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Conjunctive use of groundwater and surface water in the Burdekin delta area

Ahmed Hafi
Australian Bureau of Agricultural and Resource Economics

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One of the key economic issues faced by the managers of the two water boards in the Burdekin River Delta is the allocation of surface water between immediate use on farm for irrigation and storage in the aquifer for future use. Because of the significance of the interaction between surface water and groundwater and the return flow externalities within the delta, policies on surface water and groundwater need to be determined simultaneously. A model is formulated of the dynamic system of surface water and groundwater in the delta with water demand, groundwater extraction cost and stochastic recharge and surface water availability. The optimal pumping/artificial recharge policies for each state of the joint surface water and groundwater system is derived for the two hydrogeologically different parts of the delta and for each area this policy is applied over a large number of years to derive the expected development over time of extraction/artificial recharge and the state of the groundwater system. The implications of the optimal pumping/artificial recharge policy for any review of existing allocations of surface water and groundwater are discussed.

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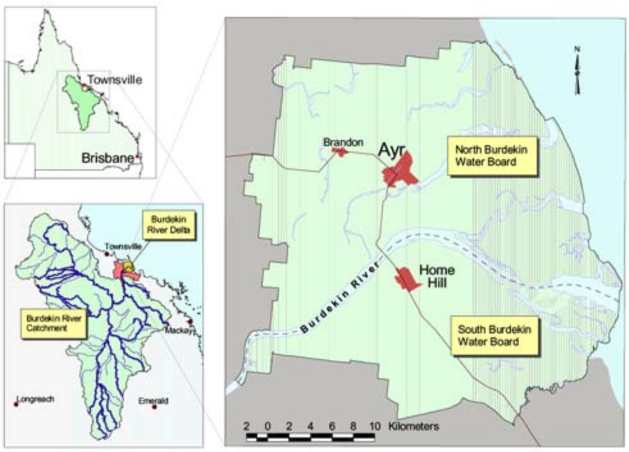
ABARE
Innovation in Economic Research
GPO Box 1563 Canberra 2601 Australia
Telephone +61 2 6272 2000 • Facsimile +61 2 6272 2001
www.abareconomics.com

Introduction

The bulk of the recent expansion in the Australian sugar industry took place in the Burdekin district of north Queensland (figure 1). The Burdekin district currently accounts for about 25 per cent of the Australian sugar production from about 35,000 ha of cane grown in the Burdekin delta and 25,000 ha of cane grown in the Burdekin River Irrigation Area (BRIA). The Burdekin delta overlies a shallow groundwater aquifer, which is hydrologically linked to environmentally sensitive wetland, waterways, estuaries and the Great Barrier Reef. The use of land and water resources in the Burdekin delta has generated increased community awareness of the sustainability of these ecosystems that depend on the underlying aquifer. Two autonomous water boards namely the Northern Burdekin Water Board (NBWB) and the Southern Burdekin Water Board (SBWB), that were created in 1965 manage the water resources in the delta. These water boards divert water from the Burdekin River to a channel and lagoons network to be used for artificial recharge of the aquifer and irrigation supplies to sugarcane farms to supplement the groundwater pumped from the aquifer. In addition to agricultural use, the aquifer also supplies potable water for three towns in the delta.

Figure 1 The location of the Burdekin River Delta



Source: McMahon et al 2001

One of the key economic issues for the management of water resources in the delta is the determination of the optimum level of groundwater pumping for a given state of surface water availability, groundwater stocks and water use efficiency. In addressing this issue the water boards need to decide on the allocation of available surface water between artificial recharge and on-farm irrigation. In determining the optimal groundwater pumping levels, the groundwater stocks need to be managed in such a way as to prevent the water table from rising to a level, that could cause waterlogging in some areas and from falling to a level at which seawater could flow into deeper aquifers, particularly in the coastal areas.

This paper first discusses briefly the characteristics and the current use of land and water resources in the Burdekin delta leading to identification of relevant economic issues in their use. Second, a model, which incorporates the dynamic process of the evolution of the linked surface and groundwater system under different water use regimes, is developed. Third, the use of the model in identifying optimal strategies to conjunctively use groundwater and surface water resources to maximise their economic value is demonstrated for a given set of parameters.

The Burdekin delta

Land resources

In the Burdekin Delta, only 40 per cent of the total agricultural area of 92,500 ha is irrigated with sugarcane accounting for 95 per cent of the irrigated crop area in 2001 (Bristow et al). The northern part of the delta accounts for around two third of the sugar cane area. The bulk of the unirrigated agricultural area in the delta is under pasture, which is used for cattle grazing. Soils in the delta are categorised in to low, medium and high permeability groups. Approximately a third of the land in the delta has low permeable soils (Arunakumaren, et al., 2000).

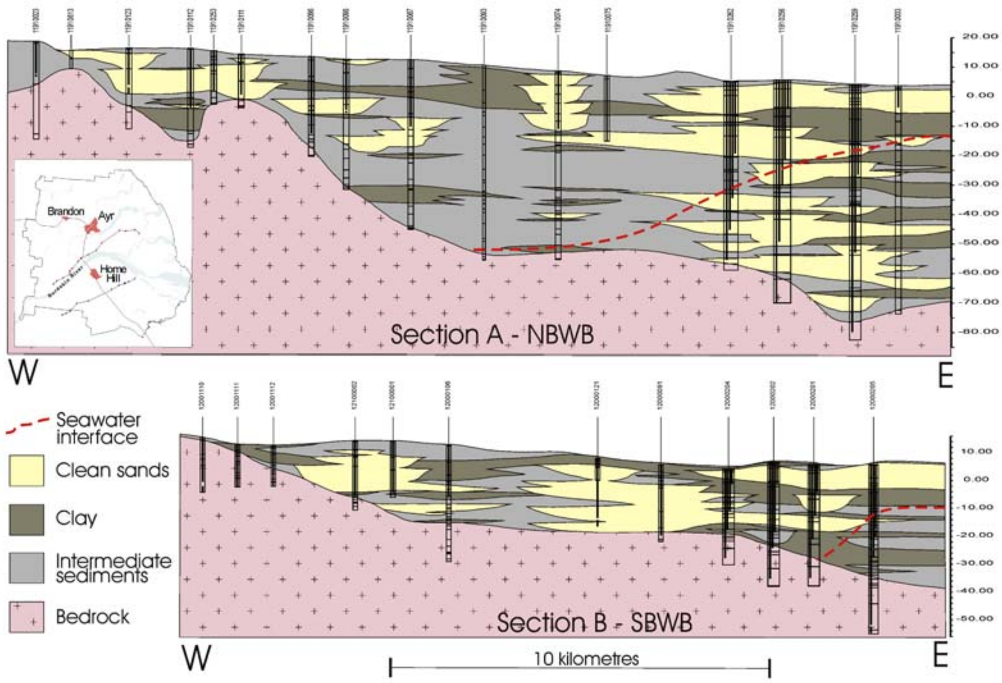
Surface water resources

Surface water is pumped from the Burdekin River and then diverted to channels to deliver to recharge pits and channel intrusion areas and irrigation farms. The channel system also delivers water to the natural waterways, gullies and lagoons. The aquifer and the extensive channel, gullies and lagoon system together are used as a low cost storage of surface water diverted and for most of the rainfall runoff in the area. However, during hot summer conditions, evaporation from open channels leads to significant water losses. When the water diverted from the River is too turbid to be used in recharge pits or in excess of recharge capacities it is made available to growers for supplementary irrigation. The primary aim of the two water boards is to ensure adequate groundwater stocks of good quality to meet pumping demands and the provision of surface water to farmers is only a secondary objective. However, recently the turbidity of water diverted from Burdekin Falls Dam has made the diverted water unsuitable for artificial recharge; consequently increased allocation for irrigation and reliance on irrigation return flows to recharge the aquifer has become an option. Despite that the supply of surface water for irrigation is a secondary aim, there have been times that water boards have increased the seasonal volume of surface water supplied to prevent excessive draw down of the aquifer.

