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# Conformational effects on the torsional barriers in m-methylanisole studied by microwave spectroscopy 

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#### Abstract

The microwave spectrum of $m$-methylanisole (also known as 3-methylanisole, or 3-methoxytoluene) was measured using a pulsed molecular jet Fourier transform microwave spectrometer operating in the frequency range of $2-26.5 \mathrm{GHz}$. Quantum chemical calculations predicted two conformers with the methoxy group in trans or cis position related to the ring methyl group, both of which were assigned in the experimental spectrum. Due to the internal rotation of the ring methyl group, all rotational transitions introduced large A-E splittings up to several GHz , which were analyzed with a newly developed program, called aixPAM, working in the principal axis system. There are significant differences in the $V_{3}$ potential barriers of $55.7693(90) \mathrm{cm}^{-1}$ and $36.6342(84) \mathrm{cm}^{-1}$ determined by fitting 223 and 320 torsional components of the cis and the trans conformer, respectively. These values were compared with those found in other $m$-substituted toluenes as well as in $o$ - and $p$-methylanisole. A comparison between the aixPAM and the XIAM code (using a combined axis system) was also performed. Published by AIP Publishing. https://doi.org/10.1063/1.5016273


## I. INTRODUCTION

Rotational transitions of a molecule with internal rotation of a methyl group feature splittings into torsional components and can no longer be treated using a rigid-rotor model. A number of programs have been developed to treat this problem; among them are IAM, ${ }^{1} X I A M,{ }^{2} B E L G I-C_{s}{ }^{3}$ and BELGI-C ${ }_{1},{ }^{4}$ Erham, ${ }^{5}$ and RAM36. ${ }^{6}$ All of them use a rigid frame-rigid top model $^{7}$ as a theoretical background, which is supplemented by a number of higher order (effective) terms, including centrifugal distortion of both the frame and the top in the Hamiltonian. On the other hand, they differ from each other by their methods which are named after the coordinates in use; e.g., the XIAM code uses the combined axis method; Erham sets up and solves an Effective Rotational HAMiltonian; and BELGI$C_{s}, B E L G I-C_{1}$, as well as RAM36 use the rho axis method (RAM). The numerical techniques are also different: The Hamiltonian matrices are pre-diagonalized, small matrix elements are neglected, and the matrices are truncated at a given limit.

The program XIAM was mostly used in our previous investigations on molecules of different chemical classes, such as methyl butyrate, ${ }^{8} \mathrm{~N}, \mathrm{~N}$-diethylacetamide, ${ }^{9}$ and methyl isobutyl ketone, ${ }^{10}$ undergoing internal rotation because it is user-friendly and offers a reasonable compromise of accuracy and speed of the calculations. On the other hand, XIAM has its weaknesses in treating internal rotations with low barrier heights, e.g., in the cases of 3-pentin-1-ol, ${ }^{11}$ allyl acetate, ${ }^{12}$

[^0]and N -ethylacetamide, ${ }^{13}$ where in the fits, standard deviations much larger than measurement accuracy have been observed.

For numerous molecular systems, we compared fits carried out with XIAM and BELGI in its $\mathrm{C}_{\mathrm{s}}$ or $\mathrm{C}_{1}$ version, where in almost all the cases, BELGI yielded a lower standard deviation than XIAM with the same data set. ${ }^{14-16}$ There were some discussions about the reasons for this observation. A frequently used argument is that only a limited number of parameters can be fitted using XIAM in comparison with BELGI and that the torsional interactions between different $\mathrm{v}_{\mathrm{t}}$ states are not taken into account explicitly. Higher order coupling terms between the internal rotation and the overall rotation cannot be easily implemented in the XIAM code to improve the quality of the fit. Sometimes, it is also argued that even if there were more effective terms in XIAM, the standard deviation would still be worse than with BELGI. The reason is that XIAM uses a twostep diagonalization procedure, whereby only one $J$-block is considered in the second diagonalization step and some matrix elements are thus neglected. This truncation could additionally affect the accuracy of the fit.

These discussions induced the authors to write another internal rotation program, called aixPAM, where aix stands for Aix-la-Chapelle, the French name of Aachen where the program was written. m-methylanisole (MMA) was chosen for the first application of aixPAM since two conformers with rather extensive data sets have been recently measured and assigned and because the XIAM fits of these data have not achieved measurement accuracy.

Our recent studies on the microwave spectra of $o$-methylanisole (OMA) ${ }^{17}$ and $p$-methylanisole (PMA) ${ }^{18}$ have
shown that the ring methyl group undergoes internal rotation with quite different barrier heights of about $444 \mathrm{~cm}^{-1}$ and $50 \mathrm{~cm}^{-1}$, respectively, whereas the effect arising from the methoxy methyl group is negligible. It is interesting to compare the respective torsional barriers when the ring methyl group is in $m$-position, because depending on the relative position of the substituents, considerable effects on the torsional barrier height can be observed. There are studies in the literature on the isomers of fluorotoluene, ${ }^{19-21}$ cresol,,${ }^{22-24}$ and methylbenzaldehyde, ${ }^{25,26}$ stating very different barrier heights for the $o$-, $m$-, and $p$-isomers.

A comparison with the barriers of other toluene derivatives with a methyl group in $m$-position is also very interesting. Some previous investigations ${ }^{17,18,26}$ have shown that the $o$ - and $p$-isomers often possess only one stable conformer because of the sterical hindrance of the $o$-isomer and the structural symmetry of the $p$-isomer. In $m$-substituted toluenes, two conformers are often observed, ${ }^{25}$ making these isomers ideally suited to study conformational effects on torsional barriers.

## II. THEORETICAL SECTION

## A. Quantum chemical calculations

## 1. Conformational analysis

Before recording the microwave spectrum, quantum chemical calculations were performed to determine the possible conformers of MMA.

