ANNALES DE L'I. H. P., SECTION A

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Annales de l'I. H. P., section A, tome 27, nº 4 (1977), p. 425-434
<http://www.numdam.org/item?id=AIHPA_1977__27_4_425_0>

Ann. Inst. Henri Poincaré, Section A :
Vol. XXVII, n° 4, 1977, p. 425-434. *Physique théorique.*

Conformal transformations in superspace

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ABSTRACT. — In this work the spinor extension of the conformal algebra is investigated. The transformation law of superfields under the conformal coordinate inversion R defined in the superspace is derived. Using R-technique, the superconformally covariant two-point and three-point correlation functions are found.

1. INTRODUCTION

During the last years the possibilities of extending Poincaré invariance in particle physics are widely discussed. The theory of conformal invariance [*1-7*] is a typical example. As a space-time symmetry group, the conformal group is the most general group which leaves the light cone invariant. Many interesting ideas in quantum field theory, as well as many theoretical developments in the theory of strong interaction have been formulated and investigated by means of the conformal group. A noteworthy fact is that the requirements of invariance under conformal transformations allow us to define the two-point and three-point correlation functions almost unambiguously [*8-12*].

In the recent time many attentions are paid to the spinor extension of the Poincaré group [*13-16*], and the associated symmetry is called supersymmetry. A characteristic feature of supersymmetry is that it allows us to combine together bosons and fermions into finite irreducible multiplets. Supersymmetry has much enriched the elementary particle physics—the simplest supersymmetric models possess many unexpected and attractive properties due to the relations between the Green functions of bosons and fermions.

A problem naturally arises—the spinor extension of the conformal group. This problem has been examined first in the papers of Wess and Zumino [*15*], Ferrara [*17*], Dondi and Sohnius [*18*], and the author [*19*], and also in refs. [*20-22*]. In this work we consider this problem in more details, using the coordinate inversion transformation R defined in the superspace. The content of the paper is arranged as follows. Section 2 is devoted to the superconformal algebra, its realization and the transformation law of the superfields. In section 3 we consider the coordinate inversion transformation R in superspace, the transformation law of superfields under this operation. In section 4, using the R-technique, we obtain the superconformally covariant two-point and three-point correlation functions.

2. SUPERCONFORMAL TRANSFORMATIONS

The superconformal algebra consists, besides of the conformal generators $M_{\mu\nu}$, P_μ, K_μ and D, of two Majorana bispinor generators S_α, Q_α, and a chiral charge E. They obey the following commutation relations

$$(2.1)\qquad [M_{\mu\nu}, S_\alpha] = -\frac{1}{2}(\sigma_{\mu\nu}S)_\alpha, \qquad [M_{\mu\nu}, Q_\alpha] = -\frac{1}{2}(\sigma_{\mu\nu}Q)_\alpha$$

$$(2.2)\qquad \{S_\alpha, S_\beta\} = -(\gamma_\mu C)_{\alpha\beta}P^\mu$$

$$(2.3)\qquad \{Q_\alpha, Q_\beta\} = -(\gamma_\mu C)_{\alpha\beta}K^\mu$$

$$(2.4)\qquad \{S_\alpha, Q_\beta\} = -\frac{1}{2}(\sigma_{\mu\nu}C)_{\alpha\beta}M^{\mu\nu} - iC_{\alpha\beta}D - \frac{3}{2}i(\gamma_5 C)_{\alpha\beta}E$$

$$(2.5)\qquad [D, S_\alpha] = \frac{i}{2}S_\alpha$$

$$(2.6)\qquad [D, Q_\alpha] = -\frac{i}{2}Q_\alpha$$

$$(2.7)\qquad [P_\mu, Q_\alpha] = -(\gamma_\mu S)_\alpha$$

$$(2.8)\qquad [K_\mu, S_\alpha] = -(\gamma_\mu Q)_\alpha$$

$$(2.9)\qquad [S_\alpha, E] = -i(\gamma_5 S)_\alpha$$

$$(2.10)\qquad [Q_\alpha, E] = i(\gamma_5 Q)_\alpha$$

$$(2.11)\qquad [P_\mu, S_\alpha] = [K_\mu, Q_\alpha] = [P_\mu, E] = [M_{\mu\nu}, E] = [K_\mu, E] = [D, E] = 0$$

Other commutators including the conformal generators only are well-known and missed here. Operating on the functions defined in the superspace (x_μ, θ_α) [*16*], these generators can be realized as follows:

$$(2.12)\qquad M_{\mu\nu} = i(x_\mu\partial_\nu - x_\nu\partial_\mu) - \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta\theta_\beta\frac{\partial}{\partial\theta_\alpha}$$

$$(2.13)\qquad P_\mu = i\partial_\mu$$

$$(2.14)\qquad K_\mu = i(2x_\mu x^\nu\partial_\nu - x^2\partial_\mu) + \frac{i}{8}(\bar\theta\theta)^2\partial_\mu + \frac{1}{2}\bar\theta\theta(\gamma_\mu\theta)_\alpha\frac{\partial}{\partial\theta_\alpha} + i(\widehat{x}\gamma_\mu\theta)_\alpha\frac{\partial}{\partial\theta_\alpha}$$

