Honam Mathematical J. **38** (2016), No. 1, pp. 59–69
http://dx.doi.org/10.5831/HMJ.2016.38.1.59

CONFORMALLY FLAT NORMAL ALMOST CONTACT 3-MANIFOLDS

Jong Taek Cho

Abstract. We classify conformally flat Kenmotsu 3-manifolds and classify conformally flat cosympletic 3-manifolds.

1. Introduction

Let M be a smooth manifold of odd dimension $m = 2n + 1$. Then M is said to be an *almost contact manifold* if its structure group of the linear frame bundle is reducible to $\mathrm{U}(n) \times \{1\}$. This is equivalent to existence of an endomorphism field φ, a vector field ξ and a 1-form η satisfying

$$\varphi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \tag{1}$$

where I denotes the identity transformation. Then M admits a Riemannian metric g satisfying

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2}$$

for any vector fields X, Y on M. Such (φ, ξ, η, g) is called an *almost contact metric structure*. In case that their automorphism groups have the maximum dimension $(n + 1)^2$, they are classified by the following three classes ([26]): (i) Sasakian space forms, that is, complete, simply connected, normal contact Riemannian manifolds of constant holomorphic sectional curvature; (ii) $\mathbb{R} \times F(k)$ or $\mathbb{S} \times F(k)$, the product spaces of a line $\mathbb{R}$ or a circle $\mathbb{S}$ and a complex space form $F(k)$; (iii) warped product spaces $\mathbb{R} \times_{ce^t} \mathbb{C}E^n$ (c a positive constant) of a real line and a complex Euclidean space. Each class has been intensively developed. For the class (i), the contact metric structure including Sasakian structure has been investigated by many authors (cf. [2]). The geometric property of

Received September 24, 2015. Accepted January 25, 2016.
2010 Mathematics Subject Classification. 53B20, 53C25.
Key words and phrases. almost contact 3-manifold, conformally flatness.
This study was financially supported by Chonnam National University, 2015.

(ii) is represented as the so-called *cosymplectic structure*. The Riemannian products of a real line or a circle and a Kählerian manifold admit such a structure. Extending the model (iii), Kenmotsu [13] introduced another class, which is expressed (locally) by a warped product space of an open interval and a Kählerian manifold. We call such a manifold M a *Kenmotsu manifold* and its almost contact metric structure is called a *Kenmotsu structure*. It is worth noting that every orientable Riemannian 2-manifold N admits a Kählerian metric. Taking a product metric or a warped product metric with a warping function $\rho(t) = c\exp t$, respectively on the product space $\mathbb{R} \times N$, then we have a cosymplectic or a Kenmotsu 3-manifold, respectively. A Sasakian, a cosymplectic, or a Kenmotsu manifold holds the *CR-integrability*, and moreover the *normality*.

On the other hand, M. Okumura [19] proved that a conformally flat Sasakian manifold of dimension ≥ 5 is of constant curvature $+1$. S. Tanno [25] extended the result to the dimension 3. Very recently, D-h. Yang and the present author [10] develop the study of conformally flat contact 3-manifolds. A confomally flat cosymplectic manifold M^{2n+1} is locally flat for $n > 1$ ([11]). In [13] it was proved that a conformally flat Kenmotsu manifold M^{2n+1} is of constant curvature -1 for $n > 1$. In this paper, we study the case of dimension 3. In Section 3, we classify a conformally flat Kenmotsu 3-manifold (Theorem 3.3) and we classify a conformally flat cosymplectic 3-manifold in Section 4 (Theorem 4.2). In Section 5, we treat real hypersurfaces in a complex projective space or a complex hyperbolic space. Due to a result in [14], we know that there are no conformally flat real hypersurfaces in a non-flat complex space form $\widetilde{M}_{n+1}(c)$ $(c \neq 0)$, $n \geq 2$. Then, we show that a totally η-umbilical real hypersurface in $P_2\mathbb{C}$ or $H_2\mathbb{C}$ cannot be conformally flat (Proposition 5.6).

