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Connectivity of ad hoc wireless networks: an alternative to graph-theoretic approaches

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Abstract Connectivity in wireless ad hoc and sensor networks is typically analyzed using a graph-theoretic approach. In this paper, we investigate an alternative communication-theoretic approach for determining the minimum transmit power required for achieving connectivity. Our results show that, if there is significant multipath fading and/or multiple access interference in the network, then graph-theoretic approaches can substantially underestimate the minimum transmit power required for connectivity. This is due to the fact that graph-theoretic approaches do not take the route quality into consideration. Therefore, while in scenarios with line-of-sight (LOS) communications a graph-theoretic approach could be adequate for determining the minimum transmit power required for connectivity, in scenarios with strong multipath fading and/or multiple access interference a communication-theoretic approach could yield much more accurate results and, therefore, be preferable.

Keywords Ad hoc wireless networks · Sensor networks · Connectivity · Power control · Transmission range assignment

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1 Introduction

Connectivity is a crucial property in ad hoc wireless networking and graph theory [1–14]. If connectivity is achieved, any pair of nodes in the network can communicate with each other. Network connectivity is tightly coupled with the transmit power of each node: if this power is too low, then the network might be disconnected. While the use of a very high transmit power can make the network connected, it is undesirable because this may create an excessive amount of interference. In addition, using a high transmit power also shortens a node's lifetime. Thus, determining the right amount of transmit power that should be used by each node is an important and challenging task, especially when the topology of the network is random. This paper investigates network connectivity from a novel physical layer-oriented viewpoint [15].

The notion of connectivity commonly used by most researchers is essentially based on graph theory [13]. More specifically, the network is said to be connected if there exists a multi-hop path joining every pair of nodes in the network. This notion of connectivity is perfectly suitable for networks with highly reliable communication links, such as fiber optic networks and the Internet. However, caution should be exercised when using this notion of connectivity for an ad hoc *wireless* network, where the communication channels are error-prone. It is important to realize that although there may be a multi-hop path connecting source and destination, for a given delay requirement, the communication between them may not be possible. In fact, since the wireless links are susceptible to errors, the route bit error rate (BER) deteriorates as the number of hops in the route increases. Consequently, the performance may be unacceptable although there exists a sequence of links from source to destination. In other

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words, a physical layer-oriented quality of service (QoS), in terms of maximum tolerable BER at the end of a multi-hop route, may not be satisfied.

As opposed to following the conventional graph theoretic notion of connectivity, in this paper we investigate a new perspective on network connectivity, where a BER-based QoS at the end of a multi-hop route is considered. It will be shown that this communication-theoretic notion of connectivity provides a more complete viewpoint than the graph-theoretic one, as the characteristics of the wireless communication channel are also taken into consideration. It not only indicates that there are paths between source and destination, but also specifies whether these paths are of good quality. We point out that a similar physical layer-oriented notion of connectivity is also considered in our earlier works [16, 17]. While the main focus in [16, 17] is on networks with *regular topology*, in this paper we extend our approach to networks with *random* topology, and we explicitly (for the first time, to the best of our knowledge) compare communication-theoretic and graph-theoretic approaches.

Whether nodes randomly deployed over a geographical area will be able to form a single connected network depends on their transmit powers. The relationship among transmit power, interference, and transmission distance is explained in detail in [18, 19]. Ideally, each node may use a different transmit power, which may be adjusted on a link-by-link basis. However, due to the absence of a central controller in an ad hoc wireless network with flat architecture, performing power control on a link-by-link basis is a complicated and cumbersome task. A simpler solution, which is more viable for implementation, is to have all nodes use a *common* transmit power. This is desirable in sensor networks where nodes are relatively simple and it is difficult to adjust the transmit power after deployment. In this paper, we determine the minimum common transmit power required in order to achieve and maintain network connectivity from a communication-theoretic perspective, and we discuss its relation with the value predicted by a graph-theoretic approach. In order to validate our analytical results, we also present NS-2-based simulation results on the connectivity of realistic Zigbee wireless networks.

The rest of this paper is organized as follows. Details about the network model are given in Sect. 2. The notions of connectivity, from graph-theoretic and communication-theoretic perspectives, are described in Sects. 3 and 4, respectively. The minimum transmit power for connectivity is analyzed in Sect. 5. Numerical results are presented and discussed in Sect. 6. In Sect. 7, we provide a discussion of the global implication of our results. In Sect. 8, we comment on related work. Finally, conclusions are given in Sect. 9.

