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CONFORMAL MAXWELL THEORY AS A SINGLETON FIELD THEORY ON ADS$_5$, IIB THREE BRANES AND DUALITY

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ABSTRACT. We examine the boundary conditions associated with extended supersymmetric Maxwell theory in 5-dimensional anti-De Sitter space. Excitations on the boundary are identical to those of ordinary 4-dimensional conformal invariant super electrodynammics. Extrapolations of these excitations give rise to a 5-dimensional topological gauge theory of the singleton type. The possibility of a connection of this phenomenon to the world volume theory of 3-branes in IIB string theory is discussed.

I. Introduction

Recent developments in the theory of p-branes, and duality interconnections in M theory and string theories, have brought renewed interest in higher dimensional supergravity theories and their extensions [1][2]. In recent times attention has been called to an intriguing connection, between the horizon geometry of certain black p-brane configurations and the world volume dynamics of the brane degrees of freedom [3]. Further relations between Anti-de Sitter supergravities and brane backgrounds, using S- and T-duality, have been also investigated [42].

This connection follows from earlier considerations [4][5][6], where it was pointed out that the asymptotic horizon geometry of certain p-branes in d dimensions, typically on an $\mathrm{ADS}_{p+2} \times S_{d-p-2}$ background, admits a superalgebra that is identical to the superconformal algebra of the corresponding world-volume $p+1$ dimensional field theory, the latter describing the world volume degrees of freedom when gravity (or substringy effects) is decoupled. * Moreover, the horizon geometry of the p-branes has twice the number of supersymmetries carried by the brane background far from the horizon [6][7][8]. In the cases discussed below the relevant superalgebras admit 32 supersymmetries.

The most prominent examples are the M-theory (11 dimensional supergravity [9]) chiral five-brane, [10] where the underlying superconformal symmetry is $Osp(6,2/4)$ [11], with bosonic isometry $O(6,2) \times USp(4)$, and the Dirichlet three-brane of the IIB superstring, where the underlying superconformal symmetry is $SU(2,2/4)$ [11][12][13][14][15], with bosonic isometry $SU(2,2) \times SU(4) \approx O(4,2) \times O(6)$.

These conformal symmetries are the same as the isometries of the corresponding asymptotic horizon background, i.e. $AdS_7 \times S_4$ and $AdS_5 \times S_5$ respectively. This mathematical coincidence was the origin of a proposed duality [3] between supergravity around the horizon background and superconformal brane dynamics. It is a kind of strong-weak coupling duality in which the fundamental supergravity degrees of freedom (supergravitons) are expected to emerge as bound states in the non-perturbation regime of the corresponding (world-volume) theory.

It is the aim of the present work to further investigate this idea, also reminiscent of

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* The full nonlinear Born-Infeld type action is presumably conformally invariant if coupled to the appropriate anti-De Sitter background [3][10].

previous speculations [5][6][16][17][18] on the interpretation of the conformal world-volume degrees of freedom as "boundary" degrees of freedom of certain field theories formulated in anti-De Sitter space. The latter have the unconventional property that they admit "topological" unitary representations [19][20][21][38], the singletons (or their ramifications) that do not correspond to local degrees of freedom in the bulk, but to interactions purely localized on the $p+1$-dimensional boundary of AdS_{p+2}.

In this context it was also proposed [4][5][16][18] that 2-brane dynamics (the fundamental brane of M-theory) should correspond to singleton field theories in $\mathrm{AdS}_4 \times S_7$, with underlying superalgebra $OSp(8/4)$. The corresponding supergravity is gauged $SO(8)$ in $d=4$ [22]. Similarly, 1-branes corresponding to type IIA, type IIB and heterotic fundamental strings were interpreted [17] as singleton field theories of the 3-dimensional conformal group $SO(2,2) \approx Sp(2,\mathbf{R}) \otimes Sp(2,\mathbf{R})$ with superconformal algebras given respectively by $Osp(8/2)_c \otimes Osp(8/2)_s, Osp(8/2)_c \otimes Osp(8/2)_c$ and $Osp(8/2)_c \otimes Sp(2,\mathbf{R})$.

In the present situation the corresponding supersingleton degrees of freedom are the (2,0), $d=6$ tensor multiplet and the $N=4$, $d=4$ vector multiplet ($U(N)$ Yang-Mills for N three branes). * It was already shown in [21], that compactification of 11 dimensional supergravity on S_4 down to seven dimensions [23], gives rise to a spectrum of unitary representations containing a singleton representation, called doubleton by in [21], which precisely correspond to a self-dual 2-tensor, a Weyl spinor and 5 scalar degrees of freedom. It is crucial to the present work that these degrees of freedom are purely topological in the bulk, they do not allow local interaction in AdS_7 but only on its six-dimensional boundary [10], where they are conformal.

