(1)

Interim report on ONR grant N00014-92-J-1561 Conference: Three Decades of Numerical Linear Algebra at Berkeley

James Demmel
Computer Science Division and Mathematics Department
University of California
Berkeley, CA 94720
demmel@cs.berkeley.edu

April 30, 1993

A conference in numerical linear algebra was held at the Mathematical Sciences Research Institute at the University of California at Berkeley on October 17, 1993. The occasion was the 60th birthdays of Professors Beresford Parlett and Velvel Kahan. There were approximately 100 attendees. In addition to Demmel, Prof. James Bunch of UC San Diego and Dr. Horst Simon of NASA-Ames were co-organizers. Here is the list of speakers:

- James Bunch, University of California, San Diego, "Three Decades of Numerical Linear Algebra at Berkeley".
- G. W. Stewart, University of Maryland, "On the Perturbation of Matrix Factorizations".
- John Reid, Rutherford Appleton Laboratory, England, "Taking Advantage of Sparsity within 2x2 Pivots When Solving Symmetric Indefinite Sets of Linear Equations".
- Horst Simon, NASA Ames, "Spectral Algorithms-A New Approach to Some Discrete Optimization Problems in Scientific Computing".
- Larry Nazareth, Washington State University, "The Newton and Cauchy Perspectives on Computational Nonlinear Optimization".

- James Ortega, University of Virginia, "Solution of Nonlinear Poisson-type Equations".

- Anne Greenbaum, Courant Institute, NYU, "Matrices that Generate the Same Krylov Residual Spaces".

- Bahram Nour-Omid, Scopus, Berkeley, "Ordered Modified Gram-Schmidt".

- David Scott, Intel, "A High Performance Out-of Core Dense Equation Solver for the Intel Parallel Supercomputer".

- Scott Baden, University of California, San Diego, "The Role of Heuristics in Parallel Computation of Scientific Problems".

- Peter Tang, Argonne National Laboratory, "Recent Advances in Rank-Revealing QR Factorization".

- James Demmel, University of California, Berkeley, "Recent progress in parallel algorithms for the nonsymmetric eigenproblem".

The speakers and others submitted papers for a special issue of the Journal of Numerical Linear Algebra with Applications (JNLAA) dedicated to Parlett and Kahan. These papers were sent out for the standard refereeing process. Most have been accepted, and several are still out for review and revision. We attach a list of these papers, abstracts and their status below. When the refereeing process is complete and the journal published, we will forward copies to ONR as requested.

Titles.txt

JNLAA

SPECIAL ISSUE DEDICATED TO PARLETT AND KAHAN

AUTHORS	TITLE
(1) Demmel, Higham, and Schreiber	Block LU Factorization
(2) Elsner, He, and Mehrmann	Minimization of the norm, the norm of the inverse and the condition number of a matrix by completion
(3) Fierro and Bunch	Orthogonal Projection and Total Least Squares
(4) Ikramov	On the Computational Aftermath of Matrix Consimilarity and Concommutativity Relations II
(5) Z. Jia	The Convergence of Krylov Subspace Methods
(6) Paige, Parlett, and van der Vorst	Approximate Solutions and Eigenvalue Bounds frc Krylov Subspaces
(7) Van Huffel and Park	Efficient Algorithms for Bordered Band Matrices
(8) Arbenz and Golub	Matrix Shapes Invariant under the QR Algorithm
(9) Borges, Frezza, and Gragg	Some Inverse Problems for Jacobi and Arrow Matr
(10) Druskin and Knizhnerman	Krylov Subspace Approximation of Eigenpairs and Matrix Functions in Exact and Computer Arithmet
(11) C-T Pan and Peter Tang	Bounds on Singular Values Revealed by QR Factorizations
(12) Nour-Omid, Dunbar and Woodbury	Ordered Modified Gram-Schmidt Orthogonalization
(13) Batterson	Dynamical Analysis of Numerical Systems
(14) R-C Li	Solving Secular Equations Stably and Efficientl
(15) Z. Bai	Progress in the Numerical Solution of the Nonsymmetric Eigenvalue Problem
(16) Nazareth	Trust Regions based on Conic Functions in Linear and Nonlinear Programming
(17) Edelman and Mascarenhas	On Parlett's Matrix Norm Inequality
(18) Dan Szyld	Regions of Convergence of the Rayleigh Quotient Iteration Method
(19) Walden, Karlson, and Sun	Optimal Backward Perturbation Bounds for the Linear Least Squares Problem
(20) G. W. Stewart	On the Solution of Block Hessenberg Systems

April 6, 1993

(1)
accepted

BLOCK *LU* FACTORIZATION

JAMES W. DEMMEL
Computer Science Division and Mathematics Department
University of California
Berkeley
CA 94720
U.S.A.
(na.demmel@na-net.ornl.gov).

