Conflicts and Negotations

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Abstract. Conflicts analysis and resolution play an important role in business, governmental, political and lawsuits disputes, labor-management negotiations, military operations and others. In this paper we show how the conflict situation and development can be represented and studied by means of conflict graphs. An illustration of the introduced concepts by the Middle East conflict is presented.

Keywords: Conflicts analysis; Conflict resolution; Decisions analysis; Rough sets.

1 Introduction

Conflict analysis and resolution play an important role in many domains [1,2,5,6,11,12,13] and stimulated research on mathematical models of conflict situations [1,3,4,7,8,10,11].

This paper is devoted to conflict analysis.

We start our consideration by presenting basic ideas of conflict theory, proposed in [8,10].

Next we introduce conflict graphs to represent conflict structure. These graphs can be very useful to study coalitions and conflict evolution.

2 Anatomy of Conflicts

In a conflict at least two parties, called *agents*, are in dispute over some *issues*. In general the agents may be individuals, groups, companies, states, political parties etc.

Before we start formal considerations let us first consider an example of the Middle East conflict, which is taken with slight modifications from [1].

The example does not necessarily reflect present-day situation in this region but is used here only as an illustration of the basic ideas considered in this paper.

In this example there are six agents

1 – Israel,
2 – Egypt,

G. Wang et al. (Eds.): RSKT 2006, LNAI 4062, pp. 12–27, 2006.

3 – Palestinians,
4 – Jordan,
5 – Syria,
6 – Saudi Arabia,

and five issues

a – autonomous Palestinian state on the West Bank and Gaza,
b – Israeli military outpost along the Jordan River,
c – Israeli retains East Jerusalem,
d – Israeli military outposts on the Golan Heights,
e – Arab countries grant citizenship to Palestinians who choose to remain within their borders.

The relationship of each agent to a specific issue can be clearly depicted in the form of a table, as shown in Table 1.

In the table the attitude of six nations of the Middle East region to the above issues is presented: -1 means, that an agent is against, 1 means favorable and 0 neutral toward the issue. For the sake of simplicity we will write $-$ and $+$ instead of -1 and 1 respectively.

Table 1. Data table for the Middle East conflict

U	a	b	c	d	e
1	$-$	$+$	$+$	$+$	$+$
2	$+$	0	$-$	$-$	$-$
3	$+$	$-$	$-$	$-$	0
4	0	$-$	$-$	0	$-$
5	$+$	$-$	$-$	$-$	$-$
6	0	$+$	$-$	0	$+$

Each row of the table characterizes uniquely the agent, by his approach to the disputed issues.

In conflict analysis primarily we are interested in finding the relationship between agents taking part in the dispute, and investigate what can be done in order to improve the relationship between agents, or in other words how the conflict can be resolved.

3 Conflicts and Information Systems

Tables as shown in the previous section are known as *information systems*. An information system is a table rows of which are labeled by *objects* (*agents*), columns – by *attributes* (*issues*) and entries of the table are *values of attributes* (*opinions, beliefs, views, votes, etc.*), which are uniquely assigned to each agent and attribute, i.e. each entry corresponding to row x and column a represents opinion of agent x about issue a.

Formally an *information system* can be defined as a pair $S = (U, A)$, where U is a nonempty, finite set called the *universe*; elements o U will be called *objects* (*agents*) and A is a nonempty, finite set of *attributes* (*issues*) [9].

Every attribute $a \in A$ is a total function $a : U \to V_a$, where V_a - is the set of *values* of a, called the *domain* of a; elements of V_a will be referred to as *opinions*, and $a(x)$ is opinion of agent x about issue a.

The above given definition is general, but for conflict analysis we will need its simplified version, where the domain of each attribute is restricted to three values only, i.e. $V_a = \{-1, 0, 1\}$, for every a, meaning *against, neutral* and *favorable* respectively. For the sake of simplicity we will assume $V_a = \{-, 0, +\}$. Every information system with the above said restriction will be referred to as a *situation*.

An information system contains *explicit* information about the attitude of each agent to issues being considered in the debate, and will be used to derive various *implicit* information, necessary to conflicts analysis.

In order to express relations between agents we define three basic binary relations on the universe: *conflict, neutrality* and *alliance*. To this end we need the following auxiliary function:

$$\phi_a(x, y) = \begin{cases} 1, & \text{if } a(x)a(y) = 1 \text{ or } \quad x = y, \\ 0, & \text{if } a(x)a(y) = 0 \text{ and } x \neq y, \\ -1, & \text{if } a(x)a(y) = -1. \end{cases}$$

This means that, if $\phi_a(x, y) = 1$, agents x and y have the same opinion about issue a (are *allied* on a); $\phi_a(x, y) = 0$ means that at least one agent x or y has neutral approach to issue a (is *neutral* on a), and $\phi_a(x, y) = -1$, means that the two agents have different opinions about issue a (are in *conflict* on a).