(2.15) $$D = -ix_\mu\partial^\mu - \frac{i}{2}\theta_\alpha\frac{\partial}{\partial\theta_\alpha}$$

(2.16) $$E = -i(\gamma_5\theta)_\alpha\frac{\partial}{\partial\theta_\alpha}$$

(2.17) $$S_\alpha = -i\frac{\partial}{\partial\bar{\theta}^\alpha} + \frac{1}{2}(\gamma_\mu\theta)_\alpha\partial^\mu$$

(2.18) $$Q_\alpha = \frac{i}{2}\bar{\theta}\theta\frac{\partial}{\partial\bar{\theta}^\alpha} + \frac{i}{2}(\gamma_5 C)_{\alpha\beta}\bar{\theta}\gamma_5\theta\frac{\partial}{\partial\theta_\beta} + \frac{i}{4}(\gamma_5\gamma^\mu C)_{\alpha\beta}\bar{\theta}\gamma_5\gamma_\mu\theta\frac{\partial}{\partial\theta_\beta} - (\widehat{x}C)_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} + \frac{i}{2}(\widehat{x}\gamma_\mu\theta)_\alpha\partial^\mu + \frac{1}{4}\bar{\theta}\theta(\gamma_\mu\theta)_\alpha\partial^\mu$$

Consider now the transformation laws of the field operators under the superconformal transformations. From (2.12)-(2.18) we note that the point ($x = 0$, $\theta = 0$) remains unchanged under the homogeneous Lorentz transformations $M_{\mu\nu}$, the scale transformation D, the special conformal transformation K_μ, the Q- and E-transformations. These transformations form the little group of the superconformal group. According to any given representation of this little group we can define the entire action of the generators of the superconformal group on the field operators $\varphi_A(x, \theta)$. This is done by the method of the theory of induced representations [*5, 23*] in the following manner.

Let

(2.19) $$[M_{\mu\nu}, \varphi_A(0, 0)] = -(\Sigma_{\mu\nu}^{(\varphi)}\varphi(0, 0))_A$$

(2.20) $$[D, \varphi_A(0, 0)] = -(\Delta^{(\varphi)}\varphi(0, 0))_A$$

(2.21) $$[K_\mu, \varphi_A(0, 0)] = -(K_\mu^{(\varphi)}\varphi(0, 0))_A$$

(2.22) $$[Q_\alpha, \varphi_A(0, 0)] = -(q_\alpha^{(\varphi)}\varphi(0, 0))_A$$

(2.23) $$[E, \varphi_A(0, 0)] = -(e^{(\varphi)}\varphi(0, 0))_A$$

where $\Sigma_{\mu\nu}$, Δ, K_μ, q_α, e are some matrices obeying the analogous commutation relations as those for $M_{\mu\nu}$, D, K_μ, Q_α, E. We are to find the commutation rule $[J, \varphi_A(x, \theta)]$ for the elements J of the superconformal algebra and the field operator $\varphi_A(x, \theta)$ (*). Choose the basis in the index space in such a way, that the operators P_μ and S_α do not act on the indices, *i. e.*

(2.24) $$[P_\mu, \varphi_A(x, \theta)] = -i\partial_\mu\varphi_A(x, \theta)$$

(2.25) $$[S_\alpha, \varphi_A(x, \theta)] = i\frac{\partial}{\partial\bar{\theta}^\alpha}\varphi_A(x, \theta) - \frac{1}{2}(\gamma_\mu\theta)_\alpha\partial^\mu\varphi_A(x, \theta),$$

and, therefore

(2.26) $$\varphi_A\left(x + a + \frac{i}{2}\bar{\varepsilon}\gamma\theta, \theta + \varepsilon\right) = e^{iaP - i\bar{\varepsilon}S}\varphi_A(x, \theta)e^{-iaP + i\bar{\varepsilon}S}$$

(*) For definiteness, let $\varphi_A(x, \theta)$ be a Bose operator.