The same analysis was carried out for the case of S_5 compactification of IIB, 10-dimensional supergravity on five-dimensional anti-de Sitter space [38]. This is the supergravity theory that corresponds to the horizon geometry of the IIB three-brane [3]. The corresponding sueprgravity theory is gauged 5-dimensional supergravity with $SU(4)$ gauge symmetry [25].

In this case the super-singleton multiplet (called $SU(2,2/4)$ doubleton in[38]), corre-

* More precisely, the singleton field theory corresponds to the Goldstone multiplet on the branes [6][10]. For a $U(N)$ theory, this is the $U(1)$ Maxwell multiplet of the $N=4, d=4$ superconformal algebra, describing the center of mass motion of the 3-brane.

sponds to the $N = 4$ super-Yang-Mills multiplet.

In the present paper we will extend this analysis, showing in particular that the $N = 4$ vector multiplet does indeed have the property that its degrees of freedom are purely topological, in the sense that they do not lead to local interaction on AdS_5, but only to interactions on the 4-dimensional boundary where the action of $O(4, 2)$ becomes the conformal group of 4-dimensional field theories [24].

Our analysis will essentially consist of a proof that conformal invariant Maxwell theory can be interpreted as a topological "singleton" field theory in AdS_5. It is done by Fourier analysis of the propagator, as in earlier studies of 4-dimensional singletons [19].

The paper is organized as follows: In Section II we recall the theory that is dual, in the sense of [3], to the world volume 3-brane theory in the conformal limit. This is a gauged, $d = 5$ maximal supergravity [25] with 32 supersymmetries. This theory can be defined *per se* or can be viewed as coming from the IIB string, when compactified on a five-sphere down to five dimensions [25][3]. In Section III we give preliminaries of field equations in anti-De Sitter space times. In Section IV we formulate conformal Maxwell theory and its relation to a singleton field theory in AdS_5. In Section V we consider superconformal and $N = 4$ extended superconformal Maxwell theory and their interpretation as singleton field theories in AdS_5. The paper ends with a paragraph of conclusions and suggestions for future investigations.

II. Gauged Supergravity in anti-De Sitter Space.

Gauged $N = 8$ supergravity in five dimensions was constructed by Gunaydin, Romans and Warner in 1984 [25]. It lives in anti-De Sitter space-time $O(4, 2)/O(4, 1)$ and admits 32 supersymmetries. It can be viewed as the gauge theory of the superalgebra $SU(2, 2/4)$, the same superalgebra as that of its "dual", namely $N = 4, d = 4$ super-Yang-Mills theory.

We assume, following [3], that the supergraviton multiplet arises as a bound state in the dual superconformal theory. Such bound states form a multiplet, which corresponds to a unitary massless representation of $SU(2, 2/4)$ containing 1 graviton, 8 gravitini, 15 gauge bosons, 12 antisymmetric tensors, 48 spin-$\frac{1}{2}$ fermions and 42 scalars. Note that there is a relation [25][26] between the gauge group $SU(4)$ and the 5-dimensional duality group $E_{6(6)}$ of toroidal compactification [27]. Indeed $SU(4)$ is the subgroup of the maximal compact subgroup $USp(8)$ in $E_{6(6)}$, on which the reduction of the 27-dimensional representation of $USp(8)$ contains the adjoint representation. Indeed $27 \to 15 + 6 \times 2$ as $USp(8) \to SU(4)$ (this corresponds to the embedding $8 \to 4 + \bar{4}$). Note that the theory has a potential [25] for the 42 scalars, which decomposes as follows under $SU(4)$ [43]: $42 \to 20 + 10 + \bar{10} + 1 + \bar{1}$. This is consistent with the fact that 20 and 10 are "massive', since their v.e.v. is driven to zero by $SU(4)$ symmetry, while the singlet is massless since its quadratic fluctuation is absent from the potential. (This analysis is similar to gauged $N = 8$, $d = 4$ supergravity in the $SO(8)$ invariant vacuum.)