By rotating the entire $-\mathrm{OCH}_{3}$ group about the $\mathrm{C}_{4}-\mathrm{O}_{11}$ bond (for atom numbering, see Fig. 1), we calculated a potential curve, where the dihedral angle $\beta=\angle\left(\mathrm{C}_{5}, \mathrm{C}_{4}, \mathrm{O}_{11}, \mathrm{C}_{12}\right)$ was varied in a grid of $10^{\circ}$, while all other geometry parameters were optimized. If not stated otherwise, calculations were carried out at the B3LYP/6-311++G(d,p) level of theory using the Gaussian09 program package. ${ }^{27}$ In almost all the recent investigations, we applied the MP2/6-311++G(d,p) level. However, for molecules containing aromatic rings like phenetole, ${ }^{28}$ 2,5-dimethylthiophene,,${ }^{29}$ 2-acetyl-5-methylfuran, ${ }^{30}$ as well as the isomer OMA ${ }^{17}$ of MMA, harmonic frequency


FIG. 1. Molecular structures of the cis (I) and the trans (II) conformer of MMA optimized at the B3LYP/6-311++G(d,p) level of theory. The dihedral angles are defined as $\alpha=\angle\left(\mathrm{C}_{3}, \mathrm{C}_{2}, \mathrm{C}_{16}, \mathrm{H}_{18}\right), \beta=\angle\left(\mathrm{C}_{5}, \mathrm{C}_{4}, \mathrm{O}_{11}, \mathrm{C}_{12}\right)$, and $\gamma=\angle\left(\mathrm{C}_{4}, \mathrm{O}_{11}, \mathrm{C}_{12}, \mathrm{H}_{13}\right)$.
calculations yielded one imaginary vibrational mode, which is a bending vibration of the phenyl ring. It is well known that the MP2/6-311++G(d,p) level of theory often yields an imaginary frequency for stable planar ring systems, which has been reported for benzene and arenes. ${ }^{31}$ This effect did not occur in calculations using the B3LYP method. Variation of the methods and basis sets in our previous studies on OMA ${ }^{17}$ and PMA ${ }^{18}$ confirms that the structural parameters do not change significantly and the rotational constants are calculated with sufficient accuracy at the B3LYP/6-311++G(d,p) level.

The potential curve as a function of $\beta$ is given in Fig. 2; the Fourier coefficients of the potential function are given in Table S1 (supplementary material). Calculations yield two possible conformers of MMA: the energetically more favorable trans conformer at $\beta=0^{\circ}$ and the cis conformer at $\beta=180^{\circ}$. Their fully optimized structures are illustrated in Fig. 1; the Cartesian coordinates are given in Table S2 (supplementary material). Frequency calculations state that both of them are true minima with an energy difference of only $0.40 \mathrm{~kJ} / \mathrm{mol}$. We thus expect to observe both conformers in the experimental spectrum.

Optimizations predict the rotational constants $A=2756.8$ $\mathrm{MHz}, B=1286.6 \mathrm{MHz}$, and $C=887.0 \mathrm{MHz}$, as well as the dipole moment components $\mu_{\mathrm{a}}=-0.08 \mathrm{D}, \mu_{\mathrm{b}}=-1.64 \mathrm{D}$, and $\mu_{\mathrm{c}}=0.00 \mathrm{D}$ for the cis conformer. The respective values for the trans conformer are $A=3537.9 \mathrm{MHz}, B=1116.2 \mathrm{MHz}$, and $C=857.7 \mathrm{MHz}$, and $\mu_{\mathrm{a}}=0.67 \mathrm{D}, \mu_{\mathrm{b}}=0.73 \mathrm{D}$, and $\mu_{\mathrm{c}}=0.00 \mathrm{D}$. Accordingly, for cis-MMA, only b-type transitions are expected in the microwave spectrum, while $a$-type transitions should also be observable for trans-MMA. Finally, anharmonic frequency calculations were carried out to obtain the centrifugal distortion constants.

## 2. Methyl internal rotations

Potential energy scans generated barrier heights of the ring and the methoxy methyl groups by varying the dihedral


FIG. 2. The potential energy curve of MMA obtained by rotating the methoxy group about the $\mathrm{C}_{4}-\mathrm{O}_{11}$ bond by varying the dihedral angle $\beta=\angle\left(\mathrm{C}_{5}, \mathrm{C}_{4}, \mathrm{O}_{11}, \mathrm{C}_{12}\right)$ in a grid of $10^{\circ}$ at the B3LYP/6-311++G(d,p) level of theory. The relative energies with respect to the lowest energy conformation with $E=-386.1953453$ hartree are given. Two stable $\mathrm{C}_{\mathrm{s}}$ conformers found at $\beta=0^{\circ}$ (trans) and $180^{\circ}$ (cis) differ by only $0.40 \mathrm{~kJ} / \mathrm{mol}$ in energy.
angles $\alpha=\angle\left(\mathrm{C}_{3}, \mathrm{C}_{2}, \mathrm{C}_{16}, \mathrm{H}_{18}\right)$ and $\gamma=\angle\left(\mathrm{C}_{4}, \mathrm{O}_{11}, \mathrm{C}_{12}, \mathrm{H}_{13}\right)$ in a grid of $10^{\circ}$, corresponding to rotations about the $\mathrm{C}_{2}-\mathrm{C}_{16}$ and $\mathrm{O}_{11}-\mathrm{C}_{12}$ bonds, respectively. All other geometry parameters were optimized. Due to the three-fold symmetry of the methyl groups, a rotation of $120^{\circ}$ was sufficient. The potential energy curves showing the rotation of the ring methyl group are presented in Fig. 3; the corresponding Fourier coefficients are available in Table S1 (supplementary material). For the cis conformer, we found $V_{3}$ potentials of $50.13 \mathrm{~cm}^{-1}$ for the ring methyl group and $1085.35 \mathrm{~cm}^{-1}$ for the methoxy methyl group. The respective values for the trans conformer are 32.77 $\mathrm{cm}^{-1}$ and $1079.11 \mathrm{~cm}^{-1}$.