NICHOLAS J. HIGHAM
Nuffield Science Research Fellow
Department of Mathematics
University of Manchester
Manchester
M13 9PL
UK
(na.nhigham@na-net.ornl.gov).

ROBERT S. SCHREIBER
RIACS
NASA Ames Research Center
Moffett Field
CA 94035
U.S.A.
(na.schreiber@na-net.ornl.gov).

ABSTRACT

Many of the currently popular "block algorithms" are scalar algorithms in which the operations have been grouped and reordered into matrix operations. One genuine block algorithm in practical use is block *LU* factorization, and this has recently been shown by Demmel and Higham to be unstable in general. It is shown here that block *LU* factorization is stable if A is block diagonally dominant by columns. Moreover, for a general matrix the level of instability in block *LU* factorization can be bounded in terms of the condition number $\kappa(A)$ and the growth factor for Gaussian elimination without pivoting. A consequence is that block *LU* factorization is stable for a matrix A that is symmetric positive definite or point diagonally dominant by rows or columns as long as A is well-conditioned.

Keywords: block algorithm, LAPACK, level 3 BLAS, iterative refinement, *LU* factorization, backward error analysis, block diagonal dominance.

AMS(MOS) subject classifications. primary 65F05, 65F25, 65G05.

1 Introduction

Block methods in matrix computations are widely recognised as being able to achieve high performance on modern vector and parallel computers. Their performance benefits have been investigated by various authors over the last decade (see, for example, [11, 14, 15]), and in particular by the developers of LAPACK [1]. The rise to prominence of block methods has been accompanied by the development of the level 3 Basic Linear Algebra Subprograms (BLAS3)—a set of specifications of Fortran primitives for various types of matrix multiplication, together with solution

(2)
accepted

Minimization of the norm, the norm of the inverse and the condition number of a matrix by completion

L. Elsner C. He V. Mehrmann

Fakultät für Mathematik

Universität Bielefeld

Postfach 8640

D–4800 Bielefeld, FRG

Abstract

We study the problem of minimizing the norm, the norm of the inverse and the condition number with respect to the spectral norm, when a submatrix of a matrix can be chosen arbitrarily. For the norm minimization problem we give a different proof than that given by Davis/Kahan/Weinberger. This new approach can then also be used to characterize the completions that minimize the norm of the inverse. For the problem of optimizing the condition number we give a partial result.

Keywords: condition number, norm of a matrix, matrix completion, dilation theory, robust regularization of descriptor systems

1 Introduction

We study the following optimization problem: Given integers $n, m, N > n, m$ and matrices $A \in \mathbb{C}^{n,m}, B \in \mathbb{C}^{n,N-m}, C \in \mathbb{C}^{N-n,m}$, find $X \in \mathbb{C}^{N-n,N-m}$ such that the matrix

$$W(X) = \begin{bmatrix} A & B \\ C & X \end{bmatrix} \tag{1.1}$$

satisfies

$$\begin{aligned} \operatorname{cond}(W(X)) &= \min_{Z \in \mathbb{C}^{N-n,N-m}} \{\|W(Z)\| \|W(Z)^{-1}\|\} \\ &= \min_{Z \in \mathbb{C}^{N-n,N-m}} \{\operatorname{cond}(W(Z))\}. \end{aligned} \tag{1.2}$$

(3)
accepted

Orthogonal Projection and Total Least Squares

Ricardo D. Fierro and James R. Bunch

Department of Mathematics
University of California, San Diego
La Jolla, CA 92093

Dedicated to Beresford Parlett and William Kahan
on the occasion of their 60th birthdays.

ABSTRACT. When the overdetermined system of linear equations $AX \approx B$ has no solution, compatibility may be restored by an orthogonal projection method. The idea is to determine an orthogonal projection matrix P by some method M such that $[\tilde{A}\ \tilde{B}] = P[A\ B]$, and $\tilde{A}X \approx \tilde{B}$ is compatible. Since both A and B are allowed to be perturbed, denote by X_M the minimum norm total least squares solution by method M (the TLS-M solution) to $\tilde{A}X = \tilde{B}$. In this paper conditions for compatibility of the lower rank approximation and subspace properties of $\tilde{A}$ in relation to the nearest rank-k matrix to A are discussed. We find upper and lower bounds for the difference in the TLS-M solution X_M and the SVD-based TLS-SVD solution X_{SVD} and also provide a perturbation result for the SVD-based TLS method. These results suggest a new algorithm for computing a total least squares solution based on a rank revealing QR factorization and subspace refinement. Numerical simulations are included to illustrate the conclusions.

