

## Connecting low energy leptonic $CP$ violation to leptogenesis

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It was commonly thought that the observation of low energy leptonic  $CP$ -violating phases would not automatically imply the existence of a baryon asymmetry in the leptogenesis scenario. This conclusion does not generically hold when the issue of flavor is relevant and properly taken into account in leptogenesis. We illustrate this point with various examples studying the correlation between the baryon asymmetry and the  $CP$ -violating asymmetry in neutrino oscillations and the effective Majorana mass in neutrinoless double beta decay.

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Leptogenesis [1] is a simple mechanism to explain the baryon number asymmetry (per entropy density) of the Universe  $Y_B = (0.87 \pm 0.02) \times 10^{-10}$  [2]. A lepton asymmetry is dynamically generated and then converted into a baryon asymmetry due to  $(B + L)$ -violating sphaleron interactions [3,4] which exist in the standard model (SM). A simple model in which this mechanism can be implemented is the “seesaw” (type I) [5], consisting of the SM plus three right-handed (RH) Majorana neutrinos. In thermal leptogenesis [6] the heavy RH neutrinos are produced by thermal scatterings after inflation and subsequently decay out-of-equilibrium in a lepton number and  $CP$ -violating way, thus satisfying Sakharov’s constraints [4]. At the same time the smallness of neutrino masses suggested by oscillation experiments [7] can be ascribed to the seesaw mechanism where integrating out heavy RH Majorana neutrinos generates mass terms for the left-handed flavor neutrinos which are inversely proportional to the mass of the RH ones.

Establishing a connection between the  $CP$ -violation in low energy neutrino physics and the  $CP$ -violation at high energy necessary for leptogenesis has received much attention in recent years [8] and is the subject of the present paper. In the case of three neutrino mixing,  $CP$ -violation at low energy is parameterized by the phases in the Pontecorvo-Maki-Nagakawa-Sakata (PMNS) [9] lepton mixing matrix  $U$ . It contains the Dirac phase  $\delta$  and, if neutrinos are Majorana particles, two Majorana phases  $\alpha_{21}$  and  $\alpha_{31}$  [10]. The Dirac phase  $\delta$  enters in the probability of neutrino oscillations. The corresponding  $CP$ -asymmetry is given by the difference between the oscillation probability for neutrino and antineutrinos,  $\Delta P = P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \propto J_{CP}$  where the rephasing invariant  $J_{CP} = \text{Im}(U_{e1}U_{e2}^*U_{\mu 1}^*U_{\mu 2})$  [11] is proportional to  $\sin 2\theta_{13} \sin \delta$ . This implies that the observation prospects of  $CP$ -violation in future long-baseline experiments depend on the true value of  $\sin 2\theta_{13}$ . Present studies indicate that a wide range of values of the  $\delta$  phase could be tested in superbeam and

betabeam experiments if  $\sin^2 2\theta_{13} \simeq \text{few} \times (10^{-3} - 10^{-2})$ , or in a future neutrino factory even if  $\sin^2 2\theta_{13}$  is as small as  $10^{-4}$ . The two Majorana  $CP$ -violating phases enter only processes at low energy in which the lepton number is violated by two units. The most sensitive of these processes is neutrinoless double beta decay, which is currently under intensive experimental search [12]. The decay rate is a function of the effective Majorana mass  $\langle m_\nu \rangle = (m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2)$  which depends on the type of neutrino mass spectrum. Typically, one can consider the normal hierarchical (NH) ( $m_1^2 \ll m_2^2 \simeq \Delta m_{21}^2 \ll m_3^2 \simeq \Delta m_{31}^2$ ), inverted hierarchical (IH) ( $m_3^2 \ll m_1^2 \simeq m_2^2 \simeq \Delta m_{21}^2$ ), and quasidegenerate (QD) ( $m_1^2 \simeq m_2^2 \simeq m_3^2 \gtrsim \Delta m_{21}^2$ ) spectra. Here  $\Delta m_{21}^2$  and  $\Delta m_{31}^2$  are the mass square differences which drive the solar and the atmospheric neutrino oscillations, respectively, and  $m_i$  ( $i = 1, 2, 3$ ) are the light neutrino masses. One Majorana phase can, in principle, be observed although this represents a challenge. For a detailed discussion see Refs. [13,14].