In what follows we will define three basic relations R_a^+, R_a^0 and R_a^- over U^2 called *alliance, neutrality* and *conflict* relations respectively, and defined as follows:

$$R_a^+(x, y) \text{ iff } \phi_a(x, y) = 1,$$

$$R_a^0(x, y) \text{ iff } \phi_a(x, y) = 0,$$

$$R_a^-(x, y) \text{ iff } \phi_a(x, y) = -1.$$

It is easily seen that the alliance relation has the following properties:

(i) $R_a^+(x, x)$,
(ii) $R_a^+(x, y)$ implies $R_a^+(y, x)$,
(iii) $R_a^+(x, y)$ and $R_a^+(y, z)$ implies $R_a^+(x, z)$,

i.e., R_a^+ is an *equivalence* relation for every a. Each equivalence class of alliance relation will be called *coalition* on a. Let us note that the condition (iii) can be expressed as "friend of my friend is my friend".

For the conflict relation we have the following properties:

(iv) non $R_a^-(x, x)$,
(v) $R_a^-(x, y)$ implies $R_a^-(y, x)$,

(vi) $R_a^-(x,y)$ and $R_a^-(y,z)$ implies $R_a^+(x,z)$,
(vii) $R_a^-(x,y)$ and $R_a^+(y,z)$ implies $R_a^-(x,z)$.

Conditions (vi) and (vii) refer to well known sayings “enemy of my enemy is my friend” and “friend of my enemy is my enemy”.

For the neutrality relation we have:

(viii) none $R_a^0(x,x)$,
(ix) $R_a^0(x,y) = R_a^0(y,x)$ (symmetry).

Let us observe that in the conflict and neutrality relations there are no coalitions.

The following property holds $R_a^+ \cup R_a^0 \cup R_a^- = U^2$ because if $(x,y) \in U^2$ then $\Phi_a(x,y) = 1$ or $\Phi_a(x,y) = 0$ or $\Phi_a(x,y) = -1$ so $(x,y) \in R_a^+$ or $(x,y) \in R_a^0$ or $(x,y) \in R_a^-$. All the three relations R_a^+, R_a^0 and R_a^- are pairwise disjoint, i.e., every pair of objects (x,y) belongs to exactly one of the above defined relations (is in conflict, is allied or is neutral).

For example, in the Middle East conflict Egypt, Palestinians and Syria are allied on issue a (autonomous Palestinian state on the West Bank and Gaza), Jordan and Saudi Arabia are neutral to this issue whereas, Israel and Egypt, Israel and Palestinians, and Israel and Syria are in conflict about this issue.

This can be illustrated by a *conflict* graph as shown in Figure 1.

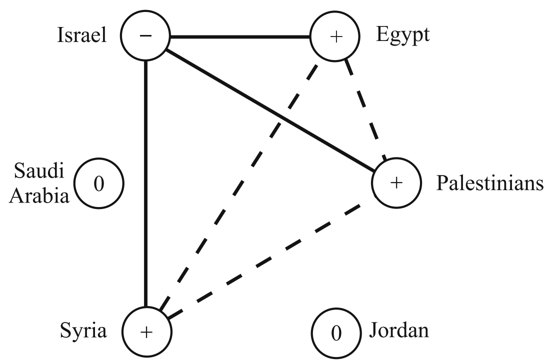

Fig. 1. Conflict graph for attribute a

Nodes of the graph are labelled by agents, whereas branches of the graph represent relations between agents. Besides, opinion of agents $(0, -, +)$ on the disputed issue is shown on each node. Solid lines denote conflicts, dotted line – alliance, and neutrality, for simplicity, is not shown explicitly in the graph.

Any conflict graph represents a set of facts. For example, the set of facts represented by the graph in Figure 1 consists of the following facts:

R_a^-(Israel, Egypt), R_a^-(Israel, Palestinians), R_a^-(Israel, Syria),
R_a^+(Egypt, Syria), R_a^+(Egypt, Palestinians), R_a^+(Syria, Palestinians),
$R_a^0(\text{Saudi Arabia}, x)$, $R_a^0(\text{Jordan}, x)$,
$R_a^0(x,x)$ for $x \in \{Israel, Egypt, Palestinians, Jordan, Syria, SaudiArabia\}$.