With the help of (2.26) we can write

$$[J, \varphi_A(x, \theta)] = e^{ixP - i\bar{\theta}S}[J', \varphi_A(0, 0)]e^{-ixP + i\bar{\theta}S} \tag{2.27}$$

where we denote

$$J' \equiv e^{-ixP + i\bar{\theta}S} J e^{ixP - i\bar{\theta}S} \tag{2.28}$$

Using the commutation relations (2.1)-(2.11), we find:

$$M'_{\mu\nu} = M_{\mu\nu} + x_\mu P_\nu - x_\nu P_\mu + \frac{i}{2}\bar{\theta}\sigma_{\mu\nu}S - \frac{i}{4}\varepsilon_{\mu\nu\lambda\rho}\bar{\theta}\gamma_5\gamma^\lambda\theta . P^\rho \tag{2.29}$$

$$\begin{aligned} K'_\mu = {} & K_\mu - 2x_\mu D + 2x_\mu x^\nu P_\nu - x^2 P_\mu + 2x^\nu M_{\mu\nu} \\ & + \frac{i}{4}\varepsilon_{\mu\nu\lambda\rho}\bar{\theta}\gamma_5\gamma^\nu\theta . M^{\lambda\rho} - \frac{i}{2}\varepsilon_{\mu\nu\lambda\rho}x^\nu\bar{\theta}\gamma_5\gamma^\lambda\theta . P^\rho - \frac{1}{8}(\bar{\theta}\theta)^2 P_\mu \\ & - \bar{\theta}\gamma_\mu\hat{x}S - \frac{i}{2}\bar{\theta}\theta . \bar{\theta}\gamma_\mu S + i\bar{\theta}\gamma_\mu Q + \frac{3i}{4}\bar{\theta}\gamma_5\gamma_\mu\theta . E \end{aligned} \tag{2.30}$$

$$D' = D - x^\mu P_\mu + \frac{1}{2}\bar{\theta}S \tag{2.31}$$

$$\begin{aligned} Q'_\alpha = {} & Q_\alpha - \frac{i}{2}(\sigma_{\mu\nu}\theta)_\alpha M^{\mu\nu} + \frac{i}{2}\bar{\theta}\theta(\gamma_\mu\theta)_\alpha P^\mu + (\hat{x}\gamma_\mu\theta)_\alpha P^\mu \\ & - \theta_\alpha D - \frac{3}{2}(\gamma_5\theta)_\alpha E - \frac{1}{2}\bar{\theta}\theta S_\alpha + \frac{1}{2}\bar{\theta}\gamma_5\theta(\gamma_5 S)_\alpha \\ & + \frac{1}{4}\bar{\theta}\gamma_5\gamma_\mu\theta(\gamma_5\gamma^\mu S)_\alpha + i(\hat{x}S)_\alpha \end{aligned} \tag{2.32}$$

$$E' = E + \bar{\theta}\gamma_5 S - \frac{i}{2}\bar{\theta}\gamma_5\gamma_\mu\theta . P^\mu \tag{2.33}$$

By inserting (2.29)-(2.33) into (2.27) and taking into account (2.19)-(2.23) we get, after some manipulations:

$$[M_{\mu\nu}, \varphi(x, \theta)] = -\left\{ \Sigma^{(\varphi)}_{\mu\nu} + i(x_\mu\partial_\nu - x_\nu\partial_\mu) - \frac{1}{2}(\sigma_{\mu\nu}\theta)_\alpha \frac{\partial}{\partial\theta_\alpha} \right\} \varphi(x, \theta) \tag{2.34}$$

$$\begin{aligned} [K_\mu, \varphi(x, \theta)] = - \Big\{ & k^{(\varphi)}_\mu - 2x_\mu\Delta^{(\varphi)} + 2ix_\mu x^\nu\partial_\nu - ix^2\partial_\mu + 2x^\nu\Sigma^{(\varphi)}_{\mu\nu} \\ & + \frac{i}{8}(\bar{\theta}\theta)^2\partial_\mu + i(\hat{x}\gamma_\mu\theta)_\alpha\frac{\partial}{\partial\theta_\alpha} + \frac{1}{2}\bar{\theta}\theta(\gamma_\mu\theta)_\alpha\frac{\partial}{\partial\theta_\alpha} \\ & + \frac{i}{4}\varepsilon_{\mu\nu\lambda\rho}\bar{\theta}\gamma_5\gamma^\nu\theta . \Sigma^{(\varphi)\lambda\rho} + \frac{3i}{4}\bar{\theta}\gamma_5\gamma_\mu\theta . e^{(\varphi)} + i\bar{\theta}\gamma_\mu q^{(\varphi)} \Big\} \varphi(x, \theta) \end{aligned} \tag{2.35}$$

$$[D, \varphi(x, \theta)] = -\left\{ \Delta^{(\varphi)} - ix^\mu\partial_\mu - \frac{i}{2}\theta_\alpha\frac{\partial}{\partial\theta_\alpha} \right\} \varphi(x, \theta) \tag{2.36}$$

