

# CONFLICTS AND DECISIONS

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## Abstract

Conflicts are one of the most characteristic attributes of human nature and study of conflicts is of greatest importance both practically and theoretically. Conflict analysis and resolution play an important role in business, governmental, political and lawsuits disputes, labor-management negotiations, military operations and others. Many formal models of conflict situations have been proposed and studied.

In this paper we outline a new approach to conflict analysis, which will be illustrated by voting analysis in conflict situations.

## 1 Introduction

Conflicts are one of the most characteristic attributes of human nature and study of conflicts is of greatest importance both practically and theoretically. Conflict analysis and resolution play an important role in business, governmental, political and lawsuits disputes, labor-management negotiations, military operations and others. To this end formal models of conflict situations are necessary. Many theoretical models of conflict situations have been proposed and studied, e.g., Casti, 1989, Coombs et al, 1988, Maeda et al, 1999, Nakamura 1999, Pawlak, 1998 and Roberts, 1976.

Conflict analysis seems to be important for decision making. Rough set based decision support plays important role in decision theory, see e.g. Slowinski, 1995. In this paper we outline new approach to conflicts analysis, which will be illustrated by voting analysis in conflict situations.

We start our considerations with a very simple illustrative example. Next basic concepts of the proposed approach will be defined and studied.

## 2 An example

Consider a parliament containing 500 members grouped in four political parties denoted A, B, C and D. Suppose the parliament discussed certain issue (e.g. membership of the country in European Union) and the voting result is presented in column *voting* in Table 1, where +, 0 and – denoted *yes*, *abstention* and *no* respectively. The column *support* contains the number of voters for each option.

Table 1: Voting result

<i>Fact</i>	<i>Party</i>	<i>Voting</i>	<i>Support</i>
1	A	+	200
2	A	0	30
3	A	-	10
4	B	+	15
5	B	-	25
6	C	0	20
7	C	-	40
8	D	+	25
9	D	0	35
10	D	-	100

Our task is to find difference between parties in view of voting result. The difference can be interpreted as a degree of conflict between parties and can be expressed as a number between 0 and 1. To this end we employ some ideas given in [6], where the conflict relation is understood as distance relation in a certain metric space.

In what follows we will formulate the problem more precisely. We will start our consideration from the concept of an information system and a decision table.

### 3 Information systems and decision tables

An information system is a data table, whose columns are labeled by attributes, rows are labeled by objects of interest and entries of the table are attribute values.

Formally, by an *information system* we will understand a pair  $S = (U, A)$ , where  $U$  and  $A$ , are finite, nonempty sets called the *universe*, and the set of *attributes*, respectively. With every attribute  $a \in A$  we associate a set  $V_a$ , of its *values*, called the *domain* of  $a$ . Any subset  $B$  of  $A$  determines a binary relation  $I(B)$  on  $U$ , which will be called an *indiscernibility relation*, and defined as follows:  $(x, y) \in I(B)$  if and only if  $a(x) = a(y)$  for every  $a \in B$ , where  $a(x)$  denotes the value of attribute  $a$  for element  $x$ . Obviously  $I(B)$  is an equivalence relation. The family of all equivalence classes of  $I(B)$ , i.e., a partition determined by  $B$ , will be denoted by  $U/I(B)$ , or simply by  $U/B$ ; an equivalence class of  $I(B)$ , i.e., block of the partition  $U/B$ , containing  $x$  will be denoted by  $B(x)$ .

If  $(x, y)$  belongs to  $I(B)$  we will say that  $x$  and  $y$  are *B-indiscernible* (*indiscernible with respect to B*). Equivalence classes of the relation  $I(B)$  (or blocks of the partition  $U/B$ ) are referred to as *B-elementary sets* or *B-granules*.

If we distinguish in an information system two disjoint classes of attributes, called *condition* and *decision attributes*, respectively, then the system will be called a *decision table* and will be denoted by  $S = (U, C, D)$ , where  $C$  and  $D$  are disjoint sets of condition and decision attributes, respectively.

Thus the decision table determines decisions which must be taken, when some conditions are satisfied. In other words each row of the of the decision table specifies a decision rule which determines decisions in terms of conditions.