Below conflict graphs for the remaining attributes are shown.

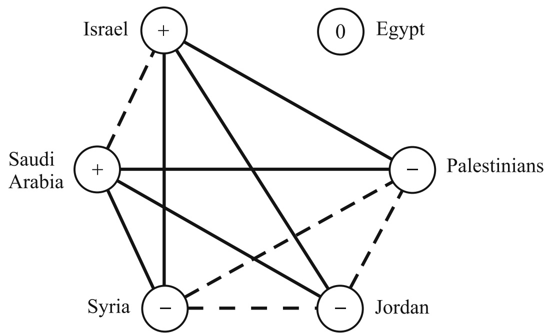

Fig. 2. Conflict graph for attribute b

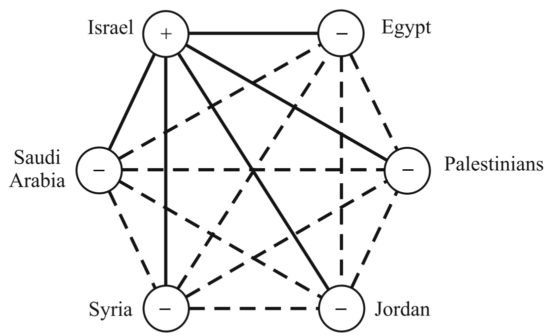

Fig. 3. Conflict graph for attribute c

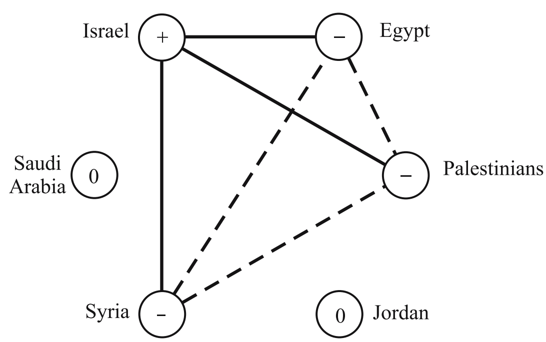

Fig. 4. Conflict graph for attribute d

4 Coalitions

Let $a \in A$. If there exists a pair (x, y) such that $R_a^-(x, y)$ we say that the attribute a is *conflicting* (agents), otherwise the attribute is *conflictless*. The following property is obvious.

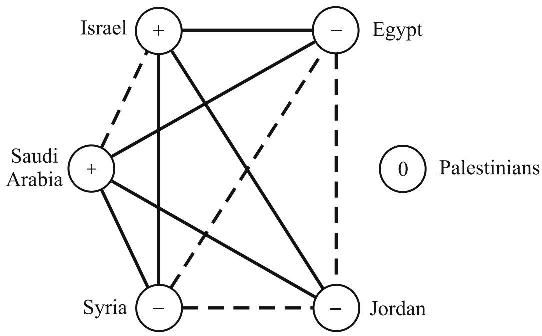

Fig. 5. Conflict graph for attribute e

If a is a conflicting attribute, then the relation R_a^+ has exactly two equivalence classes X_a^+ and X_a^-, where $X_a^+ = \{x \in U : a(x) = +\}$, $X_a^- = \{x \in U : a(x) = -\}$, $X_a^0 = \{x \in U : a(x) = 0\}$ and *$X_a^+ \cup X_a^- \cup X_a^0 = U$. Moreover $R_a^-(x, y)$ iff $x \in X_a^+$ and $y \in X_a^-$ for every $x, y \in U$.*

The above proposition says that if a is conflicting attribute, then all agents are divided into two coalitions (blocks) X_a^+ and X_a^-. Any two agents belonging to two different coalitions are in conflict, and the remaining (if any) agents are neutral to the issue a.

It follows from the proposition that the graph shown in Fig. 1 can be presented as shown in Fig. 6, called a *coalition* graph.

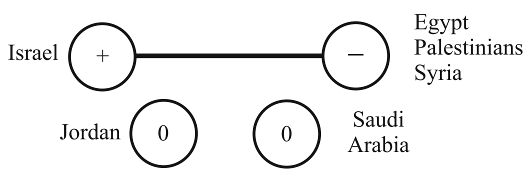

Fig. 6. Coalition graph for attribute a

Coalition graphs for the remaining attributes are given in Fig. 7.

Careful examination of coalition graphs (Fig. 7) generated by various attributes (issues) gives deep insight into structure of the Middle East conflict and offers many hints concerning negotiations between agents.

