June, 2011

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Proceedings of the 19th International Input-Output Conference, Alexandria, VA June 13-17, 2011
Eds. Michael L. Lahr and Klaus Hubacek

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Abstract

Supply chain risk management has become a popular topic among both practitioners and researchers, and different models have been proposed to help companies prepare for and react to a disruption in their supply chain. However, less attention has been given to exploring the regional economic impact of these strategies following a disruptive event. This paper examines three different strategies a firm may choose in order to continue production operations should one of its suppliers suffer a disruption. A firm may: (i) maintain inventory, (ii) purchase from an alternate supplier, and/or (iii) substitute for another type of input. We integrate the firm's risk management strategy into a risk-based input-output model that quantifies the economic impact of each strategy. We deploy the model with a data-driven case study addressing the multi-state impacts of a supply shortage at a single firm.

Keywords

supply chain disruption, input-output, constant elasticity of substitution, production function, dynamic analysis

1 Introduction

Supply chains in a globalized environment can be particularly vulnerable to disruptions (Christopher 2005). Disruptive events can lead to supply shortages and cause production delays, factory closures, and lost revenue. For example, the recent earthquake and tsunami in Japan in 2011 created supply shortages in the automobile industry. Ford was forced to close temporarily some of its factories because of parts shortages, and Toyota's production is not expected to return to normal until December 2011 (Hirsch and Hsu 2011, Trudell 2011). In 1997, the cellphone manufacturer Ericsson lost market share to Nokia because of Ericsson's failure to respond quickly to a supply shortage from a key supplier (Sheffi 2005). In general, firms that suffer from supply chain disruptions average a 10 percent decrease in their stock value (Hendricks and Singhal 2003, 2005).

A firm can potentially deploy a number of different strategies to avoid these types of consequences from a supply disruption, and this work will focus on three of those strategies. First, the firm can build up inventory of its inputs so that if a supplier is disabled, the firm can use the inventory to maintain production levels. Second, the firm can buy from multiple suppliers. Third, the firm can potentially substitute a similar product to be used in place of the constrained supply. Finding the strategy or combination of strategies that optimizes a firm's profit has been a popular topic among supply chain researchers. These models include calculating the optimal level of inventory a firm should maintain if its supply is uncertain (Song and Zipkin 1996), determining the trade-off between maintaining inventory and purchasing from an alternate supplier (Tomkin 2006), deploying game theory to model both a supplier and a firm's decision making (Babich 2006, Schmitt et al. 2010), and modeling a supply chain network where both demand and supply-side risk are present (Nagurney 2006).

Although supply chain experts and production managers have extensively examined how a firm can prepare for and respond to supply disruptions, much less analysis has been done on how an individual firm's decisions impact the economy in which it operates. An input-output (I-O) model that describes the linkages among different economic sectors can serve as a useful tool for such an analysis. Barker and Santos (2010a, 2010b) deploy the Inoperability Input-Output Model (IIM) to demonstrate how a firm's inventory level can mitigate the interdependent impact of disruptive events. We build on that approach by first developing a model where a firm optimizes its profit by selecting the level of production and the quantity of supplies it requires. If the firm faces a supply shortage, we describe how the firm's production may be forced to a reduced level of production and how inventory, an alternate supplier, and the capacity to substitute can alter both the new production level and the dynamics of the situation. We incorporate this dynamic production model into the Multiregional IIM (Crowther and Haimes 2010) in order to analyze the interdependent production losses caused by the supply shortage and to quantify the extent to which mitigation strategies reduce the regional production losses. We tailor the Multiregional IIM to measure specifically the interdependent impacts of production losses within a supply chain.

The remainder of the paper is structured as follows: Section 2 describes the firm's production model by first characterizing the equilibrium points and exploring the dynamics of moving between equilibria caused by the onset of a disruption. Section 3 integrates the production model with the Multiregional IIM. In Section 4, we develop a case study exploring a hypothetical supply shortage that impacts the southern United States.