It was commonly accepted that the future observation of leptonic low energy  $CP$ -violation would not automatically imply a nonvanishing baryon asymmetry through leptogenesis. This conclusion, however, was shown in [15–17] not to hold universally. The reason is based on a new ingredient recently accounted for in the leptogenesis scenario, lepton flavor [15–18]. The dynamics of leptogenesis is usually addressed within the ‘one-flavor’ approximation, where Boltzmann equations are written for the abundance of the lightest RH neutrino and for the total lepton asymmetry. However, this approximation is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. Supposing that leptogenesis takes place at temperatures  $T \sim M_1$ , where  $M_1$  is the mass of the lightest RH neutrino, the ‘one-flavor’ approximation only holds for  $M_1 \gtrsim 10^{12}$  GeV. In this range all the interactions mediated by the charged lepton Yukawa couplings are out of equilibrium and there is no notion of flavor. One is allowed to perform a rotation in

flavor space to store all the lepton asymmetry in one flavor, the total lepton number. However, at  $T \sim M_1 \sim 10^{12}$  GeV, the interactions mediated by the charged tau Yukawa coupling come into equilibrium followed by those mediated by the charged muon Yukawa coupling at  $T \sim M_1 \sim 10^9$  GeV and the notion of flavor becomes physical. Including the issue of flavor can significantly affect the result for the final baryon asymmetry [15–17]. Thermal leptogenesis is a dynamical process, involving the production and destruction of RH neutrinos and of the lepton asymmetry that is distributed among distinguishable flavors. The processes which wash out lepton number are flavor dependent, e.g. the inverse decays from electrons can destroy the lepton asymmetry carried by, and only by, the electrons. The asymmetries in each flavor are therefore washed out differently, and will appear with different weights in the final formula for the baryon asymmetry. This is physically inequivalent to the treatment of washout in the one-flavor approximation, where the flavors are taken indistinguishable, thus obtaining the unphysical result that inverse decays from all flavors are taken to wash out asymmetries in any flavor (that is, e.g., an asymmetry stored in the first family may be washed out by inverse decays involving the second or the third family).

When flavor is accounted for, the final value of the baryon asymmetry is the sum of three contributions. Each term is given by the  $CP$  asymmetry in a given flavor  $\alpha$  properly weighted by a washing out factor induced by the lepton  $\alpha$  violating processes. Taking into account the flavor dependence one may show that observing low energy  $CP$ -violating phases automatically implies, barring accidental cancellations, generation of the baryon asymmetry. Before going into details though, let us summarize why this conclusion is not possible in the ‘one-flavor’ approximation. The starting point is the Lagrangian of the SM with the addition of three right-handed neutrinos  $N_i$  ( $i = 1, 2, 3$ ) with heavy Majorana masses  $M_i$  and Yukawa couplings  $\lambda_{i\alpha}$ . Working in the basis in which the Yukawa couplings for the charged leptons are diagonal, the Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{M_i}{2} N_i^2 + \lambda_{i\alpha} N_i \ell_\alpha H + \text{H.c.} \quad (1)$$

Here  $\ell_\alpha$  indicates the lepton doublet with flavor ( $\alpha = e, \mu, \tau$ ) and  $H$  is the Higgs doublet whose vacuum expectation value is  $v$ . For the time being, we assume that right-handed neutrinos are hierarchical,  $M_{2,3} \gg M_1$  so that restricting to the dynamics of  $N_1$  suffices.

