CONDITION BASED MAINTENANCE USING PROPORTIONAL HAZARDS MODEL

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A THESIS

IN

THE CONCORDIA INSTITUTE

FOR

INFORMATION SYSTEMS ENGINEERING

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF MASTER OF APPLIED SCIENCE (QUALITY SYSTEM

ENGINEERING) AT CONCORDIA UNIVERSITY

MONTRÉAL, QUÉBEC, CANADA

JULY 2009

Your file Votre référence
ISBN: 978-0-494-63089-1
Our file Notre référence
ISBN: 978-0-494-63089-1

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

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Entitled: **Condition Based Maintenance Using Proportional Hazards Model**

and submitted in partial fulfillment of the requirements for the degree of

Master of Applied Science in Quality System Engineering

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ABSTRACT

Condition Based Maintenance Using Proportional Hazards Model

Bai Rong Wu

Condition-based maintenance (CBM) is an advanced maintenance strategy in which maintenance actions are scheduled based on both the age data and condition monitoring information. Proportional Hazards Model (PHM) is a powerful statistical tool for estimating the equipment failure rate under condition monitoring. Effective CBM using PHM can decrease the overall maintenance costs by reducing unnecessary scheduled preventive maintenance actions.

In CBM using PHM, main optimization objectives including minimizing maintenance costs and maximizing equipment reliability typically conflict to each other. But the reported research only focuses on single-objective. In this thesis, we propose a multiple-objective CBM optimization approach based on physical programming, which can systematically balance the tradeoff between the optimization objectives and find the optimal solution that best represents the decision maker's preference on the objectives.

In CBM using PHM, the accuracy of parameter estimation greatly affects the accuracy of the model in representing and predicting the equipment health condition. Traditional optimization methods such as Newton's methods are inaccurate because they can only find local optimal value in parameter estimation. In this thesis, we develop an approach

based on Genetic Algorithms (GA) for PHM parameter estimation and this approach can improve the accuracy of parameter estimation significantly.

To illustrate the proposed approaches, we conduct two case studies using real-world vibration monitoring data, shearing pump bearings in a food processing plant and Gould pump bearings at Canadian Kraft Mill. The proposed approaches contribute to the general knowledge of condition based maintenance, and have the potential to greatly benefit various industries.

Acknowledgments

I would like to express my appreciation to many people who provide help for this thesis.

First of all, I would like to express my deepest gratitude to my supervisor, Dr. Zhigang (Will) Tian. His warm encourage has brought me confidence in thesis research. He gives me systemic and precise guidance in research and has always been very patient. In addition, his enthusiasm in academic research has also inspired me for further exploration.

I wish to thank all the faculty and people at Concordia Institute for Information Systems Engineering, especially the professors who give fantastic courses. I have obtained abundant knowledge from these courses and benefited a great deal for my research.

I am also very grateful to the Center for Maintenance Optimization and Reliability Engineering in Department of Mechanical and Industrial Engineering at the University of Toronto. In this thesis, two pump bearing cases are used to demonstrate the proposed approaches and the key data sets are provided by this Center.

I also want to thank my family including my parents, my husband and my little daughter for encouraging and supporting my study and research at all time.

Table of Contents

List of Figures

List of Tables

Acronyms

CBM	Condition Based Maintenance
PHM	Proportional Hazards Model
PP	Physical Programming
GA	Genetic Algorithms
MLM	Maximum Likelihood Method
PPM	Parts per Million
PIM	Proportional Intensity Model
AHM	Additive Hazards Model
ALT	Accelerated Life Testing
EHR	Extended Hazard Regression
ELHR	Extended Linear Hazard Regression
PHL	Proportional Hazard Linear
MDP	Mixture of Dirichlet Processes
MOS	Metal-Oxide-Semiconductor
PM	Preventive Maintenance

CO	Corrective Operation
DM	Decision Maker
OVO	One vs. Others
PCA	Preliminary Correlation Analysis
OMDEC	Optimal Maintenance Decision Inc.
C-MORE	Center for Maintenance Optimization and Reliability Engineering

Chapter 1

Introduction

1.1. Introduction to Condition Based Maintenance

With the rapid growth of modern technology, maintenance plays a more and more important role in many industries. In some industries such as aerospace industry and energy industry, reliability and maintenance are one of the most critical issues since a tiny failure may result in inestimable loss even fatal disaster. In recent decades, people pay more attention to research in maintenance and reliability. Maintenance is defined as "all activities aimed at keeping an item in, or restoring it to, the physical state considered necessary for the fulfillment of its production function." (Jardine & Tsang, 2006). Traditional maintenance technique is basically breakdown maintenance, also called corrective maintenance, reactive maintenance and unplanned maintenance. It is limited to repair actions or item replacement caused by failures. The predominant characteristic of early maintenance is reactive since it only reacts to faults or failures.

A more recent maintenance technique is time-based preventive maintenance (also called planned maintenance). It is proactive maintenance, which sets schedules to inspect or perform preventive maintenance instead of just reacting to failures. One time-based preventive maintenance method is constant-interval based preventive replacement method, in which failure replacements are performed immediately after failures occur and preventive replacements are performed at constant intervals, say every 6 months. The optimization problem is to find the optimal preventive replacement interval to minimize the total expected replacement cost in the long run. Another time-based preventive maintenance method is the age-based replacement method, in which preventive replacements are performed when the component reaches a pre-specified age, and the optimization problem is to find the optimal preventive replacement age. The time-based maintenance technique is an improvement compared to early maintenance techniques, but at the same time makes the cost of preventive maintenance higher and higher. Eventually, preventive maintenance cost has become a heavy financial burden of many industrial companies. Therefore, more effective maintenance approaches such as condition based maintenance (CBM) are being adopted to solve the problem of high preventive maintenance cost, and to prevent unexpected failures at the same time.

CBM is a maintenance process which decides maintenance actions using the information collected through condition monitoring. It is based on the understanding that a piece of equipment goes through multiple degraded states before failure. The health conditions can be monitored and predicted, and optimal maintenance actions can be scheduled for preventing equipment breakdown and minimizing total operation and maintenance costs. (Tian et al., 2009) CBM attempts to avoid unnecessary maintenance tasks by taking maintenance actions only when there is evidence that the failure is approaching.

CBM has been widely used in many fields, such as aerospace industry, mining industry, petroleum industry, and power generation industry. CBM may use condition monitoring data collected from oil analysis, vibration analysis, fuel consumption, environmental conditions, and so on, to make maintenance decision. Oil analysis is the spectrometric analysis of metal particles in oil samples generally gathered from an engine's or transmission's lubricating oil, while vibration analysis means the spectral analysis of vibration signals collected at certain positions on rotating equipments, etc.

There are three key steps in CBM process: data acquisition, data processing and maintenance decision-making step, as shown in Figure 1. Data acquisition step is to collect the data related to system health. Data processing step is to process and analyze the acquired data. In maintenance decision-making step, effective maintenance policies will be obtained based on the analyzed information (Jardine et al., 2006).

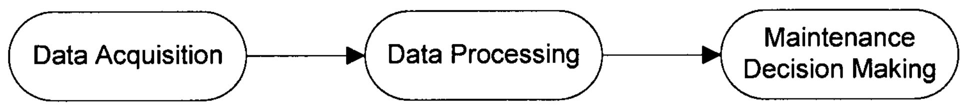


Figure 1 CBM process steps

A CBM program consists of two main categories of maintenance techniques: diagnostics and prognostics. Diagnostics focus on faults detection, isolation and identification when they occur, while prognostics attempts to predict faults or failures before they occur. Diagnostics is posterior event analysis and prognostics is prior event analysis. Prognostics is apparently more effective than diagnostics since prognostics endeavors to prevent faults or failures, or at least has prepared spare parts and planned human resources ready for the problems, and thus avoids additional unplanned maintenance cost. Nevertheless, diagnostics cannot be neglected for the reason that prognostics is impossible to be 100% sure to predict faults and failures. Besides, diagnostic can help improve prognostics in the way that diagnostic information can be useful for preparing more accurate event data and hence building better CBM model for prognostics. In addition, diagnostic information can be used as valuable feedback information for system redesign. A CBM program can be used to do both diagnostics and prognostics, or either one of them. And the above three CBM steps should be followed regardless of what the objective of a CBM program is.

A CBM optimization approach using proportional hazards model (PHM) has been developed, and has also been developed into the CBM optimization software EXAKT (Banjevic et al., 2001). EXAKT has been successfully implemented in many industries, including mining industry, food processing industry, utility industry, manufacturing industry, and so on. The main idea of CBM optimization approach using PHM is to determine an optimal replacement policy for minimizing long-run replacement cost. In this approach, the maintenance cost is calculated based on PHM and a risk threshold control limit policy. PHM is a valuable statistical procedure to estimate the risk of failure of a component or equipment when it is under condition monitoring. CBM using PHM can significantly decrease maintenance cost by reducing the number of unnecessary scheduled preventive maintenance operations. The basics of PHM will be summarized in Chapter 2 and the principles of CBM optimization approach using PHM will be discussed in Chapter 3 and Chapter 5.

1.2. Research Motivation

In CBM optimization, major objectives such as maximizing reliability and minimizing maintenance costs are often conflicting to each other. The previous research can only deal with single optimization objective. Sometimes minimizing cost is the only optimization objective while in the other times maximizing reliability, or minimizing failure probability, is the only optimization objective. When maintenance cost is set as the optimization objective, reliability may be used as a constraint. However, the disadvantage of single-objective optimization is that we cannot systematically investigate the tradeoff between the optimization objectives and find the optimal solution that best represents the preference of decision maker (DM) on the optimization objectives. To handle this situation, we propose a multi-objective CBM optimization approach based on the physical programming method. Thus, all the critical objectives can be systematically balanced and the global optimal maintenance policy can be determined.

In CBM using PHM, fitting the PHM is a vital step and the effectiveness of the optimal maintenance policy greatly depends on the accuracy of parameter estimation. Traditional optimization methods, such as the BFGS Quasi-Newton method, are currently used to perform optimization in parameter estimation using maximum likelihood method (MLM). These methods are also used in the commercial software EXAKT which are widely used in many industries. Nevertheless, traditional optimization methods have an evident limitation, that is, only local optimization value can be found using these methods. In our research we discover genetic algorithm (GA) is a very powerful optimization approach with two key advantages. (1) Global optimization ability. GA has been recognized as one of the most effective approaches in searching for the global optimal solution. (2) Flexibility in modeling the problem. GA has no strict mathematical requirements, such as derivative requirement, on the objective functions and constraints. The only requirement is that the objective functions and constraints can be evaluated in some way. (Tian & Zuo, 2006) In this thesis, we apply GA to solve the optimal problem in parameter estimation using maximum likelihood methods thus to improve the accuracy of parameter estimation. With accurate PHM parameters, we will be able to build an accurate model representing the relationship between the failure rate and the age and condition monitoring measurements, based on the event and inspection data we have collected. Thus, we can accurately evaluate the costs and reliability corresponding to a certain CBM policy, and find the optimal policy through optimization.

1.3. Research Contributions

In this thesis, we concentrate on the study of CBM optimization using PHM. The contributions of this thesis are summarized as follows.

- We propose a multi-objective CBM optimization approach based on physical programming, a multi-objective optimization method which has been demonstrated to be very effective in various fields. The physical programming method is an effective approach to capture the decision makers' preferences on the objectives by eliminating the iterative process of adjusting the weights of the objectives, and it is easy to use since decision makers just need to specify physically meaningful boundaries for the objectives. Using the proposed approach, the multi optimization objectives involved in CBM optimization, such as minimizing maintenance cost and maximizing reliability, can be systematically balanced and the optimal solution can be achieved.

- We develop an approach based on genetic algorithms for PHM parameter estimation. While the existing parameter estimation method reported in the literature can only achieve local optimal values, the proposed GA approach has much better global optimization capability. Our results show that the proposed GA approach can improve the accuracy of parameter estimation significantly. Accurate cost and reliability assessment of the CBM policy can be achieved because of the use of more accurately estimated PHM parameters, which is significant for obtaining the optimal CBM policy and for budget allocation.

- Case studies are conducted using real-world vibration monitoring data, which is collected from shearing pump bearings in a food processing plant and from bearings on a group of Gould pumps at a Canadian Kraft pulp mill company. These case studies have demonstrated the effectiveness of the proposed approaches.

1.4. Thesis Organization

The rest of this thesis is organized as follows:

- In Chapter 2, we conduct a detailed literature review on the PHM, and give a brief introduction to some advanced PHM.

- In Chapter 3, we introduce the basic principles of PHM based CBM optimization approach including PHM construction, assumptions and implementing procedures of the approach.

- In Chapter 4, we investigate the utilization of the physical programming approach to transform a PHM based CBM multi-objective optimization problem into single-objective problem, thus we can systematically balance the objectives and determine the optimal policy. We also conducted a real world case study to illustrate the approach.

- In Chapter 5, we explore the approach of applying of GA to improve the accuracy of parameter estimation using maximum likelihood method. A real world case study is also given to illustrate the proposed approach.

- In Chapter 6, we conduct another real world experiment to further demonstrate that GA can actually improve parameter estimation significantly.

- Finally, in Chapter 7, we draw a conclusion from our research and present several directions of future work.

Chapter 2

Literature Review on Proportional Hazards Model

Nomenclature

t : age of equipment

Z : $(z_1,...,z_k)$, vector of the covariates

γ_i : covariate coefficient for covariate i

$\lambda_0(\cdot)$: baseline hazard rate

Since the proportional hazards model (PHM) was introduced in 1972 by D. R. Cox, it has been utilized in many fields such as biomedicine (Xu & Gamst, 2007, Dahlberg & Wang, 2007, Kumar & Energi Ab, 1999), politics (Box-Steffensmeier & Zorn, 2001), transportation, crime and so on. It is especially widely used in the field of biomedicine and thousands of papers related to this topic can be found. But the research and application of applying PHM in maintenance field have not yet come to maturity. From 1990s, interest in applications of the PHM in this field has greatly increased and it has begun to be adopted in maintenance in diverse areas, such as aircraft engines, machine tools, power transmission cables, etc. These applications can be classified into two main categories: maintenance optimization and reliability analysis. Applications in maintenance optimization are using PHM to combine the age data with the condition monitoring data thus to determine the optimal maintenance policy. Maintenance optimization can improve system reliability and reduce overall maintenance costs. Applications in reliability analysis focus on applying PHM in the measurement and prediction of the reliability of components and systems considering different operation conditions. The components can be mechanical components, electronics components, software and other types.