2 Production model

2.1 Equilibrium analysis

In order to measure the impacts of a supply shortage and how a firm reacts to such a shortage, we first develop a firm-level production model to determine the production level and the quantity of each input. It is assumed that some amount of input material is required for the firm's production. The production model in Eq. (1) assumes a firm determines the level of production, y, that maximizes its profit (Chambers 1988). The firm's production function $f(z_1, z_2)$ requires some number of units of two types of input, z_1 and z_2, with per-unit costs of w_1 and w_2, respectively. Although a firm will undoubtedly require more than two inputs, this simplified production function enables the firm to potentially trade off between the two inputs. The final parameter is the selling price for a unit of the firm's production, $p(y)$, which we assume is a function of the number of units produced by the firm. This price or demand function, $p(y)$, is decreasing in y to reflect the inverse relationship between price and quantity.

$$\begin{array}{rl} \max & p(y)y - w_1 z_1 - w_2 z_2 \\ \text{st} & y = f(z_1, z_2) \\ & z_1, z_2 \geq 0 \end{array} \tag{1}$$

Several production functions could be deployed, including the Cobb-Douglas, Leontief, and translog production functions (Heathfield and Wibe 1987), which differ in how they treat the ability to substitute among inputs. Because we are interested in analyzing if a firm can substitute one input for another if supply is constrained, this paper will use the constant of elasticity substitution (CES) production function (Arrow et al. 1961). The CES production function assumes the elasticity of substitution, or the extent to which two goods may be substituted, remains constant between the two inputs (Chambers 1988). The elasticity of substitution is represented with coefficient $\sigma > 0$. For mathematical convenience, the parameter $\rho = \frac{\sigma - 1}{\sigma}$ represents the elasticity of substitution in the CES function. Because $\sigma > 0$, it follows that $\rho < 1$. The closer ρ is to unity, the easier it is for the firm to substitute one input for another, and likewise, the closer ρ is to $-\infty$, the more difficult this substitution.

The CES production function is given by Eq. (2), where $0 < \alpha < 1$ is a distribution parameter between the two inputs, and $\Phi > 0$ is a scaling parameter. These parameters are assumed to be fixed for a given firm.

$$[f(z_1, z_2)]^\rho = \Phi \left[\alpha z_1^\rho + (1-\alpha) z_2^\rho\right] \tag{2}$$

Under equilibrium conditions, the firm uses the CES production function to solve the optimization problem in Eq. (1) and determine the optimal level of production, y^*, and optimal level of each input, z_1^* and z_2^* that maximizes its profit. CES production functions have been used for computable general equilibrium models, which are generally more flexible than standard I-O models because of their ability to include supply-side multipliers and price effects, in addition to demand-side effects (Shoven and Whalley 1992, West 1995, Rose and Guha 2004). Capital and labor are the two inputs used most frequently for production functions (Arrow et al. 1961, Brookshire and McKee 1992, West 1995) although models have included other factors of production such as energy (Rose and Guha 2004, Rose and

Liao 2005), land (Rhee 1991), seeds and fertilizers used by the agricultural industry (Keneda 1982), and investment in information technology (Dewan and Min 1997).

If a supply chain disruption occurs, we assume the supplier of input z_1 can only deliver at most $\bar{z}_1 < z_1^*$ to the firm. We assume the firm can buy more of input z_1 at a per-unit cost of $\tilde{w}_1 > w_1$ from a different supplier. If this supply shortage lasts indefinitely, the firm eventually chooses a new production level as determined by the optimization problem in Eq. (3) where $z_1 \leq \bar{z}_1$ is the amount the firm purchases from the original supplier and $\tilde{z}_1$ is the amount the firm purchase from the alternate and more expensive supplier.

$$\begin{array}{rl} \max & p(y)y - w_{z1}z_1 - \tilde{w}_1\tilde{z}_1 - w_2z_2 \\ \text{st} & y = f(z_1 + \tilde{z}_1, z_2) \\ & z_1 \leq \bar{z}_1 \\ & z_1, \tilde{z}_1, z_2 \geq 0 \end{array} \tag{3}$$

Because $w_1 < \tilde{w}_1$, the firm buys $\bar{z}_1$ at the price of w_1 before buying any of $\tilde{z}_1$ at the price of $\tilde{w}_1$.

The substitution parameter ρ, the price function $p(y)$, and the difference between w_1 and $\tilde{w}_1$ determine if the firm purchases more of z_2, buys z_1 at a more expensive price, or produces less in reaction to a supply shortage. In general, the closer ρ is to unity, the more of z_2 the firm purchases, and the closer ρ is to $-\infty$, the more likely the firm purchases $\tilde{z}_1$ or simply produces less.

Therefore, we have two different equilibrium points: (i) under normal operating circumstances as determined by Eq. (1) whose solution is (y^*, z_1^*, z_2^*) and (ii) with a supply shortage as given by Eq. (3) whose solution is $(\tilde{y}^*, \bar{z}_1^*, \tilde{z}_1^*, \tilde{z}_2^*)$. Because the total cost of the firm's inputs may increase when a supply shortage occurs, the most the firm will produce under the second equilibrium is at the level of production at the first equilibrium. Mathematically, $y^* \geq \tilde{y}^*$, and usually, the firm will produce less at the second equilibrium.