(6)
accepted, after final revision

APPROXIMATE SOLUTIONS AND EIGENVALUE BOUNDS FROM KRYLOV SUBSPACES

CHRIS C. PAIGE
Computer Science
McGill University
Montreal, H3A 2A7
Canada

BERESFORD N. PARLETT
Mathematics Department
University of California
Berkeley, CA 94720

HENK A. VAN DER VORST
Mathematical Institute
University of Utrecht
Budapestlaan 6
Utrecht, the Netherlands

The first and third authors dedicate this to Beresford Parlett and Velvel Kahan — not only in recognition of their exceptional abilities in general, and contributions to this topic in particular, but also for the friendship, ideas and encouragement they have given us, and above all for the impeccable character and style they have both consistently exhibited.

ABSTRACT

Approximations to the solution of a large sparse symmetric system of equations are considered. The conjugate gradient and minimum residual approximations are studied without reference to their computation. Several different bases for the associated Krylov subspace are used, including the usual Lanczos basis. The zeros of the iteration polynomial for the minimum residual approximation are given the name *harmonic* Ritz values and are characterized in several ways and, in addition, attractive convergence properties are established. The connection of these harmonic Ritz values to Lehmann's optimal intervals for eigenvalues of the original matrix appears to be new.

Keywords: Krylov subspace; Lanczos process; symmetric matrix; conjugate gradients; minimum residual; Lehmann intervals.

(7)
accepted

EFFICIENT REDUCTION ALGORITHMS FOR BORDERED BAND MATRICES

SABINE VAN HUFFEL
ESAT Laboratory, Department of Electrical Engineering, Katholieke Universiteit Leuven, 3001 Heverlee, Belgium.

HAESUN PARK
Computer Science Department, University of Minnesota, Minneapolis, MN 55455, U.S.A.

Dedicated to Beresford Parlett and William Kahan on the occasion of their 60th birthdays.

Abstract

Various plane rotation patterns are presented, which provide stable algorithms for reducing a b-band matrix bordered by p rows and/or columns to $(b+p)$-band form. These schemes generalize previously presented $O(N^2)$ reduction algorithms for matrices of order N, $b = 1$, and $p = 1$ to the reduction of more general b-band, p-bordered matrices where $b \geq 1$ and $p \geq 1$. Moreover, by splitting the matrix into two similarly structured submatrices and chasing nonzeros to the corners in two directions, the newly proposed patterns reduce the number of required rotations and hence the computational cost by one half compared to the other existing one-way chasing algorithms. Symmetric, as well as more general matrices, are considered. An example of the first type is the symmetric arrowhead matrix that arises in solving inverse eigenvalue problems. Examples of the second type are found in updating the singular value decomposition (SVD) and the partial SVD.

Keywords: arrowhead matrix, band matrix, inverse eigenvalue problem, Givens rotations, singular value decomposition, updating.

1 Introduction

A matrix of order N is said to be *b-band* or *b-diagonal* if it consists of b diagonals. If p trailing rows are attached, then the resulting matrix $A_{(N+p)\times N}$ is called a

(9)
accepted

Some Inverse Eigenproblems for Jacobi and Arrow Matrices

CARLOS F. BORGES
Code Ma/Bc
Naval Postgraduate School
Monterey, CA 93943

RUGGERO FREZZA
DEI, Univ. di Padova
via Gradenigo 6/A
35131 PADOVA - ITALY

W.B. GRAGG
Code Ma/Gr
Naval Postgraduate School
Monterey, CA 93943

ABSTRACT

We consider the problem of reconstructing Jacobi matrices and real symmetric arrow matrices from two eigenpairs. Algorithms for solving these inverse problems are presented. We show that there are reasonable conditions under which this reconstruction is always possible. Moreover, it is seen that in certain cases reconstruction can proceed with little or no cancellation. The algorithm is particularly elegant for the tridiagonal matrix associated with a bidiagonal singular value decomposition.

Keywords: Jacobi matrix, Arrow matrix, inverse problem.