2. Preliminaries

All manifolds in the present paper are assumed to be connected and of class C^∞. First, we review briefly conformally flat manifolds. Let (M^m, g) be an m-dimensional Riemannian manifold. Denote by R its Riemannian curvature tensor defined by

$$R(X,Y)Z = \nabla_X(\nabla_Y Z) - \nabla_Y(\nabla_X Z) - \nabla_{[X,Y]}Z$$

for any vector fields X, Y, Z on M. The *Weyl conformal curvature tensor* is defined by

$$W(X,Y)Z = R(X,Y)Z - \frac{1}{m-2}\Big[\rho(Y,Z)X - \rho(X,Z)Y + g(Y,Z)SX - g(X,Z)SY - \frac{r}{m-1}\big(g(Y,Z)X - g(X,Z)X\big)\Big]$$

and the *Schouten tensor* of type $(1,1)$ is defined by

$$LX = \frac{1}{m-2}\big(SX - \frac{r}{2(m-1)}X\big)$$

for any vector fields X, Y, Z on M, where S denotes the Ricci operator, ρ the Ricci curvature tensor of type $(0,2)$ and r the scalar curvature. A Riemannian manifold (M^m, g) is said to be *conformally flat* if it is conformally related to the Euclidean metric in the *local sense*. Then the following facts are well-known. For $m \geq 4$, M^m is conformally flat if and only if the Weyl conformal curvature tensor W vanishes. For $m = 3$, the manifold is conformally flat if and only if the Schouten tensor L is a Codazzi tensor, that is, $g((\nabla_X L)Y, Z) = g((\nabla_Y L)X, Z)$ for any vector fields X, Y, Z on M.

Now, we return to almost contact Riemannian geometry. Let $(M; \varphi, \xi, \eta, g)$ be an almost contact Riemannian manifold (equipped with an almost contact Riemannian structure (φ, ξ, η, g)). The *fundamental 2-form* Φ is defined by $\Phi(X, Y) = g(X, \varphi Y)$ for any vector fields X, Y on M. An almost contact Riemannian manifold M is said to be *normal* if it satisfies $[\varphi, \varphi] + 2d\eta \otimes \xi = 0$, where $[\varphi, \varphi]$ is the Nijenhuis torsion of φ.

Definition 2.1. *An almost contact Riemannian structure (φ, ξ, η, g) is said to be almost Kenmotsu if it satisfies $d\eta = 0$ and $d\Phi = 2\eta \wedge \Phi$. A normal almost Kenmotsu manifold is called a Kenmotsu manifold.*

Definition 2.2. *An almost contact Riemannian structure (φ, ξ, η, g) is said to be almost cosymplectic if it satisfies $d\eta = 0$ and $d\Phi = 0$. A normal almost cosymplectic manifold is called a cosymplectic manifold.*

Definition 2.3. *An almost contact Riemannian structure (φ, ξ, η, g) is said to be contact Riemannian if it satisfies $d\eta = \Phi$. A normal contact Riemannian manifold is called a Sasakian manifold.*

For more details about almost contact Riemannian manifolds, we refer to [2].

3. Kenmotsu 3-manifolds

Let $(M;\varphi,\xi,\eta,g)$ be a 3-dimensional Kenmotsu manifold. Then we have ([13])

$$\nabla_X\xi = X-\eta(X)\xi \tag{3}$$

for any vector field X on M. From (3), we have

$$R(X,Y)\xi = \eta(X)Y-\eta(Y)X, \tag{4}$$

and

$$S\xi = -2\xi. \tag{5}$$

Since the Weyl conformal curvature tensor W vanishes in a Riemannian 3-manifold, we have

$$\begin{aligned} R(Y,X)Z =&\rho(X,Z)Y-\rho(Y,Z)X+g(X,Z)SY-g(Y,Z)SX\\ &-\frac{r}{2}\{g(X,Z)Y-g(Y,Z)X\}\end{aligned}$$

for any vector fields X,Y,Z on M. Then together with (4) and (5) we have ([9])

Proposition 3.1. *For a Kenmotsu 3-manifold, we have*

$$S=(1+\frac{r}{2})I-(3+\frac{r}{2})\eta\otimes\xi, \tag{6}$$

where I denotes the identity transformation.