2.2 Dynamic analysis

The above analysis generates two equilibrium points, from Eqs. (1) and (3). In this subsection, we develop a model to describe how the firm's production moves from one point to another and whether the firm will ever reach the equilibrium under a supply shortage. We also detail how the firm benefits from having inventory of input z_1, which allows the firm to maintain production at the higher (baseline optimal) level of y^* for a longer period of time.

Dynamic economic analyses describe how production changes over time. This model describes production at four different points in time, assuming a supply disruption occurs at time 0. We let the time parameter τ_1 be the time at which the firm has depleted its inventory of z_1. At time τ_2, the original supplier begins to recover and the firm is able to buy more than $\bar{z}_1$, the reduced maximum supply of z_1 available at the per-unit cost of w_1. We assume that the firm is not able to produce immediately at the new equilibrium point given by $(\tilde{y}^*, \bar{z}_1^*, \tilde{z}_1^*, \tilde{z}_2^*)$, and it takes τ_3 amount of time for the firm to reach that new production level. Finally, τ_4 is the time at which the original supplier has fully recovered, and the firm can purchase all of input z_1 at w_1. Although these time parameters are indexed, no particular order of the events defined to take place from τ_1 to τ_4 is required, with the exception that $\tau_2 \leq \tau_4$.

2.2.1 No on-hand inventory maintained

We first examine the situation where the firm has no inventory, so that $\tau_1 = 0$. At time 0 when the supply disruption first occurs, the firm's production drops to $\bar{y} = f(\bar{z}_1, \bar{z}_2)$ where $\bar{z}_2$ is calculated so that ratio of inputs $\frac{\bar{z}_1}{\bar{z}_2}$ equals the ratio of inputs under normal circumstances $\frac{z_1^*}{z_2^*}$. The time it takes to reach the new equilibrium τ_3 reflects that an alternate supplier might not be immediately available or that the firm might need to adjust its production processes to substitute z_2 for z_1.

Fig. 1 shows different trajectories the firm could take to move from $\bar{y}$ to $\tilde{y}^*$. A linear path indicates that the firm's production level increases at a constant rate. A convex function indicates that the firm's production initially increases slowly and then increases very quickly as time approaches τ_3. A concave function indicates that the firm's production initially increases very quickly but then increases more slowly. The greater concavity of the trajectory of production, the less production that is lost by the firm. The trajectory does not need to be smooth, and the firm's production could remain at $\bar{y}$ until τ_3 at which point it immediately moves to $\tilde{y}^*$.

While the firm is transitioning to the new equilibrium point, the firm's original supplier is also recovering its production level. The original supplier begins to recover at time τ_2 and is fully recovered at time τ_4. The time from τ_2 to τ_4 depends on how quickly the supplier is able to recover, and the trajectory could be any type of path (i.e., linear,

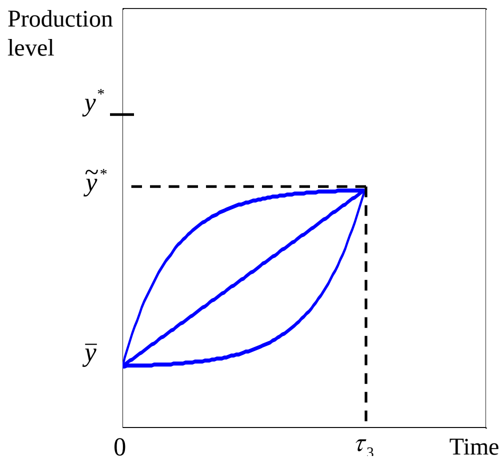

Figure 1: Possible paths to reach a new equilibrium level of production

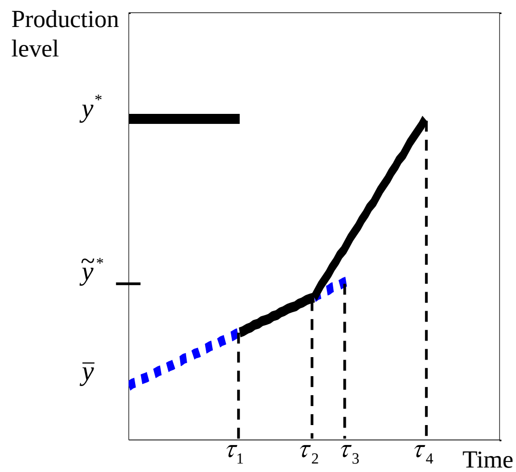

Figure 2: A firm's production level as described by the solid black line

convex, concave). If $\tau_2 < \tau_3$, the firm's production moves along the path toward $\tilde{y}^*$ until time τ_2 at which point it diverges from its current path and charts a new path toward its original equilibrium at y^* which is achieved at time τ_4. If $\tau_2 \geq \tau_3$, the firm's production reaches the new equilibrium at $\tilde{y}^*$ and remains at that level until time τ_2. At time τ_2, the production begins to transition toward the original equilibrium point which is again achieved at time τ_4. As with the path from $\bar{y}$ to $\tilde{y}^*$, the path back toward original equilibrium could take several different shapes (e.g., linear, convex, concave).