For example, let us observe that attribute c induces partition in which Israel is in conflict with all remaining agents, whereas attribute e leads to alliance of Israel and Saudi Arabia against Egypt, Jordan and Syria with Palestinians being neutral.

Ideas given in this section can be used to define *degree* of *conflict* caused by an issue a (attribute), defined as

$$Con(a) = \frac{|X_a^+| \cdot |X_a^-|}{[\frac{n}{2}] \cdot (n - [\frac{n}{2}])} = \frac{|R_a^-|}{[\frac{n}{2}] \cdot (n - [\frac{n}{2}])},$$

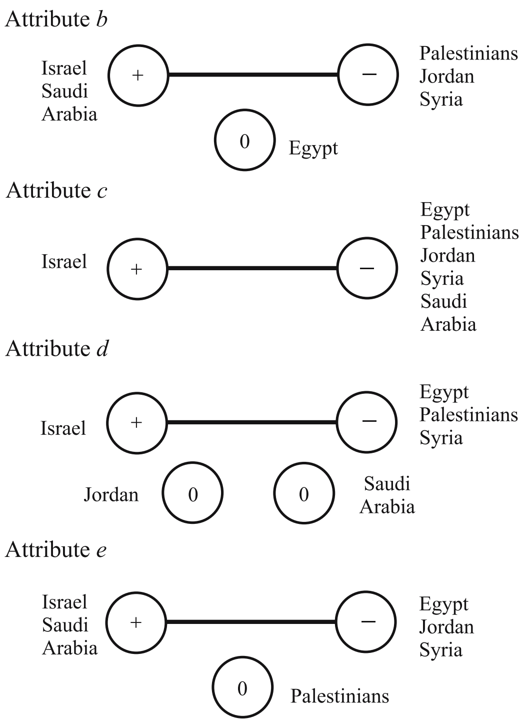

Fig. 7. Coalition graphs for attributes b, c, d and e

where $|X|$ denotes cardinality of X, n is the number of agents involved in the conflict (the number of nodes of the conflict graph) and $[\frac{n}{2}]$ denotes whole part of $\frac{n}{2}$.

For example, degree of conflict generated by the attribute b is $Con(b) = 2/3$, whereas the attribute c induces $Con(c) = 5/9$.

The degree of conflict induced be the set of attributes $B \subseteq A$, called *tension* generated by B is defined as

$$Con(B) = \frac{\sum_{a \in B} Con(a)}{|B|}.$$

Tension for the Middle East Conflict is $Con(A) \cong 0.51$.

5 Dissimilarities Between Agents

Starting point for negotiations are dissimilarities of view between agents.

In order to study the differences between agents we will use a concept of a *discernibility matrix* [14,15,16], which defines a discernibility relation between agents.

Let $S = (U, A)$, $B \subseteq A$. By a discernibility matrix of B in S, denoted $M_S(B)$, or $M(B)$, if S is understood, we will mean $n \times n$, $n = |U|$, matrix defined thus:

$$\delta_B(x, y) = \{a \in B : a(x) \neq a(y)\}.$$

Thus entry $\delta_B(x, y)$, in short, $\delta(x, y)$, is the set of all attributes which discern objects x and y.

The discernibility matrix for conflict presented in Table 2 is given below:

Table 2. Discernibility matrix for the Middle East conflict

	1	2	3	4	5	6
1						
2	a, b, c, d, e					
3	a, b, c, d, e	b, e				
4	a, b, c, d, e	a, b, d	a, d, e			
5	a, b, c, d, e	b	e	a, d		
6	a, c, d	a, b, e, d	a, b, d, e	b, e	a, b, d, e	

Each entry of the table shows all issues for which the corresponding agents have different opinions.

The discernibility matrix $M(B)$ assigns to each pair of objects x and y a subset of attributes $\delta(x, y) \subseteq B$, with the following properties:

i) $\delta(x, x) = \emptyset$,
ii) $\delta(x, y) = \delta(y, x)$,
iii) $\delta(x, z) \subseteq \delta(x, y) \cup \delta(y, z)$.

Property iii) results from the following reasoning. Let $a \notin \delta(x, y) \cup \delta(z, y)$. Hence $a(x) = a(z)$ and $a(z) = a(y)$, so $a(x) = a(y)$. W have $a \notin \delta(x, y)$.

The above properties resemble the well known properties of distance in a metric space, therefore δ may be regarded as *qualitative metric* and $\delta(x, y)$ as *qualitative distance*.

We see from Table 2 that the distance (dissimilarity) between agents 1 and 3 is the set $\delta(1, 3) = \{a, b, c, d, e\}$, whereas the distance between agents 2 and 5 is $\delta(2, 5) = \{b\}$.