Figure 3 indicates that the ring methyl group prefers different orientations in the two conformers of MMA. Taking $\mathrm{H}_{8}$ as the reference (see Fig. 1), the lowest energy conformations of cis-MMA locate two of three hydrogen atoms of the ring methyl group staggered to $\mathrm{H}_{8}$, corresponding to $\alpha=60^{\circ}, 180^{\circ}$, or $300^{\circ}$. On the contrary, trans-MMA favors a conformation with one of the hydrogen atoms eclipsed to $\mathrm{H}_{8}$, corresponding to $\alpha=0^{\circ}, 120^{\circ}$, or $240^{\circ}$.

The $V_{3}$ potential of the methoxy methyl group exceeds $1000 \mathrm{~cm}^{-1}$ in both the cis and trans conformers. Trial two-top calculations using the program XIAM predict that torsional splittings arising from the internal rotation of this methyl group are smaller than 5 kHz and thus not resolvable with the spectrometer in use.

Furthermore, a 2D potential energy surface (PES) depending on $\alpha$ and $\beta$ was calculated by varying these dihedral angles in a grid of $10^{\circ}$. The PES was parameterized with a 2D Fourier expansion based on terms representing the correct symmetry of $\alpha$ and $\beta$. The Fourier coefficients are also listed in Table S1 (supplementary material). The PES shown in Fig. 4 clearly indicates the following points: (1) two conformers of MMA exist $(\mathrm{I}=$ cis-MMA and $\mathrm{II}=$ trans-MMA), (2) conformer I is higher in energy than conformer II, and (3) the orientation of the ring methyl group depends on the position of the methoxy group.


FIG. 3. The potential energy curves for the cis (I, black) and the trans (II, blue) conformer of MMA obtained by rotating the ring methyl group about the $\mathrm{C}_{2}-\mathrm{C}_{16}$ bond. The dihedral angle $\alpha=\angle\left(\mathrm{C}_{3}, \mathrm{C}_{2}, \mathrm{C}_{16}, \mathrm{H}_{18}\right)$ was varied in a grid of $10^{\circ}$, while all the other molecular parameters were optimized at the B3LYP/6-311++G(d,p) level of theory. Relative energies with respect to the lowest energy conformations with the absolute energies $\mathrm{E}=-386.1951917$ and -386.1953453 hartree, respectively, are used.


FIG. 4. The potential energy surface (PES) depending on the dihedral angles $\alpha$ and $\beta$ of MMA calculated at the B3LYP/6-311++G(d,p) level of theory. $\alpha$ and $\beta$ were varied in a grid of $10^{\circ}$, while all the other parameters were optimized. The numbers in the color code indicate the energy (in percent) relative to the energetic minimum $E_{\min }=-386.1905225$ hartree $(0 \%)$ and the energetic maximum $E_{\max }=-386.1953453$ hartree $(100 \%)$. Note that there are significantly more colors in the lower $50 \%$ section. The PES indicates the following: (i) two conformers of MMA exist ( $\mathrm{I}=$ cis-MMA and $\mathrm{II}=$ transMMA), (ii) conformer I is higher in energy than conformer II, and (iii) the orientation of the ring methyl group is different in the two conformers.

## B. The aixPAM code

The aixPAM code is based on the rigid frame-rigid top model. We will briefly describe it here using a notation given in Ref. 32. The Hamiltonian includes the overall rotation, the internal rotation, and Coriolis-like interaction terms. It can be written as

$$
H=\frac{1}{2} \vec{P}^{\dagger} \boldsymbol{I}^{-1} \vec{P}+V
$$

with the (transposed) 4-dimensional angular momentum vector $\vec{P}^{\dagger}=\left(P_{x}, P_{y}, P_{z}, p\right)$ and the inertia tensor

$$
\boldsymbol{I}=\left(\begin{array}{cccc}
I_{x} & 0 & 0 & \lambda_{i x} I_{\alpha} \\
0 & I_{y} & 0 & \lambda_{i y} I_{\alpha} \\
0 & 0 & I_{z} & \lambda_{i z} I_{\alpha} \\
\lambda_{i x} I_{\alpha} & \lambda_{i y} I_{\alpha} & \lambda_{i z} I_{\alpha} & I_{\alpha}
\end{array}\right)
$$

and the torsional potential

$$
V=\sum_{n} \frac{V_{3 n}}{2}(1-\cos 3 n \alpha)
$$

$P_{g}$ with $g \in\{x, y, z\}$ are the Cartesian components of the overall angular momentum, $p$ is the angular momentum of the internal rotor, $I_{g}$ are the principal moments of inertia, $I_{\alpha}$ is the moment of inertia of the internal rotor, and $\lambda_{i g}$ are the direction cosines between the internal rotor axis $i$ and the principal axes of inertia $g$. Numerical inversion of $\boldsymbol{I}$ yields

$$
\frac{1}{2} \boldsymbol{I}^{-1}=\left(\begin{array}{cccc}
B_{x}^{\prime} & Z_{x y} & Z_{x z} & -Q_{x} \\
Z_{x y} & B_{y}^{\prime} & Z_{y z} & -Q_{y} \\
Z_{x z} & Z_{y z} & B_{z}^{\prime} & -Q_{z} \\
-Q_{x} & -Q_{y} & -Q_{z} & F
\end{array}\right)
$$

and the rigid frame-rigid top Hamiltonian used in aixPAM can be written as

$$
\begin{aligned}
H= & B_{J}^{\prime} P^{2}+B_{K}^{\prime} P_{z}^{2}+B_{-}^{\prime}\left(P_{x}^{2}-P_{y}^{2}\right)+\sum_{g, g^{\prime}} Z_{g g^{\prime}}\left\{P_{g}, P_{g^{\prime}}\right\} \\
& +F p^{2}-2 \sum_{g} Q_{g} P_{g} p+V
\end{aligned}
$$

with $B_{J}^{\prime}=\frac{B_{x}^{\prime}+B_{y}^{\prime}}{2}, B_{K}^{\prime}=B_{z}^{\prime}-\frac{B_{x}^{\prime}+B_{y}^{\prime}}{2}, B_{-}^{\prime}=\frac{B_{x}^{\prime}-B_{y}^{\prime}}{2}$, and the anti-commutator $\left\{P_{g}, P_{g^{\prime}}\right\}=P_{g} P_{g^{\prime}}+P_{g^{\prime}} P_{g}$.

