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Conical whirl instability of a turbulent flow hydrodynamic plain journal bearings

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Conical whirl instability of an unloaded rigid rotor supported in a turbulent flow plain hydrodynamic journal bearing has been studied theoretically, following Constantinescu's turbulent lubrication theory. The effect of Reynolds number (Re) and aspect ratio (L/D) on the stability of rotor-bearing system has been investigated. It is observed that stability improves with increase in both Re and (L/D) ratio of bearings.

In a rotor bearing system, two modes of whirl instability may occur namely translatory and conical. Cylindrical whirl in translatory mode occurs when a rigid rotor is loaded symmetrically in two bearings. Conical whirl occurs due to non-symmetric loading. For two closely spaced bearings, if the transverse moment of inertia is high, then conical whirl onset speed can occur before cylindrical whirl[1,2]. For a single bearing system, a rigid rotor in a single rigidity mounted bearing conical whirl is common particularly when transverse moment of inertia is high. Hirofumi[3] observed that the instable region of the conical mode began at much lower speed than did the cylindrical mode for a gyroscope consisting of hydrodynamic grooved journal bearing. Marsh[4] obtained the stability and gave equations of conical motion with and without gyroscopic effect, using a linearized theory. The dynamic tilt stiffness and damping coefficients of an externally pressurized porous gas journal bearings have been analysed and calculated by Rao and Majumdar[5]. A theoretical stability analysis of the conical whirl mode for a symmetric rotating shaft model has been presented by Yoshihiro *et al.*[6]. The study of stiffness and damping coefficients of a symmetric rotor bearing system, in conical vibrational mode has also been reported in the literature[7]. Guha[8,9] has presented a theoretical analysis on the conical whirl instability of an unloaded rigid rotor supported in porous oil journal bearings with tangential velocity slip on the bearing film interface. However, the effect of turbulence on conical whirl instability is not studied by the previous investigators. The analysis were mainly confined to laminar flow regimes.

The effect of misalignment on the performance of turbulent journal bearings has been presented by Safar *et al.*[10,11]. The dynamic force and moment response for turbulent flow annular pressure seals have been studied by various researchers[12-14]. Eusepi *et al.*[15] have shown experimentally that journal misalignment affects considerably the bearing performance in turbulent flow regime. An analysis for calculation of the dynamic force and moment response in turbulent flow orifice compensated misaligned hydrostatic journal bearings has been presented by Andres[16]. Theoretical investigation of the steady state performance of finite plain hydrodynamic journal bearings in turbulent regime has been made by the authors[17]. Stability analysis of a rigid rotor in turbulent finite plain hydrodynamic journal bearings has also been reported[18] in cylindrical whirl translatory mode.

In the present analysis, the conical whirl instability of a rigid rotor supported in a single bearing operating in turbulent regime is presented.

Theory and Computation

Governing equations—Fig. 1 shows schematically a hydrodynamic plain journal bearing with the coordinate system used in the analysis. The journal rotates with a steady rotational speed ω about its axis, and undergoes whirl in a conical mode with a frequency ω_p about its mean steady state position. The mean steady state position of the unloaded journal is concentric and the lubricant film in the clearance space of the bearing is turbulent.

The governing equation of flow in the film region is obtained from the following equation

$$\frac{\partial}{\partial x}\left[\frac{h^3 G_\theta}{\mu}\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[\frac{h^3 G_z}{\mu}\frac{\partial p}{\partial z}\right]=\frac{U}{2}\frac{\partial h}{\partial x}+\frac{\partial h}{\partial t} \quad \ldots (1)$$