2.2.2 On-hand inventory maintained

If the firm maintains inventory, $\tau_1 > 0$. We assume that the firm is able to produce at y^* as long as it has inventory. If $\tau_1 \geq \tau_4$, the firm has enough inventory to cover the length of the supply shortage and the firm always produces at y^*. If $\tau_3 \leq \tau_1 \leq \tau_2 \leq \tau_4$, the firm has enough inventory to cover the transition to the new equilibrium, and it will produce at y^* until time τ_1 at which point its production will immediately drop to $\tilde{y}^*$. At time τ_2, the production will begin to shift back toward y^*. If $\tau_1 \leq \tau_3$, the firm's production will remain at y^* until time τ_1 at which point it will drop to the production level of the firm at τ_1 if it did not have any inventory. Fig. 2 demonstrates an example of this last instance.

In brief, three separate time periods comprise this dynamic analysis. The length of the first period is determined by the firm's inventory, during which time the firm maintains its production at y^*. The second part occurs as the firm's production transitions toward the new equilibrium, and the production level is determined by the path connecting $\bar{y}$ and $\tilde{y}^*$. The third part occurs from time τ_2 to τ_4 as the original supplier recovers and the firm returns to its original production level at y^*. This final part is largely determined by the rate at which the supplier is able to recover its original production level.

3 Interdependent economic analysis

We integrate this dynamic production model, which focuses on the production outputs of a single firm, with a risk-based I-O model to understand the broader interdependent impacts of the firm's decisions. The Multiregional IIM enables each aspect of the firm's strategies for handling a supply shortage to be quantified within the context of a multiregional set of interconnected industry sectors.

3.1 Multiregional Inoperability Input-Output Model

The Multiregional IIM is derived from the basic Leontief (1936, 1951) I-O model. The Leontief I-O model in Eq. (4) describes the equilibrium relationship among n economic sectors in which $\mathbf{x}$ and $\mathbf{c}$, each a vector of length n, describe the production and the final demand for each sector, respectively. The $n \times n$ technical coefficient matrix $\mathbf{A}$ describes the economic interdependency in proportional terms between each sector. The Bureau of Economic Analysis (BEA 2010b) publishes I-O commodity flow data for the United States.

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{c} \Rightarrow \mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{c} \tag{4}$$

The Leontief model has been extended to a multiregional economy which contains m regions (Isard et al. 1998, Miller and Blair 2009). Let T_i^{rs} be the proportion of commodity i consumed by region s that originated from sector i in region r, and the $n \times n$ diagonal matrix $\mathbf{T}^{rs}$ describes the proportion of each commodity consumed by region s that originates in region r. The multiregional I-O model is given by Eq. (5) where $\mathbf{x}^r$ and $\mathbf{c}^r$ describe the production and final demand for each sector in a single region r, and $r = \{1, 2, \ldots, m\}$. $\mathbf{A}^r$ is the technical coefficient matrix for region r, which can be derived from national I-O accounts using regional multipliers (BEA 1997) or estimated with location quotients (Miller and Blair 2009).

$$\begin{pmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^m \end{pmatrix} = \begin{pmatrix} \mathbf{T}^{11} & \mathbf{T}^{21} & \cdots & \mathbf{T}^{m1} \\ \mathbf{T}^{12} & \mathbf{T}^{22} & \cdots & \mathbf{T}^{m2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{T}^{1m} & \mathbf{T}^{2m} & \cdots & \mathbf{T}^{mm} \end{pmatrix} \begin{pmatrix} \mathbf{A}^1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}^m \end{pmatrix} \begin{pmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^m \end{pmatrix} + \begin{pmatrix} \mathbf{T}^{11} & \mathbf{T}^{21} & \cdots & \mathbf{T}^{m1} \\ \mathbf{T}^{12} & \mathbf{T}^{22} & \cdots & \mathbf{T}^{m2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{T}^{1m} & \mathbf{T}^{2m} & \cdots & \mathbf{T}^{mm} \end{pmatrix} \begin{pmatrix} \mathbf{c}^1 \\ \mathbf{c}^2 \\ \vdots \\ \mathbf{c}^m \end{pmatrix} \tag{5}$$

The Commodity Flow Survey (Bureau of Transportation Statistics 2009) can be used to derive each $\mathbf{T}^{rs}$ matrix.