Beyond this basic model, centrifugal distortion and effective interaction terms can be added. The Hamiltonian matrix is set up in the principal axis system without

TABLE I. Some operators and their definitions in the aixPAM input file.

| Parameter | Operator ${ }^{\text {a }}$ | Definition ${ }^{\text {b }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $B_{J}^{\prime}$ | $P^{2}$ | BJ | 1.0 | P2 |
| $B_{K}^{\prime}$ | $P_{z}^{2}$ | BK | 1.0 | Pz Pz |
| $B_{-}^{\prime}$ | $P_{x}^{2}-P_{y}^{2}$ | B- | 0.5 | $\mathrm{P}+\mathrm{P}+$ |
|  |  | B- | 0.5 | $\mathrm{P}-\mathrm{P}-$ |
| $Z_{x z}$ | $\left\{P_{x}, P_{z}\right\}$ | zxz | 0.5 | $\mathrm{P}+\mathrm{Pz}$ |
|  |  | zxz | 0.5 | $\mathrm{P}-\mathrm{Pz}$ |
|  |  | zxz | 0.5 | Pz P+ |
|  |  | zxz | 0.5 | Pz P- |
| $Q_{x}$ | $-2 P_{x} p$ | Qx | -1.0 | P+ p |
|  |  | Qx | -1.0 | P-p |
| $Q_{z}$ | $-2 P_{z} p$ | Qz | -2.0 | Pz p |
| $\Delta_{J}$ | $-P^{4}$ | DJ | -1.0 | P2 P2 |
| $\Delta_{J K}$ | $-P^{2} P_{z}^{2}$ | DJK | -1.0 | P2 Pz Pz |
| $\Delta_{K}$ | $-P_{z}^{4}$ | DK | -1.0 | Pz Pz Pz Pz |
| $\delta_{J}$ | $-2 P^{2}\left(P_{x}^{2}-P_{y}^{2}\right)$ | dJ | -1.0 | P2 P+ P+ |
|  |  | dJ | -1.0 | P2 P- P- |
| $\delta_{K}$ | $-\left\{P_{z}^{2},\left(P_{x}^{2}-P_{y}^{2}\right)\right\}$ | dK | -0.5 | Pz Pz P+ P+ |
|  |  | dK | -0.5 | Pz Pz P- P- |
|  |  | dK | -0.5 | $\mathrm{P}+\mathrm{P}+\mathrm{Pz} \mathrm{Pz}$ |
|  |  | dK | -0.5 | $\mathrm{P}-\mathrm{P}-\mathrm{Pz} \mathrm{Pz}$ |
| F | $p^{2}$ | F | 1.0 | p p |
| $V_{3}$ | $\frac{1}{2}(1-\cos 3 \alpha)$ | v3 | 0.5 | e0 |
|  |  | v3 | -0.25 | e+3 |
|  |  | v3 | -0.25 | e-3 |
| $V_{6}$ | $\frac{1}{2}(1-\cos 6 \alpha)$ | v6 | 0.5 | e0 |
|  |  | v6 | -0.25 | e+3 e+3 |
|  |  | v6 | -0.25 | e-3 e-3 |
| $D_{m J}$ | $P^{2} p^{2}$ | DmJ | 1.0 | P2 p p |
| $D_{m K}$ | $P_{z}^{2} p^{2}$ | DmK | 1.0 | Pz Pz p p |
| $V_{J}$ | $P^{2}(1-\cos 3 \alpha)$ | FJ | 0.5 | P2 |
|  |  | FJ | -0.25 | P2 e+3 |
|  |  | FJ | -0.25 | P2 e-3 |
| $V_{K}$ | $P_{z}^{2}(1-\cos 3 \alpha)$ | FK | 0.5 | Pz Pz |
|  |  | FK | -0.25 | Pz Pz e+3 |
|  |  | FK | -0.25 | $\mathrm{Pz} \mathrm{Pz} \mathrm{e-3}$ |
| $V_{-}$ | $\left(P_{x}^{2}-P_{y}^{2}\right)(1-\cos 3 \alpha)$ | F- | 0.5 | $\mathrm{P}+\mathrm{P}+$ |
|  |  | F- | 0.5 | P- P- |
|  |  | F- | -0.25 | $\mathrm{P}+\mathrm{P}+\mathrm{e}+3$ |
|  |  | F- | -0.25 | $\mathrm{P}+\mathrm{P}+\mathrm{e}-3$ |
|  |  | F- | -0.25 | P-P- e+3 |
|  |  | F- | -0.25 | $\mathrm{P}-\mathrm{P}-\mathrm{e}-3$ |

[^1]pre-diagonalization. A product basis $|\sigma, k\rangle \cdot|J, K\rangle$ of free rotor functions $(2 \pi)^{-1} \exp (i(3 k+\sigma) \alpha)$ and symmetric top functions is used. Both real and complex matrix elements are allowed. The matrix size is $(2 J+1)\left(2 k_{\max }+1\right)$, with $m_{\max }=3 k_{\max }+\sigma$. The matrices are block-diagonal in the torsional state $\sigma$; therefore, one matrix is used for each $\sigma$. The matrices are quite large for high $J$ and $k_{\max }$; for example, they are of the size $697 \times 697$ for $J=20$ and $k_{\max }=8$. However, with modern computers, the time needed for diagonalizing such matrices is only a few seconds. Except for the truncation of the matrix, aixPAM does not neglect any other matrix elements and $k_{\max }$ is increased until the fit converges.