In order to prove Theorem 3.3, the following lemma is useful. Due to results in [12], we have

Lemma 3.2. *A Kenmotsu 3-manifold M is locally isometric to the warped product $\mathbb{R}\times_{ce^t}N(k)$, where $N(k)$ is a Riemannian 2-manifold of constant Gaussian curvature k if and only if grad $r\in\{\xi\}_{\mathbb{R}}$.*

Unfortunately, in a previous work (Theorem 3 in [5]) we had an incomplete result. Now, we prove

Theorem 3.3. *A conformally flat Kenmotsu 3-manifold is locally isometric to the warped product $\mathbb{R}\times_{ce^t}N(k)$, where $N(k)$ is a Riemannian 2-manifold of constant Gaussian curvature k.*

Proof. Let $(M;\varphi,\xi,\eta,g)$ be a Kenmotsu 3-manifold. Differentiating (6) covariantly and using (3) we get

(7)

$$(\nabla_YS)X=\frac{1}{2}(Yr)(X-\eta(X)\xi)-(3+\frac{r}{2})\big(g(Y,X)\xi+\eta(X)Y-2\eta(X)\eta(Y)\xi\big)$$

for any vector fields X and Y on M. Recall the formula:

$$X(r) = 2\sum_i(\nabla_{e_i}\rho)(e_i, X) \tag{8}$$

for any local orthonormal frame field $\{e_i\}$. Then we have from (7) and (8)

$$\xi(r) = -4(3 + \frac{r}{2}). \tag{9}$$

Suppose that M is conformally flat. Then, since the Schouten tensor is a Codazzi tensor, we have

$$(\nabla_X S)Y - (\nabla_Y S)X = \frac{1}{4}(X(r)Y - Y(r)X) \tag{10}$$

for any vector fields X and Y on M. Using (7) and (10), we have

(11)

$$\frac{1}{2}\big(X(r)Y-Y(r)X\big) = (Xr)\eta(Y)\xi-(Yr)\eta(X)\xi+2(3+\frac{r}{2})(\eta(Y)X-\eta(X)Y).$$

Put $X = \xi$ in (11) to get

$$\frac{1}{2}Y(r) = -2(3 + \frac{r}{2})\eta(Y) \tag{12}$$

for any vector field Y on M, where we have used (9). Then from (12) we find that $Y(r) = 0$ for any vector field $Y \perp \xi$. Using Lemma 3.2 we have that M is locally isometric to the warped product $\mathbb{R} \times_{ce^t} N(k)$, where $N(k)$ is a Riemannian 2-manifold of constant Gaussian curvature k. Conversely, the warped product $\mathbb{R} \times_{ce^t} N(k)$ is conformally flat (cf. [3]). Note that if $k = 0$ then the warped product is a space of of constant curvature -1. □

Remark 1. The present author proved ([7]) that an almost Kenmotsu 3-manifold is locally symmetric ($\nabla R = 0$) if and only if it is locally isometric to the hyperbolic space $\mathbb{H}^3(-1)$ or a product space $\mathbb{H}^2(-4)\times\mathbb{R}$.