2 Network model

We consider a scenario where N nodes are randomly and uniformly distributed over a surface with area A. The location of each node is independent of those of all the other nodes, as commonly assumed by many researchers for studying the connectivity problem [6, 7]. The ratio N/A is defined as the node spatial density and denoted by ρ_s (units: m^{-2}). With this model, for large N and A, the number of nodes n_a in a given area a can be approximated with a two-dimensional Poisson distribution [20]. The probability mass function (PMF) of the number of nodes is given by

$$\Pr(n_a = j) = \frac{(\rho_s a)^j}{j!} e^{-\rho_s a}; \quad j = 0, 1, 2, \ldots \tag{1}$$

All nodes in the network are assumed to have the same transmission range, denoted by r. This is typical for a network where devices use the same transmit power level (e.g., most commercial wireless cards have a single transmit power level [21]). As a result, any pair of nodes can directly communicate with each other if the distance between them is shorter than r. In wireless networks, having a direct link between a transmitter and a receiver means that the signal-to-noise ratio (SNR) at the receiver is above a required threshold. This implies that the BER is lower than a maximum tolerable value (e.g., 10^{-3}).

3 Connectivity from a graph-theoretic perspective

In this section, we briefly discuss network connectivity from a graph-theoretic perspective. Most of the analysis presented in this section is based on [6]. According to graph theory, a network is connected if, in the corresponding graph (with branches associated with existing links), there exists a multi-hop path from any node to any other node. A connected graph has only a single component (i.e., no isolated nodes or disconnected clusters). Therefore, a necessary (but not sufficient) condition for connectivity is that there is no isolated node in the network. In other words, for a network to be connected, each node must have at least one neighbor within its transmission range r, and the probability of this event can be computed using (1). Let U be the number of neighbors within the transmission range of a node. The probability that a node has at least one neighbor can be written as

$$\Pr(U \geq 1) = 1 - \Pr(U = 0) = 1 - e^{-\rho_s \pi r^2}. \tag{2}$$

The network will have no isolated node if *all* nodes have at least one neighbor. Since the location of each node is independent of those of the others, the probability that every node has at least one neighbor can be approximated as [6]

$$q \triangleq \Pr(\text{no isolated nodes}) \approx \left(1 - e^{-\rho_s \pi r^2}\right)^N. \quad (3)$$

This probability is computed on the basis of the implicit assumption that a generic link between any two nodes directly connected is independent of any other link in the network. In general, there is correlation among links, i.e., links cannot be treated as independent. As a limiting example, in a scenario with $N = 2$ nodes, if one node is connected, then the other automatically is. More specifically, one can show that the probability of having an isolated node tends to decrease as more nodes are being added to the network. The expression in (3) does not take this effect into account, so the actual probability of having *no* isolated node in the network will be higher than what is predicted by expression (3). In other words, the probability of network connectedness computed as in (3) is pessimistic.

Let r_0 be the minimum required transmission range such that the network will have no isolated node with probability q. From (3), r_0 can be expressed as

$$r_0 \approx \sqrt{\frac{-\ln(1 - q^{1/N})}{\rho_s \pi}}. \quad (4)$$

On the basis of the considerations carried out in the previous paragraph, the actual transmission range predicted by a more refined graph-theoretic approach would be even shorter than r_0. However, in [6, 22] it is shown, by analysis and simulations, that r_0 is a good approximation for the actual critical transmission range.

Typically, it is assumed that if each node adjusts the transmit power so that its transmission range is at least r_0, then network connectivity is achieved. From a graph-theoretic perspective, this is correct in the sense that the network graph is connected. In other words, there is a path connecting any node to any other node in the network. However, selecting the transmit power so that a node can communicate with a neighbor within the range r_0 may not be sufficient for an ad hoc wireless network scenario, as this would make the BER quality acceptable only for *one link*. In other words, it does not guarantee that the errors, accumulated in each link of the route before the data are received by the final destination, are still kept at an acceptable level. Since the wireless communication channels are error-prone, the QoS, in terms of route BER, degrades rapidly as the number of links that an information bit traverses increases. Thus, for multi-hop ad hoc wireless networks, it is important to take the end-to-end QoS into account. In the next section, we present a new perspective on connectivity, where the characteristics of wireless channels and the BER-based QoS at the end of a multi-hop route are taken into consideration.