An important feature of the aixPAM code is the possibility to add effective Hamiltonian terms from the input file. These terms are given as a sum of products of the fundamental operators $P^{2}(\mathrm{P} 2), P_{z}(\mathrm{Pz}), P_{+}=P_{x}+i P_{y}(\mathrm{P}+), P_{-}=P_{x}-$ $i P_{y}(\mathrm{P}-), p(\mathrm{p}), 1(\mathrm{e} 0), \mathrm{e}_{+}=e^{i 3 \alpha}(\mathrm{e}+3), \mathrm{e}_{-}=e^{-i 3 \alpha}(\mathrm{e}-3)$. The operator codes as they are used in the input file are given in parentheses. $P$ are the angular momenta of the overall rotation, with its components $P_{x}, P_{y}, P_{z}, p$ the angular momentum of the internal rotation about the angle $\alpha$. The operators $1, e^{i 3 \alpha}$, and $e^{-i 3 \alpha}$ are needed to code the potential operators $V_{\mathrm{n}}$ with $\mathrm{n}=3,6,9, \ldots$ Some examples of effective operators with their corresponding operator descriptions in the input file are given in Table I. We emphasize that in contrast to the BELGI code, where all parameters are in the rho axis system, all parameters in aixPAM refer to principal axis coordinates.

## III. EXPERIMENTAL SECTION

## A. Measurements

MMA, purchased from TCI, Eschborn, Germany, has a stated purity of $98 \%$ and was used without further purification. The colorless liquid with a typical aromatic smell was placed on a pipe cleaner in a stainless steel tube upstream the nozzle. Under a helium stream at a backing pressure of 2 bars, the helium-MMA mixture was expanded into the vacuum chamber. The spectra were recorded using a supersonic jet Fourier transform microwave spectrometer operating in the frequency range of $2-26.5 \mathrm{GHz} .{ }^{33}$ At first, a broadband scan from 8.5 to 16.0 GHz was carried out, where overlapping spectral segments were recorded in a step size of 0.25 MHz . All lines from the scans were remeasured in higher resolution, where they appear as doublets because of the Doppler effect. A portion of the broadband scan is shown in Fig. 5, and a typical measurement at high resolution is shown in Fig. 6. Some intense lines can be measured with an accuracy of 2 kHz , but in most cases, the line widths are larger due to unresolved splittings arising from the internal rotation of the methoxy methyl group and also due to proton spin-spin and spin-rotation coupling, resulting in a measurement accuracy of about 4 kHz . In all fits carried out in the present work, all lines were equally weighted.

## B. Spectrum assignment

Quantum chemical calculations have shown that the trans conformer is lower in energy than the cis conformer (see Sec. II A 1). Additionally, only for this conformer, $a$-type tran-


FIG. 5. A portion of the broadband scan of MMA from 8.5 to 12.5 GHz . The experimental spectrum is the upper trace. The lower trace indicates the theoretical spectrum of the cis (in blue) and the trans (in red) conformer predicted using the molecular parameters deduced from the program aixPAM.
sitions, which can often be assigned easily due to their characteristic patterns, are predicted to be observable. Therefore, we began our assignment with the spectrum of trans-MMA.

## 1. The trans conformer

As a first step, we neglected the methyl internal rotation and treated trans-MMA as an effective rigid-rotor. A spectrum containing A species transitions was predicted with the program XIAM in its rigid-rotor mode using the rotational constants given in Sec. II A 1 and was compared with the experimental broadband scan mentioned in Sec. III A. The very intense $R$-branch $J=6 \leftarrow 5$ and $7 \leftarrow 6 a$-type transitions with $K_{\mathrm{a}}=0$ and 1 were firstly identified in the scan, yielding the $B$ and $C$ rotational constants. Afterwards, $b$-type transitions were also assigned, which fixed the $A$ rotational constant. This enabled a prediction of the whole rigid-rotor spectrum with sufficient accuracy to find all the remaining A species lines in the frequency range of $2-26.5 \mathrm{GHz}$. At this stage, 183 lines were fitted with the three rotational constants $A, B$, and $C$ and five quartic centrifugal distortion constants.

As a next step, we took into account the methyl internal rotation and predicted both the A and E species transitions. The initial $V_{3}$ potential and the angle between the internal rotor axis and the principal $a$-axis were taken from quantum


FIG. 6. A typical spectrum of the $7_{07} \leftarrow 6_{16}$ transition of the cis conformer of MMA with its A and E torsional species. The A-E splitting is approximately 55 MHz . The splittings indicated by brackets are due to the Doppler effect. For each of these spectra, 50 decays were co-added.
chemical calculations. By comparing the theoretical and experimental broadband scan, the assignment was straightforward for the $a$-type $R$-branch transitions mentioned above, where the A-E splittings were less than 30 MHz . The assignment of $b$-type lines with much larger splittings (up to 3.5 GHz ) was more difficult, however, finally successful. In total, 183 A species and 137 E species lines were fitted using the program XIAM with a standard deviation of 32.1 kHz . The molecular parameters are summarized as Fit I in Table II. A list including all the fitted transitions is given in Table S 3 (supplementary material).

## 2. The cis conformer

After the trans conformer was assigned, a large number of lines remained in the scan, which belong to the cis conformer. Because only $b$-type transitions are present (see Sec. II A 1), the assignment was more difficult, nevertheless, possible by trial and error. We could measure and fit 92 A species and 131 E species transitions up to $J=13$ with the program XIAM to a standard deviation of 27.0 kHz . This fit is also given in Table II; the fitted transitions are presented in Table $S 4$ (supplementary material). We note that for both conformers, some $c$-type transitions are available in the frequency lists [Tables S3 and S4 (supplementary material)], which are all E species lines. It is known that for the E species, forbidden transitions can be observed, ${ }^{3}$ as found, e.g., in ethyl acetate ${ }^{34}$ and butadienyl acetate. ${ }^{35}$