The total lepton asymmetry per entropy density generated by the  $N_1$  decays is given by  $Y_{\mathcal{L}} \simeq (\epsilon_1/g_*)\eta(\tilde{m}_1)$ , where  $\eta(\tilde{m}_1)$  accounts for the washing out of the total lepton asymmetry due to  $\Delta L = 1$  inverse decays,  $\tilde{m}_1 = (\lambda\lambda^\dagger)_{11} v^2/M_1$ ,  $g_*$  counts the relativistic degrees of freedom and the  $CP$  asymmetry generated by  $N_1$  decays reads

$$\begin{aligned} \epsilon_1 &\equiv \frac{\sum_\alpha [\Gamma(N_1 \rightarrow H\ell_\alpha) - \Gamma(N_1 \rightarrow \bar{H}\bar{\ell}_\alpha)]}{\sum_\alpha [\Gamma(N_1 \rightarrow H\ell_\alpha) + \Gamma(N_1 \rightarrow \bar{H}\bar{\ell}_\alpha)]} \\ &= -\frac{3M_1}{16\pi} \sum_{j \neq 1} \frac{\text{Im}[(\lambda\lambda^\dagger)_{1j}^2]}{[\lambda\lambda^\dagger]_{11}} \frac{1}{M_j}. \end{aligned} \quad (2)$$

Notice, in particular, that the  $CP$  asymmetry in the ‘one-flavor approximation’ depends upon the trace of the  $CP$  asymmetries over flavors. In the basis where the charged lepton Yukawa coupling and the RH mass matrix are diagonal, the neutrino Yukawa matrix can be written as  $\lambda = V_R^\dagger \text{Diag}(\lambda_1, \lambda_2, \lambda_3) V_L$  and the low energy leptonic phases may arise from the phases in the left-handed (LH) sector, in RH sector, or from both. The  $CP$ -asymmetry can be expressed in terms of the diagonal matrix of the light neutrino mass eigenvalues  $m = \text{Diag}(m_1, m_2, m_3)$ , the diagonal matrix of the right-handed neutrino masses  $M = \text{Diag}(M_1, M_2, M_3)$  and an orthogonal complex matrix  $R = \nu M^{-1/2} \lambda U m^{-1/2}$  [19], which ensures that the correct low energy parameters are obtained.  $CP$ -violation in the RH sector is encoded in the phases of  $V_R$  and, from  $\lambda\lambda^\dagger = V_R^\dagger \text{Diag}(\lambda_1^2, \lambda_2^2, \lambda_3^2) V_R = M^{1/2} R m R^\dagger M^{1/2} / v^2$ , one sees that the phases of  $R$  are related to those of  $V_R$ . Now, summing over all flavors, one finds

$$\epsilon_1 = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}(\sum_\rho m_\rho^2 R_{1\rho}^2)}{\sum_\beta m_\beta |R_{1\beta}|^2}. \quad (3)$$

In the ‘one-flavor’ approximation a future observation of  $CP$ -violating phases in the neutrino sector does not imply the existence of a baryon asymmetry. Indeed, low energy  $CP$  phases might stem entirely from the LH sector and hence be irrelevant for leptogenesis which would be driven by the phases in  $R$ , i.e. of the RH sector.

The ‘one-flavor’ approximation rigorously holds, however, only when the interactions mediated by the charged lepton Yukawas are out of equilibrium, that is at  $T \sim M_1 \gtrsim 10^{12}$  GeV. In this regime, flavors are indistinguishable and there is effectively only one flavor, the total lepton number. At smaller temperatures, though, flavors are distinguishable: the  $\tau(\mu)$  lepton doublet is a distinguishable mass eigenstate for  $T \sim M_1 \lesssim 10^{12}(10^9)$  GeV. The asymmetry in each flavor is given by

$$\epsilon_\alpha = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U_{\alpha\beta}^* U_{\alpha\rho} R_{1\beta} R_{1\rho})}{\sum_\beta m_\beta |R_{1\beta}|^2}. \quad (4)$$