4 Connectivity from a communication-theoretic perspective

Having routes connecting every source/destination pair is desirable; however, having routes with good communication quality is also very important. Motivated by this observation, we introduce a new notion of connectivity where the quality of the routes is also taken into consideration. In particular, a network is said to be n-hop-connected if the QoS condition, in terms of route BER at the end of a generic n-hop route, can be guaranteed. Since the network connectivity is defined in terms of route BER quality, in the following subsections we first derive expressions for route and link BERs.

4.1 Route BER

In this subsection, we derive an expression for the average route BER. Let r be the transmission range of each node. For a network to be n-hop-connected, the average route BER of every n-hop route, including the worst-case route[1] where every hop in the route has the maximum length r, must meet a desired quality. In this analysis, we will evaluate the average route BER of the *worst-case* n-hop route. The average route BER of a generic n-hop route, where some hops may be shorter than r, is expected to be better than that of the worst-case route.

Given the hop length, the propagation pathloss, and the transmit power, the received signal power observed at the receiver on each link of the route can be determined [23]. However, in the presence of multiple access interference, the amount of the interference experienced by the receiver at the end of each link will vary, depending on (i) the number of interfering nodes, (ii) their activity and (iii) their positions with respect to the receiver. Consequently, the SNR and the BER at the receiving node of each link of the route will differ, because of the varying (temporally and spatially) interference signals. Since the positions of nodes in a 2-dimensional Poisson topology are homogeneous (neglecting the border effects), the positions of the interfering nodes around a receiver in any selected area are statistically the same. In addition, assuming that every node has a similar packet transmission rate, the amount of interference experienced by a receiver would also be

[1] The worst case is obviously given by a multi-hop route where all links have the maximum length, since the attenuation in each link is highest [23].

statistically the same. More details on the interference will be given in the next subsection.

Assuming that all the links of the route are independent and a bit detected erroneously at the end of a link is not corrected in the successive links (these are valid assumptions for practical link BER values [15]), the average route BER of the worst-case n-hop route can be written as

$$\overline{\mathrm{BER}}_{\mathrm{R}} = 1 - \left(1 - \overline{\mathrm{BER}}_{\mathrm{L}}\right)^n \tag{5}$$

where $\overline{\mathrm{BER}}_{\mathrm{L}}$ is the average BER at the end of a link with hop length equal to the transmission range r. The link BER is evaluated in the following subsection for various communication scenarios.

4.2 Link BER

In this subsection, we analyze the average link BER (i) in an ideal scenario, where there is no multiple access interference, and (ii) in a realistic scenario, where multiple access interference is present. In each of the scenarios, the cases with (a) strong LOS and (b) multipath Rayleigh fading will be considered. For simplicity, we assume that binary phase shift keying (BPSK) is the used modulation format. However, the analysis can be extended to other types of modulation formats as well, as will be shown in Sect. 6.3 in a scenario with M-ary quadrature amplitude modulation (MQAM). We also assume a *free space* propagation pathloss model where the pathloss exponent is equal to 2, but the analytical approach can be straightforwardly extended to other propagation models. In other words, we assume that the received power at distance r can be written as:

$$P_{\mathrm{r}} = \frac{\alpha P_{\mathrm{t}}}{r^2} \tag{6}$$

where

$$\alpha \triangleq \frac{G_{\mathrm{t}} G_{\mathrm{r}} c^2}{(4\pi)^2 f_{\mathrm{c}}^2}$$

P_{t} is the transmit power, G_{t} and G_{r} are the transmitter and receiver antenna gains, f_{c} is the carrier frequency, and c is the speed of light. In this paper, we assume that the antennas at the nodes are omnidirectional ($G_{\mathrm{t}} = G_{\mathrm{r}} = 1$), and the carrier frequency is in the unlicensed 2.4 GHz band.

