PRESERVICE TEACHERS' CONCEPTIONS ABOUT Y=X+5: DO THEY SEE A FUNCTION?

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We are studying two groups of preservice teachers' conceptions, progression and especially the concept of function in connection to y=x+5 when they are taking a course in algebra or one in calculus in their third and sixth term, respectively, in a teacher preparation program. There is a similar development in that they use to a higher degree a numerical interpretation before the course, which decreases after the course with a growth in linear and functional interpretation with the existence of two variables as a large and rather stable category. The group in their sixth term have a slightly more elaborated language and way of looking at y=x+5 than the group in the third term. For a majority of preservice teachers, in both groups, the concept of function is not evoked in connection to y=x+5.

INTRODUCTION

As parts of ongoing studies we have asked preservice teachers, in mathematics and science for school year 4-9, to answer the following question "We write y=x+5. What does that mean?" (Grevholm, 1998, 2002; Hansson, 2001). One reason to study y=x+5 is the fact that Blomhøj (1997) reported that final year students in compulsory school, age 15-16 years, have an unsatisfactory (see below) way of handling a question about how x is related to y in y=x+5. Another reason is that linear relations are common subjects that the preservice teachers are going to handle in different teaching-situations as inservice teachers. Linear relations are also often used in introductions of the function concept in later years of compulsory school.

The questions of the study are: What conceptions do preservice teachers have and what is their concept of function in connection to y=x+5? What progression can be seen between two groups in their third and subsequently their sixth term, in a teacher preparation program?

THEORETICAL FRAMEWORK

Hiebert and Carpenter (1992) present a framework for examining issues of learning and teaching with understanding. The framework is based on the assumption that individuals' knowledge is represented internally; that internal representations are structured and can be related or connected to one another to produce dynamic networks[1] of knowledge. They suggest that we think about these networks basically in terms of two metaphors, vertical hierarchy and web:

> *When networks are structured like hierarchies, some representations subsume other representations; representations fit as details underneath or within more general representations. Generalizations are examples of overarching or umbrella representations, whereas special cases are examples of details. … a network can be structured like spider's*

[1] The idea is supported by the fact that human memory, conceived as a network of entities, is a central and well founded theoretical construct in psychology and neuroscience (Anderson, 2000).

web. The junctures, or nodes, can be thought of as the pieces of represented information, and the threads between them as connections or relationships …The webs may be very simple, resembling linear chains, or they may be extremely complex, with many connections emanating from each node. (Hiebert & Carpenter, 1992, p. 67)

The two metaphors can also be mixed, resulting in additional forms of networks.

The mathematics is understood if its mental representation is part of a network of representations. Understanding grows as the networks become larger and more organized "a mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections" (p. 67). Existing networks influence the relationship that is constructed thereby helping to shape the new networks that are formed.

Some parts of the network are so tightly structured that they are accessed and applied as a whole, as a single chunk: "accessing any part of the chunk means accessing the entire network" (p. 75). Other parts, called schemata, are relatively stable internal networks that serve as templates to interpret specific events; that is abstract representations to which specific situations are connected as special cases.

Ausubel (2000) presents a hierarchical cognitive structure with similarities to the network model of Hiebert and Carpenter. He presents his theory for learning in an institutionalized setting and talks about meaningful learning and rote learning, which has consequences for the students' cognitive structures. To accomplish meaningful learning for students teachers have to activate relevant "anchoring" ideas in the learners' cognitive structures and it is necessary to build upon the learners' prior knowledge; this is what Hiebert and Carpenter call the bottom-up approach (p. 81). When meaningful learning is accomplished then:

> …eventually they [emergence of new meanings in semantic[2] memory] become, sequentially and hierarchically, part of an organized system, related to other similar, topical organizations of ideas (knowledge) in cognitive structure. It is the eventual coalescence of many of these sub-systems that constitutes or gives rise to a subject-matter discipline or a field of knowledge.
>
> Rote learning, on the other hand, obviously do not add to the substance or fabric of knowledge inasmuch as their relation to existing knowledge in cognitive structure is arbitrary, non-substantive, verbatim, peripheral, and generally of transient duration, utility, and significance. (Ausubel, 2000, p. x)

We consider what Ausubel calls meaningful learning to be similar to what Hiebert and Carpenter call learning with understanding where the dynamic network becomes larger and more organized with growing understanding; a similar phenomenon occurs in Ausubel's model:

> It is important to recognize that meaningful learning does not imply that new information forms a kind of simple bond with pre-existing elements of cognitive structure. On the contrary, only in rote learning does a simple arbitrary and nonsubstantive linkage occur with pre-existing cognitive structure. In meaningful learning the very process of acquiring information results in a modification of both the newly acquired information and of the

[2] Ausubel describes semantic memory as "Semantic memory is the ideational outcome of a meaningful (not rote) learning process as a result of which new meaning(s) emerge." (p. x).

specifically relevant aspect of cognitive structure to a specific relevant concept or proposition. (Ausubel, 2000, p. 3)

Tall and Vinner (1981) introduced the notion of concept image as "the concept image consists of all the cognitive structure in the individual's mind that is associated with a given concept", p. 151. Different parts of the concept image are evoked in different contexts and they say "we shall call the portion of the concept image which is activated at a particular time the evoked concept image", p. 152. In this paper, we see a concept image as a chunk of the knowledge structure described above and an evoked concept image as a portion of the concept image in the way of Tall and Vinner.

BLOMHØJ'S STUDY

Preservice teachers supervised by Blomhøj (1997) studied the concept of function in a group of 22 pupils that were in their final year of compulsory school (the 9th year). They asked the pupils to write their answers to the question "y= x+5, What can you say about x in relation to y?" and followed up the answers with interviews. In his report Blomhøj distributes the answers in four categories: a) answers that say that x is 5 less than y, b) answers that interpret the equation without answering the question, c) answers that say that x is 5 more than y and finally d) answers that neither interpret the equation nor answer the question.

The distribution of answers was that a) got 6, b) got 4, c) got 7 and finally the category d) got 5 answers. So category c), which is a wrong answer, includes the most answers. Moreover, the answers from the pupils often contain contradictions and more than half of the students could not give an acceptable interpretation.

METHOD AND RESULTS

The preservice teachers at Kristianstad University are studying mathematics in their first, third and sixth term and are then taking courses of a total of 30 weeks full time study where approximately one third relates to educational studies in mathematics. We studied two separate groups of preservice teachers in their third and sixth term, respectively, of a four and a half-year teacher preparation program. The first one took place in the third term where Grevholm (1998, 2002) asked a group of 38 preservice teachers to answer a questionnaire that contained the question of interest before and after a five-week course in algebra and also interviewed some of the preservice teachers. The second took place in the sixth term where Hansson (2001) replicated the first study with a group of 19 preservice teachers in connection to a five-week course in calculus. Hansson also asked them to draw a map that represented their way of thinking about y=x+5 after the course.

Grevholm created a categorization based on the preservice teachers written answers to the question "We write y=x+5. What does that mean?" The categorization arose from the data that was gathered. The categories separate answers that:

1) describe how x and y are related numerically, here called category N
2) state that there are two variables, V
3) give a table of values for y=x+5, T
4) describe the relation as a straight line, L
5) describe the relation as a function, F
6) give other specific descriptions, O

Table 1 gives the distribution of answers. Hansson used the same categories and table 2 gives the distribution[3] of answers. The tables are based on the total number of categories that the preservice teachers' answers included.

Category	N	V	T	L	F	O
Before	19 (46%)	11 (27%)	2 (5%)	3 (7%)	4 (10%)	2 (5%)
After	12 (27%)	12 (27%)	2 (5%)	8 (18%)	9 (20%)	1 (2%)

Table 1. 28 preservice teachers answered the question before and after the course in algebra; where 36 answered the questionnaire before and 31 after the course. Table from Grevholm (1998).

Category	N	V	T	L	F	O
Before	12 (38%)	14 (44%)	0	4 (12%)	2 (6%)	0
After	8 (16%)	17 (34%)	0	13 (26%)	8 (16%)	4 (8%)

Table 2. 18 preservice teachers answered the question before and after the calculus course; where 17 answered the questionnaire both before and after the course.

In the following we give an illustration of how the categorization was assessed (Fn and nF belong to the first and second study respectively):

N) F1: *y=(value of x)+5*, 8M: *y is a number that is 5 units larger than the number x*, F4: *that y is the sum of 5 and the number you decide x to be*.

V) F3: *Two unknown, x and y are variables*, 14M: *y depends on x*, M10: *Different for different people. For me it means that one x-value represents one y-value.*

T) F7: *A table of values with x-values on one line "x 5 4 3 2" and y-values on the second line "y 0 1 2 3"*[4].