## 3. The aixPAM fits

As mentioned above, aixPAM was written to check the accuracy of XIAM. It is a very flexible code in which effective terms can easily be added in the Hamiltonian. We made the following two sample calculations.
(1) We took the same MMA data set and the same set of parameters mentioned in Secs. III B 1 and III B 2 for comparative fits with XIAM and aixPAM. These aixPAM fits are given as Fit II in Table II. The $D_{p i 2}$ parameters often used in XIAM are also floated in aixPAM, where $D_{p i 2 J}$ multiplies $2\left(p_{\alpha}-\vec{\rho}^{\dagger} \vec{P}\right)^{2} \vec{P}^{2}$, $D_{p i 2 K}$ multiplies $\left\{\left(p_{\alpha}-\vec{\rho} \dagger \vec{P}\right)^{2}, P_{z}^{2}\right\}$, and $D_{p i 2-}$ multiplies $\left\{\left(p_{\alpha}-\vec{\rho}^{\dagger} \vec{P}\right)^{2},\left(P_{x}^{2}-P_{y}^{2}\right)\right\}$. The anti-commutator is defined as $\{u, v\}=u v+v u ; \vec{\rho}^{\dagger}$ is the transposed rho vector. The complete definitions in terms of aixPAM's fundamental operators are extensive and therefore given in Table S5 (supplementary material).

The standard deviations are very similar in both the fits. The one obtained from aixPAM is only slightly better than that from XIAM by about 4 kHz , indicating that the matrix elements neglected in XIAM do not significantly limit its accuracy. The observed-minuscalculated values of Fit II are listed in Table S3 (supplementary material) for trans-MMA and in Table S4 (supplementary material) for cis-MMA.
(2) We added in a second aixPAM fit some effective terms not available in XIAM. This enabled us to achieve a standard deviation close to the experimental accuracy. For cis-MMA, three additional parameters $V_{J}$ multiplying $P^{2}(1-\cos 3 \alpha), D_{m K}$ multiplying $P_{z}^{2} p_{\alpha}{ }^{2}$, and

TABLE II. Molecular parameters of $m$-methylanisole in the principal axis system obtained by the programs XIAM and aixPAM.

| Parameter ${ }^{\text {a }}$ | Unit | cis-MMA |  |  |  | trans-MMA |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Fit $\mathrm{I}^{\text {b }}$ | Fit II ${ }^{\text {c }}$ | Fit III ${ }^{\text {d }}$ | Calculation ${ }^{\text {e }}$ | Fit $\mathrm{I}^{\text {b }}$ | Fit II ${ }^{\text {c }}$ | Fit III ${ }^{\text {d }}$ | Calculation ${ }^{\text {e }}$ |
| $A^{\text {f }}$ | MHz | 2755.8636(70) | 2755.997(18) | 2755.304(73) | 2756.845 | 3529.454(40) | 3528.242(61) | 3521.824(85) | 3537.943 |
| $B^{\text {f }}$ | MHz | 1293.7072(25) | 1293.3105(60) | 1291.236(45) | 1286.597 | 1122.3350 (17) | 1122.3847(26) | 1121.498(45) | 1116.215 |
| $C^{\text {f }}$ | MHz | 890.60188(47) | 890.55131(66) | 890.88(16) | 886.992 | 861.12564(46) | 861.14249(47) | 862.060(45) | 857.656 |
| $\Delta_{J}$ | kHz | 0.0421(31) | 0.0493(26) | $0.05137(42)$ | 0.05173 | 0.0310(33) | 0.0285(29) | 0.02397(43) | 0.02326 |
| $\Delta_{J K}$ | kHz | -0.027(22) | -0.076(19) | -0.0951(31) | -0.06667 | 0.071(21) | 0.034(19) | 0.0451(28) | 0.09015 |
| $\Delta_{K}$ | kHz | 0.601(99) | $0.542(85)$ | 0.446(14) | 0.39097 | -0.478(94) | 0.299(84) | 0.954(15) | 0.57448 |
| $\delta_{J}$ | kHz | 0.0129(14) | 0.0169(12) | 0.01833 (19) | 0.01869 | 0.0091(17) | 0.0085(15) | 0.00609(22) | 0.00598 |
| $\delta_{K}$ | kHz | 0.045(16) | 0.101(13) | 0.0846(21) | 0.04213 | 0.119(57) | $0.134(51)$ | 0.0857(75) | 0.00370 |
| $D_{\text {pi2 }}$ | kHz | 93.10(87) | 97.68(85) | 157.8(59) |  | -34.80(46) | -34.64(42) | -64.99(65) |  |
| $D_{\text {pi2K }}$ | MHz | -0.2333(43) | -0.1864(43) | -0.4450(33) |  | 0.904(22) | 0.996(21) | 1.2828(85) |  |
| $D_{\text {pi2 - }}$ | kHz | 66.34(84) | 75.72(83) | 161.4(66) |  | -20.56(48) | -18.01(44) | 49.9(36) |  |
| $V_{3}$ | $\mathrm{cm}^{-1}$ | 55.784(34) | 55.469(34) | 55.7693(90) | 50.128 | 36.548(32) | 36.392(32) | 36.6342(84) | 32.768 |
| $F_{0}$ | GHz | 157.074(96) | 156.190(96) | 157.192(28) | 160.898 | 158.10(15) | 157.45(14) | 158.825(38) | 160.855 |
| $V_{J}$ | MHz | ... | ... | 1.41(12) |  | ... | ... | ... |  |
| $D_{m K}$ | MHz | $\ldots$ | $\ldots$ | 0.4122(59) |  | $\ldots$ | $\ldots$ |  |  |
| $V_{K}$ | MHz | $\ldots$ | $\ldots$ | ... |  | $\ldots$ | $\ldots$ | 9.97(12) |  |
| $V_{-}$ | MHz | $\ldots$ | $\ldots$ | 1.92(13) |  | $\ldots$ | $\ldots$ | 1.265(61) |  |
| $D_{m J}$ | MHz |  |  |  |  |  |  | 0.0491(12) |  |
| $\angle(\mathrm{i}, a)$ | deg | 51.6451(1) | 51.65520(20) | 51.6942(12) | 52.140 | 146.8215(1) | 146.83013(53) | 146.8552(14) | 146.887 |
| $\angle(\mathrm{i}, \mathrm{b})$ | deg | 141.6451(1) | 141.65520(20) | 141.6942(12) | 142.140 | 123.1785(1) | 123.16987(53) | 123.1448(14) | 123.113 |
| $\angle(\mathrm{i}, \mathrm{c})$ | deg | $90.0{ }^{\text {g }}$ | $90.0{ }^{\text {g }}$ | $90.0{ }^{\text {g }}$ | 89.996 | $90.0{ }^{\text {g }}$ | $90.0^{\text {g }}$ | $90.0{ }^{\text {g }}$ | 89.995 |
| $\mathrm{N}_{\mathrm{A}} / \mathrm{N}_{\mathrm{E}}{ }^{\text {h }}$ |  | 92/131 | 92/131 | 92/131 |  | 183/137 | 183/137 | 183/137 |  |
| $\sigma^{\text {i }}$ | kHz | 27.0 | 23.0 | 3.7 |  | 32.1 | 28.3 | 4.1 |  |