It is most important that the supergravity theory in AdS$_5$ also has a global $SU(1,1)$ symmetry, to be identified with the S-duality in the corresponding dual super Yang-Mills theory. Note that here $SU(1,1)$ is the subgroup of $E_{6(6)}$ that commutes with the gauge group $SU(4)$. This symmetry is the original $SU(1,1)$ symmetry of type IIB, 10-dimensional supergravity [28]. Note the chain decomposition [25]:

$$E_{6(6)} \to SL(6,R) \times SL(2,R) \to SO(6) \times SL(2,R) \ .$$

(Of course, the two subgroups $SL(2,R)$ and $USp(8)$ do not commute.) It is easy to identify the "composite operators" of the world-volume theory that correspond to the supergraviton in $AdS_5 \times S_5$. They are contained in the supercurrent [29], $N = 4$ multiplet that contains two $N = 4$, spin $\frac{3}{2}$ currents, giving 8 spin-3/2 gravitinos, the stress tensor giving the graviton, and 15 $SU(4)$ (R-symmetry) currents giving the anti-De Sitter gauge bosons. It also contains the scalars in the 20 of $SU(4)$ as its first components.

We also note that in the anti-De Sitter supergravity context, according to [25], one gets a quantization of the ratio k'/g^3 where k' is the 5-dimensional gravitional constant and g is the 5-dimensional gauge coupling. This is due to Chern-Simons interactions* and to the non-triviality of the 5'th homotopy group of $SU(4)$. **

In the type IIB picture the three-brane BPS mass (per unit volume) is dilaton independent (in the Einstein frame) and only depends on the Planck mass (i.e., $(\sqrt{g}\alpha')^{-1/2}$)

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* Turning things around, 5-dimensional gauged supergravity may be viewed as the supersymmetrization of the 5-dimensional Chern-Simons coupling. A similar argument applies to gauged 7-dimensional supergravity [23]. This theory lives in AdS$_7$ and has gauge group $USp(4)$. It is "dual" to the M-theory five-brane [3][10].

** Strictly speaking, the quantization would only apply if the 5-manifold were compact, and three-brane charge. This is due to the fact that the three-brane (unlike strings) is a U-duality singlet in IIB theory. Therefore, the horizon geometry depends only on the Planck scale and three-brane charge and not on the asymptotic value of the dilaton.

This is consistent with the attractor mechanism which essentially states that at the horizon, black brane physics looses memory of the initial values of moduli fields [30] [41].

III. Preliminaries on scalar fields in anti-De Sitter space.

Coordinates.

Five-dimensional anti-De Sitter space may be defined as a subspace of $\mathbf{R^6}$, more precisely of a 6-dimensional pseudo Euclidean space with signature $+,-,-,-,-,+$. Let $\{y_i\}_{i=0,1,...,5}$ be global coordinates of this space, then AdS_5 is the sub-manifold defined by the polynomial equation

$$y^2 := y_0^2 - \vec{y}^2 + y_5^2 = \rho^2, \quad \vec{y} := (y_1, ..., y_4), \tag{3.1}$$

where $\rho > 0$ is a fixed parameter. It is possible to introduce coordinates, for example $y_0, ..., y_4$, but as any set of coordinates is of necessity local it is preferable to retain the parameterization in terms of $\{y_i\}_{i=0,...,5}$. Nevertheless, it will be expedient, a little later, to introduce a particular time coordinate, t.

It will be convenient to generalize: from now on $d = p + 1$ will denote the space dimension, so that $d = 4$ in the case of present interest. This generalization will be made only in the formulas, in the text we always take $d = 4$, save explicit emphasis to the contrary.

Boundary asymptotics.

As a topological space, $\mathrm{AdS}_5 = S_3 \times S_1 \times \mathbf{R}^+$. The radial coordinate (the coordinate of $\mathbf{R}^+$) is defined by

$$\vec{y}^2 = r^2, \quad y_0^2 + y_5^2 = \rho^2 + r^2.$$

The boundary at infinity is the limit $r \to \infty$, taken for a fixed point in $S_3 \times S_1$. As the parameter ρ disappears in the limit, the boundary can be identified with the cone $y^2 = 0$.

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while AdS_5 has the topology of $S_3 \times S_1 \times R^+$. So one may question the validity of the argument given in [25].

More precisely, let f be a function on AdS; if we look at ρ as an abreviation for $\sqrt{y^2}$, then f has a unique extrapolation F to the subspace $y^2 > 0$ of pseudo-Euclidean 6-space, homogeneous of some arbitrary but fixed degree, N, say. Then, if the limit exists, we have

$$\lim_{r\to\infty} r^{-N} f = (\rho^{-N} F)|_{y^2=0}. \tag{3.2}$$

The appropriateness of this type of limit will be demonstrated below.

Wave equation.