The most important reason that the PHM is more effective than previous approaches is that it considers not only time data but also condition data which influence the health of the component or equipment. In maintenance optimization, PHM can effectively estimate the risk of failure of equipment under condition monitoring. For reliability engineering, the reliability data is collected under different conditions. For instance, in maintenance optimization, PHM takes into account the event data (failure data and suspension data) as well as inspection data (vibration data or oil analysis data such as the parts per million (PPM) of iron or lead found in oil). In reliability analysis, examples may be equipment being used by different operators or under different temperature and humidity. All the environment conditions may have influence on the reliability characteristics of the equipment and should be considered. These inspection data and environment conditions are called covariates and they cannot be ignored when we deal with the maintenance optimization and reliability analysis problems. In a word, the PHM takes into account the age data as well as the condition monitoring data; the effects of different covariates influencing the time to failure of a system can be estimated in this model. PHM has been found to be a valuable statistical procedure to estimate the risk of failure of equipment when it is under condition monitoring.

2.1. PHM basic Model and Extensional Models

a) The basic model of PHM

The basic model of PHM (Jardine & Tsang, 2006) combines a baseline hazard function with a component including all the covariates which influence the time to failure, as follows:

$$h(t, Z(t)) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{\sum_{i=1}^{m}\gamma_i z_i(t)} \tag{2-1}$$

where $h(t, Z(t)$ denotes the hazard value or failure rate and it means the conditional probability of failure at time *t*, given the values of $z_1(t), z_2(t), \ldots, z_m(t)$. The first part of this model is a baseline hazard function $\beta/\eta\,(t/\eta)^{\beta-1}$, which takes into account the age of the equipment at the inspection point of time, given the values of parameters β and η. The second part $\exp\left(\sum_{i=1}^{m}\gamma_i z_i(t)\right)$ takes into account the covariates, ($z_1(t), z_2(t), \ldots, z_m(t)$), which are the key factors influencing the health of equipment, and their associated weights, $\gamma_1, \gamma_2, \ldots, \gamma_m$.

b) Nonparametric models

The basic model of PHM which was just discussed is a parametric model. We have to identify their lifetime distribution before we can build the parametric models. But sometimes the lifetime data follows a complex lifetime distribution and cannot be easily indentified. In this situation, nonparametric models (Shyur et al., 1999, Prasad & Rao, 2002) appear to be a more effective approach, although parametric models are more effective than nonparametric models when lifetime data exactly follows a certain lifetime distribution. So the major advantage of nonparametric models is that they are totally distribution-free. The most general form of the nonparametric model used in the reliability field to analyze the lifetime data of components or equipments is given as follows (Shyur et al., 1999):

$$\lambda(t;z) = g(\beta \cdot z) \cdot \lambda_0(t) \tag{2-2}$$

and the notations are listed as follows:

$\lambda(t;z)$	: hazard rate at time t when the applied stress is z,
$\lambda_0(t)$	: baseline hazard rate at time t,
z	: covariate vector (applied stresses),
$g(\beta \cdot z)$	: relative risk function in the model.

c) Semi parametric models

Merrick et al. (2003) developed a semi parametric inference using a Mixture of Dirichlet Processes (MDP) approach, which is summarized as follows:

$$
\begin{aligned}
&(T_i \mid \theta_i, \beta, Z_i) \sim f(t_i \mid \theta_i, \beta, Z_i), \\
&(\theta_i \mid G) \sim G, \\
&(G) \sim DP(G_0, M), \\
&(\beta) \sim \pi(\beta).
\end{aligned}
\tag{2-3}
$$

where
$$f(t_i \mid \theta_i, \beta, Z_i) = \lambda_0(t_i;\theta_i) e^{\beta^T Z_i} e^{-\Lambda_0(t_i;\theta_i) e^{\beta^T Z_i}} \tag{2-4}$$

$$\Lambda_0(t_i;\theta_i) = \int_0^{t_i} \lambda_0(s;\theta_i) ds \tag{2-5}$$

The continuous function $\lambda_0(t;\theta_i)$ is the baseline failure rate where θ_i is the vector of unknown parameters specific to the i^{th} equipment. Uncertainty about the θ_i's is described by specifying a prior distribution G. If the form of G is known but the hyper parameters are unknown, this class of problems is referred to as the hierarchical Bayes problems. If the form of G is unknown, then uncertainty about G must be modeled. One way to model this uncertainty is to describe uncertainty about G by a Dirichlet process prior denoted by $G \sim DP(G_0, M)$, Where G_0 is the baseline prior and M is the strength of belief parameter. $\pi(\beta)$ denotes a parametric prior for the covariate effects β. Research work on semi parametric models can also be found in some other papers (Kobbacy et al., 1997, Kvam & Peña, 2005, Ishwaran & James, 2007, Horowitz & Lee, 2004, Cius & Nikulin, 2005, Bagdonavicius & Nikulin, 1997).

d) The proportional intensity model (PIM)

When Cox introduced PHM in 1972 he also mentioned proportional intensity model (PIM) (Lugtigheid et al., 2007, Lugtigheid et al., 2008). The difference between these two models is that the PHM is used to model the hazard function of lifetime, while the

PIM is used to model the intensity process of failures and repairs of a repairable system. Both PHM and PIM include illustrative variables. But the PHM can be deemed as a special case of the PIM because the PHM assumes that the system is renewed as "new one" after failure, while the PIM doesn't need to assume that. However, it should be noted that in many cases the term 'PHM' is used as a synonym of 'PIM'. After Cox's papers were published in 1972, the PHM was quickly and widely applied in the field of biomedicine, and in maintenance and reliability from 1980s, whereas the PIM only gained interest starting from early 1990s.

e) The additive hazards model (AHM)

Models with a hazard rate $h(t;z) = h_0(t) + g(z)$ is called additive hazards model (AHM) (Pijnenburg, 1991, Badı´A et al., 2002, Newby, 1994), where $h_0(t)$ is the baseline hazard function, and $g(z)$ is a function of explanatory variables z which does not need to be positive in order to have $h(t;z) > 0$. After considering all the restrictions such as the baseline hazard rate h_1 has been supposed to be identical in all intervals, the additive hazards model is defined as follows (Pijnenburg, 1991):

$$h(t;n(t),z_{n(t)}) = h_{1,s}(t - t_{n(t)}) + \beta_s^T z_{n(t)} \tag{2-6}$$

where $n(t)$ is the number of repairs up to time t, β_s is a vector of stratum-specific regression coefficients and $h_{1,s}$ is a stratum-specific baseline hazard function.

f) Accelerated life testing (ALT) model

Accelerated life testing (ALT) (Shyur et al., 1999, Elsayed et al., 2006, Elsayed & Zhang, 2007, Wang & Kececioglu, 2000, Zhao & Elsayed, 2005, Finkelstein, 2003, Mazzuchi et. Al, 1989) is a model built on the base of the PHM. It is used to obtain failure time data quickly under high stress levels in order to predict product life performance under design stress conditions. The assumption that covariates (applied stress) act multiplicatively on the failure time, or linearly on the log (failure time) is necessary for accelerated life testing models. Denote the failure time of a unit under a vector of covariates z by T, and the failure time under normal stress by T_0. The accelerated life testing models assume that $T_0 = e^{z'\beta}T$. Let $\lambda(t;z)$ be the hazard function at time t for z. Then the hazard function of the accelerated life testing models can be expressed in terms of a baseline hazard function $\lambda_0(\cdot)$ as follows (Elsayed et al., 2006):

$$\lambda(t;z) = \lambda_0(\exp(z'\beta)t)\exp(z'\beta) \tag{2-7}$$

The accelerated life testing models are equivalent to the class of linear models for $Y = \ln T = \ln T_0 - z'\beta$ with its error density function defined corresponding to what $\lambda_0(t_0)$ implies. Ordinary linear regression methods can be used to estimate β, but it is difficult to include censored data in these methods.

g) Extended hazard regression (EHR) model

The extended hazard regression (EHR) model is used as a general model and includes the PHM and accelerated life testing models as special cases. It can be applied to create an accelerated life testing model with different types of stress loading. It provides a full likelihood approach to the estimation of the PHM and accelerated life testing models while at the same time allowing for tests of the basic assumptions such as failure times or failure rates proportionalities. And extended hazard regression model can provide a broader framework for analysis. Generally speaking, extended hazard regression model cover a wide range of applications. For example, crossing survival curves are allowed in the accelerated life testing but not in the PHM.

The general expression of the extended hazard regression model is shown as follows (Shyur et al., 1999):

$$\lambda(t \mid Z) = g_1(\alpha^T Z)\lambda_0[g_2(\beta^T Z)t] \tag{2-8}$$

and the notations are listed as follows:

t	: inspection time
z_i	: applied stress (covariate) $i, j = 1, \ldots, k$
Z	: $(z_1, \ldots, z_k)$, vector of the covariates
$\lambda(t \mid Z)$	: hazard rate for a given Z
$\lambda_0(\cdot)$	: baseline hazard rate
$g_1(x), g_2(x)$	: positive functions, equal to 1 at $x = 0$
α_i	: regression coefficient $i, j = 1, \ldots, k$
β_i	: regression coefficient $i, j = 1, \ldots, k$
α, β	: vectors of regression coefficients

k : total number of applied stresses and their interactions

U_+ : $U_+ = U$ for $U > 0; U+ = 0$ for $U \leq= 0$

Elsayed et al. (2006) proposed a new model called the extended linear hazard regression (ELHR) model by generalizing the extended hazard regression (EHR) model and proportional hazards linear (PHL) model; here the PHL model expands the PHM in a way that considers a time by covariate interactions. The ELHR model function is described as (e.g., with one covariate)

$$\lambda(t;z) = \lambda_0(te^{(\beta_0+\beta_1 t)z}e^{(\alpha_0+\alpha_1 t)z}) \qquad (2\text{-}9)$$

h) Bivariate proportional hazards model

When the studied unit experiences more than one event or when there exists some natural grouping of subjects, the lifetime data is no longer univariate but multivariate. Multivariate failure time data is also referred to as correlated or clustered failure time data. Statistical analysis of such data needs to account for intracluster dependence. The following is a bivariate PHM using vector hazard rate. In this model the covariates under study have different effects on two components of the vector hazard rate function (Sankaran & Sreeja, 2007).

$$\lambda_i(t_i \mid t_j, \underline{z}) = \lambda_{0i}(t_i \mid t_j)e^{\underline{\beta}_i'(t_j)\underline{z}}, i, j = 1,2, i \neq j. \qquad (2\text{-}10)$$

where, $\underline{z}$ is a $p_i \times 1$ covariate vector, $\underline{\beta}_i(t_j)$ is the $p_i \times 1$ parameter vector, $\lambda_i(t_i \mid t_j, \underline{z})$ is the hazard function of the pair of lifetimes. $T = (T_1, T_2)$ given the covariate vector $\underline{z}$, and $\lambda_{0i}(t_i \mid t_j), i, j = 1,2, i \neq j$ is an unspecified baseline hazard function. When $\underline{\beta}_i(t_j)$ is a zero vector, the covariates has no effect on the hazard functions. Research on PHM with bivariate current status data can also be found in (Wang et al., 2008).

2.2. Applications of PHM

In 1980s research work focused on PHM application in maintenance optimization and reliability engineering appeared. From 1990s, interest in PHM applications in this field has greatly increased and it has begun to be adopted in industry in diverse areas, such as mining industry, automobile industry, power generation industry, semiconductor industry, papermaking industry, petroleum industry, aircraft engines industry (Jardine & M), construction industry (Metal, 2004), electronic components industry (Bendell et al., 1991), locomotive diesel engines industry (Jardine et al.) and many other industries. These applications can be classified into two main categories: condition based maintenance optimization and reliability analysis.

2.2.1. Applications in condition based maintenance optimization

Applications in maintenance optimization combine the age data with the condition monitoring data in the PHM. In these applications the effects of different covariates influencing the time to failure of the components are considered thus the optimal maintenance policy can be determined to minimize the maintenance cost. Research studies focused in PHM applications in maintenance optimization are summarized as follows:

Jardine et al. (2008) described the development of an optimal predictive maintenance program for critical pump bearings in the food processing industry. Measurements are taken in three directions for the bearings under investigation: axial, horizontal and vertical. In each of these directions, the velocity spectrum was obtained in five frequency bands. In addition, overall velocity and acceleration are also measured in the three directions. Therefore there were altogether 21 covariates in this PHM model. Significance analysis was taken to reduce the covariates and three covariates were found out to be necessary: VEL#1A (band 1 velocity in the axial direction), VEL#1V (band 1 velocity in the vertical direction), and VEL#2A (band 2 velocity in the axial direction). Assuming the inspection interval is 20 days, the transition probability matrices for the three covariates were estimated. Based on all this information, the optimal CBM replacement policy was determined. The results showed that, comparing to the failure replacement only policy, the optimal policy could achieve 84.5% of cost savings.

EXAKT (Makis & Jardine, 1992) is a commercial software widely used in industry for CBM decision making. It was developed by Optimal Maintenance Decision Inc. (OMDEC). Jardine et al. (2003) used the EXAKT software to build a condition based maintenance optimization model for the interpretation of inspection data from a nuclear reactor station. The data set included the information of 11-year period from 1990 onwards. In the nuclear reactors, hydro-dyne seals perform a vital function, and they can prevent the leakage of heavy water from the reactor. So site engineers would like to optimize the preventive seal replacement intervals in order to minimize the overall failure and maintenance costs. Therefore PHM based statistical decision methodology was applied to determine the optimal moment at which to perform proactive maintenance. In this case, two types of data were used to determine the optimal CBM policy: inspection data and events data. Inspection data is referred to the condition monitoring data (called covariates) which affect the health of each hydro-dyne seal along with the date of inspection and the corresponding working age of the seal. The event data comprises the dates and working ages at particular events, including beginning event (the installation of a new seal), failure event (the failure of a seal), and suspension event (the replacement of a seal that has not yet failed). A proportional hazards model was fitted to the data by the maximum likelihood method and the LeakRate was found out to be the only significant covariate. Finally the optimal replacing policy was determined and around 52.5% saving may be realized over the current replace-on-failure policy.