L) M6: *y=x+5 is a line that intersects the y-axis when x=-5 and intersects the x-axis when y=5*, 12F: *it can also be a straight line*, 4F: *You can also see it as an equation for a straight line that uniquely determines what the line looks like.*

F) M5: *Function y=variable x+number 5*, 3F: *y is a function of x*, 4F: *y is a function of x+5.*

O) 16F: *It is also an example of an equation…*, F7: *It is an algebraic expression*, M11: *An equation with two unknown numbers.*

Eight preservice teachers were also interviewed and tape-recorded in the first study and seven interviewed, four tape-recorded, in the second study. The interviews reveal that the students have more to say than they express in the answers of the questionnaire. In the conversation they usually give interpretations of "y=x+5" covering more of the categories N-O than in the questionnaire.

The use of concept maps

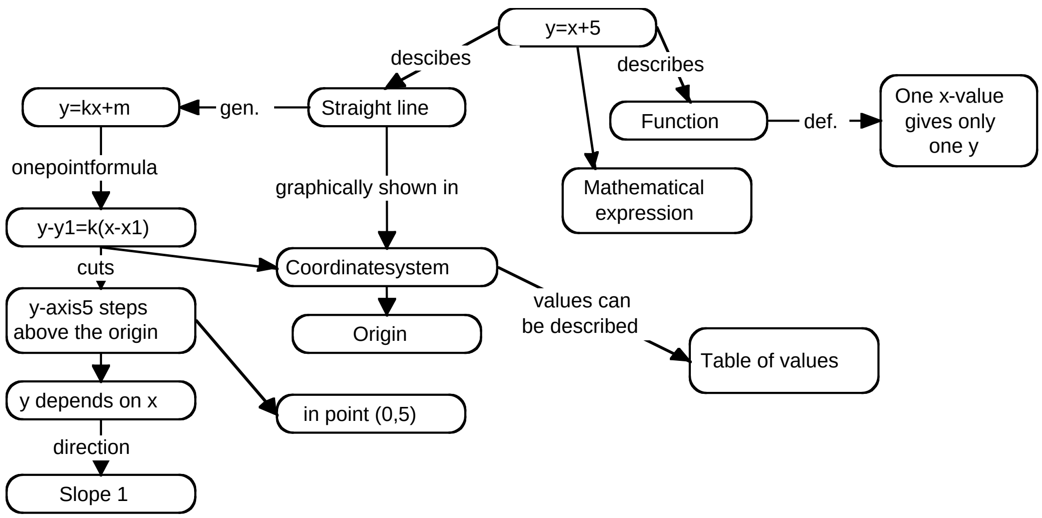
[3] The distribution of answers is adjusted compared to Hansson (2001) to become more uniform with the categorization of answers in Grevholm's studies. In Hansson (2001) the categories F and V were broader and narrower respectively. Functional thinking (Vollrath, 1986) like "y depends on x" was graded F and more specific statements like "… variables x, y …" graded V.

[4] In this case the preservice teacher puts x and y on the wrong values.

In the questionnaires students give only a few knowledge propositions to the question. Normally they give one and at most four propositions are given. In earlier research Grevholm (2000, in press) has shown that the use of concept maps is one way to get students to reveal more about their mental representations. It is intellectually more demanding to draw a concept map than to answer a question. In the concept maps students activate more concepts and more links between them than in a verbal written proposition. Inspired by this experience Hansson also decided to use concept maps in his study.

Map made by 5M. A concept map about y=x+5.

The preservice teachers in the second study had some experience of drawing mind maps and concept maps in pedagogy and biology, so drawing maps was familiar to them.



Hansson (2001) gave a lecture on how to use concept maps in mathematics education (in the way introduced by Novak and Gowin, 1984) and discussed how different maps can help visualization of knowledge and understanding and also be used as Ausubel's advance organizers[5]. At the end of the lecture, he asked the preservice teachers to draw a map about y=x+5 which they did for almost 30 minutes. One of the preservice teachers' maps, made by 5M, is shown above.

DISCUSSION

The two groups of preservice teachers we studied had a similar development of answers to the question before and after the courses they took in that there was a reduction in category N and growth in category L and F as shown in tables 1 and 2. Category V is large and quite stable in the algebra course; it is the largest category and rather stable in the calculus course. It is surprising that the preservice teachers in their sixth term before the calculus course came up with so many answers in category N; a category we judge as less advanced than the categories L and F. The ordering of categories N, V, T, L, F is a

[5] An advance organizer is a pedagogic device that helps … bridging the gap between what the learner already knows and what he needs to know if he is to learn new material most actively and expeditiously. (Ausubel, 2000, p. 11)

reflection of our view of order of more advanced levels of thinking; demanding more developed cognitive structures with the function concept having the most complex structure with connections to numerous sub concepts.

