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CONNECTION BETWEEN INELASTIC PROTON-PROTON REACTIONS AND DEEP INELASTIC ELECTRON SCATTERING*

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ABSTRACT

Following the idea that the electromagnetic and nuclear distributions behave similarly for large momentum transfers (as introduced by Wu and Yang and further developed by Abarbanel, Drell and Gilman) we examine the possibility of relating the deep inelastic electron scattering to large momentum transfer and high energy inelastic proton-proton reactions.

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A remarkable property of the elastic pp scattering is that in the large momentum transfer region the differential cross section behaves similarly to the fourth power of the electron scattering form factor,[1,2] i.e.,

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{t=0} \left(\frac{1}{\mu}\right)^4 G_M^4(t) \qquad t < M^2 \tag{1}$$

where $G_M(t)$ is the proton magnetic form factor normalized to the total magnetic moment $\mu = 2.79$ at $t = 0$, t the momentum transfer, and M the nucleon mass.

As an explanation of this fact Wu and Yang[1] have proposed that in the large t region the nuclear matter distribution is essentially the same as the electromagnetic distribution and that in elastic p-p scattering one has the overlap of the two proton distributions giving rise to $G_M^2(t)$ in the scattering matrix element.

Abarbanel, Drell and Gilman[2] have suggested an even more specific mechanism for this large t region where the scattering is deemed a consequence of an effective local four-fermion vector (or axial vector) coupling with the universal form factor $G_M^2(t)$.

Should these descriptions of the elastic p-p scattering be valid then we may easily extend their scope to make a direct comparison between the deep inelastic electron scattering and the inelastic p-p scattering which essentially amounts to replacing each elastic factor $G_M^2(t)$ by the inelastic strength factor νW_2.

The process we envision is the reaction

$$P_1 + P_2 \rightarrow \Gamma_1 + \Gamma_2 \tag{2}$$

where the hadronic state Γ_1 is a collection of particles that kinematically can be associated with P_1 and similarly the hadronic state Γ_2 can be associated kinematically with P_2 as illustrated in Fig. 1a. Further the momentum transfer $\left|\left(P_1 - P_{\Gamma_1}\right)\right|^2 = t$ is to be outside the diffraction region and reasonably large, i.e., $t > M^2$ so that we may apply directly the proposition that the inelastic p-p distribution should imitate the deep inelastic electron behavior. Thus we envisage a kind of two fire ball production but in a region of large t. Since the two groups of particles are to be well separated we assume that the resulting final state interactions have already been included by introducing the inelastic electron strength factors at both vertices.

Hence the cross section for process (2) can be expressed as an extension of Eq. (1) and we have in this case

$$\frac{d\sigma}{dm_3^2\, dm_4^2\, dt} = C\,(M^2/S^2)\; W_{\mu\nu}^{(1)}\, W_{\mu\nu}^{(2)} \qquad (t > M^2) \tag{3}$$

where the tensors[3] $W_{\mu\nu}$ are defined as

$$W_{\mu\nu}^{(1)} = W_1^{(1)}\,(g_{\mu\nu} - q_\mu q_\nu/q^2)$$

$$+ W_2^{(1)}\,(P_{1\mu}-(P_1q)\, q_\mu/q^2)(P_{1\nu}-(P_1q)\, q_\nu/q^2)$$

and $W_{\mu\nu}^{(2)}$ with $P_1 \to P_2$ and $q = P_1-P_2$. The quantities m_3 and m_4 are the masses of the states Γ_1 and Γ_2 respectively. In our application here we will restrict their values to m_3, $m_4 > 2$ GeV so that we are outside the dominant resonance region where the inelastic electron functions are especially simple.[4]

If the interaction was due to a four fermion vector (or axial-vector) coupling as proposed for the elastic case then the constant C would be given by $C = (1/\mu)^4\,(d\sigma/dt)_{t=0}$. Below we show that with this substitution integration of Eq. (3) leads to a total cross section which is quite compatible with present experiments and as such process (2) should be investigated in more detail.