Groundwater resources

The Burdekin River Delta aquifer consists of sedimentary deposits, up to 100 metres below the surface, particularly near the coastline, overlying a granite bedrock surface. An important feature of the delta aquifer is that the sediments are not continuous laterally even over short distances (figure 2). Discontinuity in impervious clay layers exposes the aquifer to infiltration of water from the surface and as a result the aquifer is generally considered unconfined. Consequently a single layer approach has been used in most hydrological studies (McMahon, et al. 2001). The water levels in shallow and deep bores were found to be correlated indicating vertical hydraulic connectivity. The water levels in these bores are also correlated with rainfall and recharge through the unsaturated zone is found to be rapid.

Figure 2 Cross sections of the Burdekin River Delta aquifers



Source: McMahon et al 2001

Water resources management

As mentioned before, surface water is distributed among growers by the water boards for supplementary irrigation when the diverted water is unsuitable for artificial recharge. An advantage of this policy is that the water boards have direct control over aquifer hydraulic head and thus salt-water intrusion and water logging. Also, storm water that would otherwise have run off is used to artificially recharge the aquifer. Artificial recharge can also be used to recharge the aquifer when surface water is plenty, to be subsequently used by pumping groundwater when surface water is not available. In the delta, artificial recharge is defined as a practice by which the volume of water entering the aquifer from recharge areas is increased by artificial means. This is achieved by letting water collect in recharge pits and natural and artificial channels that go through recharge areas. Other sources for recharge include rainfall infiltration, irrigation return flows from on farm application losses, vertical and lateral leakages from rivers and springs. In normal years, rainfall recharge from outcrop areas and discharges from flooded rivers are sufficient to recharge the aquifer. However, after several successive years of drought, the aquifer has been depleted to near sea level mainly due to pumping for irrigation and continuous discharge to the sea.

Use of water on farm

There are no comprehensive records on actual usage of groundwater, as metering has not been introduced. Both groundwater and surface water usages appear to have increased recently due to expansion of the farmed area and as evident from the increase in the number of groundwater and surface water pumps since 1981 (Bristow et al. 2000). The flood furrow is used as the predominant water application method for sugar cane. The topsoil layer of the furrows is laser-levelled to assist in even wetting. Tail water is either drained or allowed to seep back into the aquifer.

Issues

The key economic issues faced by the managers of the two water boards are the allocation of surface water captured through diversion, storm water and rainfall runoff between immediate use on farm for irrigation and storage in the aquifer for future use and the temporal allocation of water presently in storage and expected to be captured in the future. Because of the significance of the return flows in the delta, the policies on surface water and groundwater need to be determined simultaneously.

The water managers need to know the optimum level of groundwater pumping for any combination of the possible levels of surface water allocations for artificial recharge and irrigation, groundwater stocks and on-farm water use efficiency. In determining the optimal groundwater pumping levels, the groundwater stocks need to be managed in such a way as to prevent the water table from rising to a level that could cause waterlogging in some areas and from falling to a level at which seawater could flow into deeper aquifers, particularly in the coastal areas.

A model of surface water groundwater system in the Burdekin delta

Control variables and state transition equations

The surface water and groundwater system in the Burdekin delta can be modelled to have two state variables; surface water (s_t) and groundwater (h_t) and one control variable (x_t), the volume of groundwater pumped from the aquifer in year t if $x_t > 0$ or the volume of surface water used in artificially recharging the aquifer in year t if $x_t < 0$. The stochastic dynamic processes between the state and the control variables are governed by the stationary difference equations (1) — (2).

$$s_{t+1} - s_t = -s_t + \varepsilon_{t+1}^s \text{ , for } \forall \ t \tag{1}$$

$$h_{t+1} - h_t = \kappa \left[-x_t + \theta\beta^c s_t + \theta\beta^a \left[(1-\beta^c) s_t + x_t \right] + \varepsilon_{t+1}^g \right] \text{, for } \forall \ t \tag{2}$$

The control variable x_t is subject to upper and lower bounds (equation 3). The upper bound is set at the volume of water available for pumping without causing the water table to fall below the mean sea level. The lower bound in this case is negative and set at the volume of water diverted from the river after adjusting for transmission losses to ensure that the artificial recharge volume cannot exceed the volume of surface water available.

$$-(1-\beta^c)s_t \le x_t \le {h_t}/{\kappa} \tag{3}$$

Where; $\kappa = {1}/{As^y}$

s^y = specific yield of aquifer ($0 \le s^y \le 1$) per year.

A = area of the aquifer (m^2)

β^a = fraction of irrigated water deep percolated past root zone under the farms ($0 \le \beta^a \le 1$) during a year.

β^c = fraction of surface water deep percolated from

leaking channel beds ($0 \le \beta^c \le 1$) during a year.

θ = return flow coefficient or the proportion of deep percolated water recharging the aquifer ($0 \leq \theta \leq 1$) during a year.

s_t = the stock of surface water at the beginning of year *t* net of evaporation losses (ML).

h_t = the hydraulic head of the groundwater table in the aquifer at the beginning of year *t* (metres above Mean Sea Level)

x_t = if, $x_t > 0$, the volume of groundwater pumped from the aquifer in year *t* (ML); and if, $x_t < 0$, the volume of surface water used in artificially recharging the aquifer in year *t* (ML).

ε_t^s = stochastic disturbance term for surface water availability at start of year *t* (ML).

ε_t^g = stochastic recharge for aquifer at start of year *t* (ML).

The stochastic terms ε_t^s and ε_t^g are assumed to be stationary and serially independent random variables, each normally distributed around the mean. Uncertainty about weather conditions within the current year, *t*, could affect the intraseasonal demand for irrigation water for the crop planted at the beginning of the current year as well as the inflows to surface storage and recharges to aquifers in the next year, (*t+1*). However, only the influence of the latter form of uncertainty is considered in this model. For the aquifer, the product of aquifer area, A , and the specific yield, s^y can be defined as the volume of water that need to be extracted to cause a 1 metre fall in the water table. In hydrological terms, this definition is appropriate for free water table conditions where As^y volumes of water can be assumed to have drained by gravity. This can be assumed to be the case in the Burdekin delta as the aquifer is not confined under pressure.

The proportion of irrigation water diverted from the river that deep percolates from leaking channel beds (β^c) and the proportion of water reaching the farm that deep percolates on-farm (β^a) are assumed to be constant over time for simplicity. It is assumed that only a fraction (θ) of all the water deep percolated past the root zone recharges the aquifer. If $\theta = 1$, all the water that deep percolates recharges the underlying aquifer whereas if $\theta = 0$, all the water that deep percolates is 'lost' to the aquifer. The recharge or the return flow coefficient θ depends on the permeability of the soil. It is assumed that any time lag between the onset of deep percolation and completion of aquifer recharge is negligible. This may be a reasonable assumption for the alluvial unconfined aquifer in the Burdekin delta area where recharges were observed to be rapid.

Again for simplicity, it is assumed that h^{MAX} is not so high as to cause waterlogging problems. It is also assumed that the water resource managers seek an optimal pumping policy subject to h_t not falling below the seawater interface to prevent salt-water intrusion so that groundwater pumping is unlikely to affect the salt content of the aquifer water.