Lin et al. (2006) proposed the application of a principal components proportional hazards regression model in CBM optimization. They gave two examples to illustrate this application. The oil analysis data set of the first example was collected from transmissions on haul trucks in a mining company. After a series of analysis, the original 11 covariates: sodium (Na), potassium (K), iron (Fe), aluminum (Al), titanium (Ti), phosphorus (P), zinc (Zn), calcium (Ca), magnesium (Mg), molybdenum (Mo) and vanadium (V) were reduced to six significant covariates: Fe, Al, Ti, Mg, Mo and V. Three models (SW, PC_23 and PC_236) were built and compared. The final results suggested that the PHM PC_23 and the corresponding optimal replacement policy performed better than the other two models for the transmissions in this example. The other example was vibration analysis data set taken from a pulp and paper company. This data set contains event records and vibration measurements collected from water pumps every month. The pumps basically work 24 h per day, 7 days per week. Vibration signals were taken at seven different locations. For each vibration signal, the overall amplitude and the amplitude for six different frequency bands were recorded. So, altogether there are 49 covariates recorded. Preliminary correlation analysis was applied to eliminate the covariates and the 49 original covariates were transformed into 49 principal components. The final model included only one covariate, the fifth principal component (model PC_5). At the same time, a 'simple' Weibull model (model SW) without considering covariates was also built for comparative study purpose. The result of comparison of these two models indicated that the model PC_5 is a reasonably excellent model.

PHM was also utilized by Vlok et al. (2002) to determine the optimal replacement policy for a vital item which is subject to vibration monitoring. In their study they chose circulating pumps in a coal wash plant as the research case. The lifetime data was collected during a period of 2 years. Their study shows that, even with some problems in the collected data, vibration measurements can be used in proportional hazards modeling and that a useful decision policy can be obtained.

In the research by Rao & Prasad (2001), the PHM was used to analyze failure data and plan maintenance intervals for material handling equipments in mining industry, such as loaders, trucks, dozers, dumpers and etc. In this paper PHM was applied to model the repairable equipment whose performance is affected by concomitant variables. Graphical methods were used to calculate maintenance intervals.

Kobbacy et al. (1997) proposed a heuristic approach for implementing the PHM to schedule future preventive maintenance actions on the basis of the equipment's full condition history. An example based on data for four similar pumps used in four different plants was taken to illustrate their approach. This approach can be applied to repairable systems and does not require any restrictive assumption such as renewal regarding the quality of corrective work or planned maintenance. The main assumption in this approach is that lives of components following preventive maintenance or corrective operation depend on covariates values measured at points in time just before the maintenance work, and that lengths of these lives are conditionally independent. There were altogether 8 covariates: (a) age (age), (b) average preventive maintenance (PM) interval (*avintpm*), (c) total number of failures *(nofails)*, (d) total number of PMs *(pmno)*, (e) total down time of all PMs *(tdtpm)*, (f) total man hours of all PMs *(tmhpm),* (g) time since last corrective work *(tslco)*, (h) time since last PM *(tslpm)*. After detailed analysis, three covariates, *nofails*, *tslco* and *tslpm*, were selected for preventive maintenance and the model was built as: $\lambda_{pm} = \lambda_{pm,0}(t)\exp(0.034 nofails - 0.0026 tslco + 0.0028 tslpm)$; two covariates, *tmhpm* and *tslco*, were selected for corrective operation (CO) and the model was built as: $\lambda_{co} = \lambda_{co,0}(t)\exp(0.031 tmhpm - 0.0047 tslco)$. Their study results indicated a higher availability for the recommended schedule than the availability resulting from applying the optimal preventive maintenance intervals as suggested by using the conventional stationary models.

2.2.2. Applications in reliability analysis

Applications in reliability analysis are applying PHM in the measurement and prediction of the reliability of component or equipment by using covariates to describe different operating conditions. Research studies focused on PHM applications in reliability analysis are summarized as follows:

Elsayed & Chan (1990) used PHM to estimate thin oxide dielectric reliability and time-dependent dielectric breakdown hazard rates. These models are distribution free since no assumptions need to be made about the failure time distribution. However, there is a necessary assumption that the hazard rate functions for various devices when tested at various stress levels are proportional to one another. The need for proportionality can be relaxed by using time-dependent explanatory variables or stratified baseline hazard rates. In this approach, two groups of models are considered: group one ignores interactions between temperature and electric field while group two considers several forms of interaction.

Elsayed et al. (2006) applied extended linear hazard regression (ELHR) model to study the time-dependent dielectric breakdown of thermal oxides on n-type 6H-SiC using laboratory data. The ELHR model was extended from the extended hazard regression (EHR) model by generalizing the extended hazard regression model and proportional hazards linear (PHL) model; here the PHL model expands the PHM in a way that it considers covariate interactions. Their results suggest that the reliability of oxides on 6H-SiC will be satisfactory for long-term operation only if the oxide field is kept below 5 MV/cm at temperatures up to 150^0C. So their research effectively concluded SiC MOS (metal-oxide-semiconductor) devices from many high-temperature applications although SiC has a high inherent temperature capability.

Kumar et al. (1992) used PHM to examine the effects of two different designs and maintenance on the reliability of a power transmission cable of an electric mine loader. In this paper, 6 covariates were excluded out of 8 covariates and only the cable type (z_1) and the first repair (z_2) were found to have a significant effect on the hazard rate of the cable. The plotting of the estimated log-cumulative hazard rates showed that the hazard rate for the cable type B is less than the cable type A. Base on these results they suggested that cable type B be used so that unplanned interruption of production can be reduced.

The study of Prasad & Rao (2002) involved failure data of an electro-mechanical equipment in an underground coal mine. The failures due to electrical problems (z_1), compressed air (z_2) and cable fault (z_3) were found to be significant. The maximum likelihood method was used to estimate the parameters and a PHM was built with the data set. The results indicated that the failure rate due to electrical problems was 19% more than compressed air problems and 42% more than cable fault problems. Thus additional attention should be paid to reduce the failures due to electrical problems. In this paper, they gave another example of thermal power unit to study the reliability of repairable systems considering the effect of operating conditions. In this case, the failure time data was collected through a long period of four years, and the failures due to boiler (z_1), electrical (z_2) and turbine (z_3) were selected as significant covariates. A PHM model was built with the data set to optimize preventive maintenance interval in the thermal power unit.

Campean et al. (2001) presented a general PHM based methodology for automotive systems life prediction modeling. This approach aimed to establish a correlation among the degradation mechanism, the real world customer usage profile and the rig life testing. An example of development of a life model for the camshaft-timing belt was given to illustrate this approach. In this example, tooth shear fatigue mechanism led to the common cause failure mode and the covariates were found to be the tooth deflection and belt operating temperature. The contribution of building this timing-belt model is that it can directly establish a correlation between damage accumulation in real world conditions and belt life testing under laboratory conditions. Practically it can be used either as a life prediction tool for different usage profiles, or as a risk assessment tool in establishing the service interval.

In the paper of Eliashberg et al. (1997), PHM was utilized to calculate the reserve for a time and usage indexed automobile warranty. Purchased time and used mileage are selected as concomitant variables.

The PHM is also used in multi-sample reliability modeling. In the paper of Mudholkar & Sarkar (1999), the analysis of multi-sample data was illustrated using the bus motor failure. Multi-sample reliability data are often found in the monitoring of repair-reuse systems. The PHM based multi-sample reliability model follows distributions with unimodal and bathtub hazard functions, yields a broader class of monotone hazard rates, and can be analyzed and computed in a simple way. Generally, it can be used for proportional hazards modeling in comparative studies of lifetime data from several populations.

Gasmi et al. (2003) developed a PHM based statistical model of complex repairable systems. These systems are observed to operate in either loaded or unloaded mode. In most cases, a system is in loaded operation. But sometimes the system is placed in an unloaded status even though it is mechanically still running. It is assumed that the failure intensity of an unloaded operation is lower than loaded operation because the operating intensity is reduced in the unloaded mode. In their research, a PHM was used to capture this potential reduction in failure intensity due to switching of operating models. A case in the B. C. Hydro Power was used to illustrate this model. The data was collected from a specific turbine in this power station in a period of one year. Altogether 466 sojourns (the time between two actions) were recorded, of which 142 ended with failure (140 in loaded mode and 2 in unloaded mode). There were also 60 major repairs, 88 minor repairs and the remaining data were minimal repairs (the unit was stopped due to being taken off line and was restarted when needed). The purpose of building this model is to quantify the impacts of performing these repair actions on the failure intensities.

JoWiak (1992) developed an approach to utilize PHM in reliability analysis of microcomputer systems. In this approach, he examined the influence of two concomitant variables, temperature and mean daily user's exploitation time of the system, on system reliability and found that the PHM with Weibull baseline failure rate had considerable potential for estimating equipment failure rate in the presence of time-dependent and time-independent concomitant variables. He recommended that PHM should be used more frequently in this field of engineering reliability. The fully parametric PHM allows engineers to examine the relative influences of equipment age and covariates on equipment failure, including only those covariates which have a statistically significant effect on time to failure.

Ansell & Phillips (1997) used PHM to represent the repairable data from the hydrocarbon industry. The data set consisted of two parts: (1) failure data in a pipeline arising from a set of different causes; (2) information supplied on a daily basis on average temperature and the stress the system was under. Using the two covariates, stress and temperature, several models were built to fit the data set. Residuals based diagnostic techniques using PHM and graphical methods were used in this paper to interpret these repairable data.

2.3. Introduction to CBM Software EXAKT

EXAKT is a software package for CBM data pre-processing, proportional hazards modeling and maintenance decision making and is designed to address complex real world problems. EXAKT (Makis & Jardine, 1992) was developed by Optimal Maintenance Decision Inc. (OMDEC) (www.omdec.com) which is founded by the Centre for Maintenance Optimization and Reliability Engineering (C-MORE) in the Department of Mechanical and Industrial Engineering at the University of Toronto. The fundamental papers for the development of the first version of EXAKT were the papers by Makis and Jardine (1992) and Banjevic et al. (2001). This software uses available inspection and event data, such as recent oil or vibration data histories obtained from equipment condition monitoring, to build a Weibull PHM off-line, determines the optimal preventive replacement policy, and then processes the data obtained from an on-line condition monitoring system to make optimal maintenance decisions. EXAKT uses the basic model of PHM which is introduced earlier. EXAKT has been widely applied in many different industrial areas such as electrical utility industry, food process industry, mining industry, nuclear industry, defense sector, construction sector, and petrochemical industry. Its customers are found all over the world including ABB, BP Australia, DEI - Maryland, USA, SKF, Oceana Sensor Technologies, USA, Hydro One, Canada, Maritime Platforms Divisions, Australia, Cerrejon Coal, Columbia, Profertil, Argentina, PT Inco, Indonesia, Kobe Steel Co, Japan, etc.

Chapter 3

Principles of Condition Based Maintenance Using Proportional Hazards Model

Nomenclature

h: hazard value (failure rate)

t : equipment age

β, η: PHM parameters

γ_i: covariate coefficient for covariate i

$z_i(t)$: covariate i value at time t

PHM based CBM is a maintenance process which decides maintenance actions using the information collected through condition monitoring. The objective of PHM based CBM is to avoid unnecessary maintenance tasks by performing maintenance actions only when there is evidence that failure is approaching. By reducing the number of unnecessary scheduled preventive maintenance operations, the CBM optimization approach using PHM can significantly decrease maintenance cost if it is appropriately established and effectively implemented.

This chapter mainly discusses the basic principles of CBM optimization using PHM including PHM construction, assumptions and implementing procedures of the approach.

3.1. Methods

In CBM optimization process using PHM, Weibull distribution function PHM is used to model the data. PHM is a valuable statistical procedure to estimate the risk of failure of a component or equipment when it is under condition monitoring. The most important advantage of PHM is that it considers the age data as well as the condition monitoring data thus optimal maintenance actions can be effectively scheduled. The PHM function combines the baseline hazard function and the covariates together. The age of the equipment is the main variable while the condition information can be considered as a series of covariates.

$$h(t, Z(t)) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{\sum_{i=1}^{m}\gamma_i z_i(t)} \tag{3-1}$$

3.2. Assumptions

In the CBM approach using PHM, we consider a parametric PHM with Weibull baseline hazard function as the model for the hazard function. This model is also known as Weibull parametric regression model. There are two important assumptions for the CBM approach using PHM.

Assumption 1: The lifetime data of the items are independent and identically distributed and follow a Weibull distribution.

Assumption 2: An item is replaced with a new one either because of failure or of suspension, and the system is restored to a "new one" after replacement.

3.3. Procedures

There are six steps to perform CBM based on PHM, which are described as follows:

Step 1 Significance analysis

Identify the best set of covariates which are significantly influencing the hazard rate of the equipment.

Step 2 Parameter estimation.

Estimate the PHM parameters based on the inspection and event data using the maximum likelihood method.

Step 3 Transition probability matrix development.

Determine transition probability matrix based on the covariates values history for future covariates prediction.

Step 4 Cost data estimation.

Estimate the preventive cost and failure cost respectively based on the historical data.

Step 5 CBM optimization.

Perform optimization and determine the optimal risk threshold corresponding to the lowest cost per unit of time, and thus the optimal CBM policy.

Step 6 Deploy the CBM optimization policy.

Apply the optimal CBM policy in maintenance practice: perform preventive replacement when the risk $K \times h(t, z(t))$ at the given inspection point of time is greater than the optimal threshold value d^*; perform failure replacement if failure occurs between two inspection points of time.

Chapter 4

CBM Optimization Considering Multi-Objective

Nomenclature

$\bar{g}_i$: class function of design objective i

g_i : value of design objective i

$g_{i1}, \ldots, g_{i5}$: boundary values of preference ranges for design objective i

d : risk threshold value

$R(d)$: reliability function

$C(d)$: cost function

C : preventive cost

$C+K$: failure cost

CBM optimization objectives such as maximizing reliability and minimizing maintenance costs are often conflicting to each other. As mentioned in Chapter 1, the current method can only deal with single optimization objective. Either minimizing maintenance cost or maximizing reliability is set as the only optimization objective, while the other objective can only be used as a constraint. So it is difficult to systematically investigate the tradeoff between the optimization objectives and find the optimal solution that best represents the decision maker's preference on the optimization objectives.

In our research, we find out the application of the physical programming approach can deal with the multi-objective optimization problem. Physical programming is an effective multi-objective optimization approach developed in (Messac, 1996). It presents two major advantages (Tian et al., 2009): (1) it is an effective approach to capture the decision maker's preferences on the objectives by eliminating the iterative process of adjusting the weights of the objectives, and (2) it is easy to use in that decision makers just need to specify physically meaningful boundaries for the objectives.

In this chapter, we propose an approach based on physical programming to deal with the multi objectives involved in CBM optimization, that is, the cost objective and the reliability objective.

4.1. Multi-objective CBM Optimization Using Physical Programming

4.1.1. Review on physical programming

Physical programming (Messac, 1996) is a multi-objective optimization tool that explicitly incorporates the decision maker's preferences on each design metric into the optimization process. Within the physical programming procedure, the design metrics are classified into four classes: smaller is better (i.e., minimization), larger is better (i.e., maximization), center is better, and range is better. A class function is a function of a design objective. The value of a class function represents the preference of the designer on the objective function value, and the smaller the class function value is, the better. There are two types of class functions, soft class functions and hard class functions, as we can see in Figure 2 (Messac, 1996), the soft class functions are additive constituent components of the aggregate objective function (to be minimized) of the optimization model while the hard class functions only work as the constraints. For each type of class function, preference falls under four classes; three of which are shown in Figure 2.