Nevertheless since it is not known how the strong interactions could conspire to produce an effective vector (as axial vector) coupling it is also possible that the coupling strength has different values for the elastic and inelastic cases respectively.

Alternatively we may argue that even if the idea of an effective universal vector interaction is not valid the resultant overlap for the inelastic p-p distributions should follow the inelastic e-p distribution (analogously to the elastic case). In this case an expression essentially given by (3) as well as (4) and (6) below would be the relevant description without, however, a predictable value for the constant C.

Supposing (3) to be applicable then with the known[4] behavior of the inelastic electron strength functions we predict interesting distributions in the mass and momentum transfer variables in p-p inelastic scattering reactions.

To this end we use the SLAC[5] data on the ratio of the longitudinal to transverse virtual photon cross sections which yield that

$$W_1 \approx (\nu W_2) \; \nu / q^2 \Big| \quad \nu = Pq$$

and hence

$$\frac{d\sigma}{dm_3^2 \, dm_4^2 \, dt} = C \left(\nu_1 W_2^{(1)}\right) \left(\nu_2 W_2^{(2)}\right)$$

$$\times \left(\frac{1}{\left(m_3^2 + t\right)\left(m_4^2 + t\right)} - \frac{1}{st} + \frac{\left(m_3^2 + t\right)\left(m_4^2 + t\right)}{2s^2 t^2} \right) \qquad (4)$$

where $\nu_1 = (p_1 q)$, $\nu_2 = (p_2 q)$, $s = (p_1 + p_2)^2$ and $t > M^2$. Thus we expect that independent of the numerical value of the constant C, Eq. (4)(and (6) below) predicts a very definite behavior of the differential cross section in mass and momentum transfer.

If in fact (νW_2) were to remain finite as ν and $t \to s$ and $s \to \infty$ then (4) would lead to a total cross section rising as s which we take to be theoretically unacceptable.

On the other hand we observe that the function in brackets in (4) is dominated by low values of mass and t so that we may estimate the cross section by restricting νW_2 to have the constant value $\nu W_2 \approx 0.3^{(4)}$ up to $m_{3,4}^2 \approx 20\ \mathrm{GeV}^2$ and $t \approx 20\ \mathrm{GeV}^2$ and zero beyond this value. (At $s = 60\ \mathrm{GeV}^2$ allowing complete constancy of νW_2 changes the numerical value by only 10%.)

In Fig. 2 we show the numerical value and s-dependence of the total cross section resulting from integrating (4)[6] with the constant C permitted the elastic value $C = (d\sigma/dt)_{t=0}(1/\mu)^4$. Even the asymptotic value of approximately 2.6 mb for the net cross section is seen to be permissible when compared with present pp inelastic data.

For a possible direct comparison we compare our values with the recent experiments[7] on the reaction

$$PP \longrightarrow P + X \tag{5}$$

where X is an unobserved inelastic state.

This example is process (2) envisaged with one of the states Γ_1 consisting of just the elastically scattered proton as shown in Fig. 1b. In this case the corresponding $W_2^{(1)}$ would be replaced by the factor $G_M^2(t)$. For this quasi-elastic process and with the same assumption on $W_1^{(2)}$ as in (4) we have

$$\frac{d^2\sigma}{dm_4^2dt} = \frac{1}{2s\sqrt{s - 4m^2}}\left(s + m^2 - m_4^2\right)\frac{d^2\sigma}{dP_t^2dP_L}$$

$$= C'G_M^2(t)(\nu W_2)\frac{1}{2s^2}\left\{(s - 2m^2)^2 + \left(s - t - m_4^2 - m^2\right)^2\right\}/\left(t + m_4^2 - m^2\right) \tag{6}$$