We can also define distance between agents numerically, by letting

$$\rho_B(x, y) = \frac{|\delta_B(x, y)|}{|A|},$$

where $B \subseteq A$.

The following properties are obvious

1) $\rho_B(x, x) = 0$,
2) $\rho_B(x, y) = \rho_B(y, x)$,
3) $\rho_B(x, z) \leq \rho_B(x, y) + \rho_B(y, z)$,
thus the $\rho_B(x, y)$ is the distance between x and y.

For example, for the considered Middle East situation the distance function ρ_A is shown in Table 3.

Table 3. Distance function for the Middle East conflict

	1	2	3	4	5	6
1						
2	1					
3	1	0.4				
4	1	0.6	0.6			
5	1	0.2	0.2	0.4		
6	0.6	0.8	0.8	0.4	0.8	

6 Reduction of Attributes

Objects x and y are discernible in terms of the set of attributes $B \subseteq A$ (opinion) if they have different opinion on same attributes (issues) from B.

Before we start negotiations we have to understand better the relationship between different issues being discussed. To this end we define a concept of a *tr-reduct* of attributes, where $tr \in (0, 1]$ is a given threshold of discernibility [8,16].

A tr-reduct of A is any minimal subset B of A satisfying the following condition:

$$\rho_A(x, y) \geq tr \text{ if and only if } \rho_B(x, y) \geq tr.$$

One can consider objects x, y to be (B, tr)-discernible (in symbols $xDIS_{B,tr}y$) if and only if $\rho_B(x, y) \geq tr$ and (B, tr)-indiscernible (in symbols $xIND_{B,tr}y$) if and only if $\rho_B(x, y) < tr$. For any x let $\tau_{B,tr}(x)$ be a set $\{y : xIND_{B,tr}y\}$ called the (B, tr)-indiscernibility class of x. Then, $B \subseteq A$ is tr-reduct of A if and only if $\tau_{A,tr} = \tau_{B,tr}$, i.e., $\tau_{A,tr}(x) = \tau_{B,tr}(x)$ for any x (see, [16]).

Observe that for $tr = \frac{1}{|A|}$ we obtain the classical definition of the reduct.

In order to find a tr-reduct of a set A of attributes we will use ideas proposed in [15,16]. The algorithm goes as follows: every discernibility matrix $M(B)$ and a given threshold $tr \in (0, 1]$ determines a Boolean function

$$\phi(B) = \prod_{x,y \in U^2} [\delta(x, y)], \qquad (*)$$

where $[\delta(x, y)] = \sum\{\Pi C : C \subseteq \delta(x, y) \text{ is minimal such that } \rho_C(x, y) \geq tr\}$.

Each prime implicant of (*) corresponds to a tr-reduct of A preserving discernibility of objects x, y such that $\rho_A(x, y) \geq tr$.

For example, it is easy to check that if $tr = 0.1$ then sets of attributes $\{a, b, e\}$ and $\{d, b, e\}$ are the only tr-reducts of the set of attributes $\{a, b, c, d, e\}$ in the Middle East conflict. If $tr = 0.65$ then it is necessary to preserve tr-discernibility between objects $(1, 2), (1, 3), (1, 4), (1, 5), (2, 6)(3, 6), (5, 8)$ (see Table 3). This means that for each of these pairs of objects we should preserve at least 4 attributes to satisfy the requirement of discernibility. Hence, one can easily calculate that the only tr-reduct for $tr = 0.65$ is $\{a, b, d, e\}$.

Intersection of all *tr*-reducts is called the *tr-core* of attributes. The *tr*-core contains all attributes which are most characteristic for the conflict and thus cannot be eliminated in the negotiation process.

For the Middle East conflict the 0.2-core attributes are b and e.

Let us also mention that if B' is a *tr*-reduct of A and x is any object then $\tau_{B'}(x) = \tau_C(x)$ for any objects x and C such that $A \supseteq C \supseteq B'$, i.e.,extending any *tr*-reduct of A by new attributes from A does not change the indiscernibility classes.

The above properties give us clear information on how the issues are structured and their importance in negotiations.

7 Negotiations

In order to change the conflict situation we need negotiations. There are many ways to negotiate, but we will restrict our considerations only to simple methods and consider how the change of neutrality to support or objection to disputed issues of some agents change the conflict.

To this end let us consider the attitude of agents to attribute a. Suppose that Jordan changed neutrality to autonomous Palestinian State to objection then the situation is shown in Fig. 8, i.e., it leads to coalition of Israel and Jordan.