The trace over the flavors of  $\epsilon_\alpha$  coincides of course with  $\epsilon_1$ . Similarly, one may define a parameter for each flavor  $\alpha$ ,  $\tilde{m}_\alpha = |\lambda_{1\alpha}|^2 v^2 / M_1$  parametrizing the decay rate of  $N_1$  to the  $\alpha$ -th flavor and the trace  $\sum_\alpha \tilde{m}_\alpha$  coincides with the  $\tilde{m}_1$  parameter defined for the one-single flavor case. Solving the Boltzmann equations for each flavor one finds  $Y_\alpha \simeq (\epsilon_\alpha/g_*)\eta(\tilde{m}_\alpha)$  [15–17]. The way the total baryon asym-

metry depends upon the individual lepton asymmetries is a function of temperature. For instance, for ( $10^9 \lesssim T \sim M_1 \lesssim 10^{12}$ ) GeV, only the interactions mediated by the  $\tau$  Yukawa coupling are in equilibrium and the final baryon asymmetry is  $Y_B = -(12/37g_*) (\epsilon_2 \eta(0.7\tilde{m}_2) + \epsilon_\tau \eta(0.67\tilde{m}_\tau))$ , where  $\epsilon_2 = \epsilon_e + \epsilon_\mu$ ,  $\tilde{m}_2 = \tilde{m}_e + \tilde{m}_\mu$ ,  $Y_2 = Y_{e+\mu}$  [17]. As the  $CP$  asymmetry in each flavor is weighted by the corresponding wash out parameter,  $Y_B$  is generically not proportional to  $\epsilon_1$ , but depends on each  $\epsilon_\alpha$ . The dependence on the PMNS matrix elements in (4) is such that nonvanishing low energy leptonic  $CP$ -violating phases imply, in the context of leptogenesis and barring accidental cancellations, a nonvanishing baryon asymmetry [16,17].

We can go even further.  $CP$  invariance would correspond to a real matrix  $R$  provided that the  $CP$ -parities of the heavy and light Majorana neutrinos are equal to  $+i$  [20]. In this case the low energy Majorana phases vanish (modulo  $2\pi$ ) and  $\delta = 0$  (modulo  $\pi$ ).  $R$  real [16,17] corresponds to the class of models where  $CP$  is an exact symmetry in the RH neutrino sector [17]. Now, the flavor  $CP$  asymmetries and the baryon asymmetry depend exclusively on the low energy phases in the PMNS matrix. Consequently, leptogenesis is maximally connected to the low energy leptonic  $CP$ -violation. This conclusion is clear from the expression of the flavor  $CP$  asymmetries in terms of a real  $R$  matrix,  $\epsilon_\alpha \propto \sum_{\beta,\rho>\beta} \sqrt{m_\beta m_\rho} (m_\rho - m_\beta) R_{1\beta} R_{1\rho} \text{Im}(U_{\alpha\beta}^* U_{\alpha\rho})$ . Notice that  $\epsilon_1 = 0$  if  $R$  is real and  $\epsilon_\alpha = 0$  if  $R$  is real and diagonal. Once flavor effects are taken into account, a baryon asymmetry is generically generated from nonzero phases in the PMNS matrix.

To illustrate better this point, we provide two examples where the baryon asymmetry is generated uniquely by the  $CP$  phases in the PMNS matrix. We will consider the range of values ( $10^9 \lesssim M_1 \lesssim 10^{12}$ ) GeV, for which it is sufficient to consider  $\epsilon_\tau$ , being  $\epsilon_2 = -\epsilon_\tau$ . In the first example, we consider the NH spectrum. In the limit  $M_1 \ll M_2 \ll M_3$ , we obtain

$$\begin{aligned} \epsilon_\tau \simeq & \frac{3M_1}{16\pi v^2} \frac{(\Delta m_\odot^2 \Delta m_\oplus^2)^{1/4} R_{12} R_{13}}{\sqrt{\Delta m_\odot^2 / \Delta m_\oplus^2 R_{12}^2 + R_{13}^2}} \\ & \times c_{13} \left( \frac{1}{2} c_{12} \sin 2\theta_{23} \sin \frac{\alpha_{32}}{2} \right. \\ & \left. - s_{12} c_{23}^2 s_{13} \sin \left( \delta - \frac{\alpha_{32}}{2} \right) \right), \end{aligned} \quad (5)$$