${ }^{\text {a }}$ All parameters refer to the principal axis system. Watson's A reduction and $I^{r}$ representation were used.
${ }^{\mathrm{b}}$ Fit with the program XIAM.
${ }^{c}$ Fit with the program aixPAM using the same parameters as in Fit I.
${ }^{\mathrm{d}}$ Fit with the program aixPAM using three additional parameters.
${ }^{e}$ Centrifugal distortion constants obtained from anharmonic frequency calculations; all other values from geometry optimizations at the B3LYP/6-311++G(d,p) level of theory.
${ }^{\mathrm{f}}$ Derived from the linear combinations $B_{J}=1 / 2(B+C), B_{K}=A-1 / 2(B+C)$, and $B_{-}=1 / 2(B-C)$. The standard errors of $A$, $B$, and $C$ were obtained from $\sigma_{A}=\sqrt{\sigma_{B J}^{2}+\sigma_{B K}^{2}+2 \rho_{B J, B K} \sigma_{B J} \sigma_{B K}}, \sigma_{B}=\sqrt{\sigma_{B J}^{2}+\sigma_{B-}^{2}+2 \rho_{B J, B-} \sigma_{B J} \sigma_{B-}}$, and $\sigma_{C}=\sqrt{\sigma_{B J}^{2}+\sigma_{B-}^{2}-2 \rho_{B J, B-} \sigma_{B J} \sigma_{B-}}$, respectively, with the correlation coefficients $\rho_{B J, B K}$ and $\rho_{B J, B-}$.
${ }^{\mathrm{g}}$ Fixed due to symmetry.
${ }^{\mathrm{h}}$ Number of the A and E species lines.
${ }^{\text {i }}$ Standard deviation of the fit. All fits were carried out with a basis size $k_{\max }=8$.
$V_{-}$multiplying $\left(P_{x}^{2}-P_{y}^{2}\right)(1-\cos 3 \alpha)$ lowered the standard deviation to 3.7 kHz , which is the measurement accuracy. For trans-MMA, the standard deviations were reduced to 4.1 kHz by adding the parameters $V_{K}$ multiplying $P_{z}{ }^{2}(1-\cos 3 \alpha), D_{m J}$ multiplying $P^{2} p_{\alpha}{ }^{2}$, and $V_{-}$, where in both cases, the operators containing the $(1-\cos 3 \alpha)$ term led to the most significant changes of the fits. It should be noted that the RAM equivalent versions of these operators are also available in BELGI. These aixPAM fits are summarized as Fit III in Table II.

## IV. DISCUSSION

While no significant differences between Fit I and Fit II are observed, Fit III in Table II shows that aixPAM improves the standard deviations for both the trans and cis conformers of MMA, to measurement accuracy by adding three parameters in the fits. Therefore, we assumed that additional effective parameters in XIAM would allow fits with standard deviations close to those obtained with aixPAM or BELGI. The torsional barriers of the ring methyl group of $36.6342(84) \mathrm{cm}^{-1}$ (transMMA) and 55.7693(90) $\mathrm{cm}^{-1}$ (cis-MMA), according to Fit III,
are significantly different. Compared with the barrier heights of $444 \mathrm{~cm}^{-1}$ and $50 \mathrm{~cm}^{-1}$ observed for OMA ${ }^{17}$ and PMA, ${ }^{18}$ respectively, it is deduced that the $V_{3}$ potential changes for each isomer because the methyl rotor encounters different local environments. The same effect was observed in other toluene derivatives, as indicated in Table III.

The torsional barriers of the $o$-isomers are largest, because the barrier heights are dominated by steric hindrance, since the substituents are adjacent to each other in the benzene ring. The barriers decrease in $m$-isomers, because the substituents are further apart, which creates a symmetric local environment near the methyl group, even though the global frame of the molecule is still asymmetric. The $p$-isomers often possess the smallest barriers, ${ }^{18,24,26}$ because the molecule is electronically and structurally symmetric, as already discussed in Ref. 18.

In OMA, only one conformer exists, with the methoxy methyl group pointing away from the methyl group, since steric effects predominate any stabilization due to sixmembered ring interaction. ${ }^{17}$ Conversely, the distance between the substituents of MMA is large enough to negate steric hindrance, leading to two stable conformers. In PMA, the

TABLE III. Torsional barriers of toluene derivatives in $\mathrm{cm}^{-1}$.