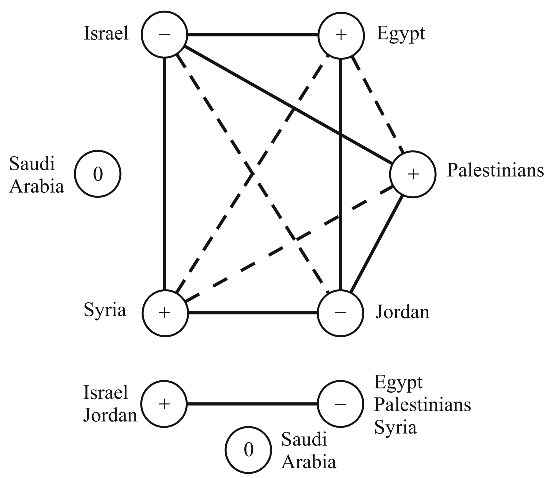

Fig. 8. Jordan objects Palestinian state

If Jordan would change neutrality to support to this issue then the conflict situation is presented in Fig. 9.

Change of attitude of Saudi Arabia from neutrality to support and objection is presented in Fig. 10 and Fig. 11 respectively.

A very interesting case is when both Jordan and Saudi Arabia change their position from neutrality to support or objection. Two most interesting cases are presented in Fig. 12 and Fig. 13.

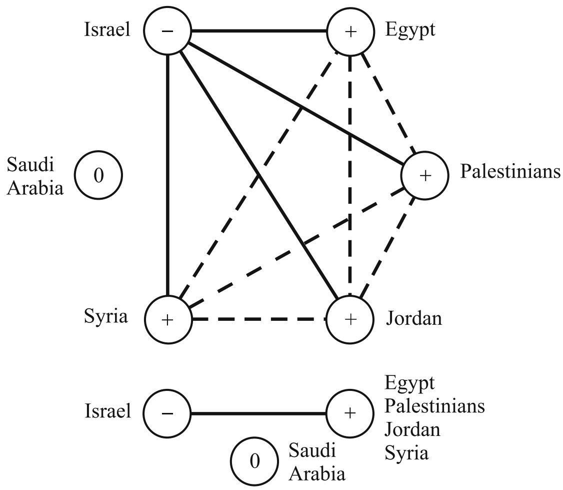

Fig. 9. Jordan supports Palestinian state

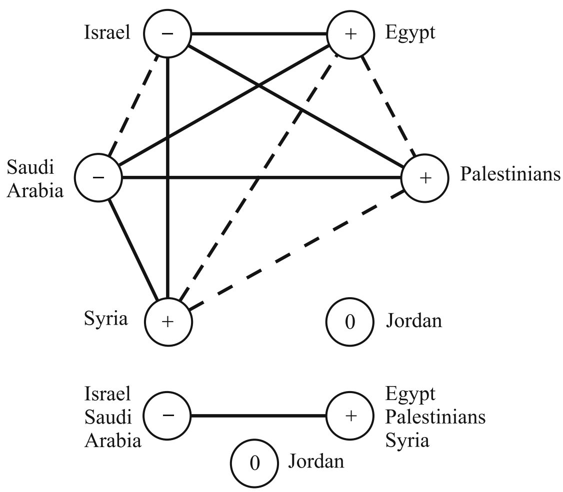

Fig. 10. Saudi Arabia objects to Palestinian state

We see from these figures that situation presented in Fig. 12 leads to conflict of Israel with all the remaining parties involved in the conflict, whereas changes as presented in Fig. 13 induce partition of agents where Israel, Jordan and Saudi Arabic are in conflict with Egypt, Palestinians and Syria.

The above information can be very useful in negotiations.

8 Conflict Graphs

In this section we will consider in more detail conflict graphs introduced in previous sections.