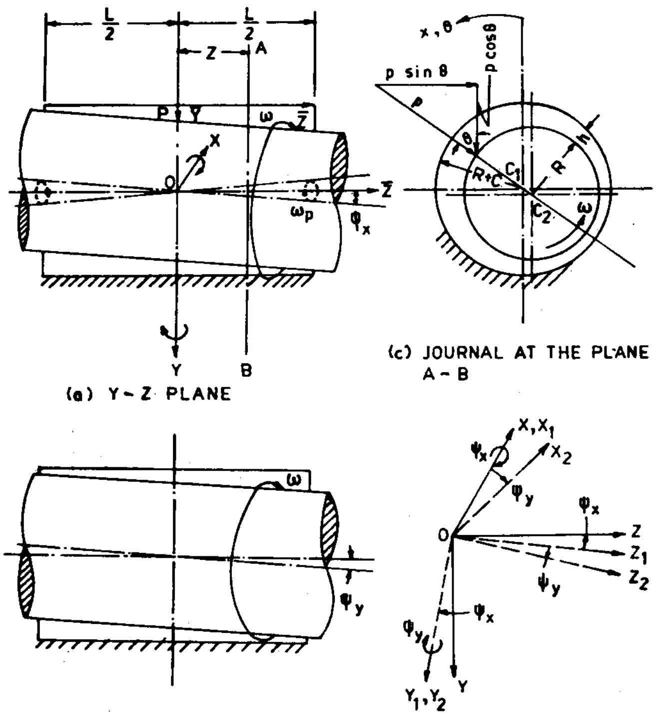

Fig. 1—Bearing geometry, coordinates and rotation angles of journal

where G_θ and G_z are turbulence coefficients. Using the following non-dimensional terms

$$p=\frac{pC^2}{\mu UR},\ \bar{z}=\frac{2z}{L},\ \bar{h}=\frac{h}{C},$$

$$D=2R,\ \theta=\frac{x}{R},\ T=\omega_p t,\ \lambda=\frac{\omega_p}{\omega}$$

the above equation can be reduced to the following non-dimensional form

$$\frac{\partial}{\partial\theta}\left[\bar{h}^3 G_\theta \frac{\partial\bar{p}}{\partial\theta}\right]+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left[\bar{h}^3 G_z\frac{\partial\bar{p}}{\partial\bar{z}}\right]=\frac{1}{2}\frac{\partial\bar{h}}{\partial\theta}+\frac{\lambda\partial\bar{h}}{\partial T} \quad \ldots (2)$$

Constantinescu[19] indicated the following values of G_θ and G_z

$$1/G_\theta=12+0.0260\,(Re^*)^{0.8265}$$
$$1/G_z=12+0.0198\,(Re^*)^{0.741} \quad \ldots (3)$$

The journal axis performs periodic motions around its steady state concentric position. These motions are pure rotation about axis OX and OY, with amplitudes $Re(\gamma e^{iT})$ and $Re(\delta e^{iT})$, respectively. In Fig. 1, these periodic rotational motions are represented by ψ_X and ψ_Y. For a first-order perturbation, which is generally valid for small amplitude oscillations, the perturbed equations for pressure in bearing clearance and the local film thickness can be expressed as:

$$\bar{p}=\left(\frac{L}{2C}\right)\gamma e^{iT}\bar{p}_1+\left(\frac{L}{2C}\right)\delta e^{iT}\bar{p}_2$$

$$\bar{h}=1+\bar{z}\left(\frac{L}{2C}\right)(\cos\theta)\,\gamma e^{iT}+\left(\bar{z}\frac{L}{2C}\right)(\sin\theta)\,\delta e^{iT} \quad \ldots (4)$$

where

$\bar{p}=\bar{p}(\theta,\bar{z},T)$

$\bar{p}_1=\bar{p}_1(\theta,\bar{z})$

$\bar{p}_2=\bar{p}_2(\theta,\bar{z})$

Substituting Eq. (4) into Eq. (2) and collecting only the first order terms for γ and δ, the following equations result.

