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Review Article

Conjugate Problems in Convective Heat Transfer: Review

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A review of conjugate convective heat transfer problems solved during the early and current time of development of this modern approach is presented. The discussion is based on analytical solutions of selected typical relatively simple conjugate problems including steady-state and transient processes, thermal material treatment, and heat and mass transfer in drying. This brief survey is accompanied by the list of almost two hundred publications considering application of different more and less complex analytical and numerical conjugate models for simulating technology processes and industrial devices from aerospace systems to food production. The references are combined in the groups of works studying similar problems so that each of the groups corresponds to one of selected analytical solutions considered in detail. Such structure of review gives the reader the understanding of early and current situation in conjugate convective heat transfer modeling and makes possible to use the information presented as an introduction to this area on the one hand, and to find more complicated publications of interest on the other hand.

1. Introduction

The heat transfer coefficient has been used in modeling convective heat transfer since the time of Newton. This coefficient is usually determined experimentally and no well-found theoretical approach was available until the last few decades. However, starting from the end of the 1960s, heat transfer problems began to be considered using a conjugate, coupled, or adjoint formulation. These three equivalent terms correspond to the problems containing two or more subdomains with phenomena describing by different differential equations. After solving the problem in each subdomain, these solutions should be conjugated. The same procedure is needed if the problem is governed by one differential equation but the subdomains consist of different substances.

For example, the heat transfer between a body and a fluid flowing past it is a conjugate problem, because the heat transfer inside the body is governed by the elliptical Laplace equation or by the parabolic differential equation, while the heat transfer inside the fluid is governed by the elliptical Navier-Stokes equation or by the parabolic boundary layer equation. The solution of such problem gives the temperature and heat flux distributions on the interface, and there is no need for a heat transfer coefficient (which can be calculated using these results). Another example of a conjugate problem is the transient heat transfer from a hot plate to a cooling, flowing past it fluid film. Although this problem is governed by one differential equation, the two parts of the plate need to be conjugated because the part covered by the film is cooler and the uncovered part is hot (see Example 4.3).

There are many conjugate problems in other fields of science. For instance, studying of subsonic-supersonic flows requires conjugation because the subsonic flow is governed by elliptic or parabolic differential equations, while the supersonic flow is governed by hyperbolic differential equation [1]. As two other examples, here are mentioned combustion processes and problems in biology. Every combustion process has two areas containing fresh and burnt gases with different properties [2]. In biology, diffusion processes usually proceed simultaneously in qualitatively different areas (e.g., in membranes) and, therefore, require a conjugation procedure [3].

Analogous mathematical problems with a similar formulation, usually called mixed problems, began to be considered much earlier than these physical systems. The most famous mixed parabolic/hyperbolic equation was investigated by Tricomi in 1923.

In this paper, a review of conjugate problems in convective heat transfer considered during the last 45–50 years is given. Starting with [4] in 1970, the author has studied systematically conjugate heat transfer over the last 40 years. Problems solved by the author, together with graduate student and colleges from the Ukrainian Academy of Science, and during his time as a Visiting Professor of University of Michigan (since 1996) have been published in many articles and in the book [5] that was the author’s doctoral thesis. Although many of these publications are originally in Russian, almost all of them have been published in English.

This survey is intended to give the reader an understanding of conjugate formulation of heat transfer problems, and to provide a guide helping to orientate in this modern approach of studying heat transfer in order to find publication of interest. In a view of these aims, the survey is organized as follows. (1) The almost two hundred publications presented here are combined into groups of works considering the same topic, for example, steady-state heat transfer, unsteady heat transfer, thermal treatment of materials, and so on. (2) An analytical solution of relatively simple conjugate heat transfer problem corresponding to each of groups and references forming this group are given in the text. Such structure of review makes possible to use the information presented as an introduction in the area of conjugate convective heat transfer on the one hand, and to find more complicated, than given examples, modern analytical, and/or numerical publications of interest on the other hand.

2. Early Works

Intense interest in studying conjugate problems in convective heat transfer arose at the end of the 1960s. During the next decade, many conjugate problems were considered. Most of the early works considered the heat transfer between a plate and a fluid using different assumptions for the velocity distribution in the thermal boundary layer.

In some articles [6–8] Lighthil’s method [9] was used, which is based on the linear velocity distribution in the thermal boundary layer. The validity of this assumption improves as the fluid Prandtl number increases. In the other limiting case of small Pr, the velocity across the thermal boundary layer can be considered to be equal to that in the external flow [10–12]. A group of early articles considered solutions of conjugate problems in the form of the power series [13–21]. In [22], a method for solving conjugate problems was developed that uses a series of negative powers of the longitudinal coordinate. Such an asymptotic series can be applied for large distances from the plate origin. In [23], this method was used to solve conjugate problems of steady-state and quasi-unsteady heat transfer from streamlined plates. Some conjugate problems were solved using well-known simple integral method [24, 25]. In the middle of 1970s, numerical solutions of conjugate heat transfer problem also were published [26, 27].

3. General Expression for the Heat Flux on A Nonisothermal Streamlined Body

In the conjugate problem, the temperature and heat flux distributions along the interface of a fluid and a body are basic unknowns. Therefore, the analytical solutions of conjugate problems rely on methods that determine the heat transfer from arbitrary nonisothermal surfaces. Solving the system of boundary layer equations for an arbitrary nonisothermal surface, one gets the equation $q_{w1} = f_1[T_w(x)]$. The other relation between the same unknowns, $q_{w2} = f_2[T_w(x)]$, can be obtained by solving conduction equation for a solid. Then, using conjugate condition $q_{w1} = q_{w2}$, the equation for the temperature field is obtained, $f_1[T_w(x)] = f_2[T_w(x)]$. Since the methods of solution of the conduction equation are well developed, the main difficulties are usually connected with solving the system of boundary layer equations. A review of such methods is given in [28].

Major part of practically important problems of flow and heat transfer for slightly viscose fluids such as water, air, or liquid metals is characterized by high Reynolds and Peclet numbers. In such a case, viscosity and conductivity are significant only in thin boundary layers and the system of Navier-Stokes and energy equations is simplified to the system of boundary layer equations. For laminar steady-state flow of an incompressible fluid with constant properties these equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - U\frac{dU}{dx} - \nu\frac{\partial^2 u}{\partial y^2} = 0, \quad (3.1)$$
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \alpha\frac{\partial^2 T}{\partial y^2} - \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 = 0.$$

The solutions of system (3.1) must satisfy the boundary conditions at the body surface and far away from the body in the external flow:

$$y = 0, \quad u = v = 0, \quad T = T_w(x), \quad y \longrightarrow \infty, \; u \longrightarrow U, \; T \longrightarrow T_\infty. \quad (3.2)$$

Because the energy equation is linear, the superposition method can be used to solve it. By using this method, the heat flux for gradientless flow on an arbitrary nonisothermal plate can be presented as an integral containing the first derivative of the plate temperature [29]:

$$q_w = h_* \left[\int_0^x f\left(\frac{\xi}{x}\right) \frac{dt_w}{d\xi} d\xi + t_w(0) \right], \quad f\left(\frac{\xi}{x}\right) = \left[1 - \left(\frac{\xi}{x}\right)^{3/4} \right]^{-1/3}, \tag{3.3}$$

where $f(\xi/x)$ is the influence function of the unheated zone of length ξ. This method was applied to solve several early considered conjugate problems [30–34].