1 Introduction

We consider the problem of reconstructing Jacobi matrices and real symmetric arrow matrices from two eigenpairs. The algorithms we present for solving these inverse problems are simple, and useful for constructing test matrices for eigenproblems. The algorithm for reconstructing Jacobi matrices was applied to the problem of model identification of reciprocal stochastic processes in [3].

2 Jacobi matrices

Let T be an unreduced real symmetric tridiagonal matrix (i.e. a Jacobi matrix)

$$T = \begin{bmatrix} \alpha_1 & \beta_1 & & & \\ \beta_1 & \alpha_2 & \beta_2 & & \\ & \beta_2 & & \ddots & \\ & & \ddots & & \beta_{n-1} \\ & & & \beta_{n-1} & \alpha_n \end{bmatrix} \tag{2.1}$$

with $\beta_i > 0$ for $i = 1, 2, \ldots, n-1$. We use the notation introduced in [13] and let $\mathbf{UST}(n)$ denote the set of $n \times n$ real unreduced symmetric tridiagonal matrices, and let $\mathbf{UST}_+(n)$ denote that subset of $\mathbf{UST}(n)$ with positive β_i.

(2)
accepted

ORDERED MODIFIED GRAM-SCHMIDT ORTHOGONALIZATION

BAHRAM NOUR-OMID
Scopus Technologies Inc.
1900 Powell St., Suite 900
Emeryville, California, U.S.A. 94608

W. SCOTT DUNBAR
Acres International Ltd.
5259 Dorchester Road
Niagara Falls, Ontario, Canada L2E 6W1

and

ALLAN D. WOODBURY
Department of Geological Engineering
Room 314, Engineering Building
University of Manitoba
Winnipeg, Manitoba, Canada R3T 2N2

ABSTRACT

The modified Gram-Schmidt algorithm is used to orthogonalize a vector, r, against a set of orthogonal vectors, u and v. By means of an error analysis, it is shown that the order in which the vectors u and v are processed can have a significant effect on the state of orthogonality of the final set of vectors. An ordered modified Gram-Schmidt algorithm is presented in which the order of orthogonalization is determined by computing the magnitude of the components of r along the vectors of an orthogonal basis and the orthogonalization proceeds in the order of descending component magnitude. The benefit of the ordered algorithm is an improved orthogonality state. Examples involving the use of the Arnoldi algorithm, in which the modified Gram-Schmidt algorithm is used to obtain a set of orthogonal basis vectors, demonstrate the effect of the order in which the orthogonalizations are performed and that faster convergence can result when the ordered algorithm is used to obtain an orthogonal set of basis vectors.

Keywords: Gram-Schmidt orthogonalization, unsymmetric matrices, Krylov subspace, Arnoldi algorithm

1. Introduction

In this paper the problem of orthogonalizing a given vector, $\mathbf{r}$, against a vector basis, $\{\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_m\}$, which is orthogonal to working precision, is addressed. In practical situations, the intent is to augment the basis with an orthogonalized and normalized $\mathbf{r}$. The modified Gram-Schmidt (MGS) algorithm is typically used for this purpose. The focus of the paper is on the order in which the vectors $\mathbf{q}_i$ are processed in MGS. Although in exact arithmetic the result is independent of order in which orthogonalizations are performed, the following illustrative example and error analysis show that in finite precision the order of orthogonalizations can have a significant effect on the state of orthogonality of the final set of vectors.

(131)
excerpted

DYNAMICAL ANALYSIS OF NUMERICAL SYSTEMS

STEVE BATTERSON

Department of Mathematics and Computer Science
Emory University
Atlanta, GA 30322 USA
sb@mathcs.emory.edu

ABSTRACT

For many years techniques from numerical analysis have been applied fruitfully to the study of dynamical systems. In this paper it is shown that the theory of dynamical systems may be applied to certain computational problems. In particular the question of global convergence of various QR algorithms can be reduced to the study of certain vector iterations derived from Schur forms of matrices. The technique is illustrated in determining the convergence behavior of normal Hessenberg matrices under the Francis and multishift QR iterations.

The purpose of this paper is to demonstrate how techniques from dynamical systems can be applied to certain problems in numerical analysis. Iterative algorithms such as Newton's method, the power method, and the QR algorithms may be viewed as dynamical systems. In this setting a variety of dynamical techniques address numerical issues of convergence. We will place previous Rayleigh quotient iteration work of Parlett and Kahan into this framework and explain how it generalizes to our results on the global convergence of Francis shifted QR.