An example of a decision table is given in Table 1. In the decision table the only condition attribute is *Party*, whereas the decision attribute is *Voting*. Each row of the table determines a decision rule.

## 4 Decision rules and decision algorithms

Every decision table describes decisions determined, when some conditions are satisfied. In other words each row of the decision table specifies a decision rule which determines decisions in terms of conditions.

Let us describe decision rules more exactly.

Let  $S = (U, C, D)$  be a decision table. Every  $x \in U$  determines a sequence  $c_1(x), \dots, c_n(x), d_1(x), \dots, d_m(x)$  where  $\{c_1, \dots, c_n\} = C$  and  $\{d_1, \dots, d_m\} = D$ .

The sequence will be called a *decision rule induced by  $x$*  (in  $S$ ) and denoted by  $c_1(x), \dots, c_n(x) \rightarrow d_1(x), \dots, d_m(x)$  or in short  $C \rightarrow_x D$ .

The number  $supp_x(C, D) = |C(x) \cap D(x)|$  will be called the *support* of the decision rule  $C \rightarrow_x D$  and the number

$$\sigma_x(C, D) = \frac{supp_x(C, D)}{|U|},$$

will be referred to as the *strength* of the decision rule  $C \rightarrow_x D$ . With every decision rule  $C \rightarrow_x D$  we associate the *certainty factor* of the decision rule, denoted  $cer_x(C, D)$  and defined as follows:

$$cer_x(C, D) = \frac{|C(x) \cap D(x)|}{|C(x)|} = \frac{supp_x(C, D)}{|C(x)|} = \frac{\sigma_x(C, D)}{\pi(C(x))},$$

where  $\pi(C(x)) = \frac{|C(x)|}{|U|}$ .

The certainty factor may be interpreted as conditional probability that  $y$  belongs to  $D(x)$  given  $y$  belongs to  $C(x)$ , symbolically  $\pi_x(D|C)$ .

If  $cer_x(C, D) = 1$ , then  $C \rightarrow_x D$  will be called a *certain decision rule* in  $S$ ; if  $0 < cer_x(C, D) < 1$  the decision rule will be referred to as an *uncertain decision rule* in  $S$ .

Besides, we will also use a *coverage factor* of the decision rule, denoted  $cov_x(C, D)$  and defined as

$$\begin{aligned} cov_x(C, D) &= \frac{|C(x) \cap D(x)|}{|D(x)|} = \frac{supp_x(C, D)}{|D(x)|} = \\ &= \frac{\sigma_x(C, D)}{\pi(D(x))}, \end{aligned}$$

where  $\pi(D(x)) = \frac{|D(x)|}{|U|}$ .

Similarly

$$cov_x(C, D) = \pi_x(C|D).$$

The strength, certainty and the coverage factors for Table 1 are given in Table 2.

Table 2: Certainty and coverage factors

<i>Fact</i>	<i>Strength</i>	<i>Certainty</i>	<i>Coverage</i>
1	0.40	0.833	0.833
2	0.06	0.125	0.623
3	0.02	0.042	0.104
4	0.03	0.375	0.353
5	0.05	0.625	0.235
6	0.04	0.333	0.412
7	0.08	0.667	0.057
8	0.05	0.156	0.143
9	0.07	0.219	0.229
10	0.20	0.625	0.571

## 5 Flow graphs and decision tables

With every decision table we associate a *flow graph*, i.e., a directed, connected, acyclic graph defined as follows: to every decision rule  $C \rightarrow_x D$  we assign a *directed branch*  $x$  connecting the *input node*  $C(x)$  and the *output node*  $D(x)$ . Strength of the decision rule represents a *throughflow* of the corresponding branch. Thus branches of the flow graph connect  $C$ -granules and  $D$ -granules of the graph.

More about flow graphs can be found in [7]

The flow graph associated with Table 1 is shown in Figure 1.