At the same early time of using conjugate approach, it was shown by multiple integration by parts that integral (3.3) may be presented in the form of the series of successive temperature derivatives of a surface temperature head distribution [35]

$$q_w = h_* \left(t_w + g_1 x \frac{dt_w}{dx} + g_2 x^2 \frac{d^2 t_w}{dx^2} + g_3 x^3 \frac{d^3 t_w}{dx^3} = \cdots \right) = h_* \left(t_w + \sum_{n=1}^{\infty} g_n x^n \frac{d^n t_w}{dx^n} \right). \tag{3.4}$$

Relations (3.3) and (3.4) are applicable only to gradientless flow on arbitrary nonisothermal plats. Considering a general case in [35, 36], the thermal boundary layer equation (3.1) was transformed by applying the Prandl-Mises-Gortler independent variables to the form

$$2\Phi \frac{\partial t}{\partial \Phi} - \varphi \frac{\partial t}{\partial \varphi} - \frac{1}{\Pr} \frac{\partial}{\partial \varphi} \left(\frac{u}{U} \frac{\partial t}{\partial \varphi} \right) = 0, \quad \Phi = \frac{1}{\nu} \int_0^x U(\eta) d\eta, \; \varphi = \frac{\psi}{\nu \sqrt{2\Phi}}, \tag{3.5}$$

$$\varphi = 0, \quad t = t_w(\Phi), \quad \varphi \longrightarrow \infty, \; t = 0. \tag{3.6}$$

Using this equation, it was proved that relations (3.3) and (3.4) remain valid for gradient flows on an arbitrary nonisothermal surface if the coordinate x is replaced by Prandl-Mises-Gortler variable Φ. Then, (3.3) and (3.4) become

$$q_w = h_* \left[\int_0^\Phi f\left(\frac{\xi}{\Phi}\right) \frac{dt_w}{d\xi} d\xi + t_w(0) \right], \quad f\left(\frac{\xi}{\Phi}\right) = \left[1 - \left(\frac{\xi}{\Phi}\right)^{c_1} \right]^{-c_2}, \tag{3.7}$$

$$q_w = h_* \left(t_w + g_1 \Phi \frac{dt_w}{d\Phi} + g_2 \Phi^2 \frac{d^2 t_w}{d\Phi^2} + g_3 \Phi^3 \frac{d^3 t_w}{d\Phi^3} = \cdots \right) = h_* \left(t_w + \sum_{n=1}^{\infty} g_n \Phi^n \frac{d^n t_w}{d\Phi^n} \right). \tag{3.8}$$

However, (3.8) is an exact solution of thermal boundary layer equation (3.5) for an external flow velocity given by a power law expression $U = Cx^m$. The coefficients g_k and the corresponding exponents c_1 and c_2 depend on the flow regime, Pr and m or on the parameter $\beta = 2m/(m+1)$. They have been calculated for laminar [35, 36] and turbulent [37] flows. For arbitrary external flow, (3.7) and (3.8) give a high accurate approximate solutions. The local coefficients g_k and exponents c_1 and c_2 in this case can be estimated using relation $\beta = 2[1-(\Phi/\mathrm{Re}_x)]$ obtained from the equality of average values of given and power law velocities [5, 36]

In fact, the expression (3.8) is an improved boundary condition of the third kind for an arbitrary surface temperature distribution [36, 38]. The case when the all derivatives are equal to zero corresponds to the isothermal surface, and expression (3.8) becomes the boundary condition of the third kind. The series containing only the first derivative presents the linear boundary condition. The series with two derivatives describes the quadratic boundary condition and so on. In general case, the series consist of infinite number of derivatives and describes arbitrary boundary conditions. Thus, such series can be considered as a general boundary condition which describes different types of surface temperature distribution. Calculations show [36] that the coefficients g_k rapidly decrease with the number, so that two or three first terms often give desired accuracy.

If the heat conduction equation is solved using general boundary condition (3.8) with the first term only, an approximate solution of the conjugate problem as with the boundary condition of the third kind is obtained. By retaining the first two terms in (3.8) and solving the heat conduction equation, a more accurate solution of the conjugate problem is obtained. This process of refining can be continued by retaining a larger number of terms in general condition (3.8). However, this entails difficulties posed by the calculation of higher order derivatives, and, therefore, the integral form of general boundary condition (3.7) is used for further approximations.

In practical calculations, it is convenient to retain the first few terms of the series and to calculate the correction term from the results of previous approximation. When in this case, the first three terms of the series are retained, the conjugate problem is reduced to a heat conduction equation for solid with the following boundary condition [38]:

$$q_w = h_* \left[t_w + g_1 \Phi \frac{dt_w}{d\Phi} + g_2 \Phi^2 \frac{d^2 t_w}{d\Phi^2} + \varepsilon(\Phi) \right], \quad \varepsilon(\Phi) = \frac{1}{h_*} \left(q_w^{\text{int}} - q_w^{\text{diff}} \right), \tag{3.9}$$

$$\frac{dt_w}{d\Phi} = \frac{\nu}{U} \frac{dt_w}{dx}, \qquad \frac{d^2 t_w}{d\Phi^2} = \frac{\nu^2}{U^2} \frac{d^2 t_w}{dx^2} - \frac{\nu^2}{U^2} \frac{dU}{dx} \frac{dt_w}{dx}. \tag{3.10}$$

Quantities q_w^{int} and q_w^{diff} are defined by integral relation (3.7) and by differential relation (3.8) in the form (3.9), respectively. The first approximation is found by assuming that the correction term $\varepsilon(\Phi) = 0$. Using the results of the first approximation makes it possible to calculate the correction term and to introduce it into condition (3.9) in order to find the second approximation. By continuing this process, the solution with the desired accuracy can be obtained.

Retaining in the boundary condition (3.9) terms with derivatives not higher than second leads to differential equations of the second order in any approximation. As a result in this case, the conjugate problem is reduced to the ordinary differential equation in the case of thin body and to the Laplace or Poisson equation for the general case. The well-known effective analytical and numerical methods can be used to solve the differential equations of this types. The outlined method is applicable to any steady-state conjugate problem for a one- or two-dimensional body. Some examples among others are given in what follows. However, this method could not be used for nonlinear problems because in that case the method of superposition is not applicable.

Example 3.1. Heat transfer from liquid to liquid through a thin plate [4].