The wave equation for a scalar field can be seen as a condition to fix the value of the second order Casimir invariant of $so(4,2)$. The vector fields

$$L_{ij} = y_i\partial_j - y_j\partial_i,$$

where $\partial_i = \pm\partial/\partial y_i$, the sign reflecting the signature, act on the manifold defined in Eq.(1.1) and generate the action on that space of the group $SO(4,2)$. The second order Casimir operator is

$$\mathcal{C} := -\frac{1}{2}\sum_{i,j} L_{ij}L_{ij} = y^2\partial^2 + \hat{N}(\hat{N}+d). \tag{3.3}$$

The sum is over all 6 values of the subscripts and $\hat{N}$ is the operator associated with the degree of homogeneity,

$$\hat{N}f = y\cdot\partial f = \sum y^i\partial_i f. \tag{3.4}$$

Quantum numbers.

An elementary particle in five-dimensional anti-De Sitter space is a unitary, irreducible, projective, highest weight representation of $SO(4,2)$. Weights are defined in terms of finite-dimensional representations of the compact subgroup $SO(4)\otimes SO(2)$. The generator of the second factor is L_{05}, the *energy.* Highest weight means lowest energy; the energy is bounded below. Since $SO(4) = SU(2)\otimes SU(2)$, a highest weight of the first factor is characterized by a pair (j_1, j_2) of positive half-integers. An elementary particle is thus characterized by an irreducible representation $D(E_0; j_1, j_2)$, where E_0 is the lowest value of the energy. The existence of highest weight representations implies a discrete energy spectrum within any unitary, irreducible representation; we normalize the energy generator,

$$\hat{E} = i(y_0\partial_5 - y_5\partial_0),$$

so that the interval is equal to unity. But the eigenvalues need not be integers. (Whence the qualification "projective" representation.) There are limitations on (E_0, j_1, j_2) that are necessary and sufficient for the representation $D(E_0, j_1, j_2)$ to be unitary:

$$E \geq \begin{cases} j_1 + j_2 + 2, & j_1 j_2 > 0, \\ j_1 + j_2 + 1, & j_1 j_2 = 0. \end{cases} \tag{2.5}$$

The highest weight modules will appear as spaces of field modes. A function associated with an eigenvalue E of the energy has the form

$$f = e^{-iEt} g(\vec{y}),$$

where $y_0 - iy_5 = \sqrt{\rho^2 + r^2} e^{it}$ defines the time t, and $\hat{E} = i\frac{d}{dt}$.

To a highest weight corresponds a value of the Casimir operator, defined in (3.3),

$$\mathcal{C} = E_0(E_0 - 4) - j_1(j_1 + 1) - j_2(j_2 + 1). \tag{3.6}$$

Therefore, the wave equation associated with $D(E_0, 0, 0)$ is

$$[y^2\partial^2 - \hat{N}(\hat{N} + d) + E_0(E_0 - d)]\varphi = 0. \tag{3.7}$$

Propagator.

Instead of solving the wave equations for the fields, it will be much more useful to calculate the propagators. The reason is that the propagator gives additional information, concerning the existence of field quantization. This information is relevant for the classical field theory as well, since Greens functions are important for the construction of solutions in the presence of external perturbations, and also in connection with questions of completeness. All this information can also be culled from the action of the conformal group on the field modes, but this leads to a discussion of indecomposable representations, a subject that we prefer to introduce later.

An invariant propagator is a function of two points labelled y and y' in AdS_5, invariant under the action of SO(4,2). Locally, it is a function K, depending on the variable,

$$z := y \cdot y' = \sum_i \pm y_i y_i'.$$

The propagator associated with the wave equation (3.7) satisfies

$$[(1-z^2)\partial_z^2 - (d+1)z\partial_z + E_0(E_0-d)]K(z) = 0. \tag{3.8}$$

The solutions, for most values of the parameter E_0, are hypergeometric series, $K(z) = z^s \times$ inverse power series. The indicial equation is

$$(-s+E_0)(s-d+E_0) = 0, \tag{3.9}$$

Usually, except for special cases to be dealt with, this signals the presence of two highest weight modules, one with minimal energy E_0 as expected, and the other with lowest energy $4-E_0$. If $E_0 > 3$, then only the first is unitary, and since this is the larger of the two, there is a propagator that is a hypergeometric series. Interpreted as a Fourier series, it is the reproducing kernel for an irreducible representation of type $D(E_0,0,0)$, and all is well. If $2 < E_0 < 3$ nothing untoward happens, except that now there are two hypergeometric series that solve the propagator equation and one may adopt either solution. (But not both!) The domain $1 < E_0 < 2$ is of course similar.