Period net benefit function

In a conjunctive surface water and groundwater system, water from surface sources such as rainfall and river are conjunctively used with aquifer water as inputs in agricultural production. For each year *t*, let $f = a - bX_t\left(1-\beta^a\right)$ denote the value of marginal product (VMP) curve for water used in irrigation where, *a* and *b* the intercept and the slope, respectively of the linear VMP curve, $X_t = \left[s_t\,(1-\beta^c) + x_t\,\right]$ is the volume of water available on farm measured in ML and $f\,[.]$ in dollars per year. Prices of agricultural produce and non-water inputs used in production are assumed to be unaffected by the actions of water users individually and collectively. However, pumping groundwater increases the pump lift and thus the unit pumping cost. For each year *t,* unit cost of groundwater pumping is defined to be equal to the unit cost of pumping, before the extraction policy was implemented, γ_0 plus the increase in the cost of lifting water since the implementation of the extraction policy due to falling hydraulic head. The cost of groundwater pumping, C_t^p is the product of the volume of water pumped, x_t and the unit cost of pumping. Artificial recharge of the aquifer with surface water has costs as the cost of construction of recharge pits need to be recovered and the beds of these infrastructures need regular maintenance and particularly cleaning to prevent siltation and vegetative growth from reducing deep percolation. The costs of groundwater pumping and artificial recharge are given in equations (4) and (5) respectively

$$C_t^p = \left[\gamma_0 + \sigma\left(h^{MAX} - h_t\right)\right]x_t, \qquad x_t \geq 0 \text{ for } \forall\ t \tag{4}$$

$$C_t^r = -p^r x_t, \qquad x_t < 0 \text{ for } \forall\ t \tag{5}$$

where, h^{MAX} is the maximum hydraulic head of the aquifer and σ the unit pumping cost per metre drop in the hydraulic head. The cost of artificial recharge C_t^r is the negative of the product of the artificial recharge volume and the unit cost, p^r. For each year, *t*, the aggregate net benefit function, NB_t is given as follows.

$$NB_t = \begin{Bmatrix} B_t - \omega s_t - x_t\left[\gamma_0 + \sigma\left(h^{MAX} - h_t\right)\right] & x_t \geq 0 \\ B_t - \omega s_t + p^r x_t & x_t < 0 \end{Bmatrix} \text{ for } \forall\ t \qquad (6)$$

where; $$B_t = \left\{a - (b/2)\left[\left(1-\beta^c\right)s_t + x_t\right]\left(1-\beta^a\right)\right\}\left[\left(1-\beta^c\right)s_t + x_t\right]\left(1-\beta^a\right)$$

ω = price of surface water at source ($/ML)

Efficient joint use of water resources

The solution of the model for the efficient joint use of groundwater and surface water resources is characterised by the maximisation of the rents summed over all the farms drawing water from the aquifer and surface water resources for irrigation and the use of surface water for artificially recharging the aquifer. In such a solution it is assumed that farmers collude to achieve the socially optimal outcome by including in their decision, the impacts of current groundwater withdrawal/artificial recharge on future groundwater levels and other forms of externalities. If farmers were to act myopically, then they would ignore the impact of their current actions on the future groundwater levels and the externalities. In other words farmers would then be assumed not to observe the state transition equations (1)—(2).

Assumption of myopic behaviour by farmers may not be appropriate as the farmers can be expected to act with some foresight with knowledge of the state transition equations. On the other hand, the assumption of a collusive behaviour by farmers to maximise their joint net benefits may also be unrealistic given incomplete private property rights over groundwater resources and a number of externalities. Even though behaviour between these two extremes seems to be more appropriate, the characteristics of an efficient solution, for all farmers in the irrigation region jointly, are examined further in this paper.

For an infinite time planning horizon, the problem for the water resource manager is to find the efficient solution $x(0)$, $x(1)$, $x(2)$, $x(3)$,……, $x(\infty)$ that maximises the expected present value function

$$V(s_0,h_0)=\begin{cases} E\left\{\sum_{t=0}^{\infty}\rho^t\left[B_t-\omega s_t-x_t\left[\gamma_0+\sigma\left(h^{MAX}-h_t\right)\right]\right]\right\} & x_t\geq 0 \\ E\left\{\sum_{t=0}^{\infty}\rho^t\left[B_t-\omega s_t+p^r x_t\right]\right\} & x_t<0 \end{cases} \text{ for } \forall\ t \quad (7)$$

subject to (1)—(3) and initial state values s_0 and h_0, where E is the mathematical expectation operator and the discount factor ρ is related to the real discount rate r by $\rho=1/(1+r)$.

Let us consider the certainty equivalent case of problem (7) after replacing ε_t^s and ε_t^g, with their long-term expected values $\bar{\varepsilon}^s$ and $\bar{\varepsilon}^g$. This leads to a time evolution of deterministic problems with the current value of the Hamiltonian function for an interior solution defined for each discrete t given in equations 8—9.

$$\begin{aligned}&\widetilde{H}\left(s_t,h_t,x_t,\lambda_t^s,\lambda_t^g\right)\\ &=B_t-x_t\left[\gamma_0+\sigma\left(h^{MAX}-h_t\right)\right]-\omega s_t\\ &+\rho\lambda_{t+1}^s\left[-s_t+\bar{\varepsilon}^s\right]\\ &+\rho\lambda_{t+1}^g\left\{\kappa\left[-x_t+\theta\beta^c s_t+\theta\beta^a\left[\left(1-\beta^c\right)s_t+x_t\right]+\bar{\varepsilon}^g\right]\right\}\end{aligned} \quad \text{for } x_t\geq 0 \text{ and } \forall\ t \quad (8)$$

$$\begin{aligned}&\widetilde{H}\left(s_t,h_t,x_t,\lambda_t^s,\lambda_t^g\right)\\ &=B_t-\left(-p^r x_t\right)-\omega s_t+\rho\lambda_{t+1}^s\left[-s_t+\bar{\varepsilon}^s\right]\\ &+\rho\lambda_{t+1}^g\left\{\kappa\left[-x_t+\theta\beta^c s_t+\theta\beta^a\left[\left(1-\beta^c\right)s_t+x_t\right]+\bar{\varepsilon}^g\right]\right\}\end{aligned} \quad \text{for } x_t<0 \text{ and } \forall\ t \quad (9)$$

For each year *t*, costate variables in current value are denoted by λ_t^s for surface water and λ_t^g for ground water. It is assumed that the Hamiltonian is concave on s_t, h_t and x_t, required sufficient conditions are met and an interior solution exists or bounds on control variables and constraints on state variables are not binding and the transversality conditions; $lim_{t\to\infty}\ \rho^t\lambda_t^s\to 0$ and $\lim_{t\to\infty}\rho^t\lambda_t^g\to 0$.

The maximum principle requires the maximisation of the deterministic problems in the Hamiltonian (8—9) for each discrete *t*. However*,* this maximisation is subject to the bounds on control variable given in equation 3. The Hamiltonian augmented with these bounds is given in the following Lagrangian expression.

$$L = \begin{cases} \tilde{H}\left(s_t, h_t, x_t, \lambda_t^s, \lambda_t^g\right) + \gamma_t\left(h_t/\kappa - x_t\right) & x_t \geq 0 \\ \tilde{H}\left(s_t, h_t, x_t, \lambda_t^s, \lambda_t^g\right) + \eta_t\left[-x_t - \left(1-\beta^c\right)s_t\right] & x_t < 0 \end{cases} \text{ and } \forall\ t \quad (10)$$

Where, γ_t and η_t are the dynamic Lagrangian multipliers for the constraints on the volumes of groundwater pumped and surface water used to artificially recharge the aquifer.