If the plate is thin ($\Delta/L \ll 1$) and its thermal resistance is comparable with that of liquids, the longitudinal conductivity of the plate is negligible. In this case, the temperature distribution across the plate thickness can be considered as linear, and, hence, the heat fluxes on both sides of the plate are taken to be equal:

$$-q_{w1} = q_{w2} = \frac{\lambda_w}{\Delta}(T_{w1} - T_{w2}). \tag{3.11}$$

Substituting q_{w1} and q_{w2} defined by (3.8) into (3.11) yields two equations:

$$-h_{*1}\left[t_{w1} + \sum_{k=1}^{\infty} g_{1k}\frac{d^k t_{w1}}{dx^k}\right] = h_{*2}\left[t_{w2} + \sum_{k=1}^{\infty} g_{k2}x^k\frac{d^k t_{w2}}{dx^k}\right], \tag{3.12}$$

$$T_{w2} - T_{w1} = \frac{h_{*1}\Delta}{\lambda_w}\left[t_{w1} + \sum_{k=1}^{\infty} g_{k1}x^k\frac{d^k t_{w1}}{dx^k}\right]. \tag{3.13}$$

As the distance from the origin of the plate grows, the boundary layer thickness increases, and the heat flux decreases so that at $x \to \infty$ the heat flux approaches zero. In this case, the plate temperature tends to a limiting value $T_{w\infty}$, which is determined from (3.12) if all derivatives of temperature are taken to be zero. This temperature and Biot number are used to form the dimensionless variables:

$$\mathrm{Bi}_* = \frac{h_*\Delta}{\lambda_w}, \qquad \theta = \frac{T_w - T_\infty}{T_{w\infty} - T_\infty}, \quad T_{w\infty} = \frac{\mathrm{Bi}_{*1}T_{\infty 1} + \mathrm{Bi}_{*2}T_{\infty 2}}{\mathrm{Bi}_{*1} + \mathrm{Bi}_{*2}}. \tag{3.14}$$

Then, (3.12), (3.13), and (3.11) defining temperature heads and heat flux become

$$\theta_1 + \sum_{k=1}^{\infty}\overline{g}_{k1}\mathrm{Bi}_*^k\frac{d^k\theta_1}{d\mathrm{Bi}_*^k} = \theta_2 + \sum_{k=1}^{\infty}\overline{g}_{k2}\mathrm{Bi}_*^k\frac{d^k\theta_2}{d\mathrm{Bi}_*^k}, \tag{3.15}$$

$$\mathrm{Bi}_{*2}(1-\theta_1) + \mathrm{Bi}_{*1}(1-\theta_2) = \mathrm{Bi}_{*1}\mathrm{Bi}_{*2}\left(\theta_1 + \sum_{k=1}^{\infty}\overline{g}_{k1}\mathrm{Bi}_*^k\frac{d^k\theta_1}{d\mathrm{Bi}_*^k}\right),$$
$$\mathrm{Bi}_K = \frac{\mathrm{Bi}_{*1}(1-\theta_1) + \mathrm{Bi}_{*2}(1-\theta_2)}{\mathrm{Bi}_{*1} + \mathrm{Bi}_{*2}}, \tag{3.16}$$

where Bi_K is the Biot number determining the overall heat transfer coefficient and coefficients $\overline{g}_k$ depend on coefficients g_k in (3.8). The boundary conditions follow: one from the fact that at the origin the temperature of each side of the plate and of the corresponding fluid should be equal, and the second from the asymptotic behavior of temperature at $x \to \infty$:

$$\mathrm{Bi}_* = \infty, \quad \theta_1 = \theta_2 = 0, \qquad \mathrm{Bi}_* \longrightarrow 0 \quad \theta_1 = \theta_2 = 1, \quad \theta_1' = \theta_2' = \theta_1'' = \theta_2'' = \cdots \longrightarrow 0. \tag{3.17}$$

Calculation in [4, 5] shows that the coefficients $\overline{g}_k$ are approximately equal for both sides of (3.15). In this case, the both dimensionless temperatures are equal, and (3.15) and (3.16) for temperature heads and heat flux can be presented in the form using another form of overall Biot number Bi_Σ:

$$\theta(1+\mathrm{Bi}_\Sigma)+\overline{g}_1\mathrm{Bi}_\Sigma^2\frac{d\theta}{d\mathrm{Bi}_\Sigma}+\overline{g}_2\mathrm{Bi}_\Sigma^3\frac{d^2\theta}{d\mathrm{Bi}_\Sigma^2}+\cdots=1,\quad \mathrm{Bi}_K=1-\theta,\ \mathrm{Bi}_\Sigma=\frac{1}{1/\mathrm{Bi}_{*1}+1/\mathrm{Bi}_{*2}}. \tag{3.18}$$

The solution of this equation with two first terms is obtained in the form

$$\theta=\frac{(-1/\overline{g}_1\mathrm{Bi}_\Sigma)\exp(-1/\overline{g}_1\mathrm{Bi}_\Sigma)}{2-(1/\overline{g}_1)}F\left[1-\left(\frac{1}{\overline{g}_1}\right),2-\left(\frac{1}{\overline{g}_1}\right),\left(-\frac{1}{\overline{g}_1\mathrm{Bi}_\Sigma}\right)\right], \tag{3.19}$$

where $F(a,b,c)$ is confluent hypergeometric function. The solution of (3.19) with three first terms which was found numerically gives practically the same data.

The results show that the following hold.

(1) The heat flux and the heat transfer coefficient depend on the temperature distributions over the plate, which depend on Prandtl number and ratio of thermal resistances of both fluids and a plate estimated by Biot number Bi_Σ.

(2) The usual method (when the heat transfer coefficient for isothermal wall and boundary condition of the third kind are used) gives satisfactory results only when Bi_Σ is close to zero (e.g., metal plate/nonmetals fluids).

(3) When the resistances are comparable (metal plate/liquid metal), the error reaches 15–20%. For the case of turbulent flow, this conjugate problem was solved in [39] where the calculation gave the maximum error of about 7%.

(4) In this case, the temperature head (temperature difference $|T_w-T_\infty|$) increases in the flow direction. Because of this, the errors are small compared to the case when the temperature head decreases in the flow direction. In the latter case, the heat flux becomes zero if the surface is enough long and then changes the direction (see Examples 3.3 and 5.2).

The same problem was solved in [31] and a similar problem for countercurrent laminar flows separated by a thin wall was considered in [40].

Example 3.2. Coupling natural heat transfer between two fluids separated by a thin vertical wall [41].

In [42] an approximation method of calculating natural heat transfer is given. For the case of a vertical plate in laminar flow the heat flux is determined as

$$q_w=C(\mathrm{Pr})\lambda\left(\frac{g\beta\mathrm{Pr}}{\nu}\right)^{1/4}\frac{[T_w(x)-T_\infty]^{5/3}}{\left\{\int_0^x[|T_w(s)-T_\infty|]^{5/3}\right\}^{1/4}}. \tag{3.20}$$

For thin plate under negligible longitudinal conduction, the heat balance equation is

$$-\lambda_{w1}\left.\frac{\partial T_1}{\partial y_1}\right|_{y_1=0}=\left(\frac{\lambda_w}{\Delta}\right)(T_{w2}-T_{w1})=\lambda_{w2}\left.\frac{\partial T_2}{\partial y_2}\right|_{y_2-0}. \tag{3.21}$$