There remains the most interesting cases. If $E_0 = 2$, then only one solution is a hypergeometric series, and the other solution is logarithmic. This means that the energy operator cannot be diagonalized, and signals the appearance of a nondecomposable representation. This is not a difficulty, since one simply rejects the logarithmic solution in favor of the other.

Finally, the case $E_0 = 1$ is also logarithmic, since $s_1 - s_2 = 2$. Only one of the two solutions is a power series, and it is the wrong one, for it propagates the set of modes that correspond to $D(3,0,0)$, with minimal energy $E_0 = 3$.

In conclusion, the case of a scalar field carrying the representation with minimal energy $E_0 = 1$ is anomalous, since its propagator is logarithmic, namely:

$$K(z) = z^{-1} + az^{-3} + ... - \log z(z^{-1} + bz^{-3} + ...).$$

The wave modes are of three types: physical modes that fall off as r^{-1} at infinity, "gauge modes" that fall off as r^{-3}, and "scalar modes" that go like either r^{-3} or $r^{-3}\log r$. Taken together, they form a nondecomposable $SO(4,2)$ module. (In a notation to be explained later, it is of the type $D(3,0,0) \to D(1,0,0) \to D(3,0,0)$.) The gauge modes form an invariant submodule with the non-logarithmic propagator and minimal energy $E_0 = 3$. The scalar modes are conjugate to the gauge modes and appear paired with them in the Fourier expansion of the propagator. (Not quite a Fourier series because of the logarithm, but an expansion in terms of generalized eigenfunctions of the energy operator.)

The gauge modes have most of the properties of an ordinary massive or massless fields in 5 dimensions. They form a complete set in the usual sense, which means that there are enough modes to make a local measurement; that is, the field can be used to find a small object. The set of physical modes, which are physical just because we are trying to use the representation with $E_0 = 1$ to do physics, is very different. It is very singular, mainly in the sense that there is very little degeneracy in the energy spectrum. In fact, the degeneracy of each energy level is precisely that of an ordinary particle in four space time dimensions and the black body radiation has the energy density $\propto T^4$ that is normal for a three-dimensional space.

This is of course what one should expect. We are looking for a field that extrapolates from the boundary to the interior, such that, on the four-dimensional boundary it describes a massless particle. In fact, the representation $D(1,0,0)$ of the conformal group of 3-dimensional Minkowski space is realized precisely as the space of states of an ordinary massless particle. What distinguishes this representation of the conformal group is that it remains irreducible when restricted to the Poincaré subgroup [31]. Here, it may be argued, we are more concerned with its reduction to $SO(3,2)$, in as much as the boundary is AdS_4 rather than Minkowski. But the situation is essentially the same, the restriction of the representation $D(1,0,0)$ to $SO(3,2)$ is the direct sum of just two irreducible representations [32].

To preserve unitarity in the presence of interactions it is necessary that the interaction be gauge invariant; that is, it must be insensitive to gauge modes. But this is a severe requirement, since gauge modes are so much more plentiful. The only thing that sets the physical modes apart is their slow decrease at infinity; precisely, the limit

$$\lim_{r\to\infty} r\varphi(y)$$

is gauge invariant. So, to be gauge invariant, the interactions can be sensitive only to the boundary value of the field; in short, the interactions take place at infinity, exclusively.

This conclusion applies in general, to all the fields examined in this paper.

There is an alternative to admitting the logarithmic modes; it consists of replacing the second order wave operator by its square [33]. This admits a propagator of the form of a hypergeometric series. The non-decomposable representation, the boundary conditions and the attendant gauge structure are exactly the same. The difference is that in this formulation it is clear what the Lorentz (physicality) condition is: it is just the second order wave equation. In the logarithmic formulation that we have chosen in this paper, in order to remain within the context of ordinary, second order wave equations, it is not clear whether the Lorentz condition has a local expression.

Similar considerations applied to AdS_{p+2} lead to the conclusion that the most interesting scalar field carries a representation with minimal energy $E_0 = \frac{p-1}{2}$. In anology with the analysis of [21], we expect that such singleton modes should appear in the Kaluza-Klein version of IIB supergravity compactified on $\mathrm{AdS}_5 \times S_5$.

IV. Conformal Maxwell theory.

All free fields, of any local field theory, satisfy after gauge fixing the same wave equation. In our case, whether the field A with components A_i is a scalar, a spinor or a vector field, it satisfies after gauge fixing the equation

$$[y^2\partial^2 - \hat{N}(\hat{N}+d) + E_0(E_0-d)]A_i = 0,\ i = 1,..,n\ . \tag{4.1}$$

The field takes values in an n-dimensional $SO(4,2)$ module. Therefore, it transforms as a subrepresentation of $D_{\mathrm{orb}} \otimes D_{\mathrm{spin}}$, where the second factor is n-dimensional and the first factor is of the type, containing $D(E_0, 0, 0)$, examined in the preceding section.