The IIM expands on the Leontief model by describing how a disruption in one or more economic sectors propagates to other sectors (Santos and Haimes 2004, Santos 2006). The vector $\mathbf{q}$ of length n represents the inoperability of each sector, the elements of which measure the extent to which a sector is (in proportion form) not productive. The vector $\mathbf{c}^*$ describes the reduction in final demand generated by the disruption. Each element in the interdependency matrix $\mathbf{A}^* = [\text{diag}(\mathbf{x})]^{-1}\mathbf{A}[\text{diag}(\mathbf{x})]$ describes the proportion of additional inoperability experienced by one sector due to inoperability in another sector. Eq. (6) presents the IIM under equilibrium conditions.

$$\mathbf{q} = \mathbf{A}^*\mathbf{x} + \mathbf{c}^* \Rightarrow \mathbf{q} = [\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* \tag{6}$$

If an economic sector's production is degraded due to a disruptive event and/or reduced demand, that sector will require fewer inputs from other sectors. Consequently, the other sectors also experience inoperability or lost production due to a reduction in the intermediate demand for their products and services.

The Multiregional IIM in Eq. (7) describes the lost production among different sectors and different regions where $\mathbf{q}^r$ and $\mathbf{c}^{*r}$ are the inoperability and perturbed demand for each sector in region r, $\mathbf{A}^{*r}$ is region r's inoperability interdependency matrix, and $\mathbf{T}^{*rs} = [\text{diag}(\mathbf{x}^r)]^{-1}\mathbf{T}^{rs}[\text{diag}(\mathbf{x}^s)]$ (Crowther and Haimes 2010).

$$\begin{pmatrix} \mathbf{q}^1 \\ \mathbf{q}^2 \\ \vdots \\ \mathbf{q}^p \end{pmatrix} = \begin{pmatrix} \mathbf{T}^{*11} & \mathbf{T}^{*21} & \cdots & \mathbf{T}^{*m1} \\ \mathbf{T}^{*12} & \mathbf{T}^{*22} & \cdots & \mathbf{T}^{*m2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{T}^{*1m} & \mathbf{T}^{*2m} & \cdots & \mathbf{T}^{*mm} \end{pmatrix} \begin{pmatrix} \mathbf{A}^{*1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{*2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}^{*m} \end{pmatrix} \begin{pmatrix} \mathbf{q}^1 \\ \mathbf{q}^2 \\ \vdots \\ \mathbf{q}^m \end{pmatrix} + \begin{pmatrix} \mathbf{T}^{*11} & \mathbf{T}^{*21} & \cdots & \mathbf{T}^{*m1} \\ \mathbf{T}^{*12} & \mathbf{T}^{*22} & \cdots & \mathbf{T}^{*m2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{T}^{*1m} & \mathbf{T}^{*2m} & \cdots & \mathbf{T}^{*mm} \end{pmatrix} \begin{pmatrix} \mathbf{c}^{*1} \\ \mathbf{c}^{*2} \\ \vdots \\ \mathbf{c}^{*m} \end{pmatrix} \tag{7}$$

The expression $\mathbf{Q}^r = [\mathbf{q}^r]^\mathsf{T}\mathbf{x}^r$ calculates the total production loss in region r. We have previously deployed the Multiregional IIM to analyze the interdependent impacts of inland waterway port disruptions (Pant et al. 2011, MacKenzie et al. 2011).

3.2 Multiregional inoperability due to a supply shortage

The Multiregional IIM from Eq. (7) and the firm's production level described in Section 2 can be combined to calculate production losses in other sectors not directly impacted by a supply shortage. We assume that the firm that experiences a supply shortage reduces its demand for other inputs according to the proportions governed by the Multiregional IIM. We also assume that the supplier is outside of the regions whose economies are being analyzed, and the supplier's specific inoperability is not included in the Multiregional IIM. If a firm in sector i experiences a supply shortage and its production levels follow the model of Section 2, sector i's total inoperability q_i can be calculated from time 0 to the resumption of full production at time τ_4. This calculation is given by Eq. (8) where $y^*_{[0,\tau_4]}$ is the firm's total production from 0 to τ_4 with no supply shortage, $y_{[0,\tau_4]}$ is the firm's total production during this time period with a supply shortage as described in Section 2, and $x_{i,[0,\tau_4]}$ is sector i's total production with no supply shortage over this time period.

$$q_i = \frac{y^*_{[0,\tau_4]} - y_{[0,\tau_4]}}{x_{i,[0,\tau_4]}} \tag{8}$$

A firm's size relative to the size of sector i is given by $y^*_{[0,\tau_4]} \Big/ x_{i,[0,\tau_4]}$.