(1) Class-1: Smaller-Is-Better (SIB),

(2) Class-2: Larger-Is-Better (LIB),

(3) Class-3: Center-Is-Better (CIB),

(4) Class-4: Range-Is-Better (RIB).

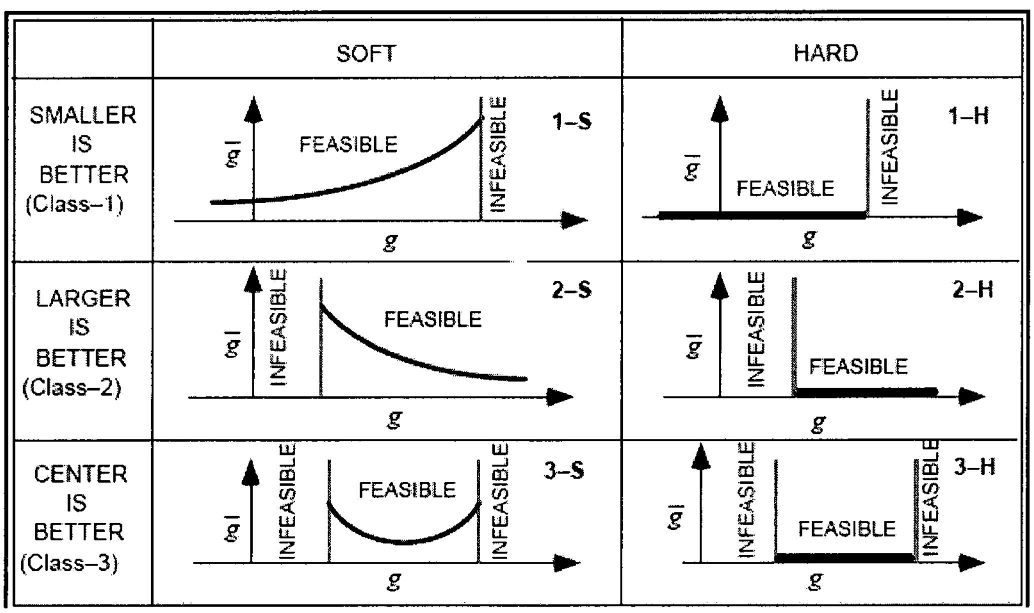
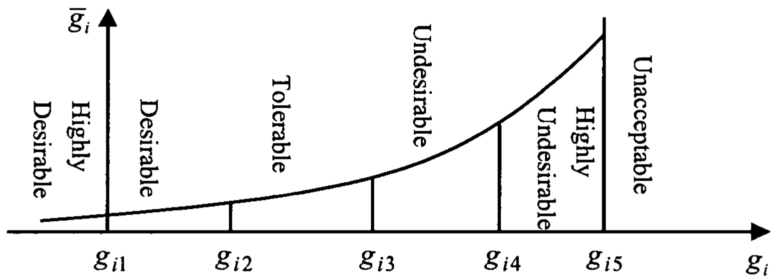


Figure 2 Class functions in physical programming

For Classes 1-H or 2-H, the designer must provide a single scalar value that defines the boundary of the feasible space; for Class 3-H two values are needed. For soft class functions, five values are required for Classes 1-S or 2-S while nine values are needed for Classes 3-S. Consider for example the case of class-1 soft class function (class 1-S), the qualitative meaning of the preference function is depicted in Figure 3. The value of the design metric, g_i, is on the horizontal axis, and the corresponding class function, $\bar{g}_i$, is on the vertical axis. A lower value of the preference function is better than a higher value thereof.



Class 1-S

Figure 3 Qualitative meaning of soft class function

Physical programming allows the decision maker to express ranges of different levels of preference with respect to each design metric with more flexibility and specificity than by simply declaring minimize, maximize or equal to. For Class 1-S, shown in Figure 3, the ranges are defined as follows.

Highly desirable range: $g_i \leq g_{i1}$,

Desirable range: $g_{i1} \leq g_i \leq g_{i2}$, Tolerable range: $g_{i2} \le g_i \le g_{i3}$,

Undesirable range: $g_{i3} \le g_i \le g_{i4}$,

Highly undesirable range: $g_{i4} \le g_i \le g_{i5}$,

Unacceptable range: $g_i \ge g_{i5}$.

The parameters g_{i1} through g_{i5} are physically meaningful constants associated with each design metric i. What the design maker needs to do in the physical programming framework is just to specify the values of the parameters g_{i1}, g_{i2}, g_{i3}, g_{i4}, and g_{i5} for each design metric i, and the class function can be completely determined by these parameters.

The range limits define the intra-criteria preference, while the "One vs. Others" criteria rule (OVO rule) describe the inter-criteria preference. Suppose there are two options: (1) full reduction for *one* criterion across a given preference range, say, the tolerable range; (2) full reduction for all the other criteria across the next better range, say, the desirable range. The OVO rule decides that option (1) is preferred over option (2). For example, assume that we have four criteria to be minimized, criterion 1 to 4. We say that the reduction of criterion 1 from the right boundary to the left boundary of the tolerable range is preferred over the reductions of criterion 2, 3, and 4 all from the right boundary to the left boundary of the desirable ranges. The OVO rule is built into the generated class function of each criterion.

4.1.2. Physical programming based multi-objective CBM optimization model

In physical programming based multi-objective optimization approach, there are two optimization objectives, cost and reliability. The cost objective class function is an increasing function, as shown in Figure 4. The lower the cost, the better it is. The values in the figure are just to qualitatively illustrate the cost class function. The reliability class function is a decreasing function of reliability value, as shown in Figure 5. The higher the reliability, the better it is.

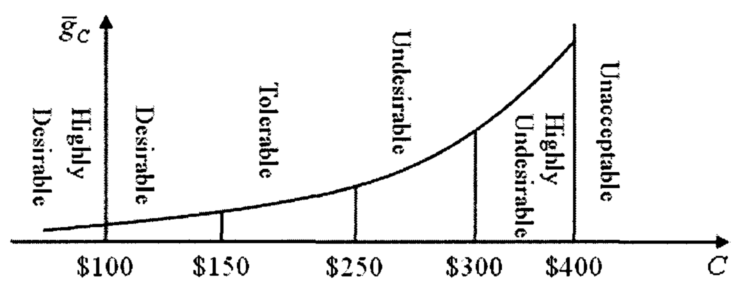


Figure 4 The cost objective class function

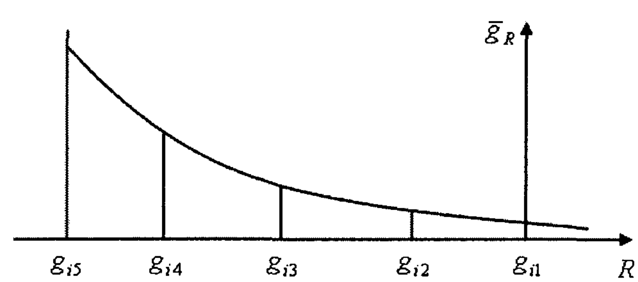


Figure 5 The reliability objective class function

The physical programming approach transforms a multi-objective optimization problem into a single-objective optimization model. The soft class functions of design objectives are combined into the aggregate objective function *f*, which is to be minimized; *d* is the risk threshold value and it is the design variable in the optimization model. The physical programming based optimization model for CBM optimization problem is given as:

$$
\begin{aligned}
&\min f(d) = \log_{10}\{\frac{1}{2}[\bar{g}_R(R(d)) + \bar{g}_c(C(d))]\} \\
&s.t. \\
&R \geq R_0, C \leq C_0 \\
&d > 0
\end{aligned}
\qquad (4\text{-}1)
$$

The cost and reliability values with respect to a given risk threshold value *d* can be calculated using the method developed by Banjevic et al. (2001). The objective function values are used to further calculate the corresponding class functions. The aggregate objective function can then be calculated and optimized to find the optimal risk threshold value *d**. Therefore multi-objective optimization problem can be formulated as a single-objective optimization problem.

4.2. Case Study

In this section, an example of condition based maintenance of shear pump bearings in a food processing plant will be used to illustrate the proposed approach, as shown in Figure 6 and Figure 7. Details of this case can be found in (Banjevic et al., 2001). The objective is to find an optimal condition based replacement policy to minimize the long-run expected replacement cost per unit of time, and to improve reliability, given the condition monitoring data (vibration data) and replacement histories.

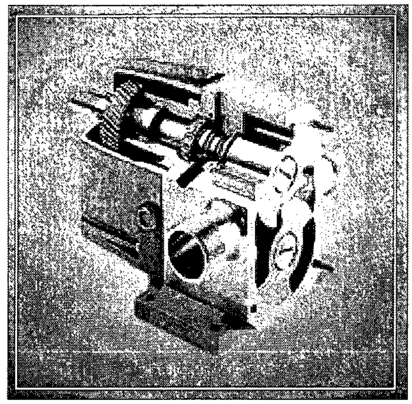


Figure 6 Shear pump

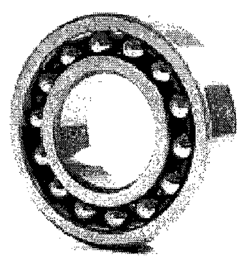


Figure 7 Bearing

In this case, totally 21 (=3+3*5+3) vibration measurements were collected using accelerometers, including vibration data in axial, horizontal and vertical directions for the overall velocity (3), velocities in 5 bands (3*5=15) and acceleration in three directions (3). There are 25 histories in the recorded data, including 13 failure replacements (ended with failure) and 12 preventive replacements (ended with suspension). Using the software EXAKT (Banjevic et al., 2001), the significance analysis was performed, and three significant covariates were identified: VEL#1A (band 1 velocity in the axial direction), VEL#1V (band 1 velocity in the vertical direction), and VEL#2A (band 2 velocity in the axial direction). The PHM parameters can thus be estimated using EXAKT, which are $\eta = 1584, \beta = 4.992, \gamma_1 = 5.831, \gamma_2 = 36.55, \gamma_3 = 24.05$, and the resulting hazard function is given as follows:

$$
\begin{aligned}
h(t, Z(t)) &= \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{(\gamma_1 z_{1A}(t)+\gamma_2 z_{2A}(t)+\gamma_3 z_{1V}(t))} \\
&= \frac{4.992}{1584}\left(\frac{t}{1584}\right)^{4.992-1} e^{(5.831 z_{1A}(t)+36.55 z_{2A}(t)+24.05 z_{1V}(t))}
\end{aligned} \tag{4-2}
$$

To calculate the cost and reliability measure, we need to specify the transition probability matrix and EXAKT can be used to estimate the transition probability matrices for the three covariates. The transition probability matrix indicates the probabilities of a covariate in different ranges at the next inspection time given its current range. Assuming the inspection interval is 20 days, the transition probability matrices for covariate VEL#1A, VEL#2A VEL#1V are shown in Table 1, Table 2 and Table 3 respectively. These matrices can be used to predict future covariate values. Take the transition probability matrix for VEL_1A as an example, if the covariate value falls in the first range (0 to 0.035266) at current inspection point, then at next inspection point of time the probability of covariate value falling in the same range is 0.765522.

Table 1 Transition probability matrix for covariate VEL#1A

VEL_1A	0 to 0.035266	0.035266 to 0.2519	0.2519 to 1.08821	1.08821 to 2.51648	Above 2.51648
0 to 0.035266	0.765522	0.214501	0.0187137	0.00123314	3.01141e-005
0.035266 to 0.2519	0.0419512	0.809202	0.134907	0.0134952	0.000445182
0.2519 to 1.08821	0.00436408	0.160862	0.683157	0.144277	0.00734044
1.08821 to 2.51648	0.000138356	0.00774194	0.0694142	0.838071	0.0846349
Above 2.51648	0	0	0	0	1

Table 2 Transition probability matrix for covariate VEL#2A

VEL_2A	0 to 0.018036	0.018036 to 0.047428	0.047428 to 0.11356	0.11356 to 0.394788	Above 0.394788
0 to 0.018036	0.579321	0.371903	0.0459901	0.00269327	9.27563e-005
0.018036 to 0.047428	0.0852781	0.731248	0.168793	0.0140551	0.000625567
0.047428 to 0.11356	0.0114118	0.182657	0.691931	0.10703	0.00696988
0.11356 to 0.394788	0.0023802	0.0541698	0.381196	0.499451	0.0628029
Above 0.394788	0.000559654	0.0164605	0.169477	0.428769	0.384734

Table 3 Transition probability matrix for covariate VEL#1V

VEL_1V	0 to 0.027284	0.027284 to 0.08616	0.08616 to 0.213246	0.213246 to 0.353974	Above 0.353974
0 to 0.027284	0.718524	0.241787	0.0374241	0.00223751	2.65316e-005
0.027284 to 0.08616	0.11716	0.662394	0.202472	0.0177508	0.000223795
0.08616 to 0.213246	0.0185641	0.207274	0.665316	0.107402	0.00144433
0.213246 to 0.353974	0.00428184	0.0701032	0.414337	0.503996	0.0072825
Above 0.353974	0.00365418	0.0636111	0.401023	0.524133	0.00757917

After the transition probability matrices are obtained, we need to estimate the preventive replacement cost and failure replacement cost. Based on the expertise and previous experience the preventive replacement cost (*C*) is estimated to be $1,800, and the failure replacement cost (*C*+*K*) is $16,200, so *K* can be calculated to be $14,400.

To use physical programming, we need to specify the boundary values for each objective to indicate the preferences on the objectives. Suppose the specified boundary values for cost and reliability are given as follows:

Cost: $$[g_{C1}, g_{C2}, g_{C3}, g_{C4}, g_{C5},] = [8,10,12,15,20] \tag{4-3}$$

Reliability: $$[g_{R1}, g_{R2}, g_{R3}, g_{R4}, g_{R5},] = [0.99,0.98,0.95,0.90,0.80] \tag{4-4}$$

Using the optimization function in Malab optimization toolbox, the optimal solution can be obtained as follows:

$$d^* = 10.79\$/day, C^* = 10.48\$/day, R^* = 0.9915 \tag{4-5}$$

So the optimal threshold value d^* is 10.79$/day, which means if the risk, that is penalty cost K times the hazards value ($K \times h(t, Z(t))$), exceeds 10.79$/day at the inspection point of time, the preventive replacement should be performed. With the optimal maintenance policy, the average maintenance cost is estimated to be 10.48$/day and the reliability of the component is around 0.9915. From these results we can see that the optimal cost falls into the tolerable range, and the optimal reliability is in the highly desirable range. The optimization results can reflect the designer's preferences on the objectives, and the tradeoff between the two design objectives.