4.2.1 *Ideal scenario (no interference)*

The ideal scenario without interference could correspond to the case where a medium access control (MAC) protocol is effective at suppressing the interference noise (e.g., use of code division multiple access, CDMA, with perfectly orthogonal spreading codes). Since there is no multiple access interference, we assume that the only source of noise is the Gaussian thermal noise. In the case with strong LOS and no multipath fading, one can show that $\overline{\mathrm{BER}}_{\mathrm{L}}$ can be expressed as [24]

$$\overline{\mathrm{BER}}_{\mathrm{L}}^{\mathrm{LOS}} = Q\left(\sqrt{\frac{2\alpha P_{\mathrm{t}}/r^2}{P_{\mathrm{therm}}}}\right) \tag{7}$$

where $Q(x) \triangleq \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \mathrm{d}u$ is the standard Q-function, $P_{\mathrm{therm}} = F\,k\,T_0\,R_{\mathrm{b}}$ is the thermal noise power, F is the noise figure, $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann's constant, T_0 is the room temperature ($T_0 = 300$ K), and R_{b} is the transmission data-rate.

In the case with multipath Rayleigh fading and perfectly coherent demodulation, it can be shown that the average link BER can be expressed as [24]

$$\overline{\mathrm{BER}}_{\mathrm{L}}^{\mathrm{Ray}} = \frac{1}{2}\left(1 - \sqrt{\frac{2\sigma_{\mathrm{f}}^2 \alpha P_{\mathrm{t}}/r^2}{P_{\mathrm{therm}} + 2\sigma_{\mathrm{f}}^2 \alpha P_{\mathrm{t}}/r^2}}\right). \tag{8}$$

where $2\sigma_{\mathrm{f}}^2$ is the mean square value of the fading coefficient with the following Rayleigh probability density function (PDF):

$$f_X(x) = \begin{cases} \frac{x}{\sigma_{\mathrm{f}}^2} e^{-\frac{x^2}{2\sigma_{\mathrm{f}}^2}} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The value of σ_{f}^2 directly affects the average power of the received signal. Basically, the SNR increases as the value of σ_{f}^2 increases [24].

4.2.2 *Realistic scenario (with interference)*

To evaluate the impact of multiple access interference, we analyze the network in its "saturated" state, i.e., in the steady-state regime where nodes always have packets to send. The amount of interference depends on the MAC protocol in use. For simplicity, we consider a very simple random access MAC protocol where each node transmits packets according to a Poisson transmission distribution. In other words, the time interval between two consecutive packet transmissions is exponentially distributed with mean $1/\lambda$, where λ is an average packet transmission rate (units: pck/s). The non-constant time interval between consecutive packet transmissions can be regarded as an interference reduction mechanism, which is commonly used in many random access MAC protocols (e.g., a random backoff time is used before each packet transmission in the IEEE 802.11 MAC protocol to avoid collisions [25]). Scheduling-based MAC protocols, such as opportunistic scheduling, are not considered in this paper, but their use may lead to an interesting extension of the current work [26].

Let I be a random variable denoting the multiple access interference power. Knowledge of the PDF of I is required in order to evaluate the exact average link BER. However, it is analytically difficult to obtain the exact expression of the PDF of I, and a closed-form expression may not exist. As a result, we use an approximation which allows one to evaluate the average link BER more easily. According to this approximation, we assume that the average link BER can be expressed as a function of the *average* interference power. The accuracy of this approximation will be validated with simulations.

The average interference power affecting a communication link can be evaluated as follows. In order to keep the interference analysis manageable, we first characterize the area within which the interfering nodes contribute a significant amount of interference to the receiver. Normally, the interfering range of a node is longer than its transmission range. In this paper, as typically done in a simulation study, we assume the interfering range to be approximately twice that of the transmission range [27].[2] Therefore, we define the *effective interference region* to be the circular area with radius $2r$ centered at the receiver. A pictorial example of the effective interference region is shown in Fig. 1. Let Z be a random variable denoting the distance between an interfering node and the receiver, and let $f_Z(z)$ be the PDF of Z. The average interference power from a single interfering node can be written as[3]

$$\mathbb{E}[P_{\text{int}}^{\text{single}}] = \int_0^1 \alpha P_t f_Z(z)\mathrm{d}z + \int_1^{2r} \frac{\alpha P_t}{z^2} f_Z(z)\mathrm{d}z. \quad (10)$$

According to the considered 2-dimensional Poisson node distribution, it can be shown that the PDF of Z is

$$f_Z(z) = \begin{cases} \frac{z}{2r^2} & 0 < z \leq 2r \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