If only a single firm in the m regions experiences a supply shortage, the direct inoperability due to the supply disruption is given by the $nm \times 1$ vector $\mathbf{q}_{nm}(0) = [0,\ldots,0,q_i,0,\ldots,0]^{\mathsf{T}}$. Because of the inoperability of the firm, the firm's immediate suppliers will experience a decline in the intermediate demand for their products and services normally purchased by the firm. The resulting inoperability, $\mathbf{q}_{nm}(1)$, in those suppliers due to the firm reducing its demand is given by Eq. (9). (Although the model remains a multiregional model, we replace the large matrices encompassing multiple regions in Eq. (7) with $\mathbf{T}^*_{nm\times nm}$ and $\mathbf{A}^*_{nm\times nm}$ for notational simplicity.)

$$\mathbf{q}_{nm}(1) = \mathbf{T}^*_{nm\times nm}\mathbf{A}^*_{nm\times nm}\mathbf{q}_{nm}(0) \tag{9}$$

Although a portion of the final demand may ultimately not be filled, the supply shortage does not directly perturb the final demand, resulting in $\mathbf{c}^*_{nm} = 0$.

The first echelon or tier of suppliers consequently also purchases less from its suppliers. This effect continues down the entire supply chain, and Eq. (10) generates the total inoperability $\mathbf{q}_{\text{total}}$ due to a supply shortage in a single firm.

$$\mathbf{q}_{\text{total}} = \sum_{k=0}^{\infty} \left(\mathbf{T}^*_{nm\times nm}\mathbf{A}^*_{nm\times nm}\right)^k \mathbf{q}_{nm}(0) = \left(\mathbf{I} - \mathbf{T}^*_{nm\times nm}\mathbf{A}^*_{nm\times nm}\right)^{-1} \mathbf{q}_{nm}(0) \tag{10}$$

If the reduced demand due to a supply shortage in the firm only reaches down a few echelons, the upper limit on the summation could be any integer to reflect the number of impacted echelons in the supply chain. However, because each element of the matrix product $\mathbf{T}^*_{nm\times nm}\mathbf{A}^*_{nm\times nm}$ is less than one, the inoperability of echelons for $k > 4$ is usually close to zero.

4 Case study

4.1 Problem parameters

The case study explores a supply shortage that initially impacts a single firm operating in the state of Texas, and we analyze the inoperability and production losses that occur in Texas and the neighboring states of Oklahoma, Arkansas, and Louisiana. The supply shortage occurs in the computer and electronic products, which is categorized under the North American Industry Classification System (NAICS) as industry code 334. The purchasing firm can be in one of seven economic sectors: agriculture, mining, utilities, construction, manufacturing, wholesale trade, and retail trade. The firm can potentially substitute electrical and component equipment (NAICS 335) when the supply of computer and electronic products is constrained. From the Commodity Flow Survey (Bureau of Transportation Statistics 2009), we estimate that computer and electronic products cost 2.11 times more than electrical and component equipment. The parameters for the different firms' CES production functions are estimated using linear regression as described in Bosworth (1976) where the annual BEA (2010b) use tables from 1998 to 2009 serve as our data set.

Table 1 reveals that although none of the seven sectors are easily able to substitute electrical and component equipment for computer and electronic products, the construction and manufacturing sectors have the most difficulty. The optimal ratios between the two inputs only depends on the relative prices of the inputs, α, ρ, and Φ and do not depend on the total amount of quantity produced. Under normal operating circumstances, manufacturing, wholesale trade, and retail trade purchase the most computer and electronic products compared to electrical equipment.

Estimating a demand or price function, $p(y)$, for each firm presents more of a challenge. We first assume that each firm faces a linear demand function. The BEA's (2010a) price and quantity indices for each of the seven economic sectors are used to estimate the slope of the demand function, which indicates the degree to which the price of a commodity decreases as a firm increases its production. Finally, we calculate the intercept by assuming that the firm produces one percent of the total production in Texas for that economic sector.