where  $c_{ij} = \cos\theta_{ij}$  and  $s_{ij} = \sin\theta_{ij}$ . Only the Majorana phase  $\alpha_{32} = \alpha_{31} - \alpha_{21}$  plays a role being the contribution of  $m_1$  negligible. With these expressions, it is straightforward to compute the final baryon asymmetry solving the flavored Boltzmann equations of Ref. [17]. In the IH case, a similar expression holds for  $\epsilon_\tau$ , but is suppressed for real  $R$  with respect to the one in the NH case by a factor  $\sim (\Delta m_\odot^2 / \Delta m_\oplus^2)^{3/4}$ , leading generically to an asymmetry which is small. A sufficiently large asymmetry can be

recovered in the case of purely imaginary product  $R_{11}R_{12}$  or in the supersymmetric version of leptogenesis [20]. In the expression (5) the dominant contribution comes from the Majorana  $CP$ -violating phase, while the effects due to  $\delta$  are suppressed by  $\sin\theta_{13}$ . The Majorana phase  $\alpha_{32}$  appears in the expression for the effective Majorana mass  $\langle m_\nu \rangle$ . The baryon asymmetry depends also on the combination  $\sin\theta_{13} \sin\delta$ , which enters in the  $CP$ -asymmetry measurable in future long-baseline oscillation experiments.

We consider the tribimaximal mixing case and take  $c_{23} = s_{23} = 1/\sqrt{2}$ ,  $s_{12} = 1/\sqrt{3}$ . In Fig. 1 we show the correlation between the baryon asymmetry and the  $CP$  invariant  $J_{CP}$  for a given choice of the parameters and varying the Dirac phase  $\delta$ . Most values of  $J_{CP}$  consistent with the observed baryon asymmetry lie well within the sensitivity reachable by superbeam and betabeam experiments and future neutrino factory. In Fig. 2 we show the correlation between  $Y_B$  and  $\langle m_\nu \rangle$  in the case of IH light neutrino mass spectrum and purely imaginary product  $R_{11}R_{12}$  (see Ref. [20] for details).

The second example we discuss is for QD neutrinos. To avoid excess of fine-tuning, we choose quasidegenerate RH neutrino masses as well,  $M_1 \sim M_2 \sim M_3$ ; all RH neutrinos contribute to the baryon asymmetry. The washing out of a given flavor is parametrized by  $\tilde{m}_\alpha = \sum_j |\lambda_{j\alpha}|^2 v / M_1$ . For  $R$  real, it is approximately the same for all flavors,  $\tilde{m}_\alpha \sim m$ . Again, for ( $10^9 \lesssim 4 < M_1 \lesssim 10^{12}$ ) GeV and  $R$  real,  $\epsilon_2 = -\epsilon_\tau$ . If we consider the case in which  $M_1 \simeq M_2 \lesssim M_3$ , the total  $CP$  asymmetry in the third flavor  $\epsilon_\tau$  is resonantly enhanced when the decay rate  $\Gamma_{N_2} \sim (M_2 - M_1)$  and [20]

$$\begin{aligned} \epsilon_\tau \simeq & \frac{1}{2m^2} (\Delta m_\odot^2 R_{11} R_{21} - \Delta m_\oplus^2 R_{13} R_{23}) \sum_{\rho>\beta} (R_{1\rho} R_{2\beta} \\ & - R_{1\beta} R_{2\rho}) \text{Im}(U_{3\beta} U_{3\rho}^*). \end{aligned} \quad (6)$$