Recall that electrodynamics in Minkowski space is carried by a vector field that satisfies, after gauge fixing, the wave equation and the Lorentz condition. The space of solutions of this pair of equations includes the transverse physical modes and the longitudinal gauge modes. But this does not allow for the construction of a quantized field, for no invariant propagator can be constructed from these modes alone. Nor can we drop the gauge modes since, though they form an invariant subspace, there is no invariant complement of physical modes. The remedy is to drop, temporarily, the Lorentz condition, and to make use of all the modes that solve the wave equation; that is, the entire representation $D_{\rm orb} \otimes D_{\rm spin}$.

In the last paragraph we had in mind the Poincaré group and its representations. It has been known, for a very long time, that Maxwell's equations are conformally invariant. In particular, the wave equation is invariant, and the solutions carry a representation of $SO(4,2)$. Unfortunately, though the wave equation is invariant, the wave operator is not, so to construct a conformally invariant propagator one must do something radical. One way that it can be done is to introduce an additional scalar field. And the simplest way to get support for this idea is to pass to Dirac's formulation of (the conformal completion of) Minkowski space as a cone in six dimension; this is, as we have seen, the same as the boundary of our space AdS_5.

Dirac's idea [34] was to develop a formalism that makes conformal invariance manifest; the next step is therefore to replace the four-dimensional vector field of ordinary electrodynamics by a six-dimensional vector field. Of course, all six components of this field should satisfy (after gauge fixing) the scalar wave equation (4.1). Therefore, to start with, we are looking at the representation $D_{\rm orb} \otimes D_6$. Since we are on the cone, where the Casimir operator does not qualify as a wave operator, we try to use the invariant operator ∂^2, but something must be said about the meaning of this operator.

The problem is that the operators of partial differentiation with respect to y_i are not tangential to the cone $y^2 = 0$. Therefore, the operator ∂^2 is not likely to be defined on the cone. But it turns out that this last operator is nevertheless well defined if the field is homogeneous of degree -1. This ties in nicely with the considerations of Section 2. The equation

$$\partial^2 A_j = \sum_{i=1,\ldots,5} \pm\partial_i^2 A_j = 0 \tag{4.2}$$

is meaningful and conformally invariant. In terms of local coordinates ∂^2 is the four-dimensional d'Alembertian. The most disquieting feature is that the field has six components instead of four. It is therefore to be expected that subsidiary conditions of the type $y \cdot A = 0$ or $\partial \cdot A = 0$ must be imposed. This cannot destroy the circumstance that the conformal group acts on the space of solutions, for each condition projects on an invariant submodule. But it can destroy the quantizability of the theory. In fact, quantization requires the retention of all components; subsidiary conditions can be imposed only on the states, just as in the conventional formulation. This is because the entire representation

$$D_{\text{boundary}} = D(1,0,0) \otimes D_6$$

is nondecomposable; it has no direct *summand.* To be precise,

$$D_{\text{boundary}} \approx D(1,\tfrac{1}{2},\tfrac{1}{2}) \to [D(2,1,0) \oplus D(2,0,1) \oplus \text{Id}] \to D(1,\tfrac{1}{2},\tfrac{1}{2}).$$

The arrows have the following meaning. $A = B \to C$ means that the big representation A has a subrepresentation C with a complement B, and that there is no invariant complement. The physical modes are in the center; note the inclusion of a zero mode, $\text{Id} = D(0,0,0)$, the gauge modes are on the right and the conjugate scalar modes are on the left. Except for the presence of the zero mode, and the fact that the spaces of gauge and scalar modes are larger than usual, this is exactly the structure of ordinary electrodynamics, as a Poincaré module. (The two representations in the middle remain irreducible when restricted to the Poincaré subgroup.)

Incidentally, the Lorentz (physicality) condition is

$$y \cdot A = 0. \tag{4.3}$$

This analysis of conformal Maxwell theory may be found in [35].

