CONFORMAL INVARIANCE AND APPLICATIONS TO STATISTICAL MECHANICS

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Editors

Claude Itzykson
Hubert Saleur
Jean-Bernard Zuber

Service de Physique Theorique
Commissariat à l' Energie Atomique
CEN Saclay, 91191 Gif sur Yvette
Cedex, France

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PREFACE

This collection of reprints on recent developments which lie on the borderline between string theory and two dimensional statistical physics is a testimony of the interplay between methods and ideas pertaining to these two domains with very different goals. Seemingly unrelated approaches converge towards a deeper understanding of critical two-dimensional field theory. Among them are exactly solved (or partly solved) models, the implications of the renormalization group and reparametrization invariance in the field theoretic formulation of the dual model, not to mention current algebra or finite size effects. On the other hand infinite Lie algebras and their representations are emerging as new tools to investigate the consequences of conformal invariance and to enlighten the significance of many apparently scattered results.

This diversity of sources justifies the present endeavour to select some significant contributions and to organize them in such a fashion as to enable an interested reader to get an introduction to the subject as well as a general overview. Such a choice is also to a certain extent arbitrary, the more so given the close connections with related fields. Even in the limited range which we assigned ourselves we listed some five hundred recent articles, without pretending to be exhaustive. It was thought that keeping the volume to a reasonable size would help the reader if he can spot through the bibliography some other work relevant to his or her interest. Due to our lack of expertise some trends are perhaps not as fully represented as they deserve. This is presumably the case for example of the relation to integrable models.

We have divided the material into ten sections, each one with a short reading guide. The latter is no substitute for a careful study of the reprinted papers, as well as of many others, hopefully listed in the bibliography, which could not be included.

Saclay, February 1, 1988

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