The author was originally trained in the theory of dynamical systems and later developed an interest in eigenvalue computation. This paper is directed at a numerical analysis audience. It is the author's intention to persuade the audience of a fruitful interdisciplinary connection between his interests.

1. The Iterative Algorithm-Dynamical Systems Paradigm

This section answers the following three questions:

(i) What is a dynamical system?

(ii) Describe a central problem in dynamical systems.

(iii) What computational questions can be placed in the setting of question ii?

While there are several possibilities we define a (discrete) dynamical system to be a map from a topological space M into itself. In most cases the space will have additional structure (e.g. manifold) and the map have some degree of smoothness. For the present

(5)

THE CONVERGENCE OF KRYLOV SUBSPACE METHODS FOR LARGE UNSYMMETRIC LINEAR SYSTEMS

ZHONGXIAO JIA

Fakultät für Mathematik

Universität Bielefeld

email: jia@math5.mathematik.uni-bielefeld.de

4800 Bielefeld 1, Germany

Abstract

The convergence of both FOM and GMRES for solving large unsymmetric linear systems is still a hard problem. Saad and some others have partially solved it, mainly for the case where the coefficient matrix A is diagonalizable and its spectrum lies in the open right (left) half plane. In this paper, we will focus on the convergence problem of both FOM and GMRES in the case where the coefficient matrix A is defective. When the spectrum of A either lies in the open right (left) half plane, namely, $(A + A^H)/2$ is positive (negative) definite, or is on the real axis, we establish the related theoretical error bounds and reveal the intrinsic relationships between the convergence speed and the spectrum of A. Besides, we make the two more notes for the case where A is diagonalizable. The results show that FOM and GMRES are likely to converge slowly when either A is defective or the Jordan basis of A is ill-conditioned. Meanwhile, we point out that most results derived in this paper are also suitable for the convergence analysis of some other methods, e.g. GCR etc.

Keywords: unsymmetric linear systems, Krylov subspace, the Chebyshev polynomials, diagonalizable, defective, derivatives.

(10)
with minor
revisions

KRYLOV SUBSPACE APPROXIMATION OF EIGENPAIRS AND MATRIX FUNCTIONS IN EXACT AND COMPUTER ARITHMETIC

Vladimir DRUSKIN[(*)], Leonid KNIZHNERMAN[(**)]

[(*)] Schlumberger Doll Research, Old Quarry Road, Ridgefield, CT 06877-4108, the U.S.A. Telephone (203) 431-54-71, telefax (203) 438-38-19, E-mail DRUSKIN@sdr.slb.com.

[(**)] Central Geophysical Expedition, Russia, 123298, Moscow, Narodnogo Opolcheniya St., house 40, build. 3. Telephone (095) 192-81-56, telex 411758 ATLAS SU, E-mail KNIZHNER@seismos.msk.su.

Abstract.

Many researchers are now working on computing the product of a matrix function and a vector, using approximations in the Krylov subspace. We review our results on the analysis of one interpretation of that approach for symmetric matrices, which we call Spectral Lanczos Decomposition Method (SLDM).

There has been proved a general convergence estimate, relating SLDM error bounds with those obtained through approximation of the matrix function by a piece of its Chebyshev series. Thus, we arrived at effective estimates for matrix functions, arising when solving parabolic, hyperbolic and elliptic partial derivative equations. We concentrate on the parabolic case, where we obtain estimates, indicative of the superconvergence of SLDM. For this case a symbiosis of SLDM and splitting is also considered and some numerical results are presented.

Further, we implement our general estimates for getting convergence bounds of Lanczos approximations to eigenvalues in the internal part of the spectrum that, unlike Kaniel-Saad estimates, are independent of the set of eigenvalues, located between the required one and the nearest spectrum bound.

We consider an extension of our general estimate for the case of the simple Lanczos method (without reorthogonalization) in the computer arithmetic, which shows that for a moderate dimension of the Krylov subspace the results, proved for the exact arithmetic, are stable to roundoff.

We also touch upon the Arnoldi method.

§ 1. Introduction

This paper is a review of the authors' results on analysis of Krylov approximations to a matrix function by a vector and of the Lanczos method, partially published in the former Soviet Union.