Using the dimensionless variables, one obtains

$$q_w^* = \frac{q\Delta}{\lambda_w(T_{1\infty} - T_{2\infty})} = \left\{\left[\left(\frac{\lambda_1}{\lambda_w}\right)\left(\frac{\overline{\Delta}}{\xi_1}\right)\mathrm{Nu}_{x1}\right]^{-1} + 1 + \left[\left(\frac{\lambda_2}{\lambda_w}\right)\left(\frac{\overline{\Delta}}{\xi_2}\right)\mathrm{Nu}_{x2}\right]^{-1}\right\}^{-1},$$

$$\theta = \frac{(T_w - T_\infty)}{(T_{1\infty} - T_{2\infty})}, \tag{3.22}$$

$$\frac{\mathrm{Nu}_x}{\mathrm{Ra}^{1/4}} = \frac{(q^* \xi^{1/4})}{(\lambda/\lambda_w\ )(\Delta/L)(\mathrm{Ra}\theta^5)^{1/4}}.$$

Here q_w^* is the overall heat transfer coefficient, $\overline{\Delta} = \Delta/L$, and $\xi = x/L$. The temperature distribution depends on two parameters:

$$P_1 = \left(\frac{\lambda_1}{\lambda_w}\right)\left(\frac{\Delta}{L}\right)\mathrm{Ra}_1^{1/4}, \qquad P_2 = \left(\frac{\lambda_1}{\lambda_2}\right)\left(\frac{\mathrm{Ra}_1}{\mathrm{Ra}_2}\right)^{1/4}. \tag{3.23}$$

The conclusions are given as follows.

(1) The agreement between the calculated and experimental data obtained by authors is reasonable. The largest difference (20%) is for brass plate.

(2) The natural convection heat transfer coefficients on the two sides of the wall are interdependent. However, agreement for the local heat transfer along the wall can be obtained by using average values of temperature and usual procedure.

(3) For ordinary fluids, except liquid metals, the thermal interaction is moderate. Additional analysis is needed for higher thermal conductivity fluids and/or for systems where two-dimensional heat conduction effects are pronounced.

Later, similar coupling problems were considered [43–46] and numerical solution was obtained also [47].

Example 3.3. Heat transfer between a flowing agent and a thin plate heated on one end and insulated on the other [5, 38].

Using the balance equation for thin plate and (3.3) for the heat flax yields the equation for the thin plate temperature:

$$\frac{d^2T_w}{dx^2} - \frac{2q_w}{\lambda_w\Delta} = 0,$$

$$\frac{d^2T_w}{dx^2} - \frac{2h_*}{\lambda_w\Delta}\left(T_w + g_1 x\frac{dT_w}{dx} + g_2 x^2 + \frac{d^2T_w}{dx^2}\cdots\right) = 0. \tag{3.24}$$

Equation (3.24) was solved for laminar and turbulent flows using the first three terms of the series (3.3). The results were refined by (3.9) written for $\Phi = Ux/\nu$ with correction $\varepsilon(x)$. In this case, one correction gives satisfy accuracy.

This example clearly demonstrated the role of temperature head gradient. Note that the results strongly depend on the flow direction. When the flow arrives at the heated end, the temperature head decreases, however, in opposite case, when the flow arrives at the insulated end, the temperature head increases. In the case of increasing temperature head, the heat transfer coefficients are higher, and in the other case they are lower than on an isothermal surface [36, 38]. As it follows from series (3.3) or (3.8), this is true in general because the first coefficient g_1 in these series is positive, and, hence, positive derivatives of the temperature head lead to increasing and negative ones results in decreasing heat fluxes. This conclusion is confirmed by many calculation results.

In the case in question, the local heat fluxes decrease quickly and become nearly zero at the isolated end in the first case, when the temperature head decreases in flow direction, while in the second case with increasing temperature head the heat flux first decreases and then increases. The total heat release is larger in the first case because there are large temperature heads in the starting length of the plate with high heat transfer coefficients, while in the other case the temperature heads at the beginning are small when the heat transfer coefficients are high and vise verse at the final part of plate. The quantitative results depend on relationship between flow and plate thermal resistances determining as always in conjugate problems by Biot or as in this case by modified Biot number $\mathrm{Bi} = h_{L*}L^2/\lambda_w\Delta$. For laminar flow and $\mathrm{Bi} = 4$ the heat fluxes in both cases differ by 40%, while for the turbulent flow, this difference is only 18% [38]. Note that using the conventional method based on the boundary condition of the third kind and heat transfer coefficient for isothermal wall leads to results independent of the flow direction.

Example 3.4. Heat transfer from an elliptical cylinder with uniformly distributed heat sources of strength q_v [48].

This problem is considered as an example of using the method of reducing a conjugate problem to heat conduction equation for the body in the case of gradient flow. The heat conduction equation and the symmetry and surface boundary conditions are

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{q_v}{\lambda_w} = 0, \qquad -\lambda_w \frac{\partial T}{dn}\bigg|_w = q_w, \qquad \frac{\partial T}{dy}\bigg|_{y=0} = 0. \tag{3.25}$$

This problem is best to solve in elliptical coordinates (u, v) which are related to the Cartesian ones (x, y) by the following expressions:

$$x = c\,chu \cos v, \quad y = c\,shu \sin v \quad c = \sqrt{a^2 - b^2}, \tag{3.26}$$

where a and b are major and minor semiaxis and *chu* and *shu* are hyperbolic function. In elliptical coordinates a half ellipse is mapped into a rectangle, one side of which corresponds to the surface of the semiellipce and other three to the axes of symmetry. Then, relations (3.25) become

$$\begin{gathered}\frac{\partial^2 \theta}{\partial u^2} + \frac{\partial^2 \theta}{\partial v^2} = Q_v\left(sn^2 u + \sin^2 v\right), \qquad \frac{\partial \theta}{\partial v}\bigg|_{u=l} = -\sqrt{1 - \frac{c^2}{a^2}\cos^2 v}\,\frac{q_w a}{\lambda_w T_\infty}, \\ \frac{\partial \theta}{\partial u}\bigg|_{u=0} = \frac{\partial \theta}{\partial v}\bigg|_{v=0} = \frac{\partial \theta}{\partial v}\bigg|_{v=\pi} = 0.\end{gathered} \tag{3.27}$$

Here $\theta = (T - T_\infty)/T_\infty$, $Q_v = -q_v c^2/\lambda_w T_\infty$, $l = (1/2)\ln[(a+b)/(a-b)]$ are dimensionless temperature difference, power of the internal sources, and the value of the coordinate u corresponding to the surface of the ellipse.