4. Cosymplectic 3-manifolds

For a cosymplectic manifold M, we have (cf. [11])

$$\nabla\xi = 0.$$

From this, we easily get $R(X, Y)\xi = 0$ and $S\xi = 0$. Then we have ([8])

Proposition 4.1. *For a cosymplectic 3-manifold, we have the Ricci operator:*

$$S = \frac{r}{2}(I - \eta \otimes \xi). \tag{13}$$

Theorem 4.2. *A conformally flat cosymplectic 3-manifold is locally isometric to a product space $\mathbb{R} \times N(k)$, where $N(k)$ is a Riemannian 2-manifold of constant Gaussian curvature k.*

Proof. Let $(M; \varphi, \xi, \eta, g)$ be a cosymplectic 3-manifold. Differentiating (13) covariantly and using $\nabla \xi = 0$ we get

$$(\nabla_Y S)X = \frac{1}{2}(Yr)(X - \eta(X)\xi) \tag{14}$$

for any vector fields X and Y on M. Then a similar computation as in the proof of Theorem 3.3 yields

$$\xi(r) = 0. \tag{15}$$

Suppose that M is conformally flat. Then, we have

$$(\nabla_X S)Y - (\nabla_Y S)X = \frac{1}{4}(X(r)Y - Y(r)X) \tag{16}$$

for any vector fields X and Y on M. Using (14) and (16), we have

$$\frac{1}{2}\big(X(r)Y - Y(r)X\big) = (Xr)\eta(Y)\xi - (Yr)\eta(X)\xi. \tag{17}$$

Put $X = \xi$ in (17) to get

$$Y(r) = 0 \tag{18}$$

for any vector field Y on M, where we have used (15). From (14) and (18), we find that the Ricci operator is parallel (for the Levi-Civita connection). Then M is locally symmetric, and hence M is locally isometric to a product space $\mathbb{R} \times N(k)$, where $N(k)$ is a Riemannian 2-manifold of constant Gaussian curvature k (cf. [22]). $\square$

The following examples are conformally flat almost cosymplectic 3-manifolds which are neither normal nor of constant scalar curvature.

Example. ([11]) On $M = \{(x, y, z) \in \mathbb{R}^3(x, y, z) : \ z \neq 0\}$, define $\eta = dz$ and $\xi = \frac{\partial}{\partial z}$. Take a global frame field

$$e_1 = \frac{1}{z}\frac{\partial}{\partial x}, \ e_2 = \frac{z}{e^{ax}}\frac{\partial}{\partial y}, \ e_3 = \xi,$$

$a \in \mathbb{R}$ and define a Riemannian metric g such that $\{e_1, e_2, e_3\}$ is orthonormal with respect to it. Moreover, we define φ by $\varphi e_1 = e_2$, $\varphi e_2 =$

$-e_1$ and $\varphi\xi = 0$. Then, the fundamental 2-form is given by $\Phi = 2e^{ax}dx \wedge dy$, and (φ, ξ, η, g) is an almost cosymplectic structure, that is, $d\eta = 0$ and $d\Phi = 0$. But $\nabla\xi \neq 0$, and hence it is not cosympletic. For a simpler one, we take $a = 0$. Then we find that $\{e_1, e_2, e_3\}$ daiagonalizes the Ricci operator, that is,

$$Se_1 = \frac{1}{z^2}e_1,\ Se_2 = -\frac{1}{z^2}e_2,\ S\xi = -\frac{2}{z^2}\xi,$$

and the scalar curvature $r = -\frac{2}{z^2}$. And then we have that the Schouten tensor L is a Codazzi tensor. For more details, see [11].

5. Real hypersurfaces in $P_2\mathbb{C}$ or $H_2\mathbb{C}$

Let $\widetilde{M} = \widetilde{M}_{n+1}(c)$ be a complex space form of constant holomorphic sectional curvature c, M be a real hypersurface of $\widetilde{M}$ and N be a unit normal vector field of M in $\widetilde{M}$. We denote by $\tilde{g}$ and J a Kählerian metric tensor and its complex structure tensor on $\widetilde{M}$, respectively. For any vector field X tangent to M, we put

$$JX = \varphi X + \eta(X)N, \quad JN = -\xi, \tag{19}$$

where φX is the tangential part of JX, φ a (1,1)-type tensor field, η is a 1-form, and ξ is a unit vector field on M. The induced Riemannian metric on M is denoted by g. Then by properties of $(J, \widetilde{g})$ we see that the structure (φ, ξ, η, g) is an almost contact metric structure on M. Indeed, we can deduce (1) and (2) from (19).