$$G_\theta\frac{\partial^2\bar{p}_1}{\partial\theta^2}+(D/L)^2\,G_z\frac{\partial^2\bar{p}_1}{\partial\bar{z}^2}=-\frac{1}{2}\bar{z}\sin\theta+i(\lambda\bar{z}\cos\theta) \quad \ldots (5)$$

$$G_\theta\frac{\partial^2\bar{p}_2}{\partial\theta^2}+(D/L)^2\,G_z\frac{\partial^2\bar{p}_2}{\partial\bar{z}^2}=\frac{1}{2}\bar{z}\cos\theta+i(\lambda\bar{z}\sin\theta)\,. \quad (6)$$

Boundary conditions—The boundary conditions for the film region are

$\bar{p}_1(\theta,\pm1)=\bar{p}_2(\theta,\pm1)=0$ at $0\leqslant\theta\leqslant 2\pi$ (ambient)

$\bar{p}_1(\theta,0)=\bar{p}_2(\theta,0)=0$ at $0\leqslant\theta\leqslant 2\pi$ (antisymmetric)

$\bar{p}_1(\theta,\bar{z})=\bar{p}_1(\theta+2\pi,\bar{z})$

$\bar{p}_2(\theta,\bar{z})=\bar{p}_2(\theta+2\pi,\bar{z}) \quad \ldots (7)$

Solving Eqs (5) and (6) satisfying the above boundary conditions by Gauss-Seidel iteration with successive over-relaxation scheme, the dynamic pressures in the film region are obtained.

Stiffness and damping characteristics—It can be shown that the four components each of stiffness and the damping coefficients can be computed from the following expressions by numerical integration using Simpson's 1/3rd rule.

$$\bar{S}_{XX}=-\mathrm{Re}\left[2\int_0^1\int_0^{2\pi}\bar{p}_1\,\bar{z}\cos\theta\,d\theta\,d\bar{z}\right]$$

$$\bar{S}_{YX}=-\mathrm{Re}\left[2\int_0^1\int_0^{2\pi}\bar{p}_1\,\bar{z}\sin\theta\,d\theta\,d\bar{z}\right]$$

$$\bar{S}_{XY}=-\mathrm{Re}\left[2\int_0^1\int_0^{2\pi}\bar{p}_2\,\bar{z}\cos\theta\,d\theta\,d\bar{z}\right]$$

$$\bar{S}_{YY} = -\mathrm{Re}\left[2\int_0^1\int_0^{2\pi} \bar{p}_2\,\bar{z}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\bar{z}\right]$$

$$\bar{D}_{XX} = -\mathrm{Im}\left[2\int_0^1\int_0^{2\pi} \bar{p}_1\,\bar{z}\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\bar{z}\right]/\lambda$$

$$\bar{D}_{YX} = -\mathrm{Im}\left[2\int_0^1\int_0^{2\pi} \bar{p}_1\,\bar{z}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\bar{z}\right]/\lambda$$

$$\bar{D}_{XY} = -\mathrm{Im}\left[2\int_0^1\int_0^{2\pi} \bar{p}_2\,\bar{z}\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\bar{z}\right]/\lambda$$

$$\bar{D}_{YY} = -\mathrm{Im}\left[2\int_0^1\int_0^{2\pi} \bar{p}_2\,\bar{z}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\bar{z}\right]/\lambda \quad \ldots (8)$$

For oil bearings the value of λ will not affect the dynamic coefficients. Hence, $\lambda = 1.0$ has been used for calculating the coefficients.

The journal bearing system is rotationally symmetric about the Z' axis since the steady state position of the unloaded journal is concentric. So the following relations should exist between response coefficients[7]

$$S_{XX} = S_{YY} \qquad D_{XX} = D_{YY}$$

$$S_{XY} = -S_{YX} \qquad D_{XY} = -D_{YX} \quad \ldots (9)$$

Equations of motion—Referring to Fig. 1d, the equations of motion of the journal for small harmonic rotational disturbances are given as[20,21]

$$I_t\ddot{\psi}_X + I_p\dot{\psi}_Y\omega + S_{XX}\psi_X + D_{XX}\dot{\psi}_X + S_{XY}\psi_Y + D_{XY}\dot{\psi}_Y = 0 \quad \ldots (10)$$

$$I_t\ddot{\psi}_Y - I_p\dot{\psi}_X\omega + S_{YY}\psi_Y + D_{YY}\dot{\psi}_Y + S_{YX}\psi_X + D_{YX}\dot{\psi}_X = 0 \quad \ldots (11)$$