The solution of (3.27) with the three last boundary conditions has the form

$$\theta_w = \frac{Q_v}{8}(ch2u + \cos 2v) + N_0 + \sum_{k=1}^{\infty} N_k chku \cos kv. \tag{3.28}$$

The constants N_k should be determined from the first boundary condition (3.27) which contains the heat flux q_w given by integral relation (3.7). To calculate the Gortler variable required for estimating this integral, the differential of the arc and the potential velocity distribution for ellipse are used

$$\begin{gathered} x = \left(a^2 + b^2\right)^{1/2} chu \cos v, \\ y = \left(a^2 + b^2\right)^{1/2} shu \sin v, \\ dS = \sqrt{a^2 \sin^2 v + b^2 \cos^2 v}, \\ U = \frac{U_\infty (a+b) \sin v}{\sqrt{a^2 \sin^2 v + b^2 \cos^2 v}}, \\ \Phi = \left[1 + \left(\frac{a}{b}\right)\right](1 - \cos v). \end{gathered} \tag{3.29}$$

Then, the expression for heat flux takes the form

$$\begin{gathered} q_w = \frac{\lambda T_\infty}{a}\left[\frac{Q_v}{4}\left(ch^2 l - J_2\right) + N_0 \sum_{k=1}^{\infty} N_k chkl (1 - kJ_k)\right], \\ J_k = \int_0^v \left[1 - \left(\frac{1 - \cos\varepsilon}{1 - \cos v}\right)^{c_1}\right]^{-c_2} \sin k\varepsilon \, d\varepsilon. \end{gathered} \tag{3.30}$$

Here ε is a new variable for integral (3.7) which is applied instead of ξ. Using (3.30) for calculating the first boundary condition (3.27) for $k+1$ points gives the system of algebraic equations determining the coefficients N_k of the temperature head series (3.28).

The basic conclusions for this example are as follows.

(1) The heat flux distribution along ellipse surface is determined mainly by the relation between flow and body resistances, $\mathrm{Bi} = \lambda_w/\lambda \mathrm{Re}^{1/2}$.

(2) The maximum ratio of temperatures obtained with and without conjugation effects is for $\mathrm{Bi} = 0.1$ which reaches an amount of 1.5.

(3) The effect of conjugation is small for $\mathrm{Bi} \geq 10$.The smaller Bi, the bigger the effect of the conjugation.

Numerical solution of this problem is considered in [49]. Several examples of other steady-state conjugate studies of heat transfer from bodies may be found in [50–60]. In [50] and [51] are considered bodies of arbitrary shape, in [52] is analyzed heat transfer from disk and in [53] from sphere. The more complicated conjugate problems of heat transfer from thick plats are investigated in [54] and [55]. Numerical solutions are presented in [56–59] and an asymptotic analysis of conjugate heat transfer from flush-mounted source is given in article [60].

Example 3.5. Flat-finned surface in a transverse flow [61].

An incompressible fluid flows along a finned surface transversely to the fins. Because the flow is normal to the fins, the eddy forms between fins in each interfin space. Assuming that the conditions in all cells are identical, the model of the problem of heat transfer in such fins is formulated using Batchelor's presentations of the boundary layer inside a cavern.

The model is considered as bilateral flow over the body representing the surfaces of the fin and of two adjacent cells. In such a model, upper and lower surfaces of the model represent both fin surfaces with increasing and decreasing temperature gradients, while the ends of the model body correspond to the ends of two adjacent cells. Thus, the model in question represents the case of countercurrent flows with complicated velocity and temperature distributions.

For thin fin, the governing equation and boundary conditions are

$$\lambda_w \Delta \frac{d^2T}{dx^2} = q_1(x) + q_2(x), \quad \left(\frac{dT}{dx}\right)_{x=0} = 0, \quad T|_{x=H} = T_0, \tag{3.31}$$

where $q_1(x)$ and $q_2(x)$ are heat fluxes from surfaces of fin, H is the height of the fin, and T_0 is the fin base temperature. Using integral formula (3.3) leads to integro-differencial equation for the fin temperature:

$$\begin{aligned}\frac{d^2\theta}{d\eta^2} = N^2\Bigg\{&\varphi(\eta)\left[\theta_H + \int_0^{\eta} f(\xi,\eta)\frac{d\theta}{d\xi}d\xi\right]\\ &+\varphi(\eta_0-\eta)\left[\theta_H + \int_0^{1} f(\xi,\eta_0-\eta)\frac{d\theta}{d\xi}d\xi - \int_0^{1} f(\eta_0-\xi,\eta_0-\eta)\frac{d\theta}{d\xi}d\xi\right]\Bigg\}\end{aligned} \tag{3.32}$$

$$\theta = \frac{T-T_b}{T_0-T_b}, \quad \theta_H = \frac{T_H-T_b}{T_0-T_b}, \quad \eta = \frac{x}{H}, \quad \eta_0 = 2+\frac{s}{H}, \quad N^2 = \frac{h_{*av}H^2}{\lambda_w\Delta}.$$

Here T_H and T_b are temperatures of the end of the fin and inside the eddy flow, which plays the role of the temperature of the external flow for the boundary layer on the fin, h_{*av} is the average heat transfer coefficient of an isothermal fin and s is the distance between fins.

Equation (3.32) was solved using the method of reduction the conjugate problem to conduction problem according to which in the first approximation, the integro-differential equation (3.32) is reduced to second order ordinary differential equation (3.9) with $\varepsilon(x) = 0$, and the next refinements are obtained by calculating the correction term $\varepsilon(x)$ applying results of previous approximation. The results show that the following hold.

(1) For $N^2 < 2$, usual and conjugate methods are in agreement. With increasing N^2, the error in the fin efficiency grows and reaches 60–70%.

(2) For $N^2 > 2$, the local characteristics obtained using the conjugate model differ significantly from those computed by simplified method. The greatest differences are observed on the front part near the base of the fin.

(3) On the back side of the fin, for $N^2 \geq 1.9$, the heat flux inversion is observed when the heat flux becomes negative and is directed toward the fin despite the temperature head remains still positive. The heat transfer coefficients in this region become negative. This effect is explained by inertia of fluid when the fluid temperature near the wall exceeds the wall temperature and the heat flux becomes directed toward the fin [29]. Because the inversion effect cannot be obtained with the use of the simplified method, neglecting the conjugation of the problem in this case yields not only quantitative errors but also leads to the qualitative incorrect results. The reason for this is that on the back side of the fin, the temperature head decreases in the flow direction.

A review of conjugate heat transfer in fins has been presented in [62].

4. Unsteady Heat Transfer Conjugate Problem

Example 4.1. Thermally thin plate with time dependent heat sources q_v and heat flaxes q_e and q_f at the end faces is streamlined by the two flows with time dependent temperatures $T_{\infty 1}$ and $T_{\infty 2}$.

In the case of quasi-steady problem formulation, (3.3) and (3.4) or (3.7) and (3.8) obtained for steady heat transfer can be used for heat transfer calculation in the flows. The reason of this is that qualitative [21] and more accurate quantitative [62] analysis shows that in the case when quasi-steady conditions are satisfied, the unsteady effects in the flow compared to that in the solid can be neglected. Such situation takes place, for example, for the pair metal plate and nonmetal fluid.