The Gauss and Weingarten formula for M are given as

$$\widetilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N,$$
$$\widetilde{\nabla}_X N = -AX$$

for any tangent vector fields X, Y on M, where $\widetilde{\nabla}$ and ∇ denote the Levi-Civita connections of $(M_n(c), \widetilde{g})$ and (M, g), respectively, and A is the shape operator field. An eigenvalue and an eigenvector of the shape operator A is called a principal curvature and a principal curvature vector, respectively. From (19) and $\widetilde{\nabla}J = 0$, we then obtain

$$(\nabla_X\varphi)Y = \eta(Y)AX - g(AX, Y)\xi, \tag{20}$$

$$\nabla_X\xi = \varphi AX. \tag{21}$$

Then we find from (20)

Proposition 5.1. *([6]) Every real hypersurface in a Kählerian manifold satisfies* $d\Phi = 0$*.*

Using (21) we have

Proposition 5.2. *([6]) There are no almost cosymplectic or almost Kenmotsu real hypersurfaces in a non-flat complex space form.*

We have the following Gauss and Codazzi equations:

$$\begin{aligned} R(X,Y)Z =& \frac{c}{4}\{g(Y,Z)X - g(X,Z)Y \\ &+ g(\phi Y,Z)\phi X - g(\phi X,Z)\phi Y - 2g(\phi X,Y)\phi Z\} \\ &+ g(AY,Z)AX - g(AX,Z)AY, \end{aligned} \tag{22}$$

for any tangent vector fields X, Y, Z on M. From (22) we get for the Ricci tensor S of type (1,1):

$$SX = \frac{c}{4}\{(2n+3)X - 3\eta(X)\xi\} + hAX - A^2X, \tag{23}$$

where $h =$ trace of A denotes the mean curvature.

R. Takagi [23], [24] classified the homogeneous real hypersurfaces of $P_{n+1}\mathbb{C}$ into six types. T. E. Cecil and P. J. Ryan [4] extensively studied a Hopf hypersurface (whose Reeb vector ξ is a principal curvature vector), which is realized as tubes over certain submanifolds in $P_{n+1}\mathbb{C}$, by using its focal map. By making use of those results, M. Kimura [16] proved the local classification theorem for Hopf hypersurfaces of $P_{n+1}\mathbb{C}$ whose all principal curvatures are constant.

Theorem 5.3. *([16]) Let M be a Hopf hypersurface of $P_{n+1}\mathbb{C}$. Then M has constant principal curvatures if and only if M is locally congruent to one of the following:*

(A_1) a geodesic hypersphere of radius r, where $0 < r < \frac{\pi}{2}$,

(A_2) a tube of radius r over a totally geodesic $P_l\mathbb{C}(1 \leq l \leq n-1)$, where $0 < r < \frac{\pi}{2}$,

(B) a tube of radius r over a complex quadric Q^n and $P_{n+1}\mathbb{R}$, where $0 < r < \frac{\pi}{4}$,

(C) a tube of radius r over $P_1\mathbb{C} \times P_{\frac{n}{2}}\mathbb{C}$, where $0 < r < \frac{\pi}{4}$ and $n(\geq 4)$ is odd,

(D) a tube of radius r over a complex Grassmann $G_{2,5}\mathbb{C}$, where $0 < r < \frac{\pi}{4}$ and $n = 8$,

(E) a tube of radius r over a Hermitian symmetric space $SO(10)/U(5)$, where $0 < r < \frac{\pi}{4}$ and $n = 14$.

For the case $H_{n+1}\mathbb{C}$, J. Berndt [1] proved the classification theorem for Hopf hypersurfaces whose all principal curvatures are constant.