The necessary conditions for optimality are derived by differentiating the Langrangian function with respect to control, state and costate variable and setting the resultant expressions equal to zero in the case of the control variable, to first difference of costate variables in the case of state variables and to first difference of state variables in the case of costate variables. The necessary conditions also include the following Kuhn-Tucker conditions to make sure that the upper and lower bounds are always in force.

$$\gamma_t\left(x_t - h_t/\kappa\right) \leq 0 \qquad \text{for } x_t \geq 0 \text{ and } \forall\ t \qquad (11)$$

$$\eta_t\left[-x_t - \left(1-\beta^c\right)s_t\right] \leq 0 \qquad \text{for } x_t < 0 \text{ and } \forall\ t \qquad (12)$$

For each year *t*, the maximand on the right hand side of equation (10) represents the profit corrected to account for intertemporal effect. The intertemporal effects of groundwater use arise as current decisions affect future profits and are measured, by the costate variable, λ_t^g. The economic and policy implications of the necessary conditions for the optimal solution are interpreted as follows.

$$\begin{matrix} \left\{a - (b)\left[\left(1-\beta^c\right)s_t + x_t\right]\left(1-\beta^a\right)\right\}\left(1-\beta^a\right) \leq \\ \left[\gamma_0 + \sigma\left(h^{MAX} - h_t\right)\right] + \rho\lambda_{t+1}^g\kappa\left(1 - \theta\beta^a\right) + \gamma_t \end{matrix}, \ (x_t \geq 0), \text{ for } \forall\ t \qquad (13)$$

If $x_t \geq 0$ and assuming interior solution ($\gamma_t = 0$), for each year, *t*, the value of marginal product of water pumped cannot exceed the unit cost of lifting water in the current year plus the discounted value of aquifer water in the next year less the discounted value of water returning to the aquifer the next year. The current cost associated with pumping 1 Ml of water consists of (a) pumping cost given by $\gamma_0 + \sigma\left(h^{MAX} - h_t\right)$ (first term on the RHS in equation 13) plus (b) the effect on future profit arising from a drop in water level given by $\rho\lambda_{t+1}^g\kappa\left(1 - \theta\beta^a\right)$ (the second term). The second cost arises due to higher pumping cost in the future (pumping cost externality) and the increased scarcity of the stock (stock externality). The term $\kappa\left(1 - \theta\beta^a\right)$ means that only $\kappa\left(1 - \theta\beta^a\right)$ Ml of each κ_k Ml pumped is lost with the amount $\kappa\theta\beta^a$ seeping back to the aquifer. The condition 13 states that the private marginal cost of water represented by the term

$\left[\gamma_0 + \sigma\left(h^{MAX} - h_t\right)\right] + \rho\lambda_{t+1}^{g}\kappa\left(1 - \theta\beta_t^{a}\right)$ for groundwater are not less than the marginal cost net of externality

$$\rho\lambda_{t+1}^{g}\kappa\left(1-\theta\beta^{a}\right) \le \left\{a-(b)\left[\left(1-\beta^{c}\right)s_t + x_t\right]\left(1-\beta^{a}\right)\right\}\left(1-\beta^{a}\right) + p^{r} - \eta_t, \; (x_t < 0), \text{ for } \forall \; t \quad (14)$$

If $x_t < 0$ and assuming an interior solution ($\eta_t = 0$), for each year, *t*, the current value of the effect on future profit arising from a rise in water level given by $\rho\lambda_{t+1}\kappa\left(1 - \theta\beta^{a}\right)$ cannot exceed the value of the forgone marginal product of additional unit of surface water now used to recharge the aquifer plus unit cost of recharge. The future profits of recharge arise due to lower pumping cost in the future (pumping cost externality) and the reduced scarcity of the stock (stock externality). The term $\kappa\left(1 - \theta\beta^{a}\right)$ means that each κ Ml of surface water withdrawn from irrigation to recharge produces a net recharge of only $\kappa\left(1 - \theta\beta^{a}\right)$ ML and the amount $\kappa\theta\beta^{a}$ represents the forgone irrigation return flow to the aquifer as the water is now withdrawn from irrigation.

$$\begin{aligned}&\left\{a-(b)\left[\left(1-\beta_t^{c}\right)s_t + x_t\right]\left(1-\beta^{a}\right)\right\}\left(1-\beta^{a}\right)\left(1-\beta^{c}\right) \le \rho\lambda_{t+1}^{s} + \omega \\ &- \rho\lambda_{t+1}^{g}\kappa\left[\theta\beta^{c} + \theta\beta^{a}\left(1-\beta^{c}\right)\right] + \eta_t\left(1-\beta^{c}\right)\end{aligned}, \; (s_t > 0), \text{ for } \forall \; t \quad (15)$$

For all interior solutions $\left(\eta_t = 0\right)$ and for each year *t*, the value of marginal product of surface water used on-farm cannot exceed the value of surface water plus the external cost of water less the value of surface water seeping down to the aquifer. Out of each Ml of surface water diverted from the source β^{c} Ml is lost in transmission, however, of this loss $\theta\beta^{c}$ Ml seeps down to the aquifer resulting a net loss of only $\beta^{c}(1-\theta)$.

$$\rho\lambda_{t+1}^{g} \le \lambda_t^{g} - x_t\sigma + \gamma_t/\kappa, \; (= \text{if } h_t > 0), \text{ for } \forall \; t \quad (16)$$

If $x_t \ge 0$, for all interior solutions $\left(\gamma_t = 0\right)$ and for each year *t*, the value of groundwater cannot exceed the value of groundwater in the previous year adjusted for the negative impact on the groundwater value of pumping in the previous year.

Application of the model

The model was applied to obtain a socially optimal extraction/recharge policy and its characteristics so that optimal pumping quotas and recharge volumes could be derived. First, for a socially optimal extraction/recharge policy, the optimal value and volume of groundwater pumped/recharged are computed for the two-dimensional space of hydraulic head (groundwater stocks) and the surface water volumes in the range between minimum and maximum levels. For given initial levels of hydraulic heads and surface water, optimal pumping quotas or recharge volumes can be fixed at levels determined by the optimal pumping/recharge policy. Second, the expected time paths for the volume of groundwater pumped and the hydraulic head (groundwater stocks) are computed for the socially optimal extraction policy implemented through optimal quotas. More details of the solution methods are given in appendix A.

Data

The data required to implement the model were collected from annual reports published by two water boards and a number of biophysical and economic studies conducted on the Burdekin delta. The recent biophysical studies on the delta were based on the results obtained from a modelling system linking the hydrology of the delta with plant-soil-water process and their interaction with groundwater (Arunakumaren 1997). The data on the hydrology and water balance of the delta were obtained from Bristow et al. (2000), McMahon, et al. (2001), NBWB (2002), SBWB (2002) and data required to estimate the VMP function for water were obtained from Qureshi, et al (2002). The data used in the mode are listed in table 1 and the data sources and the method used to develop values for model parameters are described in following two sections.

Table 1 Parameters used in the model

	Unit	*Southern*	*Northern*
As^{y}	*GL/metre*	*42.45*	*85.05*
$h^{\max}$	*metre*	*15.00*	*15.00*
$\bar{\varepsilon}_g$	*GL/year*	*85.50*	*178.50*
$\bar{\varepsilon}_s$	*GL/year*	*67.00*	*135.00*
a	*$/ML*	*173.30*	*173.30*
b	*$/ML^2*	*0.6934*	*0.4244*
β_a	$0 \le \beta_a \le 1$	*0.40*	*0.40*
β_c	$0 \le \beta_c \le 1$	*0.20*	*0.20*
θ	$0 \le \theta \le 1$	*0.90*	*0.90*
γ_0	*$/ML*	*9.11*	*9.11*
σ	*$/(ML.metre)*	*4.00*	*4.00*

Estimation of the VMP function

The data on cane yield (*y*) response to added water (*X*) with furrow irrigation used in Qureshi, et al. (2002)[1] were used to estimate a quadratic yield response function $\left\{ y = p + qX\left(1-\beta^{a}\right) + r\left[X\left(1-\beta^{a}\right)\right]^{2}\right\}$. Then assuming a sugar pool price of $325/t, a Commercial Content of Sugar (CCS) of 15 per cent, a two third share of the revenue to growers and a harvesting cost of $4.0/t, the cane producer price net of harvesting cost is estimated at $28.2/t. The benefit function (*F*) for water in $/ha was then derived after multiplying the yield response function by the estimated cane producer price net of harvesting cost and then subtracting other costs in $/ha denoted *K* $\left[F = \left\{y = p + qX(1-\beta_a) + r[X(1-\beta_a)]^2\right\} * 28.2 - K\right]$. The VMP function in $/ha was derived by differentiating this benefit function with respect to water quantity $\left[\partial F / \partial X = \left\{q(1-\beta_a) + 2r\left[(1-\beta_a)^2\right]X\right\} * 28.2\right]$ and then separate VMP functions for southern and northern areas were obtained. The term $[q(1-\beta_a) * 28.2]$ equals the intercepts of the VMP functions (*a*'s) for both areas while the slopes of these functions (*b*'s) for southern and northern areas were obtained after dividing the term $\left[2r\left[(1-\beta_a)^2\right] * 28.2\right]$ by the effective cane areas of 10,675ha and 17,440ha respectively. In deriving the effective cane areas a fallowing requirement of 20 per cent of the total cane area was assumed.