The unsteady conduction equation averaged across the thickness of a thermally thin plate has the form

$$\frac{1}{\alpha_w}\frac{\partial T_{vv}}{\partial t} - \frac{\partial^2 T_{\text{av}}}{\partial x^2} + \frac{q_{w1}+q_{w2}}{\lambda_w \Delta} - \frac{(q_v)_{\text{av}}}{\lambda_w} = 0. \tag{4.1}$$

Using the integral relation (3.3) to determine the heat fluxes results in the following equation ($\zeta = x/L$):

$$\frac{\partial\theta}{\partial \text{Fo}} - \frac{\partial^2\theta}{\partial\zeta^2} + (\text{Bi}_1+\text{Bi}_2)\zeta^{-r/s}\left[\int_0^{\zeta} f\left(\frac{\xi}{\zeta}\right)d\theta + \theta(0,\text{Fo})\right] - \frac{\text{Bi}}{\text{Bi}_1+\text{Bi}_2}\theta_\infty(\text{Fo}) + \frac{dT_{\infty 1}}{d\text{Fo}} - \overline{q}_{\text{av}} = 0,$$

$$\theta = \frac{T_w - T_{\infty 1}(t)}{T_{\infty 2}(0) - T_{\infty 1}(0)}, \quad \theta_\infty = \frac{T_{\infty 2}(t) - T_{\infty 1}(t)}{T_{\infty 2}(0) - T_{\infty 1}(0)}, \quad \text{Bi} = \frac{h_{*L}L^2}{\lambda_w \Delta}, \quad \overline{q}_{\text{av}} = \frac{q_v L^2}{T_{\infty 2}(0) - T_{\infty 1}(0)}, \tag{4.2}$$

where $T_\infty(0)$ denotes the initial temperature, and *r/s* is an exponent in the relation

$$\text{Nu}_* = C\text{Re}_x^{r/s} \tag{4.3}$$

for an isothermal surface. This exponent places an important role because as it was shown in the early work by Perelman et al. [21] that the plate temperature distribution near the origin is not analytical function of the variable x and is described by a power series in the variable $x^{1/2}$. Later [63], it was shown that in general case the plate temperature distribution at $x = 0$ is an analytical function in variable $x^{1/s}$ where s is the denominator in (4.3) for an isothermal surface. Thus, for laminar or turbulent flows, this variable is $x^{1/2}$ or $x^{1/5}$, for laminar flow with power velocity distribution with $m = 1$ or $m = 1/3$, one gets variables x or $x^{1/3}$. For a non-Newtonian fluid with the rheology exponent n_1/n_2, $s = n_1 + n_2$. For different fluids or different flow regimes on the two sides of the plate, this variable would be $x^{1/s_{11}s_2}$, so, for instance, for laminar/turbulent flow on both sides, the temperature distribution near $x = 0$ is described by series in variable $x^{1/10}$.

Equation (4.2) was solved for laminar ($r/s = 1/2$) and turbulent ($r/s = 1/5$) flows using a series of eigenfunctions [5]. Due to singularity at $\zeta = 0$, the series $\theta = \sum_{i=0}^{\infty} c_i \zeta^{i/s}$ was used. As an example, a plate symmetrically streamlined by laminar flow with isolated ends was considered. In this case, the temperature head and heat flux in the conjugate problem are less in the initial plate part and are larger in the final part of about 20–25% than these obtained as usual without conjugation effects.

Example 4.2. Thermal entrance region of a parallel duct under a sudden change of ambient temperature [64].

For the case of a fully developed velocity profile and neglected axial conduction in the wall and fluid, the energy equation and initial and boundary conditions are

$$\frac{\partial\theta}{\partial t} + u_m\left(\frac{y}{R} + \frac{y^2}{2R^2}\right)\frac{\partial\theta}{\partial x} = \alpha\frac{\partial^2\theta}{\partial y^2}, \quad x = 0,\ t > 0,\ 0 \le y \le R,\ \theta = \vartheta_0 \sin\omega t,$$
$$y = 0,\ t > 0,\ x > 0,\ \rho_w c_{pw}\Delta\frac{\partial\theta}{\partial t} - \lambda\frac{\partial\theta}{\partial y} = 0, \qquad y = R,\ t > 0,\ x > 0, \quad \frac{\partial\theta}{\partial y} = 0. \tag{4.4}$$

Here θ, ϑ_0, R, and u_m are a temperature excess, amplitude, a half height of a duct, and a mass average velocity. The analytical solution obtained using Laplace transform agrees with more accurate numerical results until $\mathrm{Fo} < 0.35$. The quasi-steady solution leads to considerable errors, especially when $\mathrm{Fo} < 0.5$. At very low values of conjugation parameter the quasi-steady result may be acceptable.

The basic result of this study is that improved quasi-state approach based on the actual velocity profile and corresponding heat flux values can be used with acceptable accuracy within and beyond the thermal entrance region of a duct. At the same time, the improved quasi-state approach based on the linear velocity profile is restricted to the entrance region only, while the standard quasi-state approach is acceptable accurate basically in a thermally developed region. The last result is expected because from general consideration, it follows that the quasi-state approximation usually fails at the beginning of the process.

Many other results were obtained for steady and unsteady conjugate heat transfer in parallel plats channels [65–73] and in circular pipes [74–89]. More complicated problems were solved in [90] for bundle of rods, in [91] for annual channel, in [92] for curved piping system, in [93] for twin-screw extruder, in [94] for radiating fluid in a rectangular channel, in [95] for film pool boiling on a horizontal tube, and in [96] for double pipe exchangers.

Example 4.3. Semi-infinite hot plate cooled by flowing liquid film [97].

The plate has two different parts that should be conjugated: the cooler part covered by film and the uncovered hotter part. The governing equations and boundary conditions are

$$\mathrm{Ls}\frac{\partial^2\Theta}{\partial\eta^2} + z\eta\frac{\partial\Theta}{\partial\eta} - z^2\frac{\partial\Theta}{\partial z} - z^2\Theta = 0 \qquad \frac{\partial^2\theta}{\partial\xi^2} - \frac{\partial\theta}{\partial z} = 0 \qquad q^+(t) = q^-(t) + q_w(t), \tag{4.5}$$

$$\begin{gathered} \eta, z = 0, \quad \Theta = 1, \qquad \eta = 0, \quad z > 0, \quad \Theta_w = (z), \\ \eta = -1, \quad z > 0, \quad \left(\frac{\partial\Theta}{\partial z}\right) = 0, \quad \xi, z = 0, \quad \theta = 0, \\ \xi, z = 0, \quad \theta = 0, \qquad \xi = 0, \quad z > 0, \quad \theta = \theta_w(z) \qquad \xi \longrightarrow \infty, \qquad z > 0, \quad \theta = 0, \\ \eta = 0, \quad q^+(t) = q^-(t) + q_w(t), \end{gathered} \tag{4.6}$$

$$\eta = \frac{x}{Ut}, \quad \xi = \left(\frac{xh}{\lambda\Delta}\right)^{1/2}, \quad z = \frac{ht}{c\rho\Delta}, \quad \Theta = \frac{T - T_f}{T_i - T_f}, \quad \theta = \frac{T - T_i}{T_f - T_i}, \quad \mathrm{Ls} = \frac{\mathrm{Bi}}{\mathrm{Pe}^2} = \frac{\lambda h}{(\rho c U)^2\Delta}. \tag{4.7}$$

Here the first and second equations (4.5) and the corresponding first and second boundary condition (4.6) are governing systems for the covered and uncovered parts of the plate, respectively. The energy balance (the third equation (4.5)) states that the heat $q^+(t)$ conducted from the dry hot region of the plate is slightly absorbed by evaporation and sputtering $q_w(t)$, at the film front, while the majority of the heat, $q^-(t)$, is transferred to the wet cooler region.