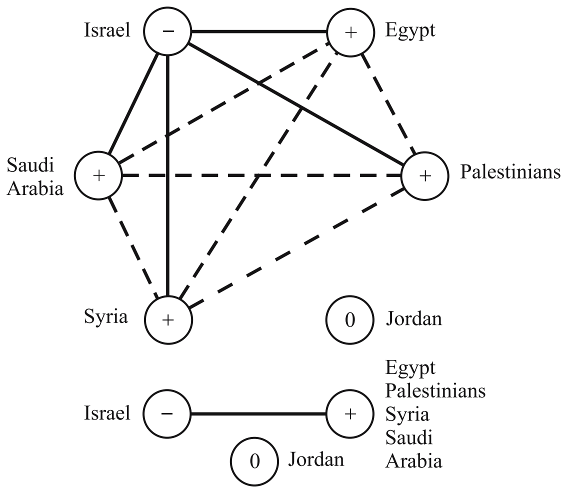

Fig. 11. Saudi Arabia supports Palestinian state

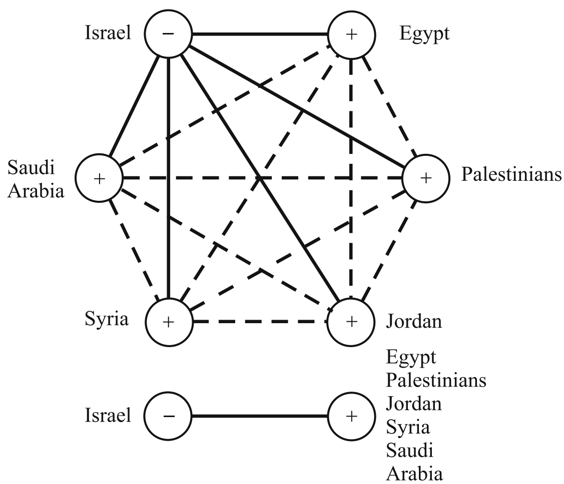

Fig. 12. Jordan and Saudi Arabia support Palestinian state

By a conflict graph we understand a set of *nodes* N (representing agents) and two sets of *branches* $\mathcal{B}^+$ and $\mathcal{B}^-$ (called alliance and conflict branches, respectively). If x, y are nodes then $(x, y) \in \mathcal{B}^+$ implies $(x, y) \notin \mathcal{B}^-$, and conversely.

We say that a conflict graph is *stable* (*consistent*) if the set of formulas defined by conditions (i)...(vii) given in Section 3 is consistent with the facts defined by the conflict graph, otherwise the conflict graph is *unstable* (*inconsistent*).

We interpret R^+ and R^- as $\mathcal{B}^+$ and $\mathcal{B}^-$, respectively and we say that if $(x, y) \in \mathcal{B}^+$ then x and y are allied, if $(x, y) \in \mathcal{B}^-$ then x and y are in conflict and if neither $(x, y) \in \mathcal{B}^+$ nor $(x, y) \in \mathcal{B}^-$ then x and y are neutral.

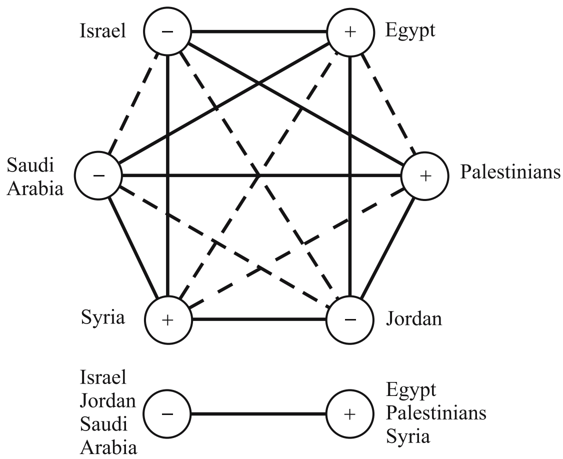

Fig. 13. Jordan and Saudi Arabia object to Palestinian state

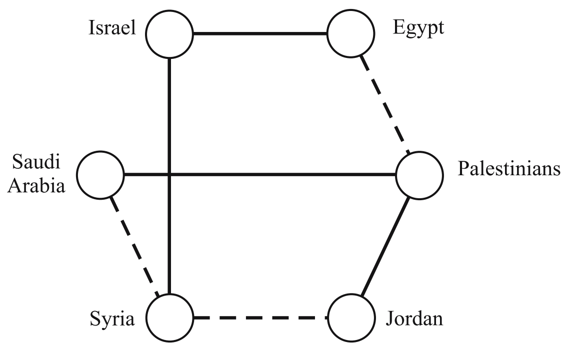

Fig. 14. Unstable conflict graph

The corresponding branches (x, y) are referred to as alliance, conflict and neutral, respectively.

Let x and y be neutral points in a conflict graph. If we replace branch (x, y) by alliance, or conflict branch, then the obtained conflict graph will be called an *extension* of the original conflict graph.

If we replace all neutral branches in a conflict graph by alliance or conflict branches then the obtained conflict graph will be called *maximal* extension.

The following is a very important property of conflict graphs:

If a conflict graph contains a loop with odd number of conflict branches there does not exist a stable maximal extension of the conflict graph.