$$\begin{aligned} [Q_\alpha, \varphi(x, \theta) = - \Big\{ & q^{(\varphi)}_\alpha - \frac{i}{2}(\sigma_{\mu\nu}\theta)_\alpha\Sigma^{(\phi)\mu\nu} - \theta_\alpha\Delta^{(\varphi)} - \frac{3}{2}(\gamma_5\theta)_\alpha e^{(\varphi)} \\ & - (\hat{x}C)_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} + \frac{i}{2}\bar{\theta}\theta\frac{\partial}{\partial\bar{\theta}^\alpha} + \frac{i}{2}\bar{\theta}\gamma_5\theta(\gamma_5 C)_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} \\ & + \frac{i}{4}\bar{\theta}\gamma_5\gamma_\mu\theta(\gamma_5\gamma^\mu C)_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} + \frac{1}{4}\bar{\theta}\theta(\hat{\partial}\theta)_\alpha + \frac{i}{2}(\hat{x}\hat{\partial}\theta)_\alpha \Big\} \varphi(x, \theta) \end{aligned} \tag{2.37}$$

(2.38) $$[E, \varphi(x, \theta) = - \left\{ e^{(\varphi)} - i(\gamma_5\theta)_\alpha \frac{\partial}{\partial\theta_\alpha} \right\} \varphi(x, \theta)$$

3. R-INVERSION

As is well-known, in studying the conformal invariance in the usual x-space it is more convenient to use the discrete transformation $x_\mu \to - \frac{x_\mu}{x^2}$, called R-inversion, rather than the special conformal transformation K_μ (see, for example, refs. [*2, 10, 24*]). Owing to the relations

(3.1) $$RP_\mu R = K_\mu$$

(3.2) $$RDR = - D$$

(3.3) $$RM_{\mu\nu}R = M_{\mu\nu},$$

the covariance (invariance) with respect to the conformal algebra, follows from the covariance (invariance) with respect to the Poincaré algebra, the scale and R-transformations, taken together. Here too, the R-inversion operator is to be defined in such a way that, acting in the superspace (x, θ), it satisfies the same relations (3.1)-(3.3), and

(3.4) $$R^2 = 1$$

as well. It can be seen that such an operator is of the form

(3.5) $$R\{x_\mu, \theta_\alpha\} = \left\{ - \frac{x_\mu}{x^2}\left(1 + \frac{1}{8}\frac{(\bar{\theta}\theta)^2}{x^2}\right), \frac{i(\widehat{x}\theta)_\alpha}{x^2} - \frac{1}{2}\frac{\bar{\theta}\theta.\theta_\alpha}{x^2} \right\}$$

By the use of (3.1) and (3.5) it is easy to write out the finite special conformal transformation:

(3.6) $$e^{icK}x_\mu = \frac{x_\mu + x^2 c_\mu}{1 + 2cx + c^2x^2} + \frac{1}{8}(\bar{\theta}\theta)^2 \frac{c^2x_\mu - (1 + 2cx)c_\mu}{(1 + 2cx + c^2x^2)^2}$$

(3.7) $$e^{icK}\theta_\alpha = \frac{1}{1 + 2cx + c^2x^2}\left[\theta_\alpha + (\widehat{c}\widehat{x}\theta)_\alpha + \frac{i}{2}\bar{\theta}\theta.(\widehat{c}\theta)_\alpha\right]$$

Now the problem is to find the transformation law of the field operator $\varphi(x, \theta)$ under R-inversion. We restrict ourselves to the representations for which

(3.8) $$k_\mu^{(\varphi)} = 0 \quad , \quad q_\alpha^{(\varphi)} = 0 \quad , \quad e^{(\varphi)} = 0 \quad , \quad \Delta^{(\varphi)} = il_\varphi$$

l_φ being the scale dimension of the field $\varphi(x, \theta)$:

(3.9) $$e^{-i\alpha D}\varphi(x, \theta)e^{i\alpha D} = \rho^{-l_\varphi}\varphi(\rho x, \rho^{1/2}\theta) \quad , \quad \rho \equiv e^\alpha$$

That will be sufficient for many physical applications. Generalizing our method [*24*] of finding U(R) in the usual x-space, we now put:

(3.10) $$U(R)\varphi(x, \theta)U^{-1}(R) = \left[x^2 - \frac{1}{8}(\bar{\theta}\theta)^2\right]^{l_\varphi} S^{(\varphi)}(x, \theta)\varphi(R(x, \theta)),$$

where $S^{(\varphi)}(x, \theta)$ is some matrix, which is to be defined through the equations (3.1)-(3.4).