Applying the superposition series in the form (3.8) or (3.4) reduces the system of equations (4.5) with unknown plate temperatures in the boundary conditions (4.6) to two infinite systems of equations and with known constant boundary conditions [97]. After solving the first two of these equations and substituting the results in balance energy equation (4.5), the ordinary differential equation for surface temperature at the moving front is obtained

$$\begin{gathered} \left[z + (g_1 - g_0)(z\pi\mathrm{Ls})^{1/2}\right]\left(\frac{d\Theta_w}{dz}\right) + \Theta_w\left[1 + g_0\left(\frac{\pi\mathrm{Ls}}{z}\right)^{1/2}\left(\frac{h_w}{h}\right)(z\pi\mathrm{Bi})^{1/2}\right] \\ - \left[1 + \left(\frac{h_w}{h}\right)(z\pi\mathrm{Bi})^{1/2}\Theta_{\mathrm{wet}}\right] = 0, \quad z = 0, \Theta_w = 1, \end{gathered} \tag{4.8}$$

where Θ_{wet} is dimesionless wetting temperature. Two approximation are calculated: the first one containing only coefficient g_0 and the second taking into account g_0 and g_1. The following results are derived.

(1) In the case of negligible heat of evaporation and sputtering at the moving front, the basic characteristics are determined only by Leidenfrost number, $\mathrm{Ls} = \mathrm{Bi}/\mathrm{Pe}^2$, the larger this number, the faster the plate cools, the lower the smallest plate temperature at the moving front, and the shorter the time required to reach the minimal temperature.

(2) In the case of significant heat of evaporation and sputtering, the plate temperature at the moving front depends on Leidenfrost number and in addition on two parameters $(h_w/h)\mathrm{Bi}^{1/2}$ and Θ_{wet}.

(3) The evaporation and sputtering affect the form of the cooling curve, $\Theta_w(z)$, but the onset time and other parameters depend only on Leidenfrost number.

Other conjugate transient heat transfer problems considering the flows past cooling bodies were studied in [98–106].

5. Thermal Treatment of Materials

Example 5.1. The infinite plate (tape) of temperature T_0 is drawing out from a slot and is pulled at velocity U_w through an agent with temperature T_∞ [107, 108]. Although the boundary layer in this case is similar to that on stationary or moving finite plate, they differ from each other. On the moving infinite plate, the boundary layer grows in the direction of the motion, as opposed to flow over the moving finite plate, on which it grows in the opposite direction of moving. It can be shown that in coordinate system attached to the moving surface, the boundary layer equations differ from the equations for the usual case of flow over a plate, but the boundary conditions are identical. These equations of a moving surface in the moving frame are unsteady, but if the coordinate system is fixed and attached to the slot, the problem becomes steady, and both boundary layer equations coincide; however, the boundary conditions differ because the flow velocity on the moving surface is not zero.

The governing equation and boundary conditions for the temperature on infinite moving plate are

$$U_w \frac{\partial T}{\partial x} = \alpha_w \frac{\partial^2 T}{\partial y^2}, \quad y = 0,\, T = T_0, \quad y \longrightarrow \infty,\, T \longrightarrow T_\infty. \tag{5.1}$$

This equation was transformed into the form (3.5) and then was solved numerically using (3.9) with correction $\varepsilon(x)$ and taking into account that in this case $\Phi = \mathrm{Re}$ and thus it is proportional to x. Three approximations were sufficient for considering a polymer film with properties $\{\lfloor(\lambda c_p\rho)_w/(\lambda c_p\rho)\rfloor = 8.5\}$ cooled by water ($\mathrm{Pr} = 6$). This problem was also solved numerically using the finite-difference method [107]. The difference between results obtained with and without conjugation is significant because the effect of nonisothermal conditions for continuously moving surfaces are much grater than for streamlined bodies [108, 109]. Hence, such problems should be considered as conjugate. The result obtained in conjugate problem agrees with experimental data [110].

Some problems of conjugate heat transfer of a continuously moving surface [111, 112] and comprehensive review [113] were published by Jaluria. In [114], conjugate heat transfer from a moving vertical plate is considered.

Example 5.2. Convective drying of a continuous material pulled through an agent [115].

The problem is described by the following system, consisting of equations and initial, symmetry, and conjugation conditions [109]

$$c_M\rho_3 \frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left(\lambda_M \frac{\partial T}{\partial y}\right) + \rho_3 \varepsilon \Lambda \frac{\partial M}{\partial t}, \qquad \rho_3 \frac{\partial M}{\partial x} = \frac{\partial}{\partial y}\left[\rho_3 \alpha_M \left(\frac{\partial M}{\partial y} + \gamma \frac{\partial T}{\partial y}\right)\right], \tag{5.2}$$

$$x = 0,\, T = T(0),\, M = M(0), \quad y = 0,\, \left(\frac{\partial T}{\partial y}\right) = 0,\, \left(\frac{\partial M}{\partial y}\right) = 0, \tag{5.3}$$

$$\frac{\rho_{\infty D}}{1-\rho_{10,\infty}}\frac{\partial\rho_{10}}{\partial y}\bigg|_{w}=\rho_3\alpha_M\left(\frac{\partial M}{\partial y}\bigg|_{w}+\delta\frac{\partial T}{\partial y}\bigg|_{w}\right),\qquad \frac{1}{\phi(T_w,u_w)}=\frac{p_s(T_w)}{p_\infty}\left[1+\frac{R_0}{R_1}\left(\frac{1}{\rho_{10,w}}-1\right)\right], \tag{5.4}$$

$$\lambda\frac{\partial T}{\partial y}\bigg|_{w}^{+}-\lambda_M\frac{\partial T}{\partial y}\bigg|_{w}^{-}=-(1-\varepsilon_w)\Lambda\alpha_M\rho_3\left(\frac{\partial u}{\partial y}\bigg|_{w}+\delta\frac{\partial T}{\partial y}\bigg|_{w}^{-}\right). \tag{5.5}$$

Conjugate conditions required to satisfy the relations between the quantities on the interface calculated from agent and from material sides. Three of such conditions are the equalities of temperatures, vapor densities, and mass fluxes $I(x)$ from both sides:

$$T_w^+(x)=T_w^-(x),\qquad \rho_{10,w}^+(x)=\rho_{10,w}^-(x),\qquad I_w^+(x)=I_w^-(x). \tag{5.6}$$

The forth condition is the heat balance on the material surface: the difference between heats incoming from coolant and absorbing by material is utilized for evaporation:

$$-q_w^+ + q_w^- = (1-\varepsilon_w)I_w^-\Lambda. \tag{5.7}$$

In these equations $\varepsilon, D, \delta, R, \phi=\rho_{1w}^-/\rho_s(T_w)$ are the coefficients of phase change, of vapor diffusion, of temperature-gradient, the gas constant, and the relative humidity. The subscripts $0, 1, 2, 3$, m.s., s, indicate: air, vapor, liquid, dry material, maximum sportive, and saturated vapor. The heat and mass fluxes are given by (3.7) and (3.8) or by (3.3) and (3.4). Aanalogous relations are used for the mass fluxes [109, 116, 117]:

$$j_w=h_{M*}\left(\rho_{10,w}-\rho_{10,\infty}+g_1x\frac{d\rho_{10,w}}{dx}+g_2x^2\frac{d^2\rho_{10,w}}{dx^2}+\cdots\right), \tag{5.8}$$

$$j_w=h_{M*}\left\{\rho_{10\cdot w}-\rho_{10,\infty}+\int_0^x\left[1-\left(\frac{\xi}{x}\right)^{C_1}\right]^{C_2}\frac{d\rho_{10,w}}{d\xi}d\xi\right\}. \tag{5.9}$$

Some general conclusions for the considering first drying period $(M > M_{\text{m.s.}}\phi=1, \varepsilon=0\}$ obtained by analyzing the system of equations are given as follows.