Being harmonic rotations, ψ_X and ψ_Y are represented by

$$\psi_X = \gamma e^{iT} \qquad \psi_Y = \delta e^{iT}$$

The two equations of motion can be expressed in non-dimensional form as follows:

$$\begin{bmatrix} (-\lambda^2\gamma_2 + \bar{S}_{XX} + i\lambda\bar{D}_{XX}) & (i\lambda J\gamma_2 + \bar{S}_{XY} + i\lambda\bar{D}_{XY} \\ (-i\lambda J\gamma_2 + \bar{S}_{YX} + i\lambda\bar{D}_{YX}) & (-\lambda^2\gamma_2 + \bar{S}_{YY} + i\lambda\bar{D}_{YY} \end{bmatrix}$$

$$\times \begin{bmatrix}\gamma \\ \delta\end{bmatrix} e^{iT} = 0 \quad \ldots (12)$$

Stability—At the threshold, Eq. (12) must allow a non-zero solution for γ and δ. To test the stability of the motion, the determinant of Eq. (12) is set to zero, giving two equations. Substitution of Eq. (9) into these two equations gives

$$\lambda^4\gamma_2^2 - \lambda^2\gamma_2(2\bar{S}_{XX} - 2J\bar{D}_{YX} + J^2\gamma_2) - \lambda^2(\bar{D}_{XX}^2 + \bar{D}_{YX}^2) + (\bar{S}_{XX}^2 + \bar{S}_{YX}^2) = 0 \quad \ldots (13)$$

$$\gamma_2 = \left(\frac{\bar{D}_{XX}\bar{S}_{XX} + \bar{D}_{YX}\bar{S}_{YX}}{\lambda^2\bar{D}_{XX} + J\bar{S}_{YX}}\right) \quad \ldots (14)$$

These two equations are solved simultaneously to find the threshold value of γ_2 and λ for different values of J. The rotor is just stable for these values of γ_2 and λ. For a given rotor-bearing system, if the numerical value of γ_2 is higher than the above predicted value, the system will become unstable. The stability of the system can be studied by the conical stability parameter γ_2.

Method of solution—Putting $J = 0$ in Eqs (13) and (14) give

$$\lambda^2\gamma_2 = \frac{\bar{D}_{XX}\bar{S}_{XX} + \bar{D}_{YX}\bar{S}_{YX}}{\bar{D}_{XX}} \quad \ldots (15)$$

$$\lambda^4\gamma_2^2 - 2\lambda^2\gamma_2\bar{S}_{XX} - \lambda^2(\bar{D}_{XX}^2 + \bar{D}_{YX}^2) + (\bar{S}_{XX}^2 + \bar{S}_{YX}^2) = 0 \quad \ldots (16)$$

Substitution of Eq. (15) into Eq. (16) gives

$$\lambda^2 = \left(\frac{\bar{S}_{YX}}{\bar{D}_{XX}}\right)^2$$

$$\text{Therefore, } \lambda = -\left(\frac{\bar{S}_{YX}}{\bar{D}_{XX}}\right) \quad \ldots (17)$$

On the right-hand side, negative sign is taken so as to make λ positive, because numerical computation gives negative value of $\bar{S}_{YX}$.

The value of λ and γ_2 can be obtained from Eqs (15) and (17), as the values of stiffness and damping coefficients are known. The value of λ for non-gyroscopic system ($J = 0$) is the initial set value for solving Eqs (13) and (14) simultaneously. A trial method gives the stability parameter for different values of J. The solution by trial method is obtained as follows: (a) Substitute the value of λ for $J = 0$ and the values of stiffness and damping coefficients into Eq. (14) to determine γ_2 for a particular value of J. (b) Substitute the values of γ_2 and those of the stiffness and damping coefficients into Eq. (13) and evaluate the left hand side, which denotes the error. If the error is not zero, choose a new value of λ and proceed until the error becomes zero. The error should be negative for a stable region[22].