By noting that the function

$$g(x, \theta) \equiv x^2 - \frac{1}{8}(\bar{\theta}\theta)^2$$

has the property

$$g(x, \theta)g(R(x, \theta)) = 1 \tag{3.11}$$

we get from equation (3.4):

$$S(x, \theta)S(R(x, \theta)) = 1. \tag{3.12}$$

Equation (3.2) gives, taking into account (3.9) and (3.11):

$$S(x, \theta)S(\rho y, \rho^{1/2}\tau) = 1 \tag{3.13}$$

where (y, τ) denotes the inversed coordinates of (x, θ):

$$(y, \tau) \equiv R(x, \theta).$$

Equations (3.12) and (3.13) together show that $S(x, \theta)$ is a homogeneous matrix of zero^th order:

$$S(\rho x, \rho^{1/2}\theta) = S(x, \theta). \tag{3.14}$$

By using (3.1), (3.12) and (2.35) $\left(\text{with } k_\mu^{(\varphi)} = 0,\ q_\alpha^{(\varphi)} = 0,\ e^{(\varphi)} = 0\right)$, after some manipulations we obtain the following equation for the matrix $S(x, \theta)$:

$$- S(y, \tau)\partial_\mu S(x, \theta) = 2iy^\nu\Sigma_{\mu\nu} - \frac{1}{4}\varepsilon_{\mu\nu\lambda\rho}\bar{\tau}\gamma_5\gamma^\nu\tau . \Sigma^{\lambda\rho} \tag{3.15}$$

This equation taken together with equations (3.12) and (3.14) is sufficient for defining completely the matrix $S(x, \theta)$ and, consequently, the transformation law of the field operators. For illustration, let us give some examples.

In the case of scalar superfield $\Sigma_{\mu\nu} = 0$, and the equations (3.12), (3.14) and (3.15) give (apart from a phase factor):

$$S(x, \theta) = 1. \tag{3.16}$$

Hence, we have

$$U(R)\phi(x, \theta)U^{-1}(R) = \delta_\phi\left[x^2 - \frac{1}{8}(\bar{\theta}\theta)^2\right]^{l\phi}\phi(R(x, \theta)) \quad , \quad |\delta_\phi| = 1 \tag{3.17}$$

For the vector superfield $V_\mu(x, \theta)$ the most general form of the matrix $S(x, \theta)$ consistent with the Lorentz covariance is

$$S_{\mu\nu}(x, \theta) = ag_{\mu\nu} + \frac{bx_\mu x_\nu}{x^2} + cg_{\mu\nu}\frac{(\bar{\theta}\theta)^2}{x^2} + \frac{dx_\mu x_\nu}{(x^2)^2}(\bar{\theta}\theta)^2 + f\varepsilon_{\mu\nu\lambda\rho}\frac{\bar{\theta}\gamma_5\gamma^\lambda\theta . x^\rho}{x^2} + \left(hg_{\mu\nu} + \frac{kx_\mu x_\nu}{x^2}\right)\frac{\bar{\theta}\theta}{(x^2)^{1/2}} \tag{3.18}$$

Equation (3.12) then gives:

(3.19) $$a = 1, \quad 2ab + b^2 = 0, \quad ac + f^2 = 0, \quad bc - f^2 + ad + bd = 0, \quad h = k = 0$$

Equation (3.15) with $(\Sigma_{\mu\nu})^\sigma_\rho = i(g_{\mu\rho}\delta^\sigma_\nu - g_{\nu\rho}\delta^\sigma_\mu)$ gives, taking into account (3.19):

(3.20) $$b = -2, \quad c = \frac{1}{4}, \quad d = -\frac{1}{4}, \quad f = \frac{i}{2}$$

Thus, we have finally:

(3.21) $$U(R)V_\mu(x, \theta)U^{-1}(R) = \delta_V\left[x^2 - \frac{1}{8}(\bar{\theta}\theta)^2\right]^{l_V} S(x, \theta)^\nu_\mu V_\nu(R(x, \theta)), \quad |\delta_V| = 1$$

(3.22) $$S_{\mu\nu}(x, \theta) = g_{\mu\nu} - \frac{2x_\mu x_\nu}{x^2} + \frac{1}{4}g_{\mu\nu}\frac{(\bar{\theta}\theta)^2}{x^2} - \frac{1}{4}\frac{x_\mu x_\nu(\bar{\theta}\theta)^2}{(x^2)^2} + \frac{i}{2}\varepsilon_{\mu\nu\lambda\rho}\frac{\bar{\theta}\gamma_5\gamma^\lambda\theta . x^\rho}{x^2}$$

The generalization for the case of tensor superfield of arbitrary rank is trivial. We have:

(3.23) $$U(R)T_{\mu_1\ldots\mu_n}(x, \theta)U^{-1}(R) = \delta_T\left[x^2 - \frac{1}{8}(\bar{\theta}\theta)^2\right]^{l_T} S(x, \theta)^{\nu_1}_{\mu_1} \ldots S(x, \theta)^{\nu_n}_{\mu_n} T_{\nu_1\ldots\nu_n}(R(x, \theta)), \quad |\delta_T| = 1$$

where the matrix $S(x, \theta)$ is the same as in (3.22).