(1) The conjugate problem is governed by four parameters: $\rho_{10,w}$, $T(0)$, $M(0)$, and T_∞.

(2) For the case of the constant material properties, the number of governing parameters reduces to two:

$$\frac{T(0)-T_\infty}{M(0)-M_{\text{m.s.}}},\qquad \frac{\rho_{10,w}-\rho_{10\infty}}{M(0)-M_{\text{m.s.}}}. \tag{5.10}$$

(3) The duration of the draying time is proportional to Δ^2.

(4) The pulled velocity determines only the distance from a slot when the material reaches a certain state.

The system (5.2)–(5.9) was solved numerically by a differential technique using the tridiagonal matrix algorithm and implicit difference scheme [115]. The calculations were performed for the following data: $p_{10,\infty} = 0.125$, $M(0) = 0.25$, $c_3 = 1500\,\text{Jkg}^{-1}\,\text{K}^{-1}$, $c_M = c_3 + c_{H_2O}M$, $\rho_1 = 800\,\text{kgm}^{-1}$, $M_{\text{m.s.}} = 0.15$ [118], $\lambda_M = 0{,}4\,\text{W/mK}$, $c\rho\lambda/(c\rho)_3\lambda_M = 6 \times 10^{-5}$, and $\alpha_M(c\rho)_3\lambda_{\text{M}} = 0.125$. Two cases of the initial material temperature ($T(0) = 70°\text{C}$ and $T(0) = 50°\text{C}$) were considered The first corresponds to higher and the second to lower temperatures than the dew-point temperature in the agent for $p_{10,\infty} = 0.125$. In the first case drying proceeds takes place from the beginning, whereas in the second case the material is first moistened, and drying begins after some time.

The results show that the following hold.

(1) The rate of the heat and mass transfer predicted in the conjugate problem are lower than those obtained by usual method. For drying, the moisture is higher, and for moistening, lower than that obtained without conjugation. The temperatures obtained in the conjugate solution are lower for drying and higher for moistening.

(2) The analysis of the temperature and concentration heads variation gives the key for understanding why both the heat and mass transfer rates predicted in conjugate problem are lower than these obtained by the usual approach using the transfer coefficients for constant heads. When the head grows, either in the direction of the flow or in time, the transfer coefficients are higher, while in the reverse case they are lower than the coefficients in the case of constant heads [5, 115]. In the process in question, the concentration heads diminish. The temperature head increases in the drying and decreases in the moistening. Due to that, the mass transfer coefficients in both cases are smaller than h_{M*}. The heat transfer coefficient is smaller than h_* for moistening and larger than h_* for drying. From system of equation follows that the concentration head is falling in any drying or moistening process of the same type. Because of this, there is always corresponding decreasing in the values of mass transfer coefficients.

(3) The analogy between the heat and mass transfer coefficients is not observed because the conjugation has little effect on the heat transfer and noticeably reduces the mass transfer. In the moistening case, when the initial temperature is higher than dew-point temperature, the ratio h_M/h_{M*} reaches 0.6, while in other case this ratio reduces to zero and then becomes negative.

(4) In the moistening case, the mass flow inversion occurs. This phenomenon is similar to the heat flux inversion [5, 115]. The mass flux reduces to zero earlier than the concentration head and the mass transfer coefficient becomes zero. After this point, the mass flux changes its sign despite the sign of concentration head remains the same ($\rho_{10,w} < \rho_{10,\infty}$). Therefore, the mass transfer coefficient is negative here. Such a pattern remains up to the point at which the concentration head vanishes and the mass transfer coefficient tend to infinity, losing its meaning.

Physically the inversion phenomenon is explained by the inertial properties of the flow, due to which the change in concentration near the wall is manifested much earlier in its immediate vicinity than far from the wall. This results in the fact that when the concentrations of the fluid on the wall and in its immediate vicinity become the same and the mass flux reduces to zero, the concentration far from the wall does not manage to become equal to that on the wall and, hence, the concentration head does not vanish [29].

Conjugate heat and mass transfer in drying was also studied in [119–128].

6. Conjugate Heat Transfer Problems for Special Applications

Numerous solutions of other conjugate problems have been published. The results obtained are applicable to: different devices and technological processes [129–156], electrical systems [157–162], building construction [163–170], and food processing [171–173].

Variety of articles outline different numerical approaches for solving conjugate heat transfer problem: finite elements, boundary elements and combined boundary-finite elements methods in [174–181], Galerkin approach in [182], unstructured hybrid scheme in [183], SIMPLE algorithm in [184], and some other means in [185–190].

7. Conclusion

The advanced modeling of convective conjugate heat transfer problems is now used extensively in different applications. Starting from simple examples during the 1965–1970s, currently this approach is used to create models of various device operation and technology processes from simple procedures to complex multistage, nonlinear processes.

The author hopes that the list of the almost two hundred publications accompanying this short survey together with analytical solutions of typical relatively simple conjugate problems give the reader an understanding for the situation in the early and current time of developing this modern technique of studying heat transfer which nowadays substitute in many cases the approach based on the empirical heat transfer coefficient used from the time of Newton. The solution of convective conjugate problems presented here allows the reader to become familiar with principles of conjugate approach, while the list of papers presented in the form of the groups considering similar problems by using different analytical and numerical methods gives the possibility to find the modern results in the area of interest.

Nomenclature

Bi: Biot number
c: Specific heat
G_k, g_k: Series coefficients
Φ: Gortler variable
Fo: Fourier number
h, h_M: Heat and mass transfer coefficients
h_L, h_w: Heat transfer coefficients at the plate end and of evaporation and sputtering
H: Height of the fin
k: Overall heat transfer coefficient
L: Length
Ls: Leidenfrost number
Nu: Nusselt number
Pe: Peclet number
Pr: Prandtl number
Ra: Rayleigh number
Re: Reynolds number
q: Heat flux
T: Temperature
$t_w = T_w - T_\infty$: Temperature head
$T - T_\infty$: Excessive temperature

U: External flow velocity
u, v: Velocity components in boundary layer or in channel
M: Moisture content
x, y: Coordinates
α: Thermal diffusivity
β: Thermal expansion coefficient
Δ: Wall thickness
λ: Thermal conductivity
Λ: Latent heat
ν: Kinematic viscosity
ρ: Density
ψ: Stream function.

Subscripts

M: Moisture
w: wall
∞: External flow
$*$: Isothermal
i: Initial
f: Flow.

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