. The total mass of freshwater and salt is unchanged upon homogenizing, so that

$$M = M_1 + M_2 \qquad M\,S = M_1\,S_1 + M_2\,S_2, \tag{59}$$

where M is the total mass and S is the salinity of the homogenized water. Return the homogenized water to the original buckets, placing the same mass M_1 back into the first bucket and mass M_2 into the second bucket.

Dependence of Density on Salinity

The dimensionless coefficients $\alpha \equiv \rho^{-1} \, \partial\rho/\partial\theta$ and $\beta \equiv \rho^{-1} \, \partial\rho/\partial S$, which are used to compute steric sea-level change (Eq. 31), measure the relative change in the in-situ density as a function of temperature and salinity. Because $\partial\rho/\partial\theta$ is generally negative, the volume of a given parcel of sea water increases as its temperature rises; this phenomenon is called thermal expansion. By analogy, since $\partial\rho/\partial S > 0$, the corresponding effect for salinity is sometimes called "haline contraction". This is a misleading analogy, because if the salinity has increased but the mass has not changed, some freshwater must have been replaced by salt, so the parcel is materially altered, unlike in the case of adding heat. If salt is added to the parcel but no mass is removed, the salinity, mass and volume of the parcel will *all* increase. The notion of "haline contraction" has led some previous authors to draw incorrect conclusions about the effect on sea level from adding freshwater to the ocean. We here aim to clarify this situation.

The change in the volume of water due to homogenization depends on the value of β. For the surface ocean, representative values are $\rho = 1028$ kg m^{-3} and $S = 0.035$, while for freshwater $\rho = \rho_{\mathrm{f}} = 1000$ kg m^{-3} and $S = 0$. Hence, a representative $\beta \approx (1/1000)(1028 - 1000)/(0.035 - 0) = 28/35 = 0.8$. This coefficient has a roughly 5% relative variation across the ocean, with most of that variation determined by temperature rather than salinity. (See Roquet et al. 2015, as well as Figure 1 in Griffies et al. 2014.) Since our concern is with salinity changes, we take β to be constant in the following.

Computing the Change in Total Volume

The change in total volume upon homogenization is the sum of the changes in the two buckets,

$$\delta V = \delta V_1 + \delta V_2. \tag{60}$$

Since the mass of water in the two buckets remains the same before and after homogenization, the volume in the two buckets is altered only due to changes in their respective densities

$$\delta \rho_n = \delta(M_n/V_n) = -(\rho_n/V_n) \, \delta V_n \Rightarrow \delta \rho_n/\rho_n = -\delta V_n/V_n, \tag{61}$$

i.e., the relative density increases as the relative volume decreases. Because the buckets have the same temperature in our experiment, relative density changes occur only through salinity changes, according to

$$\delta \rho_n/\rho_n = \beta \, \delta S_n = -\delta V_n/V_n, \tag{62}$$

in which case the change in total volume is

$$\delta V = \delta V_1 + \delta V_2 = -(V_1 \, \beta \, \delta S_1 + V_2 \, \beta \, \delta S_2). \tag{63}$$

Mass conservation for salt means that

$$\delta(M \, S) = \delta(M_1 \, S_1) + \delta(M_2 \, S_2) = 0. \tag{64}$$

Furthermore, since mass in the two buckets is unchanged, salt conservation leads to

$$M_1 \, \delta S_1 + M_2 \, \delta S_2 = 0, \tag{65}$$

so that salinity in one bucket rises while that in the other falls. Making use of this result in Eq. (63) leads to our desired expression for total volume change

$$\delta V = -\beta \, \delta S_1 \, (V_1 - V_2 \, M_1/M_2) = -\beta \, V_1 \, \delta S_1 \, (1 - \rho_1/\rho_2). \tag{66}$$

Connecting to Thickness Changes

Equation (66) provides an expression for the change in total volume upon homogenizing two buckets of water with equal Conservative Temperatures, equal pressures, but with differing salinities. To connect to sea level, assume an equal cross-sectional area, A, for the buckets, so that the volume of water is given by $V_n = A \, h_n$, where h_n is the thickness of the water in the bucket. Equation (66) then says that upon homogenization, the thickness of water changes by

$$\delta h = -\beta \, h_1 \, \delta S_1 \, (1 - \rho_1/\rho_2), \tag{67}$$

and that the total thickness of the homogenized water is given by

$$h_{\text{new}} = h_1 + h_2 + \delta h = h_2 + h_1 \, [1 - \beta \, \delta S_1 \, (1 - \rho_1/\rho_2)]. \tag{68}$$

As expected, we see that $\delta h = 0$ only when $\beta = 0$ or $\rho_1 = \rho_2$. The first is never true, and the second is not true in the general case of differing temperatures (in which case there are thermosteric changes ignored in our discussion).

An Ocean Example

To explore the oceanic implications of Eq. (68), assume bucket-1 initially has freshwater with density $\rho_1 = \rho_f$, whereas bucket-2 initially has sea water with density $\rho_2 = \rho_s = \rho_f + \rho'$. The salinity change for bucket-1 is $\delta S_1 = S$, since this bucket went from its original freshwater concentration to the homogenized sea water with salinity S. The halosteric-induced thickness change (Eq. 67) is thus given by

$$\delta h = -h_1 \, \beta \, S \, (\rho'/\rho_s) < 0. \tag{69}$$

How large is this effect? For the case of an upper ocean with salt concentration $S = 0.035$, sea-water density $\rho_s = 1028$ kg m$^{-3} \Rightarrow \rho' = 28$ kg m^{-3} and $\beta = 0.8$, we have

$$\delta h = -h_1 \times 0.8 \times 0.035 \times (28/1028) \approx -h_1 \times 7.6 \times 10^{-4}. \tag{70}$$

To within roughly 8 parts in 10^4, the change in thickness of the ocean column is nearly identical to the thickness of freshwater added to the ocean. For example, if we add one metre of freshwater into the upper ocean ($h_1 = 1$ m), then the change in sea level is equal to one metre minus the tiny amount 0.76 mm . Hence, as emphasized by Lowe and Gregory (2006), we can generally ignore the contribution to global-mean sea level from global halosteric effects.

Appendix 3: Bottom Pressure

The hydrostatic pressure at the ocean sea floor is commonly referred to as the ocean bottom pressure $\tilde{p}_b$, usually calculated as

$$\tilde{p}_b = \tilde{p}_a + g\tilde{m}, \tag{71}$$

where the first term is the atmospheric pressure at the liquid-water equivalent sea surface and the second term is the weight of the mass per unit area $\tilde{m}(\mathbf{r})$ of sea water, given by Eq. (26). This formula makes two approximations. First, hydrostatic pressure does not exactly equal the weight per unit area of the fluid above it because of the curvature of the Earth (Ambaum 2008). Second, g should appear within the vertical integral of density to obtain m (Eq. 26), because g depends on z. However, these approximations are entirely adequate for sea-level studies.

The difference in bottom pressure between two states

$$\Delta p_b = \Delta p_a + g\Delta m, \tag{72}$$

that is, the sum of the change in local atmospheric pressure and the change in the weight of the local ocean. Using Eqs. (26) and (25) to replace Δm,

$$\Delta p_b = \Delta p_a + g\rho_* \, \Delta H + g \int_F^\eta \Delta\rho(z) \, \mathrm{d}z, \tag{73}$$

where $H = \eta - F$ is the time-mean thickness of the ocean (Eq. 3). This form separates the change in pressure due to sea water into one term (the second) due to the change in the local thickness of the ocean, and another (the third) which is proportional to the local vertical-mean change in sea-water density.

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