Had we derived the operators U(R) for the superfields, we can easily find their transformation law under the special conformal transformation by the help of equation (3.1). For example, for the scalar superfield we have

(3.24) $$e^{-icK}\phi(x, \theta)e^{icK} = \left[1 + 2cx + c^2x^2 - \frac{1}{8}c^2(\bar{\theta}\theta)^2\right]^{l_\phi}\phi(x', \theta'),$$

where (x', θ') denotes the transformed value of (x, θ) and is defined by the equations (3.6) and (3.7). Consequently, for the field components appeared in the expansion of superfield $\varphi(x, \theta)$,

$$\phi(x, \theta) = A(x) + \bar{\theta}\psi(x) + \frac{1}{4}\bar{\theta}\theta F(x) + \frac{1}{4}\bar{\theta}\gamma_5\theta . G(x) + \frac{i}{4}\bar{\theta}\gamma_5\gamma_\mu\theta . A^\mu(x) + \frac{1}{4}\bar{\theta}\theta . \bar{\theta}\chi(x) + \frac{1}{32}(\bar{\theta}\theta)^2 D(x),$$

we have

$$A'(x) = (1 + 2cx + c^2x^2)^{l_\phi}A(z)$$
$$\psi'(x) = (1 + 2cx + c^2x^2)^{l_\phi - 1}(1 + \hat{x}\hat{c})\psi(z)$$
$$F'(x) = (1 + 2cx + c^2x^2)^{l_\phi - 1}F(z)$$
$$G'(x) = (1 + 2cx + c^2x^2)^{l_\phi - 1}G(z)$$

$$A'_\mu(x) = (1 + 2cx + c^2x^2)^{l_\phi - 1}\Big[\delta^\nu_\mu + \frac{2}{1 + 2cx + c^2x^2}(x_\mu c^\nu - c_\mu x^\nu$$
$$+ 2cx.x_\mu c^\nu - x^2 c_\mu c^\nu - c^2 x_\mu x^\nu)\Big]A_\nu(x)$$
$$\chi'(x) = (1 + 2cx + c^2x^2)^{l_\phi - 2}(1 + \hat{x}\hat{c})\chi(z)$$
$$- 2i(1 + 2cx + c^2x^2)^{l_\phi - 1}\hat{c}\psi(z)$$
$$D'(x) = (1 + 2cx + c^2x^2)^{l_\phi - 2}D(z) - 4l_\phi(1 + 2cx + c^2x^2)^{l_\phi - 1}c^2A(z)$$
$$- 4(1 + 2cx + c^2x^2)^{l_\phi - 2}[(1 + 2cx)c_\mu - c^2x_\mu]\partial^\mu A(z)$$

where we denote

$$Z_\mu \equiv \frac{x_\mu + x^2 c_\mu}{1 + 2cx + c^2x^2}, \qquad A' \equiv e^{-icK}Ae^{icK}$$

4. SUPERCONFORMALLY COVARIANT TWO-POINT AND THREE-POINT FUNCTIONS

In this section, using the R-technique developed in section 3, we derive the formulae for the superconformally covariant two-point and three-point correlation functions. For simplicity, we restrict ourselves to the case of scalar superfields. Let $\phi_1(x, \theta)$ and $\phi_2(x, \theta)$ be scalar superfields with dimensions l_1 and l_2, and $G_{12}(x_1, \theta_1; x_2, \theta_2)$ their two-point function:

$$(4.1) \qquad G_{12}(x_1, \theta_1; x_2, \theta_2) \equiv \langle \phi_1(x_1, \theta_1)\phi_2(x_2, \theta_2) \rangle.$$

From the translation and S-invariance it follows that:

$$(4.2) \quad \langle \phi_1(x_1, \theta_1)\phi_2(x_2, \theta_2) \rangle = \left\langle \phi_1\left(x_1 - x_2 - \frac{i}{2}\bar{\theta}_2\gamma\theta_1, \theta_1 - \theta_2\right)\phi_2(0, 0) \right\rangle$$

while D- and R-invariance requirements give, respectively:

$$(4.3) \quad G_{12}(\rho x_1, \rho^{1/2}\theta_1 ; \rho x_2, \rho^{1/2}\theta_2) = \rho^{l_1 + l_2}G_{12}(x_1, \theta_1 ; x_2, \theta_2)$$

$$(4.4) \quad G_{12}(y_1, \tau_1 ; y_2, \tau_2) = \left[x_1^2 - \frac{1}{8}(\bar{\theta}_1\theta_1)^2\right]^{-l_1}\left[x_2^2 - \frac{1}{8}(\bar{\theta}_2\theta_2)^2\right]^{-l_2}G_{12}(x_1, \theta_1 ; x_2, \theta_2)$$

where $(y, \tau) \equiv R(x, \theta)$.