Hydrological parameters

The southern area accounts for a third of the entire area of the delta estimated at 850 square kilometres with the northern area accounting for the remainder (Bajracharya. K, QDNR, personal communication, May 2002). Specific yield was estimated to range from 0.10 to 0.25 (Bajracharya, K, QDNR, personal communication, May, 2002) and a value of 0.15 for this parameter has been assumed by Bristow et al. (2000). Assuming a specific yield (s^y) of 0.15 following Bristow et al. (2000), and free water table conditions as the aquifer in the delta can be treated as unconfined the volume of water that need to be extracted to cause a 1 metre fall in the water table (As^y) is estimated at 42,450 ML for the southern area and 85,050 ML for the northern area. According to McMahon et al. (2001), the depth of the aquifer varies at different places of the delta generally increasing to 80 metres in the northern areas and 50 metres in the southern areas as one moves toward the sea. However, as one moves toward the sea, the depth of the aquifer with fresh water decreases due to increasing height of the seawater wedge. The capacity of the aquifer available for extraction is assumed to be constrained by a minimum height, below which the seawater intrusion could take place and, a maximum

[1] The author is grateful to Ejaz Qureshi for providing these data.

above which there could be water logging or high water table problems. After considering above factors, a maximum hydraulic head ($h^{\max}$) of 15 metres above mean sea level was assumed for each area. According to simulation studies of the linked surface water and groundwater system in the delta, conducted by (McMahon et al. 2001) each of the inflows to the aquifer (recharge from rain, flood, river and artificial means) and outflows from the aquifer (pumping and discharge to Burdekin River, sea and the Burdekin River Irrigation Area) fluctuated widely over the 1981 to 1995 period. For each recharge and discharge variable, the mid point of the range given in (McMahon et al. 2001) is used to estimate the annual total recharge for the entire delta. Similarly the mid point of the range for surface water pumping estimated at 102 GL/y was added to an average annual artificial recharge of 100 GL/y to estimate the total volume of surface water diversion for the entire delta. A split of 1/3:2/3 is assumed to allocate total recharge and surface water diversion volumes between the southern and northern areas.

Results and discussion

Characteristics of the socially optimal pumping/recharge policy

The model given in equation (7) is solved as explained in appendix A with the set of data and parameter values given in table 1. For each area, the solution obtained was similar in characteristics and consequently similar policy and management implications can be drawn. In order to save space, the results obtained for the southern area of the delta only are discussed in this paper.

The economic values of water resources and the optimal pumping levels for the southern part of the Burdekin River Delta were estimated for a total of 2500 combinations of initial levels of surface water diversion and hydraulic head. For each water source, 50 discrete levels of stock were used resulting in a 50x50 grid or a total of 2500 combinations of initial levels. These estimates were then plotted as a 3 dimensional surface against all combinations of initial states and the resulting value function and the optimal pumping policy are given in figures 3 and 4 respectively. In order to have a closer look at the results, the estimates were also presented in table 2 for 25 combinations of initial values of surface water diversion and hydraulic head derived using a 5x5 grid.

Figure 3 Value function

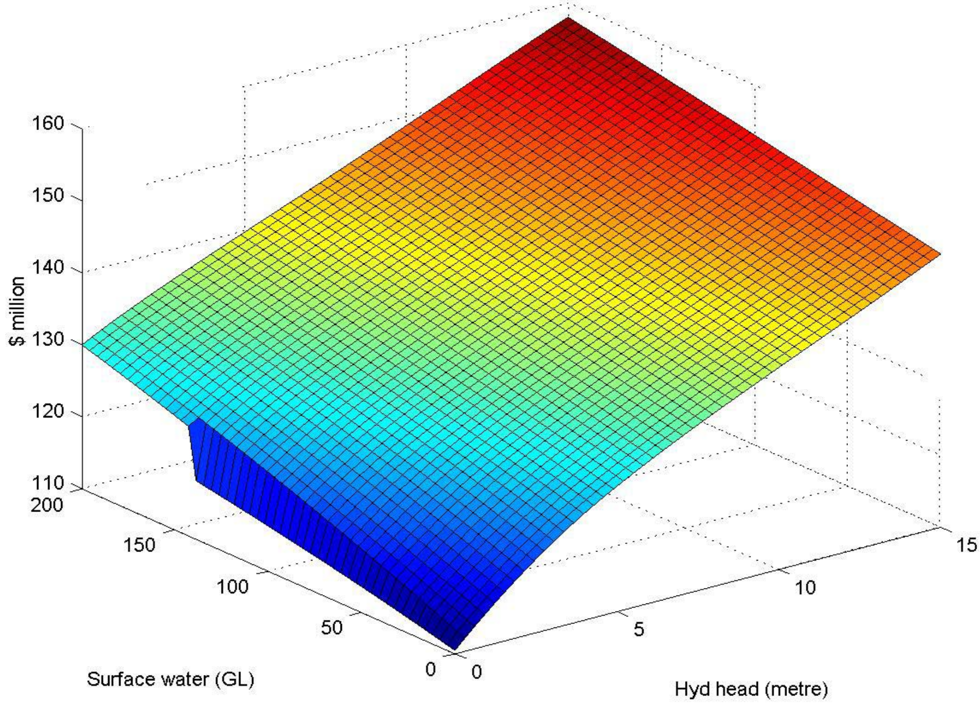
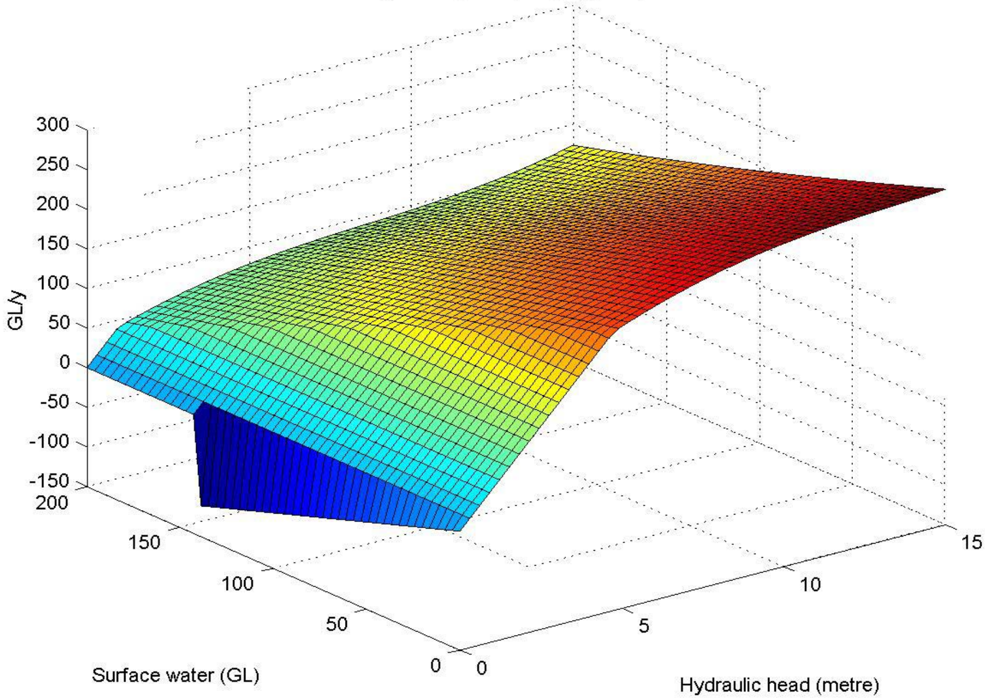