Next we will investigate another set of boundary values in which the requirement on the reliability objective has been improved:

Cost: $$[g_{C1}, g_{C2}, g_{C3}, g_{C4}, g_{C5},] = [8,10,12,15,20] \quad (4\text{-}6)$$

Reliability: $$[g_{R1}, g_{R2}, g_{R3}, g_{R4}, g_{R5},] = [0.99999,0.999,0.995,0.99,0.95] \quad (4\text{-}7)$$

Conducting the optimization, we can obtain the following optimal solution:

$$d^* = 3.4897\$/day, C^* = 11.22\$/day, R^* = 0.9973 \quad (4\text{-}8)$$

So for the new set of boundary values, the optimal risk threshold value d^* is 3.4897$/day, the optimal maintenance cost is 11.22$/day and the component reliability is 0.9973. Comparing with the previous results, when there is a higher requirement on reliability, the optimal risk threshold value d^* decreases. Both of the optimal cost and optimal reliability fall into the tolerable ranges, in order to make the best tradeoff between these two objectives. The optimization results reflect the change in the designer's preferences.

In this chapter, we investigate the application of the physical programming approach to deal with the multi-objective CBM optimization. We conduct a real world shear pump bearing case study to illustrate the proposed approach. Based on these research results we can conclude that using the proposed physical programming based multi-objective CBM optimization approach, the decision maker can systematically and effectively make good tradeoff between the cost objective and reliability objective.

Chapter 5

PHM based CBM Optimization Using Genetic Algorithms

Nomenclature

L	: likelihood value
$f(t)$	: probability density function (PDF)
$R(t)$	: reliability function
$F(t_r)-F(t_l)$	: likelihood function term for interval and left censored data
n_E	: number of exact failure data
n_R	: number of right failure data
n_I	: number of interval and left censored data
$\Phi(d)$	: average cost per unit time
d	: risk threshold level
C	: preventive replacement cost

K : penalty cost

$C+K$: failure replacement cost

$Q(d)$: failure probability

$W(d)$: expected time until replacement

Parameter estimation is very critical in PHM based CBM optimization process. The precision of parameter estimation greatly affects the accuracy of the model in representing and predicting the equipment health condition. Traditional optimization methods such Newton's methods or BFGS Quasi-Newton method can only find local optimization value. Genetic algorithms (GA) can achieve much better results since it has the advantage of global optimization ability and flexibility in modeling the problem without any strict mathematical requirements. In this chapter, we will present a GA approach for PHM parameter estimation using maximum likelihood method.

5.1. Review on Genetic Algorithms

GA is adaptive method which is developed based on the genetic processes of biological organisms. They are very effective in solving optimization problems. GA involves converting design parameters into genes. Simple parameters such as only Yes or No can be simply denoted by genes of 0 and 1, while those more complex parameters can be alphabets or numbers other than binary.

The basic idea is that natural populations evolve over many generations according to the principles of natural selection and survival of the fittest. GA is able to "evolve" solutions to real world problems by mimicking this process (Busetti).

5.1.1. Basic mechanism of GA

In GA, a population is firstly created by randomly generating a group of individuals. Each individual represents a possible solution to a given problem. The individuals in the population are then evaluated by given a score based on how well they perform at the given task. According to their fitness, individuals are selected by the rule of the higher the fitness, the higher the possibility of being selected. Two individuals are then randomly selected to exchange part of their elements to create one or more offspring; next the offspring are mutated randomly. This process continues until an appropriate solution has been found or a certain number of generations have passed, depending on the practical requirements. Generally there are five steps described as follows (Skinner):

Step 1: Population. Generate a group of individuals at random to create a population.

Step 2: Evaluation. Evaluate the individuals based on how well they perform at the given task, which is called fitness.

Step 3: Selection. The most common type of selection is *'roulette wheel selection'*. In this selection, individuals are given a likelihood of being selected which is in proportion to their fitness. And then individuals are selected randomly based on these likelihood and create a new population.

Step 4: Crossover. Two individual are then selected from the new population randomly and create offspring. The most common crossover is single point crossover. In single point crossover, a locus is chosen at which the remaining elements are exchanged from one parent to the other. An example is given as follow.

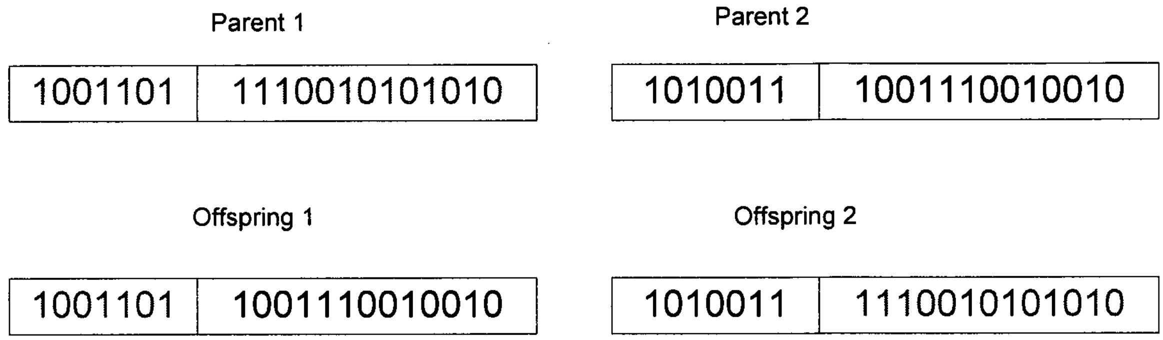


Figure 8 Crossover example

In Figure 8, we can see the offspring 1 takes first section of the chromosome from parent 1 and second section from parent 2, while the offspring 2 takes first section of the chromosome from parent 2 and second section from parent 1. In single point crossover, the point at which the chromosome is broken is selected randomly and only one crossover point exists. Crossover is not usually applied to all pairs of individuals selected, which gives each individual a chance of passing on its genes. The probability of crossover occurring is around 20% to 60%.

Step 5: Mutation. To ensure that the individuals are not all exactly the same, the selected elements can either be changed by a small amount or replace it with a new value. Only a small chance of mutation is allowed and the probability is usually between 0.5% and 10%. However, mutation is important in GA which can guarantee genetic diversity within the population. A visual example for mutation is shown in Figure 9. In this figure, we can see the 13^{th} gene ‘0’ is selected and mutated to ‘1’.

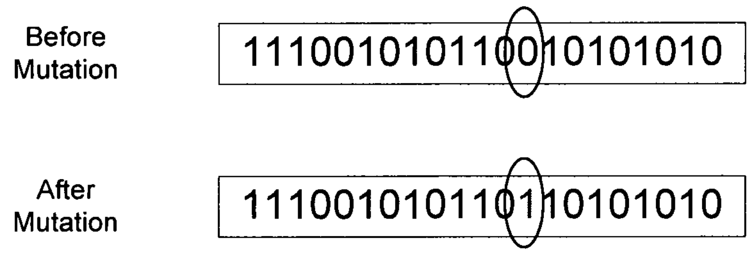


Figure 9 Mutation example

5.1.2. Strength and weakness

GA is a very powerful optimization approach with two key advantages. (1) Global optimization ability. GA has been recognized as one of the most effective approaches in searching for the global optimal solution (2) Flexibility in modeling the problem. GA has no strict mathematical requirements, such as derivative requirement, on the objective functions and constraints. The only requirement is that the objective functions and constraints can be evaluated in some way. GA is also suitable for dealing with those problems including discrete design variables. (Tian & Zuo, 2006).

Although GA is easy to implement and is powerful for solving difficult problems, it has the disadvantage of being time-consuming. GA usually requires relatively more time to achieve the global optimization value because it has to evaluate each individual in the whole population and perform the genetic process for many generations, e.g. 500 generations or more.

5.1.3. Applications

GA works effectively on quickly finding a reasonable solution to a complex problem by searching through a large and complex search space. They are most effective in a search space for which little is known. GA has been widely used in many fields such as industrial design by parameterization, scheduling, network design by construction, routing, time series prediction, database mining, control systems, artificial life systems, and molecular conformation in chemistry. If classified by technique, GA has many applications, including binary chromosomes for set membership and function optimization, real-valued chromosomes for function optimization, order-based chromosomes for optimization by construction, tree-based chromosomes for genetic programming, decision theory, database mining, etc. and domain-specific chromosomes for specialized solutions to particular problems.

5.2. Parameter Estimation Using the Maximum Likelihood Method in Reliability Analysis

Maximum likelihood parameter estimation is to determine the parameters that maximize the likelihood of obtaining the sample data.

The following is the conventional likelihood function, which is applied to reliability analysis involving only events data but no condition monitoring data.

$$L = \prod_{i=1}^{n_E} f(t_i;\theta) \cdot \prod_{i=1}^{n_R} R(t_i^+;\theta) \cdot \prod_{i=1}^{n_I} (F(t_{ir};\theta) - F(t_{il};\theta)) \qquad (5\text{-}1)$$

There are four types of data: exact failure data, right censored data, left censored data and interval censored data. Left censored data can be considered special case of interval censored data. Some examples of different types of data are given in the Figure 10 and Table 4. Unit 1 is left censored data: we know this unit failed before 100 hours but we don't know the exact failure time. Unit 2 is exact failure data since it failed exactly at 110 hours. Unit 3 is interval censored data: it is known that this unit failed in the interval between 40 hours and 140 hours but the exact failure time is unknown. Both of Unit 4 and 5 are right censored data: Unit 4 was suspended at 120 hour and Unit 5 was suspended at 180 hours when they were still working.

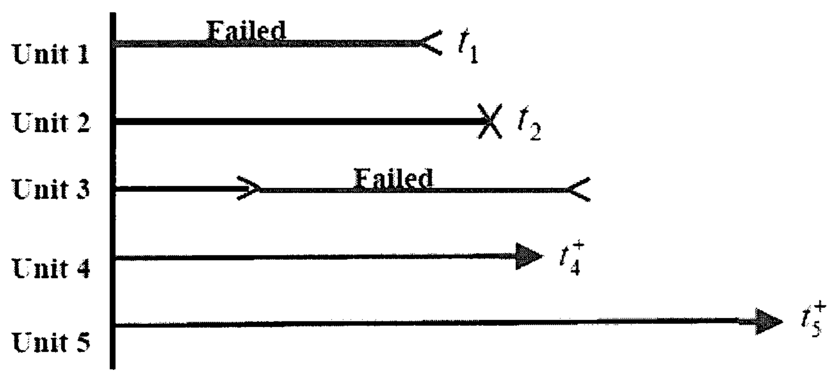


Figure 10 Examples of different types of data

Table 4 Examples of different types of data

Unit No.	**Failure Time (Hours)**	**Failure Time (Hours)**	**Data Type**
1	0	100	Left censored
2		110	Exact
3	40	140	Interval censored
4		120+	Right censored
5		180+	Right censored

5.3. Parameter Estimation Using the Maximum Likelihood Method for PHM

Condition based maintenance optimization process using PHM considers inspection data as well as events data. The effects of different covariates influencing the time to failure of equipment can also be estimated thus the model will more accurately represent the health condition of the equipment. In CBM using PHM, two types of replacements occur according to the optimal maintenance policy: failure replacement and preventive replacement. Failure replacements generally cost more than preventive replacements. Failure replacement is performed when the component is ending with failure. Preventive replacement takes place when the component still works and is ending with suspension. For example, in the case of shear pump bearings in a food processing plant which was discussed in Chapter 4, there are altogether 25 histories recorded; 13 of them are failure replacements and 12 of them are preventive replacements. In a failure or suspension history, vibration measurements were collected at different inspection points, and the vibration measurements are used as covariates in PHM. By deploying the optimal policy some of the items are replaced on schedule when they are still working while the others fail before schedule.

To estimate the parameters in PHM using maximum likelihood method, three steps are required.

Step 1: Find the likelihood function

$$L(\beta,\eta,\gamma)=\prod_{i=1}^{n_1} h(t_i,Z(t_i))\prod_{j=1}^{n_2} S(t_j,Z(t_j)) \tag{5-2}$$

where n_1 denotes the number of failure histories, and n_2 denotes the number of suspension histories. The first component $\prod_{i=1}^{n_1} h(t_i, Z(t_i))$ considers the failure histories and the second component $\prod_{j=1}^{n_2} S(t_j, Z(t_j))$ represents the suspension histories. For the failure histories, a parametric proportional hazards model can be built as follows:

$$h(t, Z(t)) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{\sum_{k=1}^{m} \gamma_k z_k(t)}, \quad \beta > 0, \eta > 0, \gamma_k > 0 \tag{5-3}$$

For the suspension histories, a survival model can be built as follows:

$$S(t, Z(t)) = e^{-H(t,Z(t))} = e^{-\int_0^t h(t,Z(t))} \tag{5-4}$$

Let k=1, 2... m, where m denotes the number of significant covariates; z_k is the k^{th} covariate value, γ_k is the corresponding weight,

i=1, 2... n_1, where n_1 denotes the number of failure histories,

j=1, 2... n_2, where n_2 denotes the number of suspension histories,

q=0, 1, 2...r_j, where r_j is the number of inspection points in the j^{th} suspension history,

$0 < t_{j0} < t_{j1} < \ldots < t_{jr_j} = t_j$ are the actual inspection points in the j^{th} suspension history.