We may write the matrix  $R$  under the form  $R = e^A$ , where

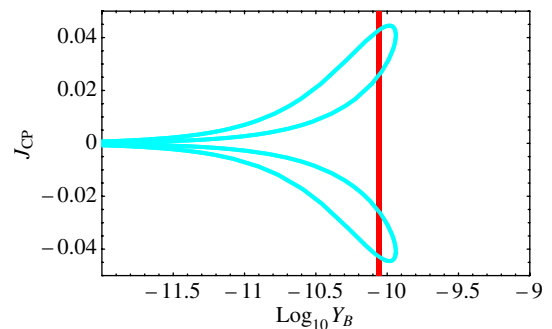


FIG. 1 (color online). The invariant  $J_{CP}$  versus the baryon asymmetry varying (in blue)  $\delta = [0, 2\pi]$  in the case of hierarchical RH neutrinos and NH light neutrino mass spectrum for  $s_{13} = 0.2$ ,  $\alpha_{32} = 0$ ,  $R_{12} = 0.86$ ,  $R_{13} = 0.5$  and  $M_1 = 5 \times 10^{11}$  GeV. The red region denotes the  $2\sigma$  range for the baryon asymmetry.

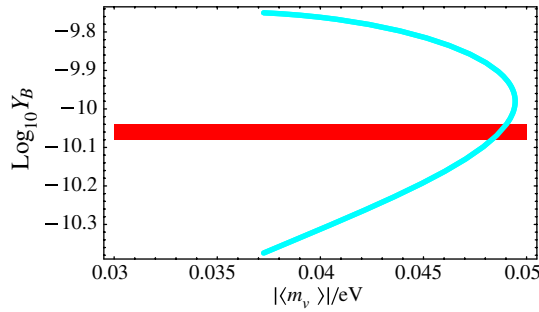


FIG. 2 (color online). The baryon asymmetry  $|Y_B|$  versus the effective Majorana mass in neutrinoless double beta decay,  $\langle m_\nu \rangle$ , in the case of Majorana  $CP$ -violation, hierarchical RH neutrinos and IH light neutrino mass spectrum, for  $\delta = 0$ ,  $s_{13} = 0$ , purely imaginary  $R_{11}R_{12}$ ,  $|R_{11}| = 1.05$  and  $M_1 = 2 \times 10^{11}$  GeV. The Majorana phase  $\alpha_{21}$  is varied in the interval  $[-\pi/2, \pi/2]$ .

$A$  is a real matrix satisfying  $A^T = -A$ . In Fig. 3, we show the correlation of the baryon asymmetry with the effective Majorana mass in neutrinoless double beta decay. A number of projects aim to reach a sensitivity to  $|\langle m_\nu \rangle| \sim (0.01-0.05)$  eV [12] and can certainly probe the region of values of  $|\langle m_\nu \rangle|$  for successful baryon asymmetry from the PMNS phases only. In particular, a direct information on the Majorana phase  $\alpha_{21}$  may come from the measurement of  $\langle m_\nu \rangle$ ,  $m$ , and  $\sin^2(\alpha_{21}/2) \simeq (1 - (|\langle m_\nu \rangle|^2/m^2)) \times (1/\sin^2 2\theta_{12})$  and might tell us if enough baryon asymmetry may be generated uniquely from the PMNS phases.

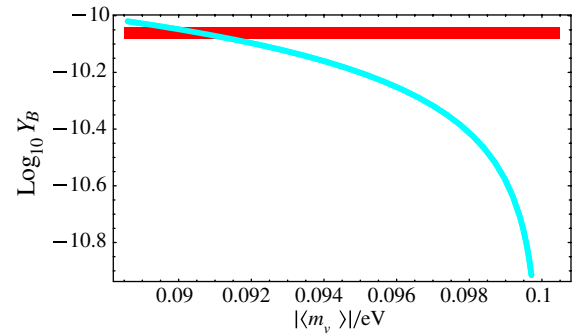


FIG. 3 (color online). The quantity  $|\langle m_\nu \rangle|$  versus the baryon asymmetry varying  $\alpha_{32}$  between 0 and  $\pi/3$  for the case of degenerate RH neutrinos and QD for light neutrinos for  $\delta = \pi/3$ ,  $s_{13} = 0.01$ ,  $M_1 = 10^{10}$  GeV and  $m = 0.1$  eV.