Figure 4 Optimal pumping policy



The optimal pumping policy given in figures 4 suggests that the optimal pumping level generally decreases with the initial hydraulic head (groundwater stocks) and when accompanied by increases in initial surface water diversion, as the marginal cost of groundwater (surface water) increases (decreases) (table 2). If the initial hydraulic head is at the sea level no water is pumped and it becomes optimal to recharge the aquifer artificially as the current value of the effect on future profits of that action exceeds the value of the forgone marginal product of surface water plus unit cost of recharge. Moving from the maximum to minimum initial hydraulic head, the optimal pumping policy surface has three distinct regions: 1- pumping volume decreasing at a slow rate, 2 - pumping volume decreasing at a faster rate and 3 - artificial recharge at sea level. The optimal pumping volumes in region 1 are interior solutions, which mean that they are within negative lower and positive upper bounds. These bounds were introduced as artificial recharge cannot exceed the surface water delivered, after adjusting for conveyance losses to recharge pits and channel intrusion areas while groundwater pumping cannot exceeds the stock of water available above sea level. The optimal pumping volumes in steeper region 2 are on the respective upper bounds with the maximum initial hydraulic head at which the upper bound becomes binding decreasing as the initial volume of surface water diverted increases. The region 3 characterises artificial recharging, which increases with the volume of surface water diverted up to a volume of 140 GL/year.

Contrary to the SBWB policy, which puts greater emphasis on using surface water diverted for artificial recharge rather than distributing it to farms for irrigation, the artificial recharge was found to be optimal only when the hydraulic head was at sea level and the initial surface water diversion was at an intermediate level. As explained before, artificial recharge can be optimal only when the current value of its effect on future profits exceeds the value of the forgone marginal product of surface water plus unit cost of recharge. The demand for surface water for artificial recharge is likely to increase as pumping cost increases with falling hydraulic head. However, increasing pumping cost also increases the demand for surface water for irrigation and thus the opportunity cost of using it in recharging aquifer. The opportunity cost of using surface water for recharging also includes the forgone value of return flow from irrigation. For each ML of surface water diverted for irrigation, β^c ML is lost in transmission and another $\beta^a(1-\beta^c)$ ML is lost when applied to crops and $\theta\left[\beta^c+\beta^a(1-\beta^c)\right]$ ML is finally seeps down to the aquifer. Taking the values used for these parameters (table 1), 47 per cent of surface water diverted seeps down to the aquifer.

For each combination of initial states, the economic values of water resources presented in figure 3 and table 2 represent the discounted values summed over an infinite time horizon. Regardless of the initial state, the stocks of groundwater are expected to reach a steady state after some time depending on its dynamic evolution. Even when initially no surface water was diverted and the hydraulic head was at sea level the water resources can be managed in the future to have a maximum economic value of estimated $ 110 million by adopting the optimal pumping policy and this policy is expected to lead the stocks of groundwater and the pumping volume to reach steady state levels after some time. The economic value of water resources increases with the initial volume of surface water diverted and the hydraulic head (groundwater stocks).

Table 2 Economic value of water resources with different pumping policies in southern area of the Burdekin River Delta

Economic value of water resources ($ million)

Surface water (GL/y)	Hydraulic head (metres from sea level) 0	3.75	7.5	11.25	15
0	110	124	133	140	148
50	114	128	136	143	150
100	116	131	139	146	153
150	126	134	141	148	155
200	130	137	144	151	158

Optimal ground water pumping policy (GL/y)

Surface water (GL/y)	Hydraulic head (metres from sea level) 0	3.75	7.5	11.25	15
0	0	159	238	263	273
50	-40	159	204	222	232
100	-80	134	168	180	193
150	0	104	128	138	156
200	0	70	87	97	122

Shadow price of groundwater ($/ML)

Surface water (GL/y)	Hydraulic head (metres from sea level) 0	3.75	7.5	11.25	15
0	99	60	44	42	42
50	87	52	41	41	42
100	72	46	39	41	41
150	59	41	38	40	39
200	49	38	38	40	37

The knowledge of the optimal pumping policy given in figures 4 may help the manager of SBWB in setting the optimal groundwater allocations for the area for a given set of hydraulic head (groundwater stocks) and surface water level. The optimal pumping policy may be applied as follows.

- For all hydraulic head levels above zero, if the initial hydraulic head and the surface water diversion in year t (h_t, s_t) are lower than the corresponding steady state levels (to be discussed shortly), the optimal groundwater allocation in that year (x_t) must be lower than the steady state allocation. In this case reduced pumping levels compared to the steady state levels in the early years contribute to building of groundwater stocks overtime.

- For all hydraulic head levels above zero, if the initial hydraulic head and the surface water diversion in year t (h_t, s_t) are higher than the corresponding steady state levels, the optimal pumping policy prescribes higher pumping levels compared to that of steady state in the initial years.

- When the hydraulic head is at sea level and if the initial surface water diversion in year t (s_t) is less than 140 GL/y, the optimal policy prescribes artificial recharge – for example using 80 GL of 100 GL of surface water diverted (table 2).

In all of the above scenarios, the optimal pumping policy when applied sequentially over time is expected to lead both the pumping and hydraulic head levels (groundwater stocks) to approach steady state levels. This is true for any combination of initial states of the resources. The optimal time paths of hydraulic head and pumping volume for 5 combinations of initial states (h_0, s_0) selected from the 5x5 grid used in table 2: (0m,0GL); (3.75m, 100Gl); (7.50m, 100GL); (11.25m,100GL) and (15.00m,100GL) are presented in figures 5 and 6. The time taken to reach the steady state depends on the initial hydraulic head and as expected the closer the initial hydraulic head to the steady state level the quicker it is reached. The optimal time paths for the combinations of initial states (0m, 100GL) given in figures 7 show that after using surface water for artificial recharge in the first year, groundwater pumping resumes and the steady state is reached in the 7th year.

It should be noted that given stochastic recharges and surface water availability there is no single steady state and the expected paths in figures 5—7 reach expected or certainty equivalent steady states. For the southern area of the Burdekin delta, the certainty equivalence policy comprises a steady state pumping level of 143 GL/year at a hydraulic head of 4 metres. These estimates show that the steady state pumping volume is significantly greater than the mean net natural recharge volume of (85.5 GL/year) as it is augmented by irrigation return flow.

For a given set of initial surface water diversion and hydraulic head (groundwater stocks), the optimal pumping quota can be set at the level determined by the optimal pumping policy. If the total annual allocation exceeds the annual optimal extraction, the water board could cut back individual allocations so that total annual use will equal the optimal annual extraction with provision for trading groundwater allocations between users. Administration of optimal quotas has some difficulties, as the optimal quotas need to be calculated for each year as they vary over time and the aquifer hydraulic heads need to be continuously monitored.

Figure 5 Expected time paths of hydraulic head with different intial states

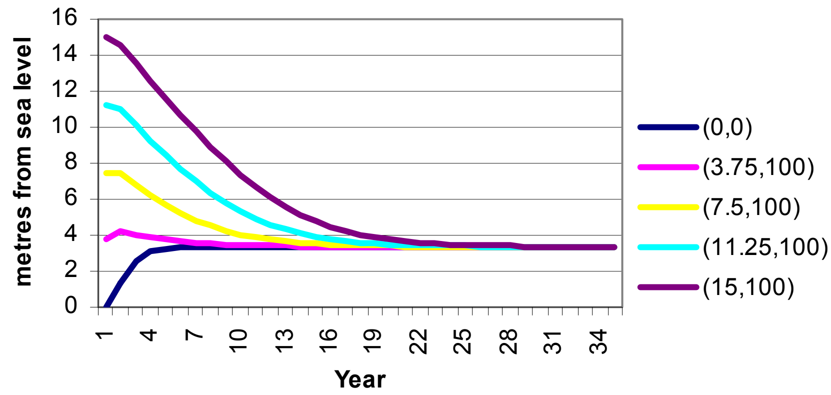


Figure 6 Expected time paths of pumping with different intial states

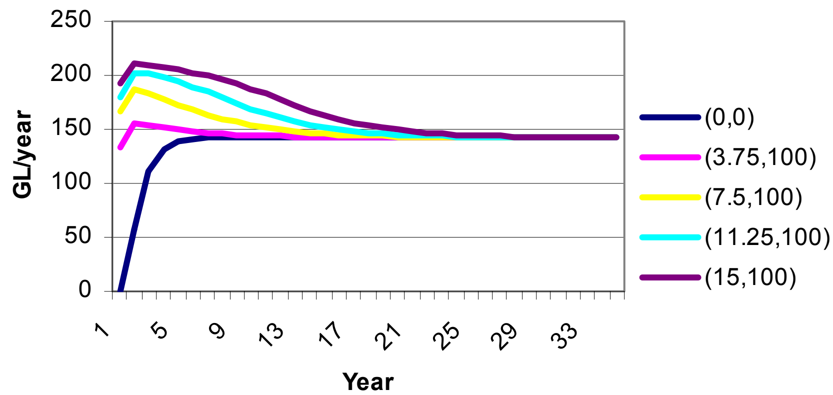


Figure 7 Expected time path of pumping with the intial state [0,100]