where m is the nucleon mass and where p_t, p_L are the transverse and longitudinal components of the outgoing proton momentum in the c.m. system. Using the elastic value of $C' = (1/\mu)^4(d\sigma/dt)_{t=0}$ Eq. (6) can be compared with experiments[7] of type (5) with the supposition that the outgoing proton suffers the elastic scattering vertex as in Fig. 1b. For this comparison not only should the momentum transfer be greater than 1 GeV^2 but more important the energy loss to the scattered proton must be chosen to be sufficiently small in order to assure that it be associated with a purely elastic vertex. A rough criterion for the proper energy loss situation to prevail is that the fraction of energy lost be smaller than the fraction of energy needed to excite the nearest inelastic state, i.e., $\Delta E/E \ll \Delta M/M \approx 1/7$. Applying this criterion to the above experiments[7] shows that in the narrow region where Eq. (6) is applicable the numerical values are in rough agreement.[8]

We note that our local four fermion interaction should correspond to isoscalar exchange since for large momentum transfer the differential cross section for $PN \rightarrow PN$ is the same as $PP \rightarrow PP$.[9] This condition applied to the elastic reactions implies that $G_M(P)/\mu_P = G_M(N)/\mu_N$ and similarly for the inelastic reactions that $W_2(P)/\mu_P^2 = W_2(N)/\mu_N^2$. Both of these results are in reasonable agreement with experimental data.[10,11]

More detailed experimental[12] comparisons with the kind of distributions considered here would be of great value especially in view of the already existing similarity between the elastic e-p scattering and elastic p-p scattering.

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6. The integration on $m_{3,4}$ starts at 2 GeV. If this lower limit is increased then there is a slight reduction in the cross section. For example with Min $(m_3, m_4) = 2.4$ GeV the cross section at $s = 60\ GeV^2$ is reduced by approximately a factor of 2.

7. J. V. Allaby et al., CERN 70-12, 23 April 1970; M. A. Abolins et al., Phys. Rev. Letters 25, 126 (1970).

8. Equation (6) may also be compared with experiments when the state Γ_2 is a specifically observed nucleon resonance. For example data for the reaction $PP \rightarrow PN^*$ (1688) (E.W. Anderson et al., Phys. Rev. Letters 19, 198 (1967)) can be compared with the inelastic electron scattering data for resonance production given in Refs. 4 and 5. Since only W_2 and not W_1 is measured by the inelastic experiment a rough comparison can be made for example at a $t = 1.7$, $s = 60\ GeV^2$ then Eq. (6) with W_1 set to zero predicts a value of $PP \rightarrow PN^*(1688)$ which is about three times smaller than the elastic PP reaction in good agreement with the above data of Anderson et al.

9. J. Cox et al., Phys. Rev. Letters 21, 645 (1968).

10. W. Bartel et al., DESY Report 69/34. See also J. Rutherglen in Proceedings of the Fourth International Electron/Photon Symposium, 1969.

11. Report No. SLAC-PUB-796, August 1970, reported at the International Conference on High Energy Physics, Kiev, 1970. See also E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

12. For the proposed colliding proton beam experiments we would expect the two fire ball structure at large t readily separated. The weak dependence on t would manifest itself as a weak dependence on the colliding beam energy for a fixed production angle. This should be observed so long as t remains at least within the range already explored by the deep inelastic electron scattering experiments.

FIGURE CAPTIONS

1. a, b Diagrams showing P-P inelastic reactions and quasi-elastic reactions respectively.

2. The integrated value of Eq. (2) subject to $\nu W_2 = 0.3$ for $4\ \mathrm{GeV}^2 < m_3^2$, $m_4^2 < 20\ \mathrm{GeV}^2$, $1 < t < 100\ \mathrm{GeV}^2$ and zero outside these limits.