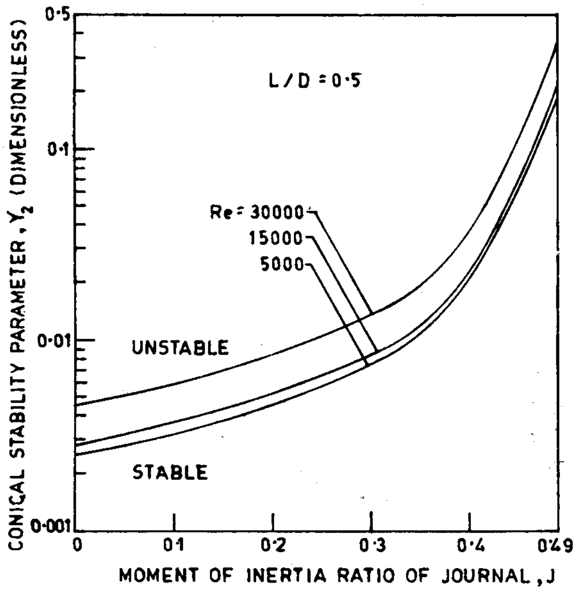

Fig. 2—Variation of conical stability parameter with moment of inertia ratio for different values of Re

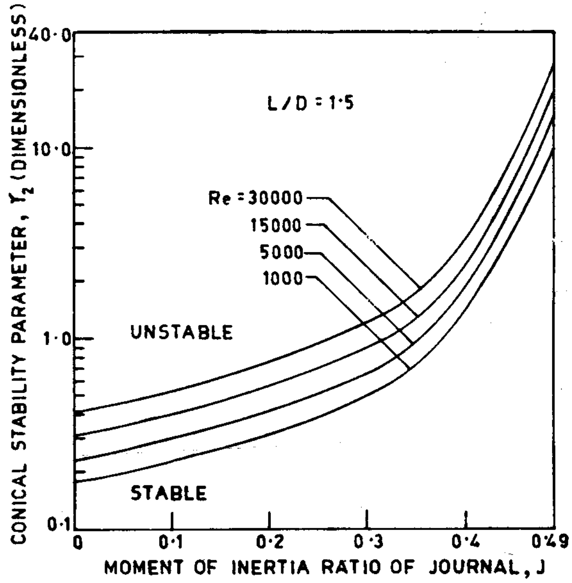

Fig. 4—Variation of conical stability parameter with moment of inertia ratio for different values of Re

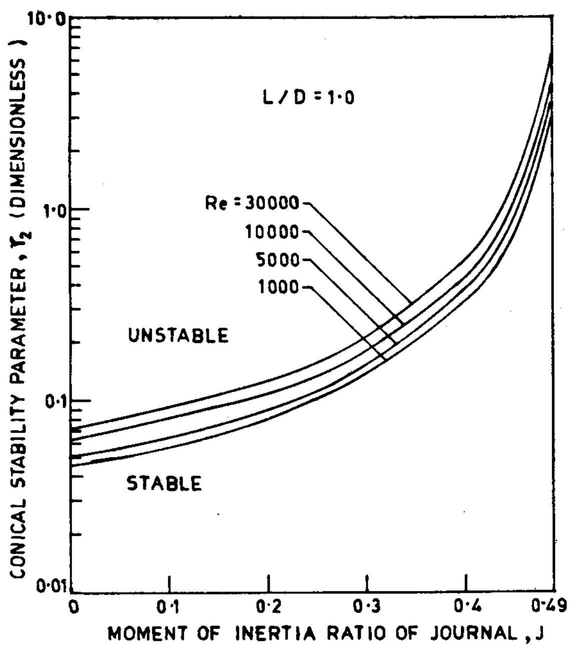

Fig. 3—Variation of conical stability parameter with moment of inertia ratio for different values of Re