Let us now go to the interior of AdS_5, trying the same strategy. That is, we start with the same big representation $D(1,0,0) \otimes D_6$ and try to implement it by imposing the wave equation (3.1) on each of the six field components. But we have seen that this does not work; we do not have a realization of $D(1,0,0)$ in terms of a scalar field. The only way is to use the logarithmic propagator for each field component and the propagator $K_{ij} = \delta_{ij} K_{\log}$ for the vector field. This means that each component of A carries the representation of the scalar singleton field, namely

$$\tilde{D}(1,0,0) := D(3,0,0) \to D(1,0,0) \to D(3,0,0),.$$

The vector field thus carries (to put it differently, the propagator is the reproducing kernel for) the monstrous representation

$$
\begin{aligned}
D_{\text{interior}} &= \tilde{D}(1,0,0) \otimes D_6 \\
&= [D(3,0,0) \otimes D_6] \to D_{\text{boundary}} \to [D(3,0,0) \otimes D_6].
\end{aligned}
$$

Thus, on top of the usual gauge structure, already somewhat amplified by the exigency of conformal invariance, we have a new instance of the singleton type of gauge structure, and four dimensional Maxwell theory turns into a five dimensional topological gauge theory.

Let us emphasize this conclusion. Maxwell theory on the boundary of AdS_5, in ordinary, four dimensional space time, can be extended to the five dimensional interior. But the extended field, though it includes extrapolations of all the physical modes, are swamped by a much more dense set of new excitations. These extra modes enter the propagator with both signs. A Gupta-Bleuler type of quantization is still possible, but now there is a new class of gauge modes and a new type of gauge transformations. There is no conformally invariant local interaction that preserves gauge invariance, and therefore there is no way to introduce interactions, except possibly on the boundary, that preserves conformal invariance. In other words, the extrapolated theory is purely topological, exactly as the prototype singleton theory in four dimensions [19][20].

V. Five-Dimensional Superconformal $U(1)$ Theory.

In this section we will first give a manifest $O(4,2)$ covariant formulation of supersymmetric $N=1$ and $N=4$ gauge theory in four dimensions Minkiwski space, conformally extende to the Dirac cone, and expressed in terms of the natural coordinates of the cone [24][36]. Then we discuss the extrapolation to the interior of AdS_5.

The $N=4$ theory can be viewed as a $N=1$ theory with three chiral multiplets added to the $N=1$ gauge multiplet, to build up the $N=4$ vector multiplet. In this formulation, the $SU(4)$, $N=4$ theory has manifest $N=1$ supersymmetry and global $SU(3)$ symmetry. Group-theoretically this corresponds to the decomposition $SU(2,2/4) \to U(2,2/1) \otimes SU(3)$. Equipping the field of the chiral multiplet with an index $\alpha (\alpha = 1,2,3)$

we have the transformation rules [36]:

$$\begin{aligned}
\delta A^\alpha &= -i\bar\chi\, y\cdot\gamma\,\psi^\alpha, \qquad\qquad \delta B^\alpha = -i\bar\chi\, y\cdot\gamma\,\psi^\alpha,\\
\delta F^\alpha &= i\bar\chi(\gamma^{ij}L_{ij}-2)\psi^\alpha, \qquad \delta G^\alpha = i\bar\chi\gamma_7(\gamma^{ij}L_{ij}-2)\psi^\alpha,\\
\delta\psi^\alpha &= -\gamma\cdot\partial(A^\alpha-\gamma_7 B^\alpha)\chi+(F^\alpha-\gamma_7 G^\alpha)\chi,\\
\delta\lambda &= -\gamma^{ij}G_{ij}\chi - D\gamma_7\chi,\\
\delta G_{ij} &= i\bar\chi(2\gamma_{ij}+y^k\gamma_{kj}\partial_i - y^k\gamma_{ji}\partial_j)\lambda,\\
\delta D &= i\bar\chi(\gamma^{ij}L_{ij}-2)\lambda.
\end{aligned}$$

Here (ψ^α,λ) are $O(4,2)$ Majorana 8-spinors, $\{y_i\}$ are the same cone variables as before, and $\{\gamma_i\}$ are six Dirac matrices. The L_{ij} are the orbital $O(4,2)$ generators. Finally, χ is an 8-dimensional, anticommuting spinor parameter corresponding to Poincaré and conformal supersymmetry.

The free-bosonic field equations on the Dirac cone are

$$\partial^2 A^\alpha = \partial^2 B^\alpha = \partial^2 A_i = 0$$

where the gauge-potential A_i ($G_{ij} = \partial_i A_j - \partial_j A_i$) is subjected to the gauge condition $\partial^i A_i = y^i A_i = 0$. The spinor field equations are

$$(\gamma^{ij}L_{ij}-2)\psi^\alpha = 0 = (\gamma^{ij}L_{ij}-2)\lambda.$$

Note that the chiral $U(1)$ charge of this ($U(2,2;1)$ algebra is 3/4 for λ and -1/4 for the ψ^α, according to [13]. This follows from the fact that the fields A^α, B^α and A_i are homogeneous of degree -1, while $F^\alpha, G^\alpha, \psi^\alpha, D$ and λ are homogeneous of degree -2.