Substituting (11) back into (10), it follows that the average interference power from a single interfering node is

$$\mathbb{E}[P_{\text{int}}^{\text{single}}] = \frac{\alpha P_t}{2r^2}\left[\frac{1}{2} + \ln(2r)\right]. \quad (12)$$

Taking multiple interfering nodes into consideration and assuming that all nodes interfere (i) *independently* from each other[4] and (ii) with the same *average* power, the total average interference power can be written as

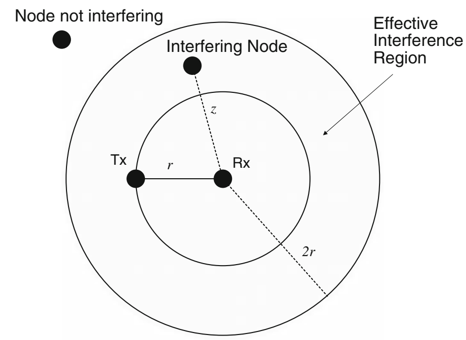

Fig. 1 Effective interference region for node Rx: it corresponds to the circle with radius $2r$ centered at node Rx

$$\mathbb{E}[P_{\text{int}}^{\text{total}}] = \mathbb{E}\left[\sum_{i=1}^{N_{\text{int}}} \mathbb{E}[P_{\text{int}-i}^{\text{single}}]\right] = \mathbb{E}[N_{\text{int}}]\mathbb{E}[P_{\text{int}}^{\text{single}}] \quad (13)$$

where $\mathbb{E}[P_{\text{int}-i}^{\text{single}}]$ is the average interference power from the ith node and $\mathbb{E}[N_{\text{int}}]$ is the average number of interfering nodes in the effective interference region. Given that there are at least 2 nodes in the effective interference region (e.g., at least the transmitter and the receiver), the average number of *interfering nodes* can be obtained by subtracting 2 from the average total number of nodes in the effective interference region. It can be shown that the average number of *interfering nodes* is

$$\mathbb{E}[N_{\text{int}}] \approx 4\pi r^2 \rho_s - 2 \quad (14)$$

where the quantity $4\pi r^2 \rho_s$ corresponds to the average total number of nodes in the effective interference region—in this case $4\pi r^2 \rho_s \geq 2$, since we assume that there are at least 2 nodes in the effective interference region.[5]

The total average interference power in (13) is computed on the basis of the assumption that all the potential interfering nodes transmit simultaneously. However, with the considered random access MAC protocol, only a fraction of the nodes will transmit at the same time. In fact, there is a probability associated with each packet transmission. Considering the previously introduced random access MAC protocol with Poisson-distributed transmission, a bit currently transmitted over a link will be interfered by the transmission from another node if the latter starts its transmission at any point within a *vulnerable interval* with length equal to L/R_b (units: s) prior to the current time instant, where L is the number of bits in a packet. In a scenario with the considered Poisson-distributed transmission scheme, it can be shown that the probability that a bit currently transmitted over a link will be interfered by another node is

[2] This assumption of interfering range is for analytical purposes only. A more conservative assumption, such that the interfering range is longer than twice the transmission range, can also be used.

[3] In computing $\mathbb{E}[P_{\text{int}}^{\text{single}}]$, for $z < 1$, we assume that the interference power is αP_t as opposed to $\alpha P_t / z^2$; otherwise, the interference power will be amplified and not attenuated.

[4] This is true for the considered random access MAC protocol because it does not use carrier-sensing and each node adds an independent random backoff time before transmitting a packet.

[5] See Appendix 1 for more details.

$$p = 1 - e^{-\frac{\lambda L}{R_b}} \tag{15}$$

where L is the packet length (in bits) and R_b is the data rate (units: bit/s). Taking this interference probability into account, the *effective* average interference power can be written as

$$\mathbb{E}[P_{int}^{eff}] = p\mathbb{E}\left[P_{int}^{total}\right] = p\mathbb{E}[N_{int}]\mathbb{E}[P_{int}^{single}]. \tag{16}$$

Note that the way the effective interference is computed is slightly different than the approach presented in [14]. In this paper, the effect of the interference reduction mechanism in the MAC protocol is also taken into account. This makes the communication possible even in an environment with a small pathloss exponent (e.g., 2). This effective average interference power will be used to approximately compute the average link BER.