Let A be a symmetric nxn matrix. It is the Lanczos method that has become accepted as a powerful tool for finding eigenpairs of A, at least, if A is large and sparse. However, the Lanczos method has got another application. Namely, let f be a function, defined on the spectral interval of A, and $\varphi\in\mathbb{R}^n$. Let us consider computation of the vector

$$u=f(A)\varphi. \tag{1}$$

Solving systems of linear equations is a problem of this

(14)
with minor revision

Solving Secular Equations Stably and Efficiently

Ren-Cang Li
Department of Mathematics
University of California at Berkeley
Berkeley, California 94720

October 16, 1992

Dedicated to B. N. Parlett and W. Kahan on the occasion of their 60th birthdays

Abstract

The way described in [2] to solve secular equations, which is the kernel of the divide and conquer method for solving symmetric tridiagonal matrix eigensystem, is analyzed. New and more efficient approaches are proposed. Some practical issues concerning implementations of these approaches are tackled in detail.

(15)
wind
research

Progress in the Numerical Solution of the Nonsymmetric Eigenvalue Problem*

Dedicated to W. Kahan and B. N. Parlett on the occasion of their 60th birthdays

Zhaojun Bai †

February 22, 1993

Abstract

With the growing demands from disciplinary and interdisciplinary fields of science and engineering for the numerical solution of the nonsymmetric eigenvalue problem, competitive new techniques have been developed for solving the problem. In this paper, we examine the start-of-the-art of the algorithmic techniques and the software scene for the problem. Some current developments are also outlined.

1 Introduction

Over several years working on the LAPACK project [2], and on algorithm and software development of the nonsymmetric eigenvalue problem and communication with a variety of users who work in diverse fields involving scientific computing, the author has seen a growing demand for the numerical solution of the nonsymmetric eigenvalue problems. Meanwhile, in numerical analysis community, since Parlett's exploratory review paper entitled "The Software Scene in the Extraction of Eigenvalues from Sparse Matrices" [34] nearly one decade ago, and with the successful development of the symmetric eigenvalue problem, many new numerical methods and analysis have been developed for the nonsymmetric eigenproblem. The aim of this essay is to review the origins of the problem and the progress of the numerical techniques over the past decade, and to share our view and expertise within scientific computing community.

The survey is by no means complete. One reason for this is that relevant articles may be found scattered throughout the scientific and engineering literature, and the task of tracking them all down is impossibly large. The author apologizes for the ignorance of some important contributions to the problem that are not mentioned here. A new book by Saad [42] is an elegant source for studying the start-of-the-art in large eigenproblem techniques. This review will only focus on the nonsymmetric eigenvalue problem in the aspects of its origins, *algorithmic* techniques, software scene and work in progress.

As defined by Parlett [34] one decade ago, there are two different user groups for the eigenproblem. One is called the *intensive* user group and the other called the *sporadic*

*This work was supported in part by NSF grant ASC-9102963 and in part by the Applied and Computational Mathematics Program, Defense Advanced Research Projects Agency, under contract DM28E04120.

†Department of Mathematics, University of Kentucky, Lexington, KY 40506, U.S.A, e-mail address: bai@ms.uky.edu

(16)

Trust Regions Based on Conic Functions in Linear and Nonlinear Programming

J.L. Nazareth*

October, 1992

Dedicated to Professors B.N. Parlett and W. Kahan
on the occasion of their 60'th birthdays

Abstract

An optimization method is developed based on ellipsoidal trust regions that are defined by conic functions. It provides a powerful unifying theory from which can be derived a variety of interesting and potentially useful optimization algorithms, in particular, conjugate-gradient-like algorithms for nonlinear minimization and Karmarkar-like interior-point algorithms for linear programming.

1 Introduction

We propose a new optimization method based on the use of trust regions (Moré [7], Ye [13], Gonzaga [5]) defined by conic functions (Davidon [1]). Our trust regions are ellipsoidal with a distinguished axis of orientation. We investigate their use in the setting of nonlinear minimization where they lead to conjugate-gradient-like algorithms, and in the setting of linear programming where they lead to Karmarkar-like interior-point algorithms.

Conic-based ellipsoidal trust regions, a natural generalization of quadratic-based trust regions, are thus shown to provide both a powerful unifying theory and a means for formulating a variety of interesting and potentially useful optimization algorithms.