So the survival value of j^{th} suspension history can be described as follows:

$$
\begin{aligned}
S(t_j, Z(t_j)) &= e^{-H(t_j, Z(t_j))} \\
&= e^{-\int_0^{t_j} h(t_j, Z(t_j))dx} \\
&= e^{-\int_0^{t_j} \frac{\beta}{\eta}(\frac{x}{\eta})^{\beta-1} e^{\sum_{k=1}^{m}\gamma_k z_k(x)} dx} \\
&= e^{-\sum_{q=0}^{r_j-1}\int_{t_{jq}}^{t_{j(q+1)}} e^{\sum_{k=1}^{m}\gamma_k z_k(x)} d\left(\frac{x}{\eta}\right)^{\beta}} \\
&= e^{-\sum_{q=0}^{r_j-1} e^{\sum_{k=1}^{m}\gamma_k z_k(t_j)} \left[\left(\frac{t_{j(q+1)}}{\eta}\right)^{\beta} - \left(\frac{t_{jq}}{\eta}\right)^{\beta}\right]}
\end{aligned}
\qquad (5\text{-}5)
$$

Thus the likelihood function in (5-2) can be expended as:

$$
\begin{aligned}
L(\beta, \eta, \gamma) &= \prod_{i=1}^{n_1} h(t_i, Z(t_i)) \prod_{j=1}^{n_2} S(t_j, Z(t_j)) \\
&= \prod_{i=1}^{n_1} h(t_i, z(t_i)) \prod_{j=1}^{n_2} e^{(-H(t_j, z(t_j)))} \\
&= \prod_{i=1}^{n_1} \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{\sum_{k=1}^{m}\gamma_k z_k(t_i)} \prod_{j=1}^{n_2} e^{-\int_0^{t_j} h(x, z_j(x)dx} \\
&= \prod_{i=1}^{n_1} \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{\sum_{k=1}^{m}\gamma_k z_k(t_i)} \prod_{j=1}^{n_2} e^{-\sum_{q=0}^{r_j-1} e^{\sum_{k=1}^{m}\gamma_k z_k(t_j)} \left[\left(\frac{t_{j(q+1)}}{\eta}\right)^{\beta} - \left(\frac{t_{jq}}{\eta}\right)^{\beta}\right]}
\end{aligned}
\qquad (5\text{-}6)
$$

Step 2: Take log of the likelihood function

$$LnL = \ln\left(\frac{\beta}{\eta}\right) + (\beta - 1)\sum_{i=1}^{n_1} \ln\left(\frac{t_i}{\eta}\right) + \sum_{i=1}^{n_1}\sum_{k=1}^{m} \gamma_k z_k(t_i) - \sum_{j=1}^{n_2} e^{\sum_{q=0}^{r_j - 1} e^{\sum_{k=1}^{m} \gamma_k z_k(t_j)} \left[\left(\frac{t_{j(q+1)}}{\eta}\right)^{\beta} - \left(\frac{t_{jq}}{\eta}\right)^{\beta}\right]}$$

(5-7)

Step 3: Perform optimization and find $\hat{\eta}, \hat{\beta}, \hat{\gamma}_1, \hat{\gamma}_2, \ldots, \hat{\gamma}_m$ to maximize LnL

Currently, traditional optimization methods such as Newton's methods are used to find the optimal parameters. Matlab optimization toolbox was developed based on traditional optimization methods and it can be used to implement these optimal problems. The commercial software EXAKT was also developed based on traditional optimization methods, and the software EXAKT is currently being used in many industries. But there is a big disadvantage of traditional optimization method, that is, it can only find the local minimum values. In our research, GA is found to have much better global optimization capability. In Section 5.6, we will conduct a case study to illustrate the parameters estimation approach using GA.

5.4. Summary of the GA Approach for PHM Parameter Estimation

From Section 5.3, the PHM parameter estimation optimization model can be summarized as follows:

$$\max \ LnL = \ln\left(\frac{\beta}{\eta}\right) + (\beta - 1)\sum_{i=1}^{n_1}\ln\left(\frac{t_i}{\eta}\right)$$

$$+\sum_{i=1}^{n_1}\sum_{k=1}^{m}\gamma_k z_k(t_i) - \sum_{j=1}^{n_2} e^{\sum_{q=0}^{r_j-1} e^{\sum_{k=1}^{m}\gamma_k z_k(t_j)}\left[\left(\frac{t_{j(q+1)}}{\eta}\right)^{\beta} - \left(\frac{t_{jq}}{\eta}\right)^{\beta}\right]} \tag{5-8}$$

s.t.

$$\beta > 0, \ \eta > 0.$$

The design variables are $\eta, \beta, \gamma_1, \gamma_2, ..., \gamma_m$, totally m+2 design variables. And the objective function to be maximized is LnL, the logarithm of the likelihood function, and it can be evaluated using Equation (5-7) based on the inspection and event data available.

In the GA approach, the decimal encoding is used, since all the design variables are taking real values. We use the roulette-wheel selection scheme, one-point cross operator with cross rate of 0.25, and even mutation operator with mutate rate of 0.1. The GA optimization process will be stopped when a certain pre-specified generation number, say 1000, is reached.

5.5. Transition Probability Matrix Development

As discussed in Chapter 4, the purpose of estimating the transition probability matrix is to predict future covariate values. It is more convenient to estimate first the transition rates and then calculate the transition probabilities. The transition rates can be estimated using occurrence/exposure rates as follows:

$$\hat{\lambda}_{ij} = \frac{n_{ij}}{A_i}, i \neq j \ , \ \hat{\lambda}_{ii} = 1 - \sum_{j=1}^{n} \lambda_{ij}, i \neq j \tag{5-9}$$

where n_{ij} is the number of all transitions from state $i \rightarrow$ state j occurred over the interval $[s_l, s_{l+1}]$ in the sample, and A_i is the total length of time that the state i is occupied over the interval $[s_l, s_{l+1}]$ in the sample.

After the transition rates are calculated, the transition probability matrix can then be calculated as follows:

$$P = \Lambda^k$$

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1i} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2i} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{j1} & \lambda_{j2} & \cdots & \lambda_{ij} \end{bmatrix} \tag{5-10}$$

where P is the transition probability matrix, Λ is the transition rates matrix, k is the inspection interval.

5.6. Optimal Maintenance Policy

The method for calculating the cost and reliability values in the CBM optimization using PHM was described in (Makis & Jardine, 1992, Banjevic et al., 2001). The basic theory of this approach can be described in the following way: if the observed risk $K \times h(t, z(t))$, $h(t, z(t))$ is hazard rate and K is penalty cost, at the given inspection point of time is greater than a certain threshold value d, preventive replacement action should be taken; otherwise operation can continue. Nevertheless, there is also possible that failure occurs between two inspection points of time. In that case, failure replacement will be performed. Thus, the objective of the CBM optimization using PHM is to find the optimal threshold value of the hazard rate to minimize maintenance cost. It is generally necessary to identify the optimal threshold if the hazard rate is an increasing or at least non-decreasing function of time t. Actually, in many cases, the covariates show increasing trend. It is unnecessary to find the optimal value if the hazard rate is not an increasing function of time t. In this approach, the expected long run average cost per unit time is a function of the threshold risk level d, which is shown as follows:

$$\Phi(d) = \frac{C(1-Q(d)) + (C+K)Q(d)}{W(d)} = \frac{C + KQ(d)}{W(d)} \qquad (5\text{-}11)$$

where $\Phi(d)$ is the expected average cost per unit time and it is a function of the threshold risk level d, C is the preventive replacement cost and $C+K$ is the failure replacement cost. $Q(d)$ is the probability that failure replacement will occur, and

$$Q(d) = P(T_d \geq T) \qquad (5\text{-}12)$$

$$T_d = \inf\{t \geq 0 : Kh(t, z(t) \geq d)\} \qquad (5\text{-}13)$$

here, T_d is the preventive time at the risk level d. $W(d)$ denotes the expected time until replacement, regardless of whether it is a preventive action or a failure replacement, that is $W(d) = E(\min\{T_d, T\})$. If the hazard rate is non-decreasing, for example if $\beta \geq 1$ and all covariates are non-decreasing and covariate parameters are positive, then the optimal risk level, d^*, can be determined with the fixed-point iteration method to get $\Phi(d^*) = \min_{d>0} \Phi(d) = d^*$. If the hazard rate is not monotonic, then the fixed-point iteration does not work, and $\min_{d>0}, \Phi(d)$ should be found by direct search. Numerically more convenient is a forward version of that procedure, which can be suitably adjusted for nonmonotonic hazard rates. Once the optimal risk level, d^*, is determined the item is replaced at the first moment t when

$$\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\exp(\gamma Z(t)) \geq \frac{d^*}{K} \tag{5-14}$$

Under a CBM using PHM optimization policy, the reliability of the component or system is defined as the probability of performing preventive replacements, which is the probability of preventing failure from occurring. Thus, the reliability objective R can be calculated using the following equation:

$$R = 1 - Q(d) \tag{5-15}$$

5.7. Case Study

In this section, a real world case of shear pump bearings in a food processing plant is used to illustrate the proposed GA based parameter estimation approach. Details of the case can be found in section 4.2.

The objective of the PHM based CBM optimization process is to find an optimal replacement policy to minimize total expected replacement cost, given the condition monitoring data (vibration data) and replacement histories. Parameter estimation is a critical step in performing the CBM optimization process and the accuracy of parameter estimation greatly influencing the effectiveness of CBM optimization.

Using the software EXAKT (Banjevic et al., 2001), the significance analysis was performed, and three significant covariates were identified: VEL#1A (band 1 velocity in the axial direction), VEL#1V (band 1 velocity in the vertical direction), and VEL#2A (band 2 velocity in the axial direction), as shown in Table 5.

Table 5 Significant analysis for shear pump bearing data

Parameter	Estimate	Sign. (*)	Standard Error	Wald	DF	p - Value	Exp of Estimate	95 % CI	
								Lower	Upper
Scale	1584	-	643.4	-	-	-	-	322.5	2845
Shape	4.992	Y	1.173	11.58	1	0.0006656	-	2.693	7.291
VEL_1A	5.831	Y	1.101	28.03	1	0	340.8	3.672	7.99
VEL_2A	36.55	Y	6.641	30.29	1	0	7.486e+015	23.53	49.57
VEL_1V	24.05	Y	5.434	19.59	1	0	2.792e+010	13.4	34.7

5.7.1. The maximum likelihood method and the parameter estimation results using EXAKT

In this case, there are 25 histories in the shear pump bearing data, including 13 failure replacements (ended with failure) and 12 preventive replacements (ended with suspension). According the procedures of maximum likelihood method, firstly, the likelihood function is identified as follows:

$$
\begin{aligned}
L(\beta,\eta,\gamma) &= \prod_{i=1}^{13} h(t_i, Z(t_i)) \prod_{j=1}^{12} S(t_j, Z(t_j)) \\
&= \prod_{i=1}^{13} h(t_i, z(t_i)) \prod_{j=1}^{12} e^{(-H(t_j, z(t_j)))} \\
&= \prod_{i=1}^{13} \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{\sum_{k=1}^{3}\gamma_k z_k(t_i)} \prod_{j=1}^{12} e^{-\int_0^{t_j} h(x, z_j(x))dx} \\
&= \prod_{i=1}^{13} \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{\sum_{k=1}^{3}\gamma_k z_k(t_i)} \prod_{j=1}^{12} e^{-\sum_{q=0}^{r_j-1} e^{\sum_{k=1}^{3}\gamma_k z_k(t_j)}\left[\left(\frac{t_{j(q+1)}}{\eta}\right)^{\beta} - \left(\frac{t_{jq}}{\eta}\right)^{\beta}\right]}
\end{aligned}
\qquad (5\text{-}16)
$$

where $i=1, 2 \ldots 13$, there are 13 failure histories.

$j=1, 2 \ldots 12$, there are 12 suspension histories.

$k=1, 2, 3$, there are 3 significant covariates, which are VEL#1A (band 1 velocity in the axial direction), VEL#1V (band 1 velocity in the vertical direction), and VEL#2A (band 2 velocity in the axial direction); z_k is the k^{th} covariate value γ_k is the corresponding weight.

$q=0, 1, 2 \ldots r_j$, r_j is the number of inspection points in the j^{th} suspension history.

$0 < t_{j0} < t_{j1} < \ldots < t_{jr_j} = t_j$ are the actual inspection points in the j^{th} suspension history.

Secondly, we may take log of the likelihood function

$$
\begin{aligned}
LnL &= \ln(\frac{\beta}{\eta}) + (\beta - 1)\sum_{i=1}^{13} \ln(\frac{t_i}{\eta}) + \sum_{i=1}^{13}\sum_{k=1}^{3} \gamma_k z_k(t_i) - \sum_{j=1}^{12} \int_0^{t_j} h(x, z_j(x)dx \\
&= \ln\left(\frac{\beta}{\eta}\right) + (\beta - 1)\sum_{i=1}^{13} \ln\left(\frac{t_i}{\eta}\right) + \sum_{i=1}^{13}\sum_{k=1}^{3} \gamma_k z_k(t_i) - \sum_{j=1}^{12} e^{\sum_{q=0}^{r_j-1} e^{\sum_{k=1}^{3} \gamma_k z_k(t_j)} \left[\left(\frac{t_{j(q+1)}}{\eta}\right)^{\beta} - \left(\frac{t_{jq}}{\eta}\right)^{\beta}\right]}
\end{aligned}
\quad (5\text{-}17)
$$

Finally, we can perform optimization and find $\hat{\eta}, \hat{\beta}, \hat{\gamma}_1, \hat{\gamma}_2, \hat{\gamma}_3$ to maximize LnL.

Currently this optimization problem is solved by traditional optimization methods such as BFGS Quasi-Newton method; the commercial software EXAKT also uses the traditional optimization method to perform optimization. Using EXAKT the parameters are estimated as $\eta = 1584, \beta = 4.992, \gamma_1 = 5.831, \gamma_2 = 36.55, \gamma_3 = 24.05$, and the PHM are built as follows:

$$
\begin{aligned}
h(t, Z(t)) &= \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{(\gamma_1 z_{1A}(t) + \gamma_2 z_{2A}(t) + \gamma_3 z_{1V}(t))} \\
&= \frac{4.992}{1584}\left(\frac{t}{1584}\right)^{4.992-1} e^{(5.831 z_{1A}(t) + 36.55 z_{2A}(t) + 24.05 z_{1V}(t))}
\end{aligned}
\quad (5\text{-}18)
$$

5.7.2. Parameter estimation results using GA

As presented previously, tradition optimization methods are limited to local optimization. To improve accuracy of parameter estimation, next we will apply GA to perform the optimization find the global optimal parameters.

In this case, there are five parameters to be estimated: η (scale parameter), β (shape parameter), γ_1 (covariate weight for VEL_1A), γ_2 (covariate weight for VEL_2A), and γ_3 (covariate weight for VEL_1V). Based on previous experience, the ranges of the parameters value can be set as follows:

η: 0-5000

β: 0.01-100

γ_1: 0.01-100

γ_2: 0.01-100

γ_3: 0.01-100

and the length of chromosome is set 20, thus each parameter length is 4 respectively. Considering the tradeoff between estimating accuracy and program running time, the population size is decided as 100 and genetic processes will be explored for 800 generations.

