Conjunctive grammars generate non-regular unary languages

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- Intuition of the semantics:
  - w is derived such production iff it is derived by each α_i
  - w is derived from $\alpha_i = N_1 \cdot N_2 \cdot \ldots \cdot N_k$ iff $w = w_1 w_2 \ldots w_k$ and w_j is derived from N_j for each j

Example

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$$\Sigma = \{a, b, c\} ,$$
$$N = \{S, B, C, E, A\}$$

$$S \rightarrow (AE)\&(BC)$$
$$A \rightarrow aA|\epsilon$$
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$$C \rightarrow cC|\epsilon$$
$$E \rightarrow bEc|\epsilon$$

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$S \to (AE)\&(BC)$	$\{a^n b^n c^n : n \in \mathbb{N}\}$
$A \to aA\|\epsilon$	a^*
$B \to aBb\|\epsilon$	$\{a^n b^n : n \in \mathbb{N}\}$
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$E \to bEc\|\epsilon$	$\{b^n c^n : n \in \mathbb{N}\}$

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- from possible extensions of CFG this keeps the meaning of language equations
- good parsing properties

Formal syntax

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A conjunctive grammar is a $\langle \Sigma, N, S, P \rangle$ where

- Σ is a finite alphabet
- N—set of non-terminal symbols
- S—starting symbol
- P—set of productions of a form

$$A \to \alpha_1 \& \alpha_2 \& \ldots \& \alpha_k, \quad \alpha_i \in (\Sigma \cup N)^*$$

Rewriting

Semantics

By term rewriting.

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Generalizes the Chomsky rewriting.

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Drawbacks

- There are more generalizations.
- Slightly problematic to handle.

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Remark

In the CFG case the only allowed operations are $\cup$ and $\cdot$.

Example revisited

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$S \to (AE)\&(BC)$ $\quad L_S = (L_A \cdot L_E) \cap (L_B \cdot L_C)$

$A \to aA|\epsilon$ $\quad L_A = \{a\} \cdot L_A \cup \{\epsilon\}$

$B \to aBb|\epsilon$ $\quad L_B = \{a\} \cdot L_B \cdot \{b\} \cup \{\epsilon\}$

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$E \to bEc \| \epsilon$	$L_E = \{b\} \cdot L_E \cdot \{c\} \cup \{\epsilon\}$	$\{b^n c^n : n \in \mathbb{N}\}$

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Negative results

- Mainly open questions

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Yes

Intuition

This should be true since regular sets are closed under

- *concatenation*
- *intersection*
- *union*

Result

Theorem (Disproving the conjecture)

Conjunctive grammars generate non-regular languages over unary alphabet.

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Theorem (Extension)

For every regular language $R \subseteq \{0, 1, \ldots, k-1\}^*$ *language*

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is a unary conjunctive language.

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Theorem (Extension)

For every regular language $R \subseteq \{0, 1, \ldots, k-1\}^*$ language

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is a unary conjunctive language. *Positional notation.*

Language

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We identify a^n with n and work with sets of integers.

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Solution

$$\begin{aligned} L_1 &= \{1 \cdot 4^n : n \in \mathbb{N}\}, \\ L_2 &= \{2 \cdot 4^n : n \in \mathbb{N}\}, \\ L_3 &= \{3 \cdot 4^n : n \in \mathbb{N}\}, \\ L_{12} &= \{6 \cdot 4^n : n \in \mathbb{N}\}. \end{aligned}$$

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Equations

$$\begin{aligned} B_1 &= (B_2 B_2 \cap B_1 B_3) \cup \{1\}, \\ B_2 &= (B_{12} B_2 \cap B_1 B_1) \cup \{2\}, \\ B_3 &= (B_{12} B_{12} \cap B_1 B_2) \cup \{3\}, \\ B_{12} &= (B_3 B_3 \cap B_1 B_2). \end{aligned}$$

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This effectively manipulates the positional notation.

What needs to be proved

- By general knowledge there is a unique ϵ-free solution.

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Example

For example L_1, the rule is

$$B_1 = (B_2 B_2 \cap B_1 B_3) \cup \{1\}$$

So we want to prove that

$$L_1 = (L_2 L_2 \cap L_1 L_3) \cup \{1\}$$

Details–what is in $B_2 B_2$

Proof.

What are the possible non-zero symbols in $B_2 B_2$?

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+ & 2 & 0\ldots0 & 0 & 0\ldots0 \\
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Either only 1 or $\{2, 2\}$. □

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The only possibility is only 1. □

First step: $ij0^*$

Theorem

For every k and $0 < i, j < k$ languages

$$\{a^n : \exists\, w \in i0^* \text{ } w \text{ read as a number is } n\}$$
$$\{a^n : \exists\, w \in ij0^* \text{ } w \text{ read as a number is } n\}$$

are unary conjunctive languages.

Idea

Done in the same way.

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- *Intersections of concatenations filter out wrong combination of leading symbols.*

Second step: any regular language

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For every k and $R \subset \{0, \ldots, k-1\}^*$

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Let $\langle \{0, \dots, , k-1\}, Q, q_0, F, \delta \rangle$ *recognizes R.*

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Let $\langle \{0, \ldots, , k-1\}, Q, q_0, F, \delta \rangle$ *recognizes* R.
We introduce nonterminal $B_{i,j,q}$ *for language*

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Information the indices carry:

- *leading symbol i*
- *second leading symbol j*
- *q—the computation of M on the rest of the word*

Productions for $B_{i,j,q}$

Example

$$B_{i,j,q} \to \left(\&_{n=1}^{4} B_{i-1,j+n} B_{k-n,x,q'}\right)$$

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 & k-n & x & \overbrace{\ldots}^{\text{state } q'} \\
+ \quad i-1 & j+n & 00\ldots0 & \\
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In particular it generates non-regular languages.
We effectively manipulate positional notation.

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- Unambiguity of the language. The construction for $R = ij0^*$ can be made unambiguous. What happens in general?

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- The result can be extended to a larger class of languages [A. Jez, A. Okhotin, CSR 2007].

Related topics and following work

- Unambiguity of the language. The construction for $R = ij0^*$ can be made unambiguous. What happens in general?
- The result can be extended to a larger class of languages [A. Jez, A. Okhotin, CSR 2007].
- Instead of grammars we can focus on sets of integers. Equations on sets of integers using $\cap$, $\cup$ and $+$ defined as

$$A + B = \{a + b : a \in A, b \in B\} .$$

  [A. Jez, A. Okhotin, TALE 2007].

Open questions

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- Unambiguity of the constructed unary languages
- Closure under complementation in the unary case.