Theorem 5.4. *([1]) Let M be a Hopf hypersurface of $H_{n+1}\mathbb{C}$. Then M has constant principal curvatures if and only if M is locally congruent to one of the following:*

(A_0) *a horosphere,*
(A_1) *a geodesic hypersphere or a tube over a complex hyperbolic hyperplane $H_n\mathbb{C}$,*
(A_2) *a tube over a totally geodesic $H_l\mathbb{C}(1 \leq l \leq n-1)$,*
(B) *a tube over a totally real hyperbolic space $H_{n+1}\mathbb{R}$.*

We call simply type (A) for real hypersurfaces of type (A_1), (A_2) in $P_{n+1}\mathbb{C}$ and ones of type (A_0), (A_1) or (A_2) in $H_{n+1}\mathbb{C}$.

Homogeneous real hypersurfaces of type (A) are characterized as follows:

Proposition 5.5. *([20],[21],[18]) Let M be a real hypersurface in a non-flat complex space form $\widetilde{M}_{n+1}(c)$ $(n \geq 1)$. Then the following conditions are mutually equivalent*:

- *M satisfies $A\phi = \phi A$;*
- *M is locally congruent to a type (A) hypersurface*;
- *the almost contact metric structure is normal.*

Let M be a real hypersurface $P_2\mathbb{C}$ or $H_2\mathbb{C}$ and suppose that its almost contact metric structure is normal. Then by Proposition 5.5, we find that M is *totally η-umbilical*, that is,

$$A = \alpha I + \beta\eta \otimes \xi,$$

where $\alpha, \beta \in \mathbb{R}$ and I denotes the identity transformation. Indeed, totally η-umbilical real hypersurfaces in $P_{n+1}\mathbb{C}$ or $H_{n+1}\mathbb{C}$ are classified in [4], [24], [17]. They are realized as a homogeneous hypersurface of type (A_1) in $P_{n+1}\mathbb{C}$ and a homogeneous hypersurface of type (A_0) and (A_1) in $H_{n+1}\mathbb{C}$. Then we prove

Proposition 5.6. *A totally η-umbilical real hypersurface M in $P_2\mathbb{C}$ or $H_2\mathbb{C}$ does not admit conformally flat structure.*

Proof. Let M be a totally η-umbilical real hypersurface in $P_2\mathbb{C}$ or $H_2\mathbb{C}$. Then from (23) we get

$$S = \lambda I + \mu\eta \otimes \xi, \tag{24}$$

where $\lambda = \frac{5c}{4} + 2\alpha^2 + \alpha\beta$, $\mu = \alpha\beta - \frac{3c}{4}$. Differentiating (24) covariantly, then using (21) we have

$$(\nabla_X S)Y = \alpha\mu\big(g(\varphi X, Y)\xi + \eta(Y)\varphi X\big) \tag{25}$$

for any vector fields X and Y on M. Suppose that M is conformally flat. Then, since r is constant, we have

$$\alpha\mu\big(2g(\varphi X, Y)\xi + \eta(Y)\varphi X - \eta(X)\varphi Y\big) = 0 \tag{26}$$

for any vector fields X, Y, Z on M. If we put $Y = \xi$, then we get $\alpha\mu = 0$. And from (25) we see that the Ricci operator is parallel. But, by a result due to [15] there are no such real hypersurfaces in $P_2\mathbb{C}$ or $H_2\mathbb{C}$. This completes the proof. □

Remark 2. Totally η-umbilical real hypersurfaces in $P_2\mathbb{C}$ or $H_2\mathbb{C}$ have *quasi-Sasakian structure*, that is, they are normal and satisfy $d\Phi = 0$.

References

[1] J. Berndt, *Real hypersurfaces with constant principal curvatures in complex hyperbolic space* J. Reine Angew. Math. **395** (1989), 132–141.

[2] D. E. Blair, *Riemannian geometry of contact and symplectic manifolds*, Second edition, Progr. Math. 203, Birkhäuser Boston, Inc., Boston, MA, 2010.