The following is the PHM built with the parameters estimated using GA:

$$
\begin{aligned}
h(t,Z(t)) &= \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{(\gamma_1 z_{1A}(t)+\gamma_2 z_{2A}(t)+\gamma_3 z_{1V}(t))} \\
&= \frac{10.52}{3396.5}\left(\frac{t}{3396.5}\right)^{10.52-1} e^{(17.89 z_{1A}(t)+33.04 z_{2A}(t)+98.85 z_{1V}(t))}
\end{aligned}
\qquad (5\text{-}19)
$$

Table 6 Likelihood value comparison between GA and EXAKT

Parameter / Method	η	β	γ_1	γ_2	γ_3	**Likelihood Value:** *LnL*	**Change**
Genetic Algorithms	3396.5	10.52	17.89	33.04	98.85	-11.5825	87.0359%
EXAKT	1584	4.992	5.831	36.55	24.05	-89.3427	

From Table 6, we can find that the logarithm of the optimal likelihood function value, LnL, is -89.3427 using EXAKT, while it is -11.5825 using GA. Since the maximum likelihood method is used to estimate the parameters, the greater the likelihood value the better. Thus we can conclude that genetic algorithms can significantly improve the accuracy of parameter estimation.

5.7.3. CBM optimization results

In this case there are altogether 25 histories including 216 inspection data points. Assuming the observation interval is 20 days, the transition probability matrices can be obtained using EXAKT, which are already displayed in section 4.2.

Based on the previous history and expert experience, the preventive replacement cost C is estimated to be \$1,800, and the failure replacement cost $C+K$ is \$16,200 for this case. Thus we have the penalty cost K equals to \$14,400.

Finally the CBM optimization policy can be determined using the estimated parameters, transition probability matrices and cost data information. Using the parameters estimated by the software EXAKT, which is $\eta = 1584, \beta = 4.992, \gamma_1 = 5.831, \gamma_2 = 36.55, \gamma_3 = 24.05$, the optimal maintenance policy is:

$$d^* = 16.4385\$/day, C^* = 10.4073\$/day, R^* = 0.9883 \tag{5-20}$$

Adhering to this optimal policy, the average preventive replacement interval is 189.1377 days. The cost versus risk threshold plot is given in Figure 11, in which the risk threshold value is given in logarithm scale. We can see the optimal maintenance cost is around 10.4073 \$/day.

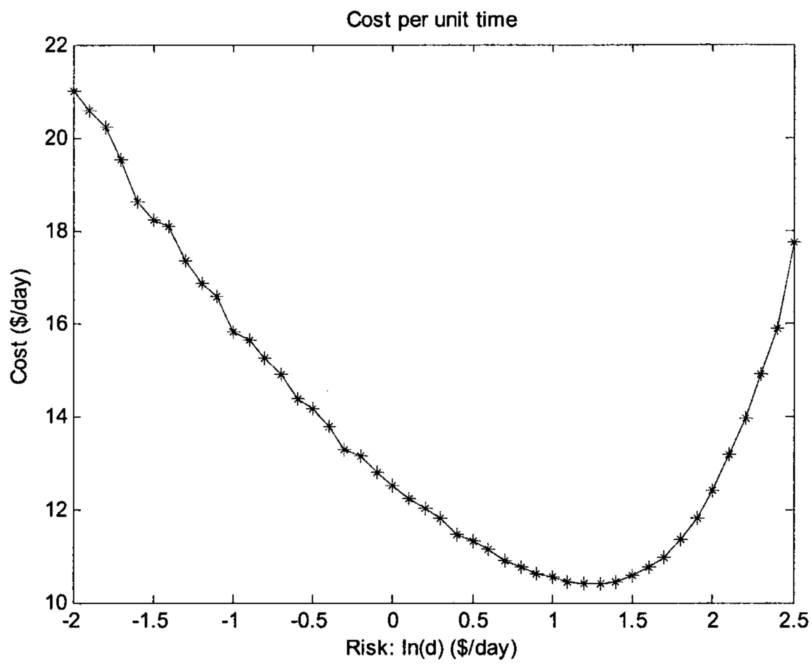


Figure 11 Cost per unit of time based on parameters estimated by EXAKT

Next we will calculate the optimal policy based on the parameters obtained using genetic algorithms, which are: $\eta = 3396.5, \beta = 10.52, \gamma_1 = 17.89, \gamma_2 = 33.04, \gamma_3 = 98.85$. The optimal maintenance policy is determined as:

$$d^* = 8.9799\$/day, C^* = 7.1481\$/day, R^* = 0.9982 \tag{5-21}$$

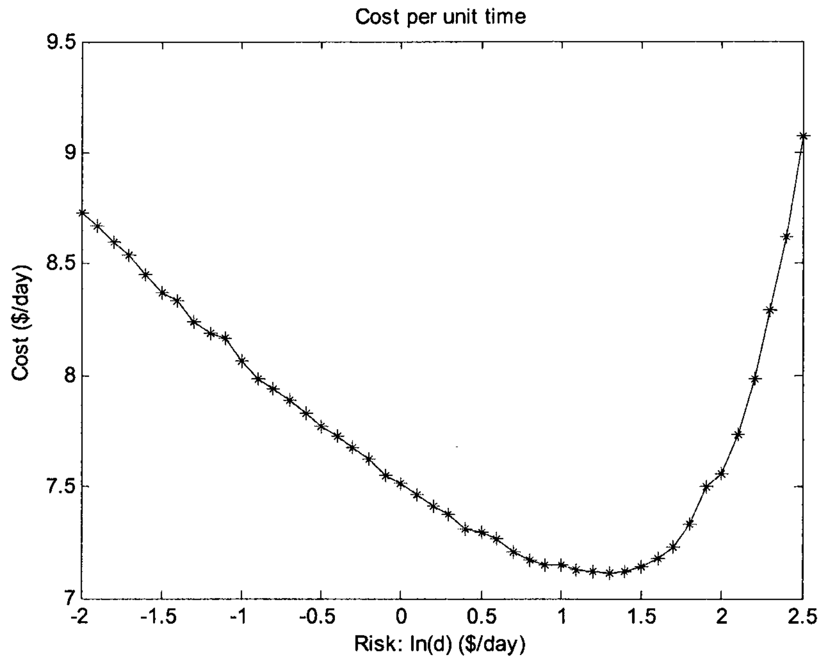


Figure 12 Cost per unit of time based on parameters estimated by GA

The cost versus risk threshold plot in this case is given in Figure 12, in which the risk threshold value is given in logarithm scale. We can see the optimal maintenance cost is around 7.1481$/day. Performing the optimal policy, the average preventive replacement interval will be 254.4501 days.

In the previous section, we conclude that GA can improve parameters estimation significantly, which means parameters estimated using GA are accurate while parameters obtained by EXAKT are inaccurate. Inaccuracy in the PHM parameters will lead to the inaccuracy in CBM policy assessment and in the CBM optimization results. In Table 7, we can see there are great differences between the CBM optimization results using GA and using EXAKT. The difference between the optimal risk threshold values is 45.37%, and the difference between the average maintenance cost values is 31.32%. Thus, we can clearly see the importance of obtaining the accurate PHM parameters using GA, because it will greatly affect the accuracy in PHM based CBM policy assessment and finding the optimal policy, and it will also affect maintenance related decisions such as budget planning.

Table 7 CBM optimization results comparison between GA and EXAKT

Resutls / Method	Risk Threshold ($/day)	Average Maintenance Cost ($/day)	Reliability	Average Replacement Interval (day)
Genetic Algorithms	8.9799	7.1481	0.9982	189.1377
EXAKT	16.4385	10.4073	0.9883	254.4501
Changes	45.37%	31.32%	1%	25.67%

In this chapter, we propose a parameter estimation approach using genetic algorithms based on the fact that genetic algorithms has the advantage of global optimization while the traditional optimization methods are limited to local optimization. A case study of shear pump bearing in a food processing plant is given to illustrate the proposed approach. We can conclude that applying genetic algorithms to perform optimization in parameter estimation using maximum likelihood method can significantly improve the accuracy of parameter estimation. It will also improve the accuracy in PHM based CBM policy assessment and finding the optimal policy, and thus enable better informed maintenance related decision making such as budget planning.

Chapter 6

Canadian Kraft Mill Pump Bearing Case Study

Nomenclature

L : likelihood value

d : threshold risk level

C : preventive replacement cost

K : penalty cost

$C+K$: failure replacement cost

In this chapter, another real world case of Gould pump bearings at Canadian Kraft Mill (Stevens) is presented to further demonstrate the proposed parameter estimation approach using GA.

6.1. Case Introduction

Canadian Kraft Mill is a large kraft pulp mill with 400 employees and it produces over 300,000 tons of kraft pulp each year. Pulp produced in this mill is used to make facial tissues, paper towels and similar products. Facing tough competition in the pulp and paper market, Canadian Kraft Mill has to focus on the key objectives of bringing costs down and production up. Condition based maintenance optimization approach using proportional hazards model can help decreasing the maintenance cost. OMDEC who has developed the commercial software EXAKT took care of this project.

Kraft Mill was confronted with a critical problem of high incidence of unpredicted failures among a small group of its fleet of Gould pumps. Hence eliminating or substantially reducing the frequency of pump failure was evidently the key objective. The units being examined were Gould 3175L pumps which were used 24/7. Bearings were critical components of these pumps, as shown in Figure 13 (Stevens). Failure of bearing definitely caused the pump failure, and bearing failure is shown in Figure 14 (Stevens).

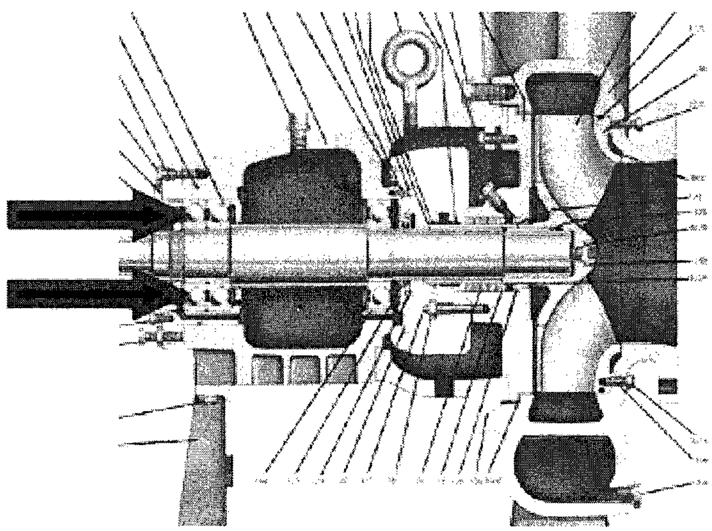


Figure 13 Gould 3175L pumps bearings (Stevens)

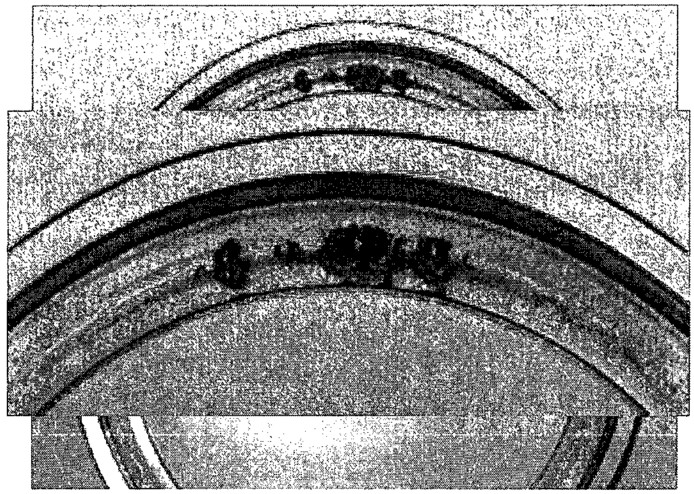


Figure 14 Gould pump bearing failure (Stevens)

Important data including event data, operating starts, out-of-service intervals and failure dates were extracted from the CMMS work history database. After sorting up these data, two categories of data were obtained, that is, event data and inspection data. There were three types of event data: beginning event, failure event and suspension event. For inspection data, 56(=8*5+8*1+8*1) vibration measurements were recorded. For each pump, seven measurements were analyzed – 5 different vibration frequency bands (8*5), and the overall vibration reading (8*1) plus the bearing's acceleration data (8*1). Altogether 36 bearing histories were examined in 8 pump locations, including 12 failures and 24 suspensions.

With all these important data collected by OMDEC ready, now we can perform the PHM based CBM optimization process and further testify the parameter estimation accuracy using GA over EXAKT.

6.2. Case Study

Using the software EXAKT, we can perform the significance analysis for the 56 vibration measurements. Only two covariates were identified to have significant influence on the health of bearings: P1H_Par5 (band 5 vibration frequency in Pump location P1H), and P1V_Par5 (band 5 vibration frequency in Pump location P1V).

Table 8 Significant analysis for Kraft Mill data

Parameter	Estimate	Sign. (*)	Standard Error	Wald	DF	p - Value	Exp of Estimate	95 % CI	
								Lower	Upper
Scale	2707	-	507.3	-	-	-	-	1713	3702
Shape	2.879	Y	0.6196	9.195	1	0.002426	-	1.664	4.093
P1H_Par5	24.87	Y	6.405	15.07	1	0.0001034	6.311e+010	12.31	37.42
P1V_Par5	42.56	Y	13.67	9.689	1	0.001854	3.039e+018	15.76	69.36

6.2.1. The maximum likelihood method and the parameter estimation results using EXAKT

In this case, 36 histories were collected from the 8 pump locations: 12 of them are failure replacements (ended with failure) and the other 24 histories are preventive replacements (ended with suspension). Using maximum likelihood method, firstly the likelihood function is defined as follows:

$$
\begin{aligned}
L(\beta,\eta,\gamma) &= \prod_{i=1}^{12} h(t_i, Z(t_i)) \prod_{j=1}^{24} S(t_j, Z(t_j)) \\
&= \prod_{i=1}^{12} h(t_i, z(t_i)) \prod_{j=1}^{24} e^{(-H(t_j, z(t_j)))} \\
&= \prod_{i=1}^{12} \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{\sum_{k=1}^{2}\gamma_k z_k(t_i)} \prod_{j=1}^{24} e^{-\int_0^{t_j} h(u, z_j(u))du}
\end{aligned}
\qquad (6\text{-}1)
$$

where $i=1, 2... 12$, 12 failure histories were recorded.

$j=1, 2... 24$, 24 suspension histories were recorded.

$k=1, 2$, 2 significant covariates are identified, which are P1H_Par5, and P1V_Par5; z_k is the k^{th} covariate value; γ_k is the corresponding weight.

$q=0, 1, 2...r_j$, r_j is the number of inspection points in the j^{th} suspension history.

$0 < t_{j0} < t_{j1} < ... < t_{jr_j} = t_j$ are the actual inspection points in the j^{th} suspension history.