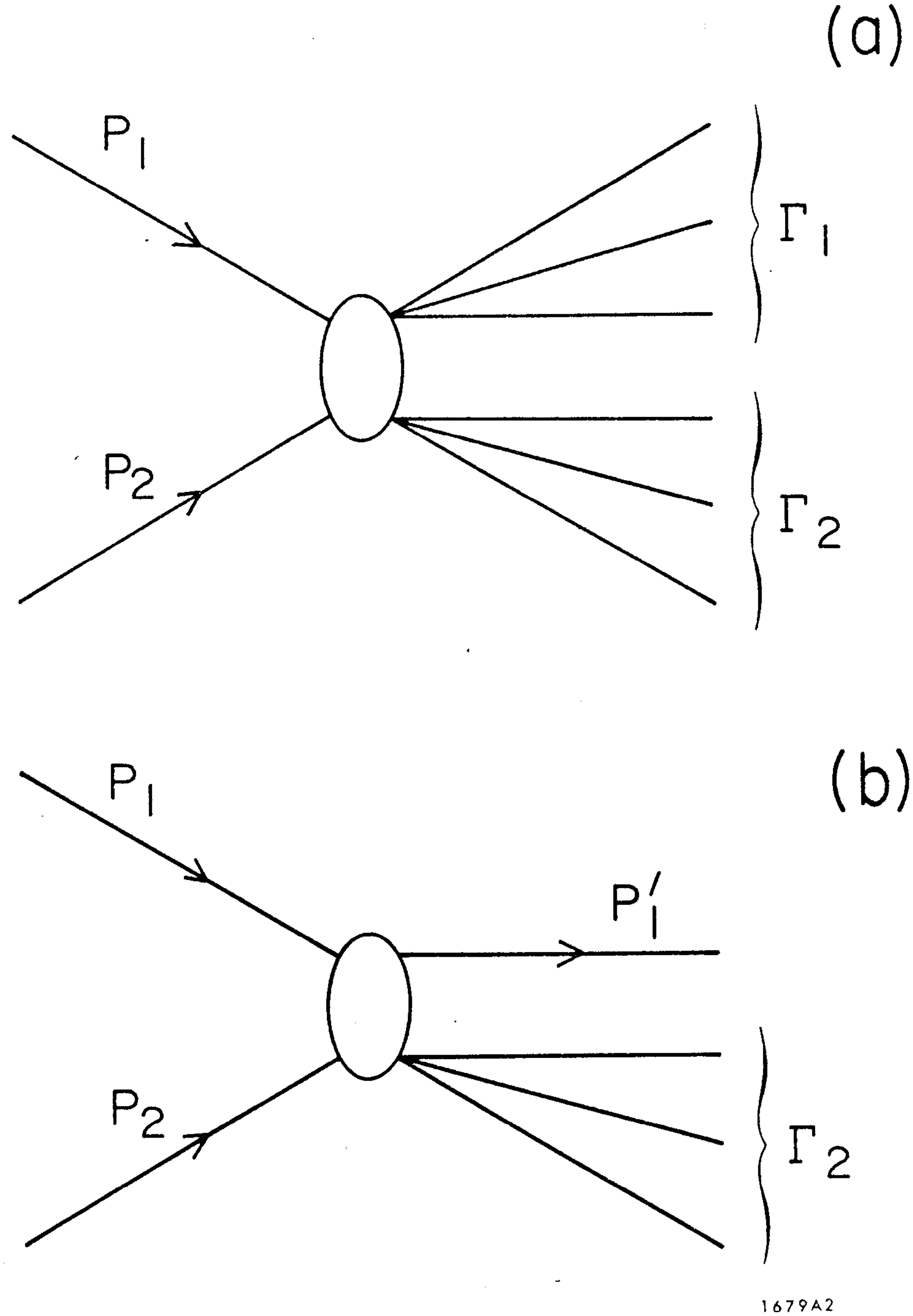
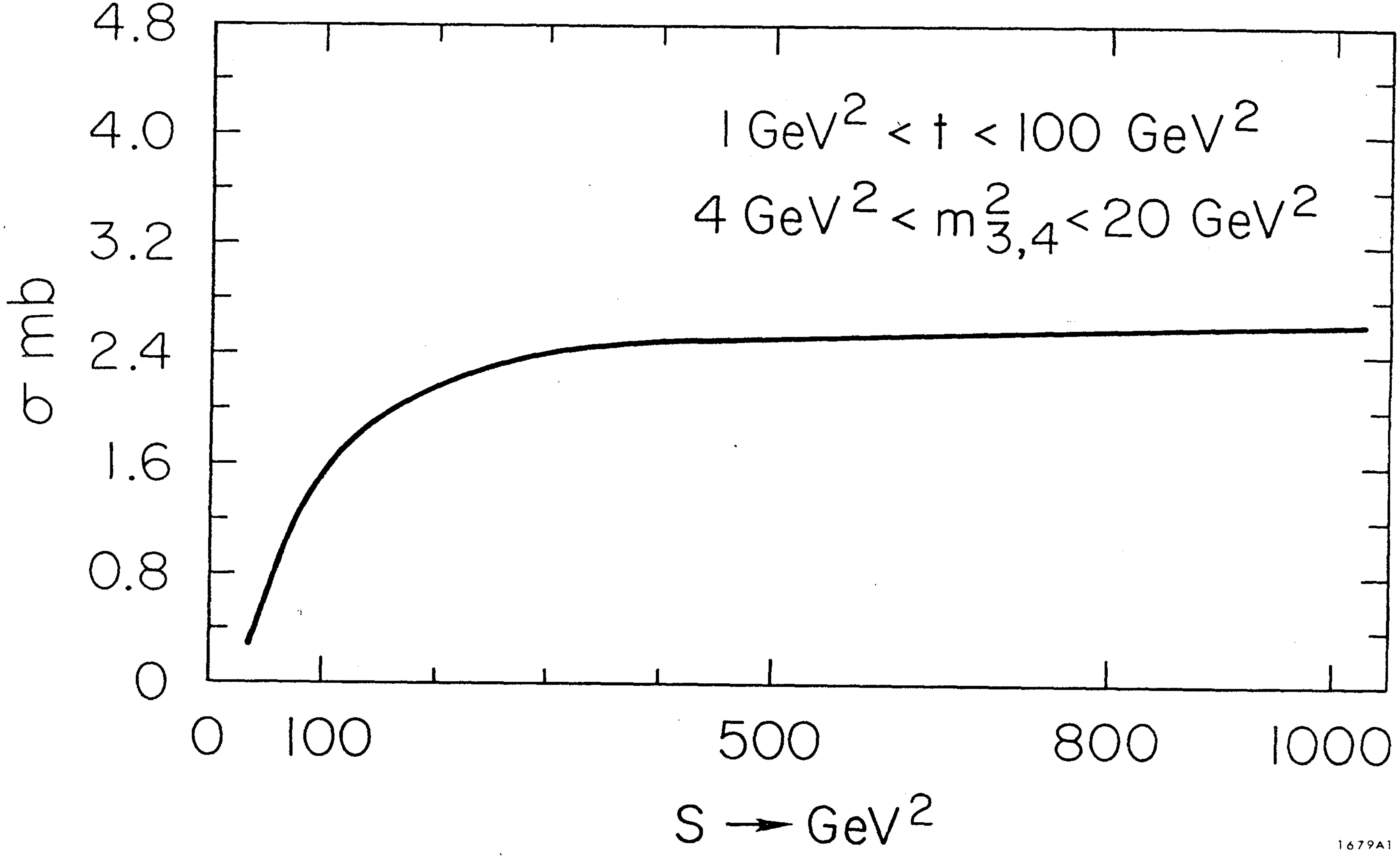


Fig. 1



1 GeV^2 < t < 100 GeV^2
4 GeV^2 < $m^2_{3,4}$ < 20 GeV^2
σ mb
4.8
4.0
3.2
2.4
1.6
0.8
0
0
100
500
800
1000
S → GeV^2
1679A1

Fig. 2