Table 1: CES parameters for computer products and electrical equipment inputs

Sector	ρ	σ	α	Φ	Optimal input ratio of computer products to electrical equipment
Agriculture	-0.288	0.776	0.444	0.123	0.471
Mining	-0.430	0.699	0.519	0.063	0.626
Utilities	-0.504	0.665	0.544	0.056	0.684
Construction	-1.302	0.434	0.058	0.020	0.216
Manufacturing	-1.538	0.394	0.849	0.017	1.469
Wholesale trade	-0.766	0.566	0.859	0.038	1.739
Retail trade	-0.215	0.823	0.729	0.360	1.217

Table 2: Demand function parameters and firm's annual production

Sector	Demand intercept (millions of dollars)	Demand slope	Firm's annual production (millions of dollars)
Agriculture	6,100	-168.5	157
Mining	15,020	-146.3	1,354
Utilities	5,250	-25.8	366
Construction	26,500	-14.8	1,122
Manufacturing	23,450	-4.2	3,862
Wholesale trade	4,530	-3.4	1,016
Retail trade	5,060	-6.7	948

Table 2 shows that in this case study, agriculture and mining face demand functions that are most sensitive to price. If a firm in these sectors changes its production levels by a small amount, the price will change by a lot. Conversely, manufacturing, wholesale trade, and retail trade face relatively flat demand functions and the firms in these sectors experience much more stable selling prices.

Under the supply disruption imagined in this case study, a firm is only able to receive 90 percent of computer and electronics products as it normally requires. (The firm's supplier is located outside the four-state region, and we ignore any reduced demand that might occur because the supplier's production has dropped.) We assume the firm's supply remains constrained at 90 percent for 15 days ($\tau_2 = 15$) and then begins to increase at a constant (linear) rate until its supply is fully restored on the 30th day ($\tau_4 = 30$). It takes the firm 90 days from the start of the disruption to rearrange its production processes to substitute electrical and component equipment and reach the new equilibrium ($\tau_3 = 90$).

In order to quantify the effects of each of the strategies (inventory, alternate supplier, and substitution), we examine each of these strategies in isolation. The first scenario is the worst case, and no inventory, alternate supplier, or substitution is available to the firm. The firm can only operate at 90 percent until day 15 at which point it can begin to purchase more from the original supplier. The second scenario analyzes inventory. The firm's purchasing pattern remains the same as in the first scenario, but it also has 10 days of inventory ($\tau_1 = 10$) on hand when the disruption occurs. In the third scenario, the firm no longer has inventory but can begin substituting electrical and component equipment at a constant linear rate. The fourth scenario introduces the possibility of an alternate supplier. The firm can no longer substitute, but it can purchase more computer and electronics products from an alternate supplier at an increased cost of 5 percent. The fifth scenario has all three strategies available (inventory for 10 days, substitution, and alternate supplier at a 5 percent increased cost).

The total number of economic sectors is 15, and only one firm in the economy experiences a supply shortage. For each instance of the supply shortage, one sector will be directly impacted, and the other 14 sectors will be indirectly impacted. We repeat this procedure for each of the 7 sectors.

Table 3: Results of supply shortages for five scenarios (lost production is in thousands of dollars)

Sector	1. Worst case		2. Inventory		3. Substitution		4. Alternate supplier		5. All strategies	
	Firm's avg. inop.	Reg. lost prod.	Firm's avg. inop.	Reg. lost prod.	Firm's avg. inop.	Reg. lost prod.	Firm's avg. inop.	Reg. lost prod.	Firm's avg. inop.	Reg. lost prod.
Agriculture	0.0750	1,592	0.0417	884	0.0747	1,585	0.0750	1,592	0.0407	863
Mining	0.0750	11,485	0.0417	6,380	0.0742	11,365	0.0750	11,485	0.0391	5,987
Utilities	0.0750	3,213	0.0417	1,785	0.0745	3,191	0.0750	3,213	0.0400	1,712
Construction	0.0750	10,896	0.0417	6,053	0.0744	10,807	0.0750	10,896	0.0397	5,764
Manufacturing	0.0750	41,167	0.0417	22,871	0.0746	40,933	0.0731	40,113	0.0353	19,389
Wholesale trade	0.0750	7,927	0.0417	4,404	0.0742	7,841	0.0707	7,472	0.0275	2,907
Retail trade	0.0750	7,982	0.0417	4,435	0.0738	7,854	0.0717	7,628	0.0307	3,262

Table 4: Direct and indirect impacts of supply shortage (thousands of dollars)