Secondly, we may take log of the likelihood function

$$LnL = \ln\left(\frac{\beta}{\eta}\right) + (\beta-1)\sum_{i=1}^{12}\ln\left(\frac{t_i}{\eta}\right) + \sum_{i=1}^{12}\sum_{k=1}^{2}\gamma_k z_k(t_i) - \sum_{j=1}^{24}\int_0^{t_j} h(x, z_j(x)dx$$
$$= \ln\left(\frac{\beta}{\eta}\right) + (\beta-1)\sum_{i=1}^{12}\ln\left(\frac{t_i}{\eta}\right) + \sum_{i=1}^{12}\sum_{k=1}^{2}\gamma_k z_k(t_i) - \sum_{j=1}^{24} e^{\sum_{q=0}^{r_j-1} e^{\sum_{k=1}^{2}\gamma_k z_k(t_j)}\left[\left(\frac{t_{j(q+1)}}{\eta}\right)^{\beta} - \left(\frac{t_{jq}}{\eta}\right)^{\beta}\right]} \quad (6\text{-}2)$$

Finally, we should perform optimization to search $\hat{\alpha}, \hat{\beta}, \hat{\gamma}_1, \hat{\gamma}_2$ to maximize LnL.

Parameters estimated by EXAKT are $\eta = 2707, \beta = 2.879, \gamma_1 = 24.87, \gamma_2 = 42.56$ and the PHM is built as follows:

$$h(t, Z(t)) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{(\gamma_1 z_{P1H}(t) + \gamma_2 z_{P1V}(t))}$$
$$= \frac{2.879}{2707}\left(\frac{t}{2707}\right)^{2.879-1} e^{(24.87 z_{P1H}(t) + 42.56 z_{P1V}(t))} \quad (6\text{-}3)$$

6.2.2. Parameter estimation results using GA

Now we will apply GA to perform optimization and search for the optimal PHM parameters. Four parameters need to be estimated in this Gould pump bearing case: η (scale parameter), β (shape parameter), γ_1 (covariate weight for P1H_Par5), γ_2 (covariate weight for P1V_Par5). The ranges of the parameters value are set as follows according to the knowledge from history:

η: 0-5000

β: 0.01-100

γ_1: 0.01-100

γ_2: 0.01-100

The length of chromosome is set as 16 and each parameter length is 4 respectively. Similar to the previous shear pump bearing case, the population size is set as 100 and genetic processes will be explored for 800 generations because of the tradeoff between estimating accuracy and program running time,

The PHM parameters estimated using GA and the corresponding hazard function is given as follows:

$$\begin{aligned} h(t,Z(t)) &= \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{(\gamma_1 z_{P1H}(t)+\gamma_2 z_{P1V}(t))} \\ &= \frac{2.2}{4999.5}\left(\frac{t}{4999.5}\right)^{2.2-1} e^{100 z_{P1H}(t)+65 z_{P1V}(t))} \end{aligned} \tag{6-4}$$

Table 9 shows the comparison of parameters estimated using EXAKT and GA.

Table 9 Likelihood value comparison between GA and EXAKT

Parameter / Method	η	β	γ_1	γ_2	Likelihood Value LnL	Change
Genetic Algorithms	4999.5	2.2	100	65	15.9410	126.45%
EXAKT	2707	2.879	24.87	42.56	-60.2576	

From the Table 9 we can clearly see, the likelihood value obtained using GA is much greater than using EXAKT. Since the maximum likelihood method is used to estimate the parameters, the greater the likelihood value the better. Hence we can further confirm on the conclusion that parameter estimation accuracy can be significantly improved by using GA.

6.2.3. CBM optimization results

This case embraces 36 histories including 12 failure histories and 24 suspension histories, and 774 inspection data points are recorded. Assuming the observation interval is 28 days, the transition probability matrices can be obtained using EXAKT as follows:

Table 10 Transition probability matrix for covariate P1H_Par5

P1H_Par5	0 to 0.007656	0.007656 to 0.014256	0.014256 to 0.05016	0.05016 to 0.136752	Above 0.136752
0 to 0.007656	0.776222	0.207444	0.0160789	0.000248641	5.96986e-006
0.007656 to 0.014256	0.049508	0.821906	0.125604	0.00288893	9.3213e-005
0.014256 to 0.05016	0.00384598	0.125887	0.830708	0.0377024	0.00185645
0.05016 to 0.136752	0.000176079	0.0085723	0.111622	0.797526	0.0821037
Above 0.136752	0	0	0	0	1

Table 11 Transition probability matrix for covariate P1V_Par5

P1V_Par5	0 to 0.00702	0.00702 to 0.01404	0.01404 to 0.043095	0.043095 to 0.13299	Above 0.13299
0 to 0.00702	0.760227	0.225498	0.0140633	0.000209625	2.15812e-006
0.00702 to 0.01404	0.051723	0.843089	0.102875	0.00228106	3.13955e-005
0.01404 to 0.043095	0.00348354	0.111098	0.847167	0.0374689	0.00078289
0.043095 to 0.13299	0.000139623	0.00662383	0.100751	0.855864	0.0366211
Above 0.13299	0	0	0	0	1

Using history information, the preventive replacement cost C is estimated to be $4,000, and the failure replacement cost $C+K$ is $12,000 for this case. Thus the penalty cost K equals to $8,000.

Eventually, the CBM optimization policy can be determined using the estimated parameters, transition probability matrices and cost data information. Using the parameters estimated by the software EXAKT, which are:

$$\eta = 2707, \beta = 2.879, \gamma_1 = 24.87, \gamma_2 = 42.56,$$

the optimal maintenance policy is obtained as:

$$d^* = 4.6940\$ / day, C^* = 9.2730\$ / day, R^* = 0.9945 \qquad (6\text{-}5)$$

With this optimal policy, the average preventive replacement interval is 44.2748 days. The cost versus risk threshold plot is given in Figure 15, in which the risk threshold value is given in logarithm scale. We can see the optimal maintenance cost is around 9.2730 $/day.

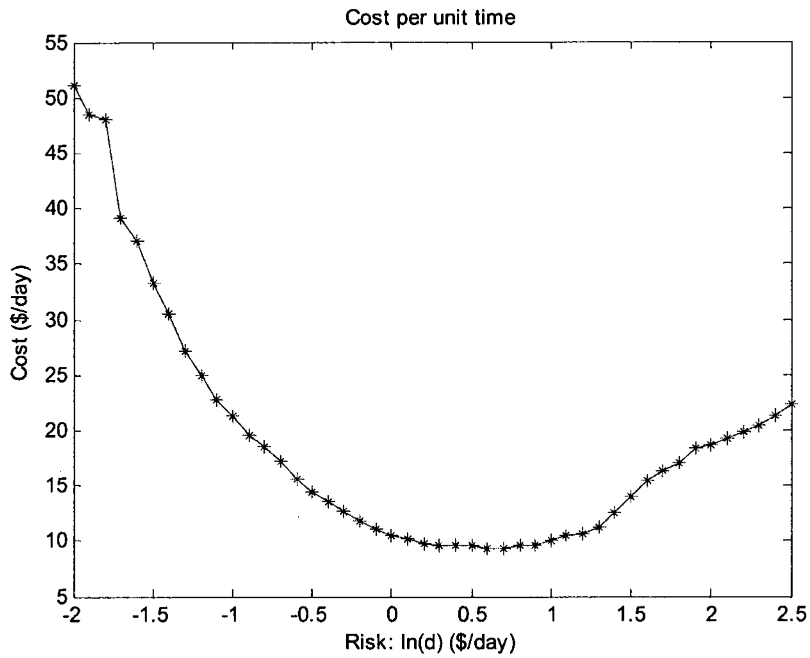


Figure 15 Cost per unit of time based on parameters estimated by EXAKT

Now we will calculate the optimal policy based on the parameters obtained using genetic algorithms, which are: $\eta = 4999.5, \beta = 2.2, \gamma_1 = 100, \gamma_2 = 65$. The optimal maintenance policy is then determined as:

$$d^* = 27.8931\$/day, C^* = 12.9444\$/day, R^* = 0.9864 \tag{6-7}$$

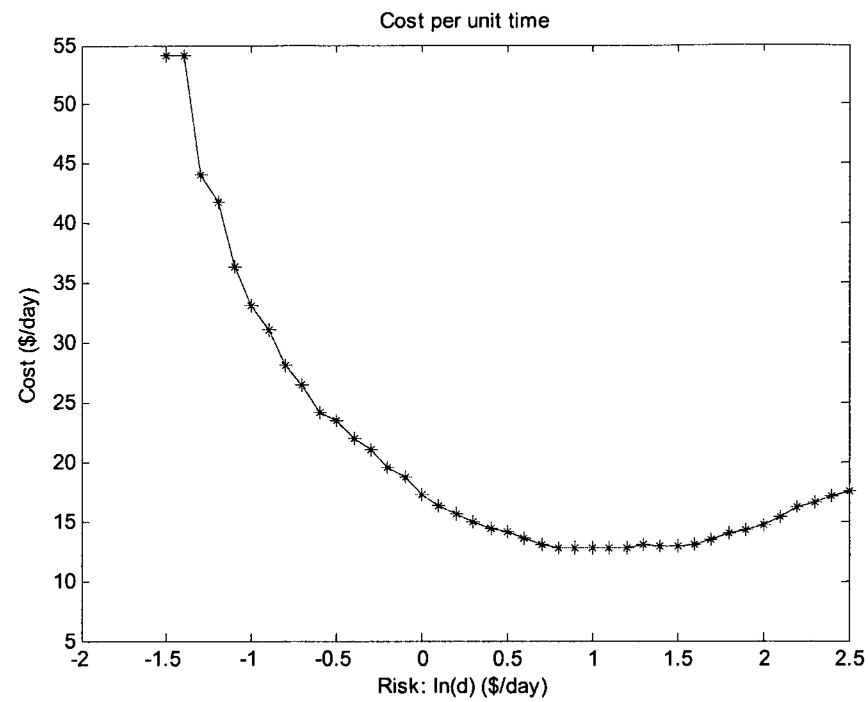


Figure 16 Cost per unit of time based on parameters estimated by GA

The cost versus risk threshold in this case is plotted in Figure 16, in which the risk threshold value is given in logarithm scale. From Figure 16, the optimal maintenance cost is shown as around 12.9444$/day. Based on the optimal policy, the average preventive replacement interval will be 43.4031 days.

In the previous section, we once again conclude that GA can improve parameters estimation significantly; therefore parameters estimated using GA are accurate while parameters obtained by EXAKT are inaccurate. The consequence of inaccurate PHM parameters is that the CBM policy assessment and the CBM optimization results will also be inaccurate. In Table 12, we can see there are great differences between the CBM optimization results using GA and using EXAKT. The difference between the optimal risk threshold values is 494.23%, and the difference between the average maintenance cost values is 39.59%. Thus, obtaining the accurate PHM parameters using GA is really vital in CBM optimization process. The reason is that the accuracy of PHM parameters will greatly affect the accuracy in PHM based CBM policy assessment and finding the optimal policy, and it will also affect maintenance related decisions such as budget planning.

Table 12 CBM optimization results comparison between GA and EXAKT

Resutls / Method	Risk Threshold ($/day)	Average Maintenance Cost ($/day)	Reliability	Average Replacement Interval (day)
Genetic Algorithms	27.8931	12.9444	0.9864	43.4031
EXAKT	4.6940	9.2730	0.9945	44.2748
Changes	494.23%	39.59%	0.81%	1.97%

In this chapter, another real world case of Gould pump bearing in Canadian Kraft Mill is conducted to further demonstrate the parameter estimation approach using genetic algorithms which is proposed in Chapter 5. This case confirms the conclusion drawn in Chapter 5, which is that applying genetic algorithms to perform optimization in parameter estimation using maximum likelihood method can significantly improve the accuracy of parameter estimation. It will also improve the accuracy in PHM based CBM policy assessment and finding the optimal policy, and thus enable better informed maintenance related decision making such as budget planning.

Chapter 7

Conclusion and Future Work

7.1. Conclusion

Condition-based maintenance (CBM) is an advanced maintenance strategy in which maintenance actions are scheduled based on both the age of the equipment and the information collected through condition monitoring. CBM process inspects the equipment's condition periodically and performs preventive replacements only when necessary to minimize maintenance cost. Proportional hazards model (PHM) is a powerful statistical tool for estimating the failure rate of a piece of equipment which considers both the age of the equipment and condition monitoring data. Hence the PHM based CBM optimization approach can represent and predict the equipment health condition more accurately, thus it is able to reduce unnecessary scheduled preventive maintenance actions hence reduce the overall maintenance costs. In this thesis, we proposed two approaches: multi-objective CBM optimization approach based on physical programming and genetic algorithms based approach for PHM parameter estimation. Both these two approaches are found to be able to improve current PHM based CBM process.

Multi-objective CBM optimization approach based on physical programming

In PHM based CBM optimization, main optimization objectives such as minimizing maintenance costs and maximizing equipment reliability are often conflicting to each other. Currently only single-objective optimization can be achieved, thus we cannot systematically investigate the tradeoff between the optimization objectives and find the optimal solution that best represents the decision maker's preference on the optimization objectives.

Physical programming presents two major advantages: (1) it is an effective approach to capture the decision maker's preferences on the objectives by eliminating the iterative process of adjusting the weights of the objectives, and (2) it is easy to use in that decision maker just needs to specify physically meaningful boundaries for the objectives. The physical programming based approach is able to transforms a multi-objective optimization problem into a single-objective optimization model. The decision maker can systematically and effectively determine an optimal balance between the cost objective and reliability objective. A real world case of shear pump bearing in a food processing plant was conducted to illustrate the proposed approach.

Genetic algorithms based approach for PHM parameter estimation

In PHM based CBM optimization, the accuracy of parameter estimation has a great influence on the effectiveness of the optimal maintenance policy. Currently only local optimal values can be obtained in parameter estimation using maximum likelihood method. Therefore the CBM optimal maintenance policy obtained based these incorrect parameters is also inaccurate, which affect obtaining the true optimal CBM policy.

GA is a very powerful optimization approach with two key advantages: (1) global optimization ability, and (2) flexibility in modeling the problem. Applying GA to solve the optimal problem in parameter estimation using the maximum likelihood method can improve the accuracy of parameter estimation significantly. It will also improve the accuracy in PHM based CBM policy assessment and finding the optimal policy, and thus enable better informed maintenance related decision making such as budget planning. A real world case of shear pump bearing in a food processing plant and another real world case of Gould pump bearing in Canadian Kraft Mill were used to validate the proposed approach.

7.2. Future Work

Based on the research elaborated in this thesis, further studies can be conducted in the following directions.

- Develop an approach using advanced PHM models in CBM optimization process to improve current approach using basic PHM model.
- Investigate the application of artificial neural network technologies in the CBM optimization using PHM.
- Conduct more experiments to further test the physical programming based approach and the genetic algorithms based approach.
- Apply the developed approaches to address CBM problems in various engineering systems, such as aircraft systems and wind energy systems.

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