Now we employ the following identity between the (x, θ)-coordinates and their inversed ones (y, τ):

$$(4.5) \quad \left(y_1 - y_2 - \frac{i}{2}\bar{\tau}_2\gamma\tau_1\right)^2 - \frac{1}{8}[(\bar{\tau}_1 - \bar{\tau}_2)(\tau_1 - \tau_2)]^2$$
$$= \left[x_1^2 - \frac{1}{8}(\bar{\theta}_1\theta_1)^2\right]^{-1}\left[x_2^2 - \frac{1}{8}(\bar{\theta}_2\theta_2)^2\right]^{-1}$$
$$\left\{\left(x_1 - x_2 - \frac{i}{2}\bar{\theta}_2\gamma\theta_1\right)^2 - \frac{1}{8}[(\bar{\theta}_1 - \bar{\theta}_2)(\theta_1 - \theta_2)]^2\right\}$$

to find out the explicit form of $G_{12}(x_1, \theta_1; x_2, \theta_2)$ satisfying the equations (4.2)-(4.4). We have:

(4.6)
$$G_{12}(x_1, \theta_1; x_2, \theta_2) = \delta_{l_1 l_2} G \left\{ \left(x_1 - x_2 - \frac{i}{2}\theta_2\gamma\theta_1 \right)^2 - \frac{1}{8}[(\bar{\theta}_1 - \bar{\theta}_2)(\theta_1 - \theta_2)]^2 \right\}^{l_1}$$

It is easy to see that the form (4.6) of G_{12} is automatically E-invariant, *i. e.* invariant under the transformation

$$(x_i, \theta_i) \to (x_i, (\cos e + \gamma_5 \sin e)\theta_i), \tag{4.7}$$

and also Q-covariant, namely it satisfies the following equation (see (2.37) with $\Sigma = 0$, $q_\alpha = 0$, $e = 0$, $\Delta = il$):

$$\sum_{i=1,2} Q_\alpha^{(i)} G_{12}(x_1, \theta_1; x_2, \theta_2) = il(\theta_{1\alpha} + \theta_{2\alpha}) G_{12}(x_1, \theta_1; x_2, \theta_2) \tag{4.8}$$

where $Q_\alpha^{(i)}$ stands for the differential operator (2.18) and acts on the variables (x_i, θ_i).

From the two-point function (4.6) of superfields we can easily obtain the two-point functions of field-components. We have:

$$\langle A(x_1)A(x_2) \rangle = G[(x_1 - x_2)^2]^l \tag{4.9}$$
$$\langle A(x_1)D(x_2) \rangle = -4lG[(x_1 - x_2)^2]^{l-1} \tag{4.10}$$
$$\langle \psi_\alpha(x_1)\bar{\psi}^\beta(x_2) \rangle = ilG[(x_1 - x_2)^2]^{l-1}(\hat{x}_1 - \hat{x}_2)_\alpha^\beta \tag{4.11}$$
$$\langle \psi_\alpha(x_1)\bar{\chi}^\beta(x_2) \rangle = -2lG[(x_1 - x_2)^2]^{l-1}\delta_\alpha^\beta \tag{4.12}$$
$$\langle F(x_1)F(x_2) \rangle = -2l(l+2)G[(x_1 - x_2)^2]^{l-1} \tag{4.13}$$
$$\langle G(x_1)G(x_2) \rangle = -2l(l+2)G[(x_1 - x_2)^2]^{l-1} \tag{4.14}$$
$$\langle A_\mu(x_1)A_\nu(x_2) \rangle = 2l(l-1)G[(x_1 - x_2)^2]^{l-1} \left\{ g_{\mu\nu} - \frac{2(x_1 - x_2)_\mu(x_1 - x_2)_\nu}{(x_1 - x_2)^2} \right\} \tag{4.15}$$
$$\langle D(x_1)D(x_2) \rangle = 16l^2(l^2 - 1)G[(x_1 - x_2)^2]^{l-2} \tag{4.16}$$
$$\langle \chi_\alpha(x_1)\chi^\beta(x_2) \rangle = 4il(l^2 - 1)G[(x_1 - x_2)]^2]^{l-2}(\hat{x}_1 - \hat{x}_2)_\alpha^\beta \tag{4.17}$$

Consider now the three-point functions. Denote

$$G_{123}(x_1, \theta_1; x_2, \theta_2; x_3, \theta_3) \equiv \langle \phi_1(x_1, \theta_1)\phi_2(x_2, \theta_2)\phi_3(x_3, \theta_3) \rangle \tag{4.18}$$

The requirements of D- and R-invariance tell that

$$G_{123}(\rho x_1, \rho^{1/2}\theta_1; \rho x_2, \rho^{1/2}\theta_2; \rho x_3, \rho^{1/2}\theta_3) = \rho^{l_1+l_2+l_3} G_{123}(x_1, \theta_1; x_2, \theta_2; x_3, \theta_3) \tag{4.19}$$

$$G_{123}(y_1, \tau_1; y_2, \tau_2; y_3, \tau_3) = \left[x_1^2 - \frac{1}{8}(\bar{\theta}_1\theta_1)^2\right]^{-l_1}\left[x_2^2 - \frac{1}{8}(\bar{\theta}_2\theta_2)^2\right]^{-l_2}\left[x_3^2 - \frac{1}{8}(\bar{\theta}_3\theta_3)^2\right]^{-l_3} . G_{123}(x_1, \theta_1; x_2, \theta_2; x_3, \theta_3) \tag{4.20}$$