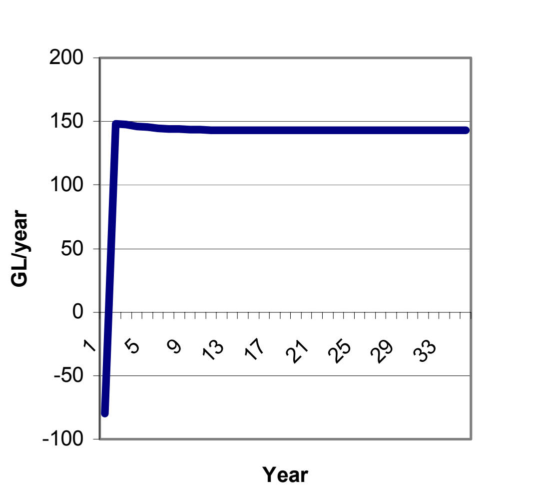
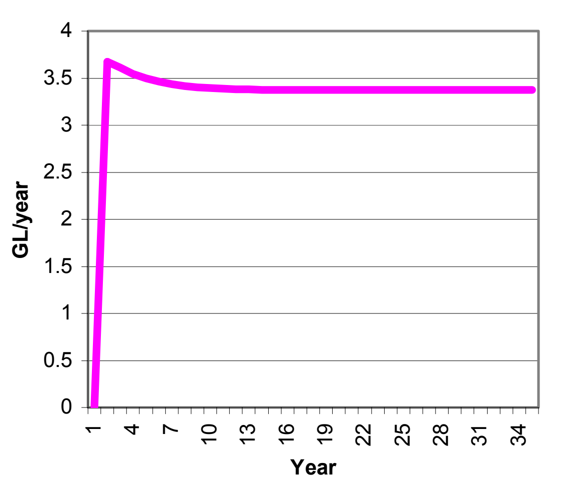


Figure 8 Expected time path of hydraulic head with the intial state [0,100]



Conclusions

In this paper we have demonstrated how a decision support tool can be used to provide water boards in the Burdekin River Delta with information on optimal pumping quotas and the allocation of surface water between immediate use on farm for irrigation and storage in the aquifer for future use that are consistent with a policy of achieving sustainable resource use. The optimal groundwater pumping quotas can be used to guide any review of existing allocations and the arrangements for their trade. The importance of taking a multiple water resource system perspective in addressing issues of conjunctive use of groundwater and surface water was highlighted. Because of the significant interaction between surface water and groundwater resources and the return flow externalities, the policies on surface water and groundwater need to be determined simultaneously. Contrary to the current SBWB policy, the optimal pumping/artificial recharge policy obtained in this study suggests that much greater emphasis should be placed on the distribution of the entire volume of surface water diverted to farm for irrigation and thereby allowing the aquifer to be recharged through return flows and the option of using surface water for artificial recharge may be deferred until when hydraulic heads fall to near mean sea level.

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Appendix A: The solution method

Socially optimal pumping/artificial recharge policy

The problem is formulated as a discrete time continuous state stochastic dynamic process on a two-dimensional state space. The value function defined on the entire two-dimensional space between the set of a given minimum and the set of given maximum levels of the state variables, $V(\cdot)$, is taken as unknown. The recursive nature in the problem defined in equation (7) is exploited in finding the optimal controls and Bellman's functional recursive equation, $V(\cdot)$, when applied to infinite time horizon is derived and used in the solution method (equation A1).

$$V(s,h)=\max_{\Omega_l \le x \le \Omega_u}\left\{\begin{array}{l}\{a-(b/2)[(1-\beta^c)s+x]\}[(1-\beta^c)s+x](1-\beta_a)\\ -\omega s - x[\gamma_0+\sigma(h^{MAX}-h)]\\ +\rho E_{\varepsilon_a,\varepsilon_b} V\left(\varepsilon^s, h+\kappa\left[\begin{array}{l}-x^g+\theta\beta^c s\\ +\theta\beta^a[(1-\beta^c)s+x]+\varepsilon^g\end{array}\right]\right)\end{array}\right\} \quad \text{(A1)}$$

Where, $V(\cdot)$ is the unknown value function in the Bellman's equation and E is the mathematical operator for expectation. It is assumed that the state space is bounded between a set of minimum, $(s^{\min},h^{\min})$ and a set of maximum, $(s^{\max},h^{\max})$, the control space is subject to bounds $\{\Omega_l \le x \le \Omega_u\}$, where $\Omega_l=-(1-\beta^c)s$ and $\Omega_u=h/k$ and the net benefit and the state transition equations (the first and the second two terms, respectively in the maximand of (A1)) are twice continuously differentiable functions.

An approximate solution to (A1) is computed using collocation method (Miranda and Feckler 2002). Assume that $\phi^s_{j_1}=[\phi^s_{1_s}(s),\phi^s_{2_s}(s),........,\phi^s_{n_s}(s)]$, and $\phi^h_{j_2}=[\phi^h_{1_h}(h),\phi^h_{2_h}(h),........,\phi^h_{n_h}(h)]$are the basis functions to approximate the univariate functions along the s domain $[s^{\min},s^{\max}]$ and the h domain $[h^{\min},h^{\max}]$, respectively with the order of n_s and n_h. For the two dimensional domain, i.e $[s,h]$ on $[s^{\min},s^{\max}]\times[h^{\min},h^{\max}]$, a set of basis functions may be constructed by taking the tensor product of the basis functions from 1 dimensional domain, that is,

$$\phi_{j_1 j_2}(s,h)=\phi^s_{j_1}(s)\otimes\phi^h_{j_2}(h) \quad \text{(A2)}$$

where, $j_1=1,2,....,n_s$ and $j_2=1,2,....,n_h$. There are a total $N=n_s n_h$ basis functions.

The collocation method consists of five steps.

First, express the function to be approximated, $V(\cdot)$, as a linear combination of basis functions $\phi_{j_1 j_2}(s,h)$, that is,

$$V(s,h) \approx \sum_{j_1=1}^{n_s} \sum_{j_2=1}^{n_h} c_{j_1 j_2} \phi_{j_1 j_2}(s,h) \qquad \text{(A3)}$$

where, $c_{j_1 j_2}$ are the $j_1 = 1,2,....,n_s, j_2 = 1,2,3,...,n_h$ coefficients which are to be estimated

Second, the function $V(\cdot)$ given in equation (A3) is approximated on a grid of collocation nodes within a given precision on tolerance[1]. The approximation is equivalent to solving the linear coefficients $c_{j_1 j_2}, j_1 = 1,2,....,n_s$ and $j_2 = 1,2,....,n_h$. There are a total of $N = n_s n_h$ coefficients and therefore, $N = n_s n_h$ nodes are required to solve these coefficients. The two dimensional collocation nodes are constructed by taking the Cartesian product of the one-dimensional nodes. Let $\lfloor s_1, s_2,........, s_{n_s} \rfloor$ and $\lfloor h_1, h_2,........, h_{n_s} \rfloor$ are the nodes for the one-dimensional state space, then $[(s,h), i = 1,2,...n_s, j = 1,2,....,n_h]$ are formed as the nodes of the 2 dimensional state space. Now for each (s,h) node chosen, the Bellman's equation given in (A1) is replaced with a system of N non-linear equations in N unknown basis functions. The stochastic expectation is approximated by a finite number of stochastic shocks.