Sector	1. Worst case		2. Inventory		3. Substitution		4. Alternate supplier		5. All strategies	
	Direct Impacts	Indirect Impacts	Dir. Imp.	Ind. Imp.	Dir. Imp.	Ind. Imp.	Dir. Imp.	Ind. Imp.	Dir. Imp.	Ind. Imp.
Agriculture	970	622	539	346	966	620	970	622	526	337
Mining	8,352	3,133	4,640	1,740	8,265	3,100	8,352	3,133	4,354	1,633
Utilities	2,258	955	1,254	531	2,242	948	2,258	955	1,203	509
Construction	6,915	3,981	3,842	2,212	6,859	3,948	6,915	3,981	3,658	2,106
Manufacturing	23,809	17,358	13,227	9,644	23,673	17,260	23,199	16,914	11,214	8,176
Wholesale trade	6,264	1,663	3,480	924	6,197	1,645	5,904	1,567	2,297	610
Retail trade	5,846	2,136	3,248	1,187	5,752	2,102	5,586	2,041	2,389	873

4.2 Results

For each scenario, we record the firm's average inoperability over the 30 days and the value of lost production in all four states across all sectors (Table 3). In this specific case study, having inventory for at least 10 days is the best strategy for both the firm and for the four-state economy regardless of which firm experiences a supply shortage. Whether substitution is preferable to buying from an alternate supplier depends on the economic sector. Firms in agriculture, mining, utilities, and construction do not purchase from the alternate supplier because even a five percent increase in the cost of computer and electronics products does not justify the lower per-unit price at which those firms would need to sell their product. Firms in manufacturing, wholesale trade, and retail trade do buy from the alternate supplier and prefer this strategy to substitution.

All firms are incentivized to substitute a little bit of electrical equipment for the computer and electronics but not to a large extent. The firm in retail trade is able to benefit the most from substitution due to an elasticity of substitution that is closer to zero and a flatter demand curve.

Implementing all three strategies provides the greatest benefit although the vast majority of this benefit is due to holding inventory. The production saved from combining all the strategies is greater than the sum of the production saved from each strategy. Here, the whole is greater than the sum of the parts. This shows how the overall production and interdependency model can begin to describe non-linear impacts despite the linear nature of I-O analysis.

The direct impacts, which is the lost production of the firm that experiences the supply shortage, exceed the indirect impacts, which are calculated by the Multiregional IIM (Table 4). This is due in part to not measuring the reduction in intermediate demand that propagates beyond the four states. Indirect impacts would even be less if the model had incorporated other firms increasing their production to replace the lost production of the firm suffering from a supply shortage. If we examine the proportion of indirect to direct losses, the indirect impacts are greatest for supply shortages in manufacturing and agricultural firms. This result indicates that these two sectors demand more production from the four states. Inoperability in firms in mining, wholesale trade, and retail trade have proportionally the smallest indirect impact.

Ninety-seven to ninety-nine percent of the production losses in this case study occur in Texas. Not only are the seven firms that suffer supply shortages all located in Texas, which means that more of their suppliers are located in Texas, but Texas is also a very big economy. Although more analysis is needed, these results suggest that a state should not be overly concerned with the indirect economic impacts from a supply shortage that occurs in a neighboring state.

5 Conclusions

This paper has integrated a firm-level production with the Multiregional IIM to quantify the regional economic impacts of a supply shortage. The CES production function creates a mechanism for a firm to trade off among ordering supplies at a higher cost, substituting a different type of product, and producing less. This production function determines two distinct equilibrium points, one without a supply shortage and the other when supply is constrained. The impacts of inventory are incorporated into the dynamic analysis and enables a firm to operate at the higher production level. The Multiregional IIM uses the firm's inoperability as calculated from the dynamic production model to quantify the impact on the firm's entire supply chain.

Although the case study results of a supply shortage impacting a firm in Texas are specific to the values of the model parameters, some general conclusions are warranted. A firm that faces a demand curve whose price is less sensitive to the quantity produced and who can more easily substitute among different inputs should decide to produce at a level close to its original production level if its supply is constrained. Unsurprisingly, production losses generally increase if firms who produce more suffer supply shortages. Because the commodity data that creates the multiregional I-O model suggest that most suppliers for any sector are located in the same state as the sector, most production losses are confined to the state whose firm experiences the initial supply shortage. If data were available to indicate the exact location of suppliers for a given firm, the results could indicate that the impacts are less concentrated in a single state.

The dynamic production model and Multiregional IIM presented in this paper demonstrate the importance of risk mitigation strategies for supply chain disruptions. We can use this modeling approach to compare the impact of different strategies on both an individual firm and the regional economy. Because inventory can play such a large role in preventing production losses, further analysis will explore a firm's decisions about maintaining inventory in advance of a supply chain disruption.

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