[3] M. Brozos-Vázquez, E. García-Río and R. Vázquez-Lorenzo, *Complete locally conformally flat manifolds of negative curvature*, Pacific J. Math. **226** (2006), 201–219.

[4] T. E. Cecil and P. J. Ryan, *Focal sets and real hypersurfaces in complex projective space*, Trans. Amer. Math. Soc. **269** (1982), 481–499.

[5] J.T. Cho, *Notes on almost Kenmotsu three-manifolds*, Honam Math. J. **36(3)** (2014), 637–645.

[6] J.T. Cho, *Notes on real hypersurfaces in a complex space form*, Bull. Korean Math. Soc. **52(1)** (2015), 335–344.

[7] J.T. Cho, *Local symmetry on almost Kenmotsu three-manifolds*, Hokkaido Math. J., to appear.

[8] J.T. Cho, *Reeb flow symmetry on almost coymplectic three-manifolds*, submitted.

[9] J.T. Cho and M. Kimura, *Reeb flow symmetry on almost contact three-manifolds*, Differential Geom. Appl. **35** (2014), 266-273.

[10] J.T. Cho and D-h. Yang, *Conformally flat contact 3-manifolds*, submitted.

[11] P. Dacko and Z. Olszak, *On conformally flat almost cosymplectic manifolds with Käherian manifolds*, Ren. Sem. Math. Univ. Pol. Torino **56(1)** (1998), 89–103.

[12] J. Inoguchi, *A note on almost contact Riemannian 3-manifolds II*, preprint.

[13] K. Kenmotsu, *A class of contact Riemannian Manifolds*, Tôhoku Math. J. **24** (1972), 93–103.

[14] U-H. Ki, H. Nakagawa and Y.J. Suh, *Real hypersurfaces with harmonic Weyl tensor of a complex space form*, Hiroshima Math. J. **20** (1990), 93–102.

[15] U.K. Kim, *Nonexistence of Ricci-parallel real hypersurfaces in $P_2\mathbb{C}$ or $H_2\mathbb{C}$*, Bull. Korean Math. Soc. **41** (2004), 699–708.
[16] M. Kimura, Real hypersurfaces and complex submanifolds in complex projective space, Trans. Amer. Math. Soc. **296** (1986), 137–149.
[17] S. Montiel, *Real hypersurfaces of a complex hyperbolic space*, J. Math. Soc. Japan **37** (1985), 515–535.
[18] S. Montiel and A. Romero, *On some real hypersurfaces of a complex hyperbolic space*, Geom. Dedicata **20** (1986), 245–261.
[19] M. Okumura, *Some remarks on space with a certain structure*, Tôhoku Math. J. **14** (1962) 135–145.
[20] M. Okumura, *Certain almost contact hypersurfaces in Kaehlerian manifolds of constant holomorphic sectional curvature*, Tôhoku Math. J. (2) **16** (1964), 270–284.
[21] M. Okumura, *On some real hypersurfaces of a complex projective space*, Trans. Amer. Math. Soc. **212** (1975), 355–364.
[22] D. Perrone, *Classification of homogeneous almost cosymplectic three-manifolds*, Differential. Geom. Appl. **30** (2012), 49–58.
[23] R. Takagi, *On homogeneous real hypersurfaces in a complex projective space*, Osaka J. Math. **19** (1973), 495–506.
[24] R. Takagi, *Real hypersurfaces in a complex projective space with constant principal curvatures I*, J. Math. Soc. Japan **15** (1975), 43–53.
[25] S. Tanno, *Locally symmetric K-contact Riemannian manifolds*, Proc. Japan Acad. **43** (1967), 581–583.
[26] S. Tanno, *The automorphism groups of almost contact Riemannian manifolds*, Tôhoku Math. J. **21** (1969), 21–38.

Jong Taek Cho
Department of Mathematics, Chonnam National University,
Gwangju 500-757, Korea.
E-mail: jtcho@chonnam.ac.kr