[1] Initial values of the value function and water pumped from or recharged to the aquifer derived for all collocation nodes are used to narrow down the search range. These initial values are derived as follows. First, from the state transition equations (1)—(2) and the first order conditions given in equations (9)—(14), the steady state levels of hydraulic head, s^* and h^*, volumes of water pumped/recharged per year, x^* and the costate variables, λ_s^* and λ_h^* are analytically solved after dropping the time t subscript. Second the parameter values given in table 1 were then used to sequentially derive the certainty equivalent (CE) steady state values of control, costate and state variables by employing analytical expressions obtained in the first step. Third, on each collocation node, a linear approximant of the state transition equations (equations 1 and 2) and a quadratic approximant of the net benefit functions (equation 6) are evaluated. First and second order Taylor series expansions around the certainty equivalent steady state values are used in deriving these approximants. Finally the linear quadratic approximant values of the value function and water pumped from each aquifer derived for all the collocation nodes are used as initial values for solving the Bellman's equation using collocation.

The continuous random variables, ε^s and ε^h representing stochastic surface water inflows and groundwater recharge, respectively in the state transition equations are replaced with m_s and m_h discrete approximants, $\varepsilon_{s_1}, \varepsilon_{s_2}, \ldots\ldots, \varepsilon_{s_m}$ and $\varepsilon_{h_1}, \varepsilon_{h_2}, \ldots\ldots, \varepsilon_{h_m}$ with associated probabilities, $w_{s_1}, w_{s_2}, \ldots\ldots, w_{s_m}$ and $w_{h_1}, w_{h_2}, \ldots\ldots, w_{h_m}$ generated using the Gaussian quadrature scheme. Basis functions are defined as Chebychev polynomial functions.

$$\sum_{j_1=1}^{n_s}\sum_{j_2=1}^{n_h} c_{j_1 j_2}\phi_{j_1 j_2}(s,h) =$$

$$\max_{\Omega_l \le x \le \Omega_u}\left\{\begin{array}{l} \{a-(b/2)[(1-\beta^c)s+x](1-\beta_a)\}[(1-\beta^c)s+x](1-\beta_a) \\ -\omega s - x[\gamma_0 + \sigma(h^{MAX}-h)] \\ +\rho\sum_{k=1}^{m}\sum_{l=1}^{m} w_{s_k} w_{h_l} \sum_{j_1=1}^{n_s}\sum_{j_2=1}^{n_h} c_{j_1 j_2}\phi_{j_1 j_2}\left(\varepsilon^s, h+\kappa\begin{bmatrix} -x^g + \theta\beta^c s \\ +\theta\beta^a[(1-\beta^c)s+x]+\varepsilon^g \end{bmatrix}\right) \end{array}\right\}$$

, for $\forall\ i$ (A4)

Third, the values for $c_{j_1 j_2}$ for all j_1, j_2 are then found by requiring the approximant to satisfy the Bellman's equation at N collocation nodes.

Fourth, once the collocation equation has been solved, a diagnostic test is performed to ensure that the computed approximant solves the Bellman's equation at any arbitrary state over the entire two dimensional state space. To do this, a residual function is defined as follows and evaluated at i' (=500) equally spaced states over the state space.

$$R(s_i, h_i) =$$

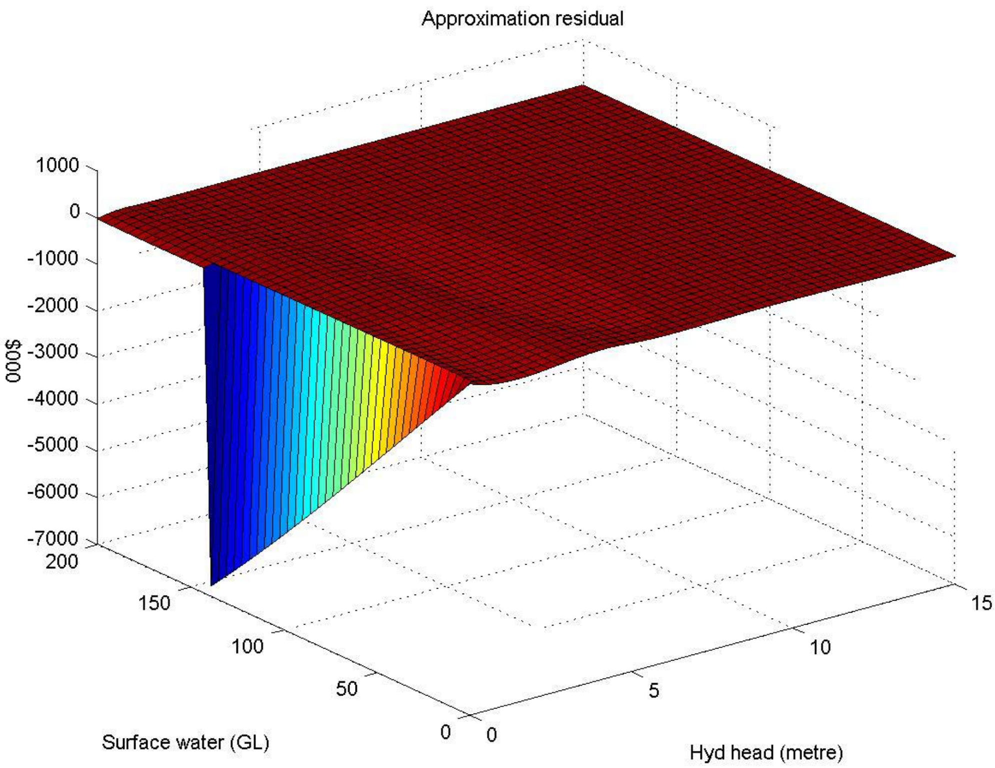
$$\max_{\Omega_l \le x \le \Omega_u}\left\{\begin{array}{l} \{a-(b/2)[(1-\beta^c)s+x](1-\beta_a)\}[(1-\beta^c)s+x](1-\beta_a) \\ -\omega s - x[\gamma_0 + \sigma(h^{MAX}-h)] \\ +\rho\sum_{k=1}^{m}\sum_{l=1}^{m} w_{s_k} w_{h_l} \sum_{j_1=1}^{n_s}\sum_{j_2=1}^{n_h} c_{j_1 j_2}\phi_{j_1 j_2}\left(\varepsilon^s, h+\kappa\begin{bmatrix} -x^g + \theta\beta^c s \\ +\theta\beta^a[(1-\beta^c)s+x]+\varepsilon^g \end{bmatrix}\right) \end{array}\right\}$$

$$-\sum_{j_1=1}^{n_s}\sum_{j_2=1}^{n_h} c_{j_1 j_2}\phi_{j_1 j_2}(s_i, h_i)$$

for $\forall\ i'$ (A5)

Fifth, for each two dimensional node, the optimal value and volume of groundwater pumped and the shadow price are computed using the approximant function.

Before discussing results it is important to make sure that the value function approximated using collocation solves the Bellman's equation at any arbitrary state chosen over the entire two-dimensional state space with a high degree of accuracy. Even though the estimated approximant function was based on only 25 nodes, it is capable of solving the value of the Bellman equation at all the 500 arbitrary points chosen between $\left[s^{\min}, h^{\min}\right]$ and $\left[s^{\max}, h^{\max}\right]$ with negligible deviation values (figure A1).



Expected time paths

For each solution, the expected paths for groundwater pumping/recharge volume and the hydraulic head are computed by performing Monte Carlo simulations with 2000 replications of a 50 year period each.