Observe that the $U(1)$ charge matrix of the four fermions (ψ^i,λ) is traceless as it must be since $U(1)\subset SU(4)$. Also the $U(1)$ charge of the (complex) triplet A^i+iB^i is $\frac{1}{2}$as it must be in order that the (non-Abelian) conformal couplings be $U(1)$ invariant [39].

It is obvious that the above free-field equations (as well as their non abelian extension to the interacting case) are $SU(4)$ covariant on the Dirac cone. In a forthcoming paper the $N=4$ manifest formulation together with its extension to AdS_5 will be given. It is now possible, nevertheless, to reach some conclusions concerning the nature of the extrapolation of super conformal $U(1)$ gauge theory to the interior of AdS$_5$.

The field equations given above must certainly remain valid in the complete theory; we mean that the equations $\partial^2 A^\alpha = 0$ and the rest have to be satisfied by the physical modes of the theory (and by the gauge modes as well). Since these field equations are to be the limits of field equations satisfied by the extrapolated fields, we take it to be evident that this extrapolation will involve exactly the same type of representations of $SO(4,2)$ that we studied in the earlier sections, though we do not know exactly how many of them will be carried by the propagator, assuming that it exists. Therefore, it is quite clear that the physical exitations, extrapolated from the boundary at infinity to the interior, are swamped by much more numerous gauge modes, that they are distinguised from the latter by their boundary conditions at infinity only, and not by any local property, and that consequently they cannot be observed. To put it somewhat differently, the full ("Gupta-Bleuler") quantization space has an indefinite metric, unitarity therefore requires that the interactions be gauge invariant in the sense that they do not engage (what we have called) the gauge modes. And since the gauge modes cannot be characterized locally, this implies that there cannot be any local interactions at all.

It is possible to calculate the smallest representation of conformal supersymmetry ($N = 1$) that contains all the physical modes and that must be involved in any complete, off-shell formulation of the theory. To describe the representation, let $D^s(E_0, j_1, j_2/c)$ denote the unique, irreducible super symmetry module with highest weight $(E_0, j_1, j_2/c)$, where the last label refers to the $u(1)$ charge. Then the total representation must contain

$$D^s(\tfrac{1}{2}, \tfrac{1}{2}, 0/\tfrac{1}{2}) \to [D^s(\tfrac{3}{2}, 0, \tfrac{1}{2}/\tfrac{1}{2}) \oplus \mathrm{Id} \oplus D^s(\tfrac{3}{2}, \tfrac{1}{2}, 0/-\tfrac{1}{2})] \to D^s(\tfrac{1}{2}, 0, \tfrac{1}{2}/-\tfrac{1}{2}).$$

As usual, physical modes are in the middle and gauge modes are at left.

We leave the complete construction of the supersingleton field theory of the $SU(2,2/4)$ algebra to a forthcoming work [40].

VI. Conclusions.

Motivated by recent attempts to relate world-volume dynamics to supergravity in the near horizon geometry, we have analyzed in some detail some features of the "dual theories" underlying 3-brane dynamics in IIB string theory. This amounts to comparing gauged $D = 5$ supergravity, with supergroup $SU(2,2/4)$ and $N = 4$, $SU(4)$ super Yang-Mills theory. On the supergravity side we find that a proposed quantization condition on the coefficient of the gauged Chern-Simons coupling [25] in the horizon geometry is equivalent to the statement that the horizon geometry of the three-brane is independent from the dilaton and only depends on the Planck scale, similar to the attractor mechanism [30].

On the world volume side, we have analyzed a superconformal $U(1)$ gauge theory and have shown that it can be formulated as a topological (singleton) field theory in AdS_5, in close parallell to what happened with the five-brane in M theory compactification [6][10][21].

For the future, we propose to complete the investigation of $N = 4$, superconformal, $U(1)$ gauge theory initiated in Section V, and to attack the problem of the relation of the interacting $SU(N)$ nonabelian $N = 4$ gauge theory with anti-De Sitter geometry. It is at present uncertain whether brane dynamics (on the world volume) can be related to singleton field theories on anti-De Sitter boundaries, but we think that further work in this direction may lead to new and interesting duality connections.

We hope the considerations presented in this paper, some of a purely speculative nature, will help to shed new light on these actual and potential connections among different aspects of non-perturative string dynamics.

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