Our examples show that the observation of effects of the  $CP$ -violating phases of  $U$  in neutrino oscillations and/or in the neutrinoless double beta decay would generically ensure a nonvanishing baryon asymmetry through leptogenesis. The value of the baryon asymmetry so generated depends also on the value of  $M_1$  and on the unknown parameters which are contained in  $R$ . We will present a more detailed analysis, including the supersymmetric generalization, in a forthcoming publication [20].

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- [1] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
- [2] D.N. Spergel *et al.* (WMAP Collaboration), astro-ph/0603449.
- [3] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, Phys. Lett. B **155**, 36 (1985).
- [4] For a review, see A. Riotto and M. Trodden, Annu. Rev. Nucl. Part. Sci. **49**, 35 (1999).
- [5] P. Minkowski, Phys. Lett. B **67**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, in Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, edited by P. Van Nieuwenhuizen and D. Freedman; T. Yanagida, in Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan, 1979, edited by A. Sawada and A. Sugamoto; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [6] See, e.g. G.F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, Nucl. Phys. **B685**, 89 (2004).
- [7] For a review see M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle, New J. Phys. **6**, 122 (2004).
- [8] A partial list: M. S. Berger and B. Brahmachari, Phys. Rev. D **60**, 073009 (1999); H. Goldberg, Phys. Lett. B **474**, 389 (2000); **506**, 1 (2001); J. Ellis, M. Raidal, and T. Yanagida, Phys. Lett. B **546**, 228 (2002); S. Davidson and A. Ibarra, Nucl. Phys. **B648**, 345 (2003); G. C. Branco *et al.*, Nucl. Phys. **B640**, 202 (2002); S.F. King, Phys. Rev. D **67**, 113010 (2003); P.H. Frampton, S.L. Glashow, and T. Yanagida, Phys. Lett. B **548**, 119 (2002); T. Endoh *et al.*, Phys. Rev. Lett. **89**, 231601 (2002); G.C. Branco *et al.*, Phys. Rev. D **67**, 073025 (2003); S. Pascoli, S. T. Petcov, and W. Rodejohann, Phys. Rev. D **68**, 093007 (2003).
- [9] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957); **34**, 247 (1958); Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
- [10] S.M. Bilenky *et al.*, Phys. Lett. B **94**, 495 (1980); J. Schechter and J.W.F. Valle, Phys. Rev. D **22**, 2227 (1980); M. Doi *et al.*, Phys. Lett. B **102**, 323 (1981).
- [11] P. I. Krastev and S. T. Petcov, Phys. Lett. B **205**, 84 (1988).
- [12] C. Aalseth *et al.*, hep-ph/0412300.
- [13] S. Pascoli, S. T. Petcov, and W. Rodejohann, Phys. Lett. B **549**, 177 (2002).
- [14] S. Pascoli, S. T. Petcov, and T. Schwetz, Nucl. Phys. **B734**, 24 (2006).
- [15] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada, and A. Riotto, J. Cosmol. Astropart. Phys. 04 (2006) 004.
- [16] E. Nardi, Y. Nir, E. Roulet, and J. Racker, J. High Energy Phys. 01 (2006) 164.
- [17] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M.

- Losada, and A. Riotto, J. High Energy Phys. 09 (**2006**) 010.
- [18] See also, R. Barbieri, P. Creminelli, A. Strumia, and N. Tetradis, Nucl. Phys. **B575**, 61 (2000); T. Endoh, T. Morozumi, and Z. h. Xiong, Prog. Theor. Phys. **111**, 123 (2004); A. Pilaftsis and T.E.J. Underwood, Nucl. Phys. **B692**, 303 (2004); S. Blanchet and P. Di Bari, hep-ph/0607330; S. Antusch, S. F. King, and A. Riotto, J. Cosmol. Astropart. Phys. 11 (2006) 011.
- [19] J. A. Casas and A. Ibarra, Nucl. Phys. **B618**, 171 (2001).
- [20] S. Pascoli, S. Petcov, and A. Riotto hep-ph/0611338.