*Professor, Department of Pure and Applied Mathematics, Washington State University, Pullman, WA 99164-3113 and Affiliate Professor, Department of Applied Mathematics, University of Washington, Seattle, WA 98195. email: nazareth@wsumath.bitnet and nazareth@amath.washington.edu

(19)

under revision

OPTIMAL BACKWARD PERTURBATION BOUNDS FOR THE LINEAR LEAST SQUARES PROBLEM

Bertil Waldén, Rune Karlson, and Ji-guang Sun *

Department of Mathematics, Linköping University, S-581 83 Linköping, Sweden

Abstract

Let A be an $m \times n$ matrix, b be an m-vector, and $\tilde{x}$ be a purported solution to the problem of minimizing $\|b - Ax\|_2$. We consider the following open problem: to find the smallest perturbation E of A such that the vector $\tilde{x}$ exactly minimizes $\|b - (A + E)x\|_2$. This problem is completely solved when E is measured in the Frobenius norm. When instead using the spectral norm of E, easily computed upper and lower bounds are given, and the optimum is found under certain conditions.

AMS Subject Classifications: 65F20

Key words: linear least squares, backward perturbations

Throughout this paper we will use the following notation. $C^{m\times n}$ denotes the set of complex $m \times n$ matrices, $C^n = C^{n\times 1}$. I is the identity matrix, and 0 is the null matrix. A^T and A^H stand for the transpose and conjugate transpose of a matrix A, respectively. $A^\dagger$ denotes the Moore-Penrose inverse of A. $\| \|_2$ denotes the Euclidean vector norm and the spectral norm, and $\| \|_F$ denotes the Frobenius norm.

Let $A \in C^{m\times n}, b \in C^m$, and let $\tilde{x}$ be a purported solution to the problem of minimizing $\|b - Ax\|_2$. Stewart [3] discovered two perturbations E of A such that the vector $\tilde{x}$ exactly minimizes $\|b - (A + E)x\|_2$. Recently, Sun [5] found a perturbation E_* (see below (10)) and showed that $\|E_*\|_F$ is an improved backward perturbation bound, but it has not been proved that the scalar $\|E_*\|_F$ is the optimal backward perturbation bound in the meaning of the Frobenius norm.

This paper gives a solution to the following open problem: to find the smallest perturbation E of A such that the vector $\tilde{x}$ exactly minimizes $\|b - (A + E)x\|_2$ (ref. [1], [3], [4, p.160-163]). We use $\mathcal{E}$ to denote the set of all perturbation matrices E of A such that $\tilde{x}$ is an exact solution to $\min_x \|b - (A + E)x\|_2$. The expression of

*The research of this author was supported by the Swedish Natural Science Research Council.

(20)

On the Solution of Block Hessenberg Systems

G. W. Stewart

Dedicated to Velvel Kahan and Beresford Parlett

ABSTRACT

This paper describes a divide-and-conquer strategy for solving block Hessenberg systems. For dense matrices the method is as efficient as Gaussian elimination; however, because it works almost entirely with the original blocks, it is much more efficient for sparse matrices or matrices whose blocks can be generated on the fly. For Toeplitz matrices, the algorithm can be combined with the fast Fourier transform to give a new superfast algorithm.

1. Introduction

This paper was motivated by the attempt to find the steady-state of Markov chains of types $M/G/1$ and $G/M/1$ (see [8] for definitions and further details). The matrix of transition probabilities of a chain of type $M/G/1$ has the block upper Hessenberg form

$$M = \begin{pmatrix} B_0 & B_1 & B_2 & B_3 & B_4 & \cdots \\ C_0 & A_1 & A_2 & A_3 & A_4 & \cdots \\ 0 & A_0 & A_1 & A_2 & A_3 & \cdots \\ 0 & 0 & A_0 & A_1 & A_2 & \cdots \\ 0 & 0 & 0 & A_0 & A_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix}. \tag{1.1}$$

The transition matrices of chains of type $G/M/1$ are block lower Hessenberg matrices, formed in analogy with (1.1). Note that after the first row and column of (1.1) are deleted, the matrix becomes block Toeplitz.

We shall return to these chains at the end of this paper. For now we are going to consider the more general problem of solving the block upper Hessenberg

(11)

Bounds on Singular Values Revealed by QR Factorizations

C.-T. Pan
Department of Mathematical Sciences
Northern Illinois University
De Kalb, IL 60115-2888

Ping Tak Peter Tang*
Mathematics and Computer Science Division
Argonne National Laboratory
9700 South Cass Ave.
Argonne, IL 60439-4801

Abstract

We introduce a pair of dual concepts: pivoted blocks and reverse pivoted blocks. These blocks are the outcome of a special column pivoting strategy in QR factorization. Our main result is that under such a column pivoting strategy, the QR factorization of a given matrix can give tight estimates on any two a priori-chosen consecutive singular values of that matrix. In particular, a rank-revealing QR factorization is guaranteed when the two chosen consecutive singular values straddle a gap in the singular value spectrum that gives rise to the rank degeneracy of the given matrix. The pivoting strategy, called cyclic pivoting, can be viewed as a generalization of Golub's column pivoting and Stewart's reverse column pivoting. Numerical experiments confirm the tight estimates that our theory asserts.