The answers in the second questionnaire were more detailed and covered more categories, and were more explicit in the group who took the calculus course. The authors could see a slightly more mature language in the answers of the second questionnaire and especially from the group who had advanced longer in the teacher preparation program, but a yet more elaborated language is desired to become successful as an inservice teacher. None of the groups used more advanced mathematics than from the curriculum of upper secondary level. The interviews indicate however that the understanding of the students is somewhat more developed than what seems to be the case in the written answers.

Examining data for individual students confirm that concepts develop slowly. More than every second student give answers in the same categories before and after the course. Other students just add an extra category. For those who keep the same categories it is notable that they after the course express themselves in a more advanced professional language for teachers. (The students were not aware of the categories that we use here. They were just asked to answer as honest and open as they could to show us their knowledge.)

There is a growth in the number of answers in category F when comparing the first and second questionnaire. But even so the majority of preservice teachers do not mention that y=x+5 is a function. Moreover, only one preservice teacher (M13) who took the algebra course gave y=x+5 any properties as a function. He wrote that it was a linear function and did so in the second questionnaire. There was also only one preservice teacher (4F) in the calculus course who, before the course, wrote that y=x+5 becomes a line. After the calculus course there were more answers mentioning that the function becomes a line (no one was referring to the line as a function graph); no other property was mentioned.

It is notable that so few preservice teachers write that y=x+5 is 'an equation' in the questionnaires. Only one preservice teacher mention it in the algebra course and four (in the second questionnaire) in the calculus course; in contrast to the concept maps where ten preservice teachers mention it. The concept of equation is one they have worked with for many years, much longer than the concept of function. It is also obvious that none of the groups is actively using the term linear. However, the concept of line is used in both groups and more frequently in connection to functions in the group studying in their sixth term.

When we look at the maps we see that they contain more information than the written answers. They are clearly more developed in the area of a straight line where they mention things like slope, intersection with the axis in a coordinate system and the equation of a straight line y=kx+m; which was also visible in the written answers.

Eight preservice teachers have function as a part of their map, but it has few links[6] connected to it (as in the map that 5M drew). One exception is 17M who writes f(x)=x+5 and gives the derivative and primitive function. Moreover, links between function and straight line is not common and only one map (14M) mentions graph and makes links between function, graph and then straight line. Other properties of functions like for example monotonity and continuity are not mentioned. The concept of equation is more explicit in the maps than in the written answers and some maps also have connections to applications and learning and teaching.

The fact that the group just took a course in calculus where the function concept makes a central part was in large not visible in the written answers or the maps they drew. It indicates that the function concept is less meaningful in the context of y=x+5. They seem to make connections with mathematical knowledge on a less advanced level than what they have worked with in their later courses in mathematics. A premature concept of function (Vollrath, 1986) is also visible in answer like "y depends on x" (14M) in category V. Even category F has answers with a less developed concept of function like "y is a function of x+5" (4F).

CONCLUSIONS

There was no indication that incorrect answers like those shown in the study by Blomhøj were frequent among the preservice teachers. There is a similar development in both groups of preservice teachers with tendencies of a numerical interpretation of y=x+5 before the course which lessens after the courses with a growth of linear and functional interpretation; with the existence of two variables as a large and rather stable category. The group of preservice teachers who had progressed further in the teacher preparation program had a slightly more developed language and flexible way of looking at y=x+5 where for example the concept of equation was more common. The maps gave valuable information about how the students look upon y=x+5 and connections between different parts of knowledge became more explicit. The function concept was not well developed in connection to y=x+5; if mentioned it did not have any properties except as a line in a few cases. Views upon the function concept, as an object with many properties, were hardly visible. This became apparent in the written answers but also clearly in the maps.

This study indicates that the preservice teachers' concept of function is not a rich cognitive structure in the evoked concept image in the context of y=x+5. It could mean that they as inservice teachers give less attention (Chinnappan & Thomas, 1999; Even, 1993; Fennema & Loef, 1992; Vollrath, 1994) to the function concept in linear relations. The fact that teachers do not give enough explicit attention to the functional aspects of linear relations can be one explanation to the results from pupils in year nine in Blomhøj's study. Niss (2001, p 43) concludes that "If it is something we want our pupils to know, understand or manage, we must make this part of an explicit and carefully designed teaching". (Our translation). If we want students (pupils) to be able to interpret a

[6] We assume the number of links is positively correlated to the concepts importance in relation to each other in the context of y=x+5.

given expression as a function this aspect must be part of the teaching that students are offered.

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