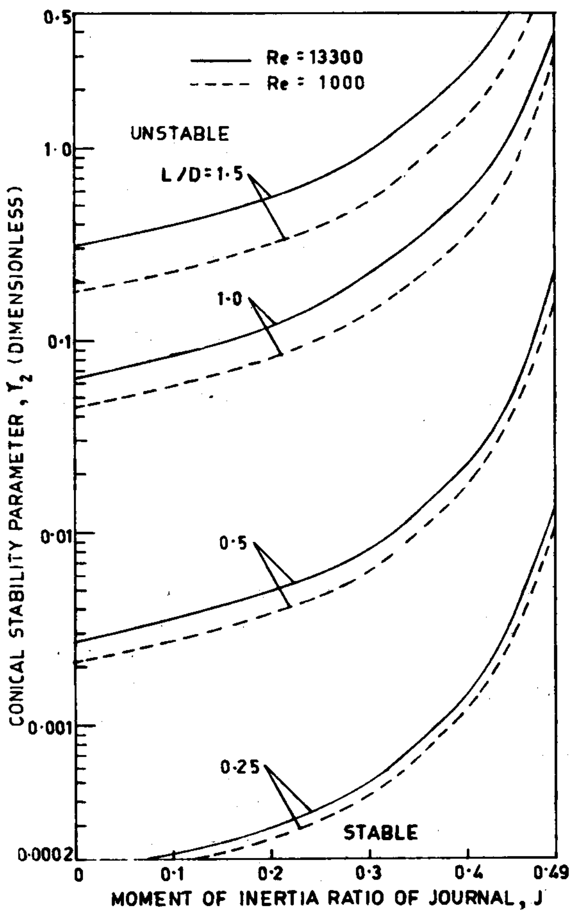

Fig. 5—Variation of conical stability parameter with moment of inertia ratio for different values of L/D

Results and Discussion

In the present analysis, a parametric study of conical stability parameter has been made for different values of Reynolds number (Re) and aspect ratio (L/D). The stability curves in terms of the conical stability parameter, γ_2 are drawn with respect to moment of inertia ratio of journal (J). The upper and lower portions of the curves represent unstable and stable regions, respectively.

Effect of Re—Figs 2-4 show the stability limit curves for different values of Re for L/D ratios of 0.5, 1.0 and 1.5. These figures clearly show that,

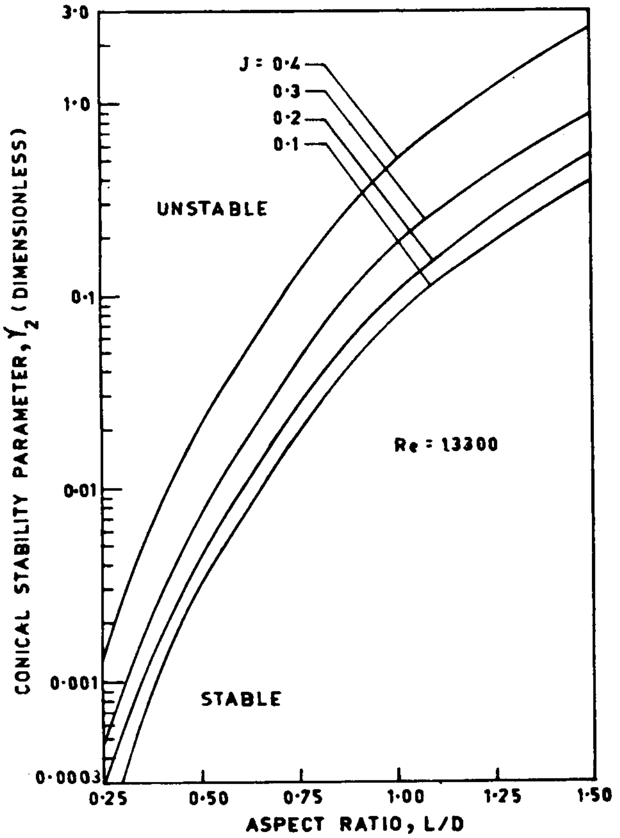

Fig. 6—Variation of conical stability parameter with aspect ratio for different values of moment of inertia ratio J

for all aspect ratios, stability improves with an increase in *Re*. Turbulent lubrication is effectively equivalent to operating with a lubricant of increased viscosity, other factors remaining same. The above improvement in stability with increase in *Re* may be due to this reason.