Our prior work suggests that the average link BER, in the case with strong LOS and no multipath fading, reaches a floor for increasing values of the node spatial density [15]. Above this route BER floor, i.e., for sufficiently low node spatial densities, it can be shown that the interference power I is accurately modeled by a Gaussian distribution [15]. This leads to the following accurate approximation for the average link BER:

$$\overline{\mathrm{BER}}_L^{LOS} \approx \max\left\{Q\left(\sqrt{\frac{2\alpha P_t/r^2}{P_{therm} + \beta_{LOS}\mathbb{E}[P_{int}^{eff}]}}\right), \mathrm{BER}_{floor}^{LOS}\right\} \tag{17}$$

where

$$\mathrm{BER}_{floor}^{LOS} \approx \frac{p}{8}\mathbb{E}[N_{int}] \tag{18}$$

and β_{LOS} is a constant. After validating this result with various simulation scenarios, we found that setting $\beta_{LOS} = 0.4$ yields a good approximation. The derivation of expression (18) for $\mathrm{BER}_{floor}^{LOS}$ can be found in Appendix 1. Observe that when $p = 0$, the expression in (17) corresponds to the case with no interference and, as expected, reduces to that given by (7). To ensure that the average link BER computed from the average interference power is accurate, we verify it with a Monte Carlo simulation in [28]. It is shown that the simulation results and the analytical results are in good agreement.

In the case with Rayleigh fading and interference, by applying the Gaussian approximation for the interference noise, it can be shown that the average link BER has the same formal expression as in the case without interference, the only modification consisting in properly taking into account the average interference power. By careful analysis and validation with simulations, in the case with Rayleigh fading and interference, one can show (we omit the details due to lack of space) that the following expression is a good approximation of the average link BER:

$$\overline{\mathrm{BER}}_L^{Ray} \approx \frac{1}{2}\left(1 - \sqrt{\frac{\kappa}{P_{therm} + 2\sigma_f^2\beta_{Ray}\mathbb{E}[P_{int}^{eff}] + \kappa}}\right) \tag{19}$$

where

$$\kappa = 2\sigma_f^2\alpha P_t/r^2 \tag{20}$$

$$\beta_{Ray} = \frac{\left(1 - 2\mathrm{BER}_{floor}^{Ray}\right)^{-2} - 1}{\frac{1}{2}\left(\frac{1}{2} + \ln 2r\right)\mathbb{E}[N_{int}]p} \tag{21}$$

and

$$\mathrm{BER}_{floor}^{Ray} \approx \frac{p}{8}\mathbb{E}[N_{int}]\left[\frac{1}{1+r^2} + \ln\left(\frac{5r^2}{1+r^2}\right)\right]. \tag{22}$$

The derivation of $\mathrm{BER}_{floor}^{Ray}$ can be found in Appendix 2. Observe that when $p = 0$, the link BER expression (19) corresponds to that in the case with no interference and coincides with (8). The validation of this link BER expression can be found in [28].

4.3 Connectivity conditions

For an ad hoc wireless network to be n-hop-connected, the average route BER of every n-hop route must be lower than the desired threshold. Let BER_R^{req} be the maximum tolerable route BER. The network will be n-hop-connected if the average route BER of every n-hop route is lower than BER_R^{req}, which means that the following condition must be satisfied:

$$\overline{\mathrm{BER}}_R \le \mathrm{BER}_R^{req}. \tag{23}$$

To guarantee that the average route BER of an n-hop route is below the desired threshold BER_R^{req}, from (5) it follows that the required average link BER must be lower than the following critical value:

$$\mathrm{BER}_L^{req} = 1 - (1 - \mathrm{BER}_R^{req})^{1/n}. \tag{24}$$

In other words, in order to make the network n-hop-connected with the required quality, the average link BER of a receiver within the critical transmission range r_0 of the transmitter must be lower than BER_L^{req}. In the case with strong LOS communications, the following condition must be satisfied:

$$\max\left\{Q\left(\sqrt{\frac{2\alpha P_t/r_0^2}{P_{therm} + \beta_{LOS}\mathbb{E}[P_{int}^{eff}]}}\right), \mathrm{BER}_{floor}^{LOS}\right\} \le \mathrm{BER}_L^{req} \tag{25}$$

whereas in the case with multipath fading, the condition to be satisfied is the following:

$$\frac{1}{2}\left(1-\sqrt{\frac{\kappa}{P_{\text{therm}}+2\sigma_{\text{f}}^2\beta_{\text{Ray}}\mathbb{E}[P_{\text{int}}^{\text{eff}}]+\kappa}}\right)\leq \text{BER}_{\text{L}}^{\text{req}} \qquad (26)$$

where $\mathbb{E}[P_{\text{int}}^{\text{eff}}]$ must be computed by substituting r_0 for r in (12) and (16).