AMS classification: 65F30, 15A23, 15A42, 15A15.

Key words and phrases: Singular value decomposition, rank-revealing QR factorization, cyclic column pivoting.

*This author was supported by the Applied Mathematical Sciences subprogram of the Office of Energy Research, U.S. Department of Energy, under Contract W-31-109-Eng-38.

On Parlett's matrix norm inequality

*Alan Edelman**
Walter F. Mascarenhas †

February, 1993

Abstract

We show that a certain matrix norm ratio studied by Parlett has a supremum that is $O(\sqrt{n})$ when the chosen norm is the Frobenius norm, while it is $O(\log n)$ for the two norm. This ratio arises in Parlett's analysis of the Cholesky decomposition. We are happy to dedicate this note to our friends Beresford and Velvel on the occasion of their sixtieth birthdays.

1 Introduction

This note bounds the quantity

$$\chi(n) \ = \sup \tau(U) \ \equiv \ \frac{\|U\|}{\|U^T + U + U^T U\|},$$

defined by Parlett [3] as the supremum over all (non-zero) upper triangular matrices U with diagonal entries $u_{ii} \geq -1$. In particular, we study $\chi_F(n)$ and $\chi_2(n)$, where the norms in $\chi(n)$ are taken to be the Frobenius norm and the two norm respectively. The quantity $\chi(n)$ arises in Parlett's analysis of the Cholesky decomposition. The term $U^T U$ in the denominator would be neglected by first order perturbation theory but, according to Parlett, it actually helps in the analysis.

An equivalent expression for $\chi(n)$ is

$$\chi(n) \ = \sup \rho(N) \equiv \frac{\|I - N\|}{\|I - NN^T\|},$$

where the supremum is over the set of n by n upper triangular matrices N with non-negative diagonals, excluding the identity matrix.

*Department of Mathematics and Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, edelman@math.berkeley.edu. Supported by the Applied Mathematical Sciences subprogram of the Office of Energy Research, U.S. Department of Energy under Contract DE-AC03-76SF00098.

†Dept. de Matematica, Universidade Estadual de Campinas, Caixa Postal 6065, Barao Geraldo, Campinas SP 13081, Brazil, walterm@dcc.unicamp.br

(18)
MATDP. [illegible]

REGIONS OF CONVERGENCE OF THE RAYLEIGH QUOTIENT ITERATION METHOD

RICARDO D. PANTAZIS* AND DANIEL B. SZYLD†

Dedicated to Professors W. Kahan and B. N. Parlett on the occasion of their sixtieth birthdays.

Abstract. Parlett and Kahan have shown in 1968 that for almost any initial vector in the unit sphere, the Rayleigh quotient iteration method converges to some eigenvalue. In this paper, the regions of the unit sphere which include all possible initial vectors converging to a specific eigenvalue are studied. It is shown that when the matrix is shifted, or multipied by a scalar, the regions do not change. In the case of three dimensions, these regions are completely characterized. It is shown experimentally that, for the three-dimensional case, the probability that an initial vector will lead to convergence to an interior eigenvalue is larger than the probability that it converges to an extreme one. It is conjectured that the same is true for matrices of any order. Experiments in higher dimensions are presented which conform with the conjecture.

Key words. Eigenvalues, eigenvectors, symmetric matrices, Rayleigh quotient iteration, convergence, basins of attraction.

AMS(MOS) subject classification. 65F15

1. Introduction. We study global convergence properties of the Rayleigh quotient iteration method for the solution of the symmetric generalized eigenvalue problem

$$Ax = \lambda Bx. \tag{1}$$

* Department of Computer Science, Duke University, Durham, North Carolina 27706-2591, USA (rdp@cs.duke.edu).

† Department of Mathematics, Temple University, Philadelphia, Pennsylvania 19122-2585, USA (szyld@euclid.math.temple.edu). The work of this author was supported by a Summer Research Fellowship at Temple University and by the National Science Foundation grant DMS-9201728.