Effect of L/D ratio—An enhancement in stability is observed with an increase in L/D ratio of bearings (Fig. 5). It is further observed that effect of turbulence, in improving the conical whirl stability, is more in a bearing with higher aspect ratios.

Fig. 6 shows a plot of conical stability parameter (γ_2) with respect to aspect ratio of the journal. It depicts that the improvement in stability of turbulent bearings, with increase in L/D ratio is very sharp for $L/D \leqslant 1.0$. Therefore, for stability of turbulent bearings, operating in conical whirl mode, the L/D ratio should be kept around 1.0.

Effect of J—Figs 2-5 clearly show that an increase in moment of inertia ratio (J) improves the stability in conical whirl.

Conclusions

The following conclusions may be drawn from the foregoing analysis: The stability is enhanced with increase in turbulence in conical whirl mode, whereas at low loads the stability is unaffected by turbulence in cylindrical whirl in translatory mode (Ref. 18). In high speed rotating machines, with bearings operating in turbulent lubrication regime, higher L/D ratio preferably L/D around 1.0 should be used for better stability in conical whirl mode. An increase in moment of inertia ratio (J) makes the journal more stable in conical whirl mode.

Nomenclature

C = mean radial clearance of bearing
D = shaft/journal diameter
D_{ij} = bearing damping coefficients (first suffix denotes the direction of moment and the second denotes the direction of angular velocity)
$\bar{D}_{ij}$ = dimensionless bearing damping coefficients $\left(\frac{8C^3 D_{ij}}{\mu R^3 L^3}\right)$
G_θ, G_z = constants in turbulent Reynolds equation
h = local film thickness
$\bar{h}$ = dimensionless film thickness (h/C)
I_p = polar moment of inertia of journal
I_t = transverse moment of inertia of journal
i = $(-1)^{1/2}$
J = inertia ratio (I_p/I_t)
L = length of bearing
p = local fluid pressure in the film region (above ambient)
$\bar{p}$ = dimensionless fluid pressure $\left(\frac{pC^2}{\mu UR}\right)$
p_1, p_2 = perturbed pressures in the film region
$\bar{p}_1, \bar{p}_2$ = dimensionless perturbed pressures
R = shaft/journal radius
Re, Re^* = mean Reynolds number, local Reynolds number ($\rho UC/\mu, \rho Uh/\mu$)
S_{ij} = bearing spring stiffness
$\bar{S}_{ij}$ = dimensionless bearing spring stiffness $\left(\frac{8C^3 S_{ij}}{\mu R^3 L^3}\right)$
t = time, s
T = dimensionless time ($T = \omega_p t$)
U = shaft surface velocity ($R\omega$)
x, y, z = circumferential, radial and axial coordinates
$\bar{z}$ = dimensionless axial coordinate ($2z/L$)

Greek symbols

θ = dimensionless circumferential coordinate (x/R)
μ = absolute viscosity of fluid
ω, ω_p = angular velocity of shaft, angular velocity of journal centre
λ = whirl ratio (ω_p/ω)
ψ_X, ψ_Y = perturbed angular rotations of journal axis about x and y axes, respectively
γ, δ = amplitude of perturbed angular rotation of journal axis about x and y axes, respectively
$\bar{\gamma}_2$ = conical stability parameter dimensionless

$$\left(\frac{8I_t C^3 \omega^2}{\mu U R^2 L^3}\right)$$

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