5 Minimum transmit power for n-hop connectivity

On the basis of the connectivity concepts discussed in the previous sections, from both graph-theoretic and communication-theoretic viewpoints, it is possible to determine the minimum transmit power required to keep the network connected with the desired quality. The following requirements specify how the transmit power should be set so that n-hop connectivity can be achieved.

- Graph-theoretic viewpoint: the transmit power should be high enough so that nodes at the critical transmission range for network connectivity, i.e., at distance r_0, can be reached.
- Communication-theoretic viewpoint:
  - in a scenario with strong LOS, the transmit power must be sufficiently high for the connectivity condition given by (25) to be satisfied;
  - in a scenario with multipath fading, the transmit power must be sufficiently high for the connectivity condition given by (26) to be satisfied.

More specifically, in the case with strong LOS communications, the maximum tolerable link BER, i.e., $\text{BER}_{\text{L}}^{\text{req}}$, needs to be higher than the link BER floor $\text{BER}_{\text{floor}}^{\text{LOS}}$; otherwise, the connectivity condition cannot be satisfied, regardless of the value of the transmit power. If $\text{BER}_{\text{L}}^{\text{req}}$ is higher than $\text{BER}_{\text{floor}}^{\text{LOS}}$, it can be shown that the minimum transmit power required to satisfy the connectivity condition given by (25) is

$$P_{\text{t}}^{\text{min,LOS}}(n)=\frac{P_{\text{therm}}r_0^2}{2\alpha}\left\{\left[Q^{-1}(\text{BER}_{\text{L}}^{\text{req}})\right]^{-2}-\frac{1}{4}\left(\frac{1}{2}+\ln 2r_0\right)\beta_{\text{LOS}}p\mathbb{E}[N_{\text{int}}]\right\}^{-1} \qquad (27)$$

where n is the number of hops in a route and $\text{BER}_{\text{L}}^{\text{req}}$ depends on n and $\text{BER}_{\text{R}}^{\text{req}}$ through (24).

Similarly, in the case with Rayleigh fading, the minimum transmit power required to satisfy the connectivity condition given by (26) is

$$P_{\text{t}}^{\text{min,Ray}}(n)=\frac{P_{\text{therm}}r_0^2}{2\sigma_{\text{f}}^2\alpha}\left[\left(1-2\text{BER}_{\text{L}}^{\text{req}}\right)^{-2}-\left(1-2\text{BER}_{\text{floor}}^{\text{Ray}}\right)^{-2}\right]^{-1}. \qquad (28)$$

For the ideal scenarios without interference, the minimum transmit powers in the strong LOS and multipath fading cases can also be obtained from (27) and (28), respectively, by simply setting the interference probability p to zero.

6 Results

Numerical results, along with their implications, are presented and discussed in this section. In all the considered scenarios, the main network parameter values used in the analysis are shown in Table 1, unless stated otherwise.

6.1 Average number of hops

The expressions for the minimum transmit powers obtained in Sect. 5 are functions of the number of hops n in a communication route. Thus, given n, the minimum transmit power required for n-hop connectivity can be determined. The next logical question that a network designer might ask is the following: what would be an appropriate value of n to be considered for a network with N nodes? In other words, given a network with N nodes, what level of connectivity should be imposed? This would certainly depend on the operative network requirements. For example, to guarantee that all routes between every source/destination pair meet the desired quality level, a very stringent requirement given by n_{max}-hop connectivity, where n_{max} is the maximum number of hops between any source/destination pair, should be imposed. To assess the connectivity in an *average sense*, we propose to use an average number of hops, denoted by n_{avg}. Given n_{avg}, one can evaluate the corresponding minimum transmit power according to (27) or (28). In the remainder of this section, we estimate the average number of hops of a multi-hop route in a network with N nodes. In order to make a fair comparison between graph-theoretic and communication-theoretic approaches,