Using again the identity (4.5), we find the following form of G_{123} satisfying equations (4.19) and (4.20):

$$(4.21)\quad G_{123}(x_1, \theta_1 ; x_2, \theta_2 ; x_3, \theta_3)$$
$$= g \left\{ \left(x_1 - x_2 - \frac{i}{2}\bar{\theta}_2\gamma\theta_1 \right)^2 - \frac{1}{8}[(\bar{\theta}_1 - \bar{\theta}_2)(\theta_1 - \theta_2)]^2 \right\}^{\frac{1}{2}(l_1+l_2-l_3)}$$
$$\cdot \left\{ \left(x_2 - x_3 - \frac{i}{2}\bar{\theta}_3\gamma\theta_2 \right)^2 - \frac{1}{8}[(\bar{\theta}_2 - \bar{\theta}_3)(\theta_2 - \theta_3)]^2 \right\}^{\frac{1}{2}(l_2+l_3-l_1)}$$
$$\cdot \left\{ \left(x_3 - x_1 - \frac{i}{2}\bar{\theta}_1\gamma\theta_3 \right)^2 - \frac{1}{8}[(\bar{\theta}_3 - \bar{\theta}_1)(\theta_3 - \theta_1)]^2 \right\}^{\frac{1}{2}(l_3+l_1-l_2)}$$

This form is evidently P- and S-invariant. Moreover, it is automatically E-invariant and Q-invariant, and therefore superconformally covariant.

REFERENCES

[1] D. Bohm, M. Flato, D. Sternheimer, J. P. Vigier, *Nuov. Cim.*, t. **38**, 1965, p. 1941.
[2] H. A. Kastrup, *Phys. Rev.*, t. **142**, 1966, p. 1060.
[3] L. Castell, *Nuov. Cim.*, t. **46** A, 1966, p. 1.
[4] A. O. Barut, H. Kleinert, *Phys. Rev.*, t. **161**, 1967, p. 1464.
[5] G. Mack, A. Salam, *Ann. of Phys.*, t. **53**, 1969, p. 174.
[6] D. I. Gross, J. Wess, *Phys. Rev.*, t. D **2**, 1970, p. 753.
[7] M. Flato, I. Simon, D. Sternheimer, *Ann. of Phys.*, t. **61**, 1970, p. 78.
[8] A. M. Polyakov, *Pisma J. E. T. P.*, t. **12**, 1970, p. 538.
[9] A. A. Migdal, *Phys. Lett.*, t. **37** B, 1971, p. 78.
[10] E. Schreier, *Phys. Rev.*, t. D **3**, 1971, p. 980.
[11] S. Ferrara, G. Parisi, *Nucl. Phys.*, t. B **42**, 1971, p. 281.
[12] Dao Vong Duc, *Teor. Mat. Phys.*, t. **20**, 1974, p. 202.
[13] I. A. Golfand, E. P. Lichtman, *Pisma J. E. T. P.*, t. **13**, 1971, p. 452.
[14] D. V. Volkov, V. P. Akulov, *Pisma J. E. T. P.*, t. **16**, 1972, p. 621.
[15] J. Wess, B. Zumino, *Nucl. Phys.*, t. B **70**, 1974, p. 39.
[16] A. Salam, J. Strathdee, *Nucl. Phys.*, t. B **76**, 1974, p. 477.
[17] S. Ferrara, *Nucl. Phys.*, t. B **77**, 1974, p. 75.
[18] P. H. Dondi, M. Sohnius, *Nucl. Phys.*, t. B **81**, 1974, p. 317.
[19] Dao Vong Duc, *J. I. N. R. preprint*, P2-8237, 1974; *Teor. Mat. Phys.*, t. **24**, 1975, p. 49.
[20] L. Corwin, Y. Ne'eman, S. Sternberg, *Rev. Mod. Phys.*, t. **47**, 1975, p. 573.
[21] V. Molotkov, S. Petrova, D. Stoyanov, *J. I. N. R. preprint* E2-8688, 1975.
[22] M. Omote, S. Kamefuchi, *Lett. al Nuov. Cim.*, t. **17**, 1976, p. 220.
[23] G. W. Mackey, *Bull. Am. Math. Soc.*, t. **69**, 1963, p. 628.
[24] Dao Vong Duc, *Teor. Mat. Phys.*, t. **20**, 1974, p. 71.

(Manuscrit reçu le 18 avril 1977)