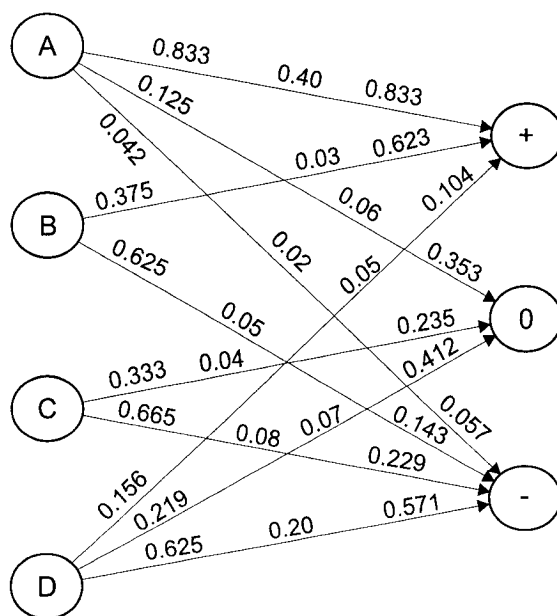


Figure 1: Flow graph

## 6 Conflict space

With every decision table having one  $n$ -valued decision attribute we can associated  $n$ -dimensional Euclidean space, where values of the decision attribute determine  $n$  axis of the space and condition attribute values (equivalence classes) determine point of the space. Strengths of decision rules are to be understood as coordinates of corresponding points. For example, for Table 1 coordinates of point A are (0.40, 0.06, 0.02) (see Figure 1). Distance  $\delta(x, y)$  between points  $x$  and  $y$  is defined as usual as

$$\delta(x, y) = \sqrt{(x_+ - y_+)^2 + (x_0 - y_0)^2 + (x_- - y_-)^2}$$

where  $x = (x_+, x_0, x_-)$ ,  $y = (y_+, y_0, y_-)$  and  $x_i, y_i$  ( $i = +, 0, -$ ) denote strength of corresponding decision rules.

Conflict space for Table 1 is shown in Figure 2.

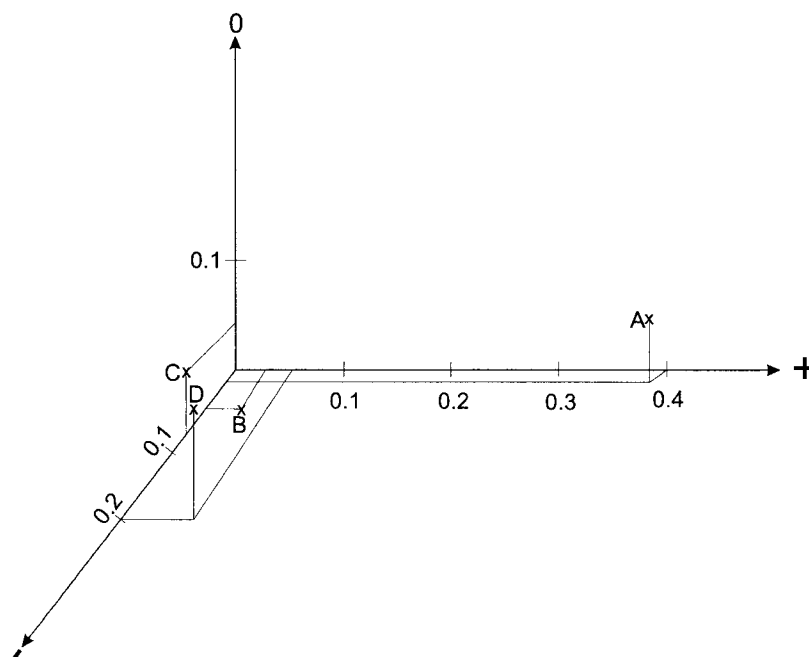


Figure 2: Conflict space

Distance between points can be also presented in a form of a graph as shown in Fig. 3.

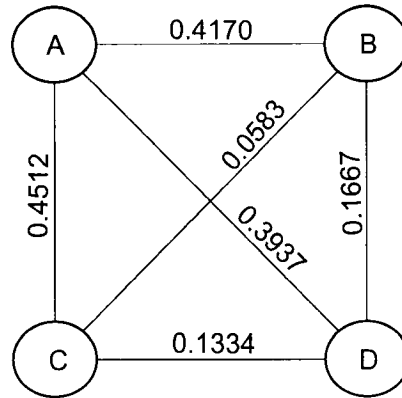


Figure 3: Conflict graph

## 7 Conclusions

In this paper we have shown a new approach to conflict analysis based on the concept of conflict space, determined by voting results of conflicted parties.

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