| Molecule | ortho | cis-meta | trans-meta | para |
| :--- | :---: | :---: | :---: | :---: |
| Tolunitrile | $187.699(3)^{36}$ | $14.1960(3)^{\mathrm{a}, 37}$ | $\ldots \mathrm{~b}^{\mathrm{b}}$ |  |
| Fluorotoluene | $227.28(2)^{19}$ | $15.8(1.1)^{\mathrm{a}, 20}$ | $4.8298(64)^{\mathrm{b}, 21}$ |  |
| ${ }^{35}$ Cl-chlorotoluene | $513.8(27)^{38}$ | $\ldots \mathrm{a}^{\mathrm{a}}$ | $4.872(14)^{\mathrm{b}, \mathrm{c}, 39}$ |  |
| Xylene | $518.3(32)^{40}$ | $4.49(14)^{\mathrm{a}, 41}$ | $\ldots$ |  |
| Cresol | $371.05(41)^{\mathrm{d}, 22}$ | $22.44(7)^{23}$ | $3.2(2)^{23}$ | $18.39(3)^{24}$ |
| Methylbenzaldehyde | $\ldots$ | $35.925(3)^{25}$ | $4.64(3)^{25}$ | $28.37^{26}$ |
| Methylanisole | $444.48(42)^{17}$ | $55.7693(90)^{\mathrm{e}}$ | $36.6342(84)^{\mathrm{e}}$ | $49.6370(1)^{18}$ |

${ }^{\text {a }}$ Only one conformer due to symmetry of the $m$-substituent.
${ }^{\mathrm{b}}$ Only $V_{6}$ potential exists due to symmetry.
${ }^{\mathrm{c}}$ Value determined by initial defect.
${ }^{\mathrm{d}}$ Value for the anti conformer. The respective value for the $\operatorname{syn}$ conformer is $669.10(51) \mathrm{cm}^{-1}$.
${ }^{\mathrm{e}}$ This work.
symmetry of the molecule allows for only one conformer. ${ }^{18}$ Only few examples are reported in the literature concerning the conformational effect on torsional barriers. The studies on $m$-cresol, $m$-methylbenzaldehyde, and MMA (this work) have explored significant differences in the $V_{3}$ potentials between the rotational conformers, where in all cases, the barrier for the trans conformer is lower. As discussed in Ref. 35, there are two factors that affect the height of a methyl rotor torsional barrier: steric hindrance and electronic configuration. In the $m$-substituted toluenes mentioned, steric effects are absent and the far distance between the two substituents implies that electronic properties are more likely responsible for the different barrier heights.

In the case of $o$-cresol, where exceptionally both anti (trans) and syn (cis) conformers exist because of the low steric hindrance of the OH group, the barrier of $371.05(41)$ $\mathrm{cm}^{-1}$ found for anti-o-cresol is also much lower than that of $669.10(51) \mathrm{cm}^{-1}$ found for syn-o-cresol. However, because the hydroxy and the ring methyl group are close in syn-o-cresol, this observation can still be explained by steric effects.

Alvarez-Valtierra et al. have already reported on the internal rotation of the ring methyl group in MMA with $V_{3}$ potentials of $57.07 \mathrm{~cm}^{-1}$ (cis-MMA) and $30.35 \mathrm{~cm}^{-1}$ (transMMA) using fluorescence spectroscopy. ${ }^{42}$ In comparison with the microwave spectroscopic data, for cis-MMA, the values are consistent within a small difference of $1.3 \mathrm{~cm}^{-1}$. For trans-MMA, the barrier height obtained from fluorescence

TABLE IV. Comparison of the rotational constants $A, B$, and $C$ (in MHz) as well as the $V_{3}$ potential (in $\mathrm{cm}^{-1}$ ) of $m$-methylanisole observed by microwave spectroscopy (Fit III in Table II) and fluorescence spectroscopy (Fit F). ${ }^{42}$

| Parameter | Fit F | Fit III | Fit III - Fit F |
| :--- | :---: | ---: | :---: |
| cis-MMA |  |  |  |
| $A$ | $2766.7(1)$ | $2755.304(73)$ | -11.4 |
| $B$ | $1297.5(1)$ | $1291.236(45)$ | -6.3 |
| $C$ | $890.7(1)$ | $890.88(16)$ | 0.2 |
| $V_{3}$ | 57.07 | $55.7693(90)$ | -1.30 |
| trans-MMA |  |  |  |
| $A$ | $3573.1(1)$ | $3521.824(85)$ | -51.3 |
| $B$ | $1124.2(1)$ | $1121.498(45)$ | -2.7 |
| $C$ | $861.1(1)$ | $862.060(45)$ | 1.0 |
| $V_{3}$ | 30.35 | $36.6342(84)$ | 6.28 |

spectroscopy differs by more than $6 \mathrm{~cm}^{-1}$ from that determined by microwave spectroscopy. ${ }^{42}$ Not only the torsional barriers but also the rotational constants do not match in studies by microwave and fluorescence spectroscopy, as indicated in Table IV. We believe that molecular parameters obtained by microwave spectroscopy are more accurate and reliable because of its higher resolution than that of fluorescence spectroscopy.

Structural parameters such as rotational constants and the angles between the internal rotor axis and the principal axes as well as the $V_{3}$ barrier height calculated at the B3LYP/6$311++G(d, p)$ level of theory agree well with the experimental values for two conformers of MMA (see Table II). Centrifugal distortion constants predicted by anharmonic frequency calculations are in the same order of magnitude with the experimental values. Therefore, we conclude that the B3LYP/6-311++G(d,p) level is sufficiently suited for optimizing the structures of MMA.

Finally, after the spectra of cis- and trans-MMA were assigned, no intense lines remain in the broadband scan (see Fig. 5), implying that ${ }^{13} \mathrm{C}$ isotopologues or water complexes are not observable for this compound under our measurement conditions.

## SUPPLEMENTARY MATERIAL

See supplementary material for the Fourier coefficients of the potential functions, Cartesian coordinates, frequency lists, definitions of some operators, and correlation matrices of the fits.

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[^1]:    ${ }^{\mathrm{a}}$ Anti-commutators are written as $\{u, v\}=u v+v u$.
    ${ }^{\mathrm{b}}$ The fundamental operators are given in the text. Example: The operator associated with $V_{3}$ translates to $0.5-0.25 e^{i 3 \alpha}-0.25 e^{-i 3 \alpha}=0.5(1-\cos 3 \alpha)$.

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