For example, conflict graph shown in Fig. 14 does not have stable maximal extension.

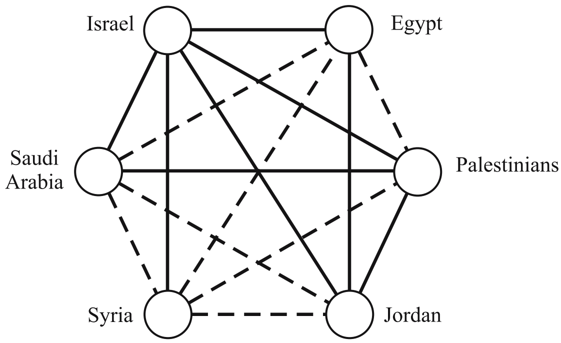

Fig. 15. Extension of graph from Fig. 14

An example of a maximal extension of the conflict graph from Fig. 14 as shown in Fig. 15 violates condition (iii).

Let us also observe that if a conflict graph is unstable then there is no consistent labelling of agents by their opinion (i.e., nodes by $+, 0, -$).

Conflict graphs can be used to study evolution of conflict situations.

Suppose we are given only partial information about a conflict situation. We assume that conflict situation can evolve only by replacing neutrality by alliance or conflict branches in such a way that stability is preserved. Thus answer to our question can be obtained by study of stable extensions of initial situation of conflict.

For example, consider initial conflict situation as shown in Fig. 16.

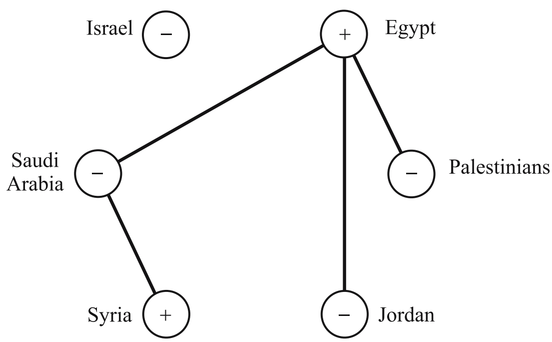

Fig. 16. Initial situation

This conflict due to assumed axioms can evolve according to patterns shown in Fig. 17.

The above methodology can be useful in computer simulation how the conflict can develop.

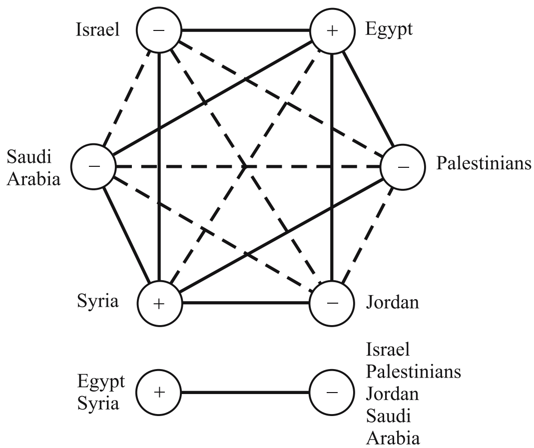

Fig. 17. Conflict evolution

9 Conclusion

The proposed attempt to conflict analysis offers deeper insight into structure of conflicts, enables analysis of relationship between parties and issues being debated. It gives many useful clues for conflict analysis and resolution. Besides, the mathematical model of conflicts considered here seems especially useful for computer simulation of conflicts in particular when negotiations are concerned.

Let us consider two examples of further investigations on the conflict analysis discussed in the paper.

– The analysis of possible extensions of partial conflict graphs (see Section 8) can also be performed using some additional knowledge (e.g., knowledge which each agent may have about the other ones). In the consequence, the number of possible extensions of a given partial conflict graph is decreasing. Hence, searching in the space of possible extensions may become feasible. In general, this additional knowledge can also help to better understand the conflict structure between agents. Observe that the analysis should be combined with strategies for revision of the generated extensions (e.g., if the constraints (i)-(iii) from Section 3 are no longer preserved for extensions). The necessity for revision follows from the fact that the additional knowledge is usually incomplete or noisy. Hence, the conflict prediction based on such knowledge may be incorrect.
– Negotiations between agents are often performed under the assumption that only partial conflict graphs and partial knowledge about possible other conflicts are available for agents. Hence, agents may have different views on possible extensions of the available partial conflict graphs. These possible extensions can be further analyzed by agents. In particular, agents involved in negotiations may attempt to avoid situations represented by some extensions (e.g., including conflicts especially undesirable or dangerous).

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