Table 1 Major network parameters used in the considered scenarios

Parameters	Value
Number of nodes (N)	100 nodes
Connectivity probability (q)	0.999
Desired route BER ($\text{BER}_{\text{R}}^{\text{req}}$)	10^{-3}
Packet length (L)	10^3 bits
Data-rate (R_{b})	10 Mb/s
Packet transmission rate (λ)	0.1 pck/s
Fading parameter (σ_{f}^2)	1
Carrier frequency (f_{c})	2.4 GHz
Room temperature (T_0)	300 K
Noise figure (F)	6 dB

we consider a simple graph-theoretic perspective for the identification of the average number of hops. The obtained average value, verified through Monte Carlo simulations, will be used also in the derivation of the minimum transmit power according to a communication-theoretic viewpoint.

The number of hops in a route between a source node and a destination node will depend on the routing protocol. One of the most common routing schemes used in ad hoc wireless networks is the shortest path routing scheme, according to which a route with the smallest number of hops between source and destination is preferred [29]. Assuming a shortest path routing scheme, the average number of hops can be estimated with the following approximation—we emphasize that this is only an approximation and not the exact computation. Consider a scenario where N nodes are distributed over a square of size $R \times R$, and the transmission range of each node is r_0. Intuitively, with a shortest path routing strategy, we expect the average number of hops to be proportional to the average route distance (not necessarily a straight line) for a random source/destination pair. The average route distance should be a function of R and, thus, the average number of hops between source and destination should be a function of R/r_0. By experimenting with various functions, our results show that the following heuristic approximation for the average number of hops is accurate:

$$n_{\mathrm{avg}} \approx \frac{\sqrt{2}R}{2r_0} \approx \sqrt{\frac{N\pi}{-2\ln(1-q^{1/N})}}. \tag{29}$$

To verify the accuracy of the approximation in (29), we validate it with simulations. In the simulations, we randomly place N nodes in a square of size $R \times R$. The transmission range of each node is set to r_0. After a specific network topology is formed, we find the shortest path for each source/destination pair. Then, the number of hops in the routes between all source/destination pairs are collected, and their arithmetic average is computed. This average number of hops is shown, as a function of N, in Fig. 2. More precisely, in this figure the average number of hops obtained from the approximation in (29) and the average number of hops obtained from the simulations are compared. The 95% confidence interval associated with each simulation point is also shown. It can be observed that there is a very good agreement between the heuristic approximation and the simulation results.

6.2 Impact of multipath fading

In Fig. 3, the minimum transmit power required for n-hop connectivity is shown as a function of the node spatial density. Three different values for n, as shown in the figure, are considered. The minimum transmit powers in the strong LOS scenario and in the scenario with Rayleigh fading are compared. In the scenario with strong LOS, it can be observed that the minimum transmit powers required for supporting n-hop connectivity in the three cases (i.e., $n = 1, 4, 8$) are not significantly different. This is due to the fact that all links are characterized by strong LOS communications. On the other hand, in the scenario with Rayleigh fading, the minimum transmit power required for n-hop connectivity varies significantly for different values of n. Considering the cases with $n = 1$ and $n = 8$, it can be observed that the minimum transmit power values in the two cases differ by more than one order of magnitude.

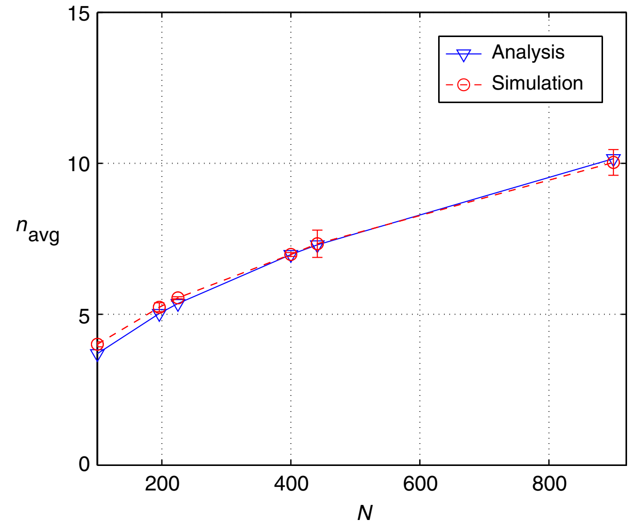

Fig. 2 The average number of hops as a function of N. The results obtained from the heuristic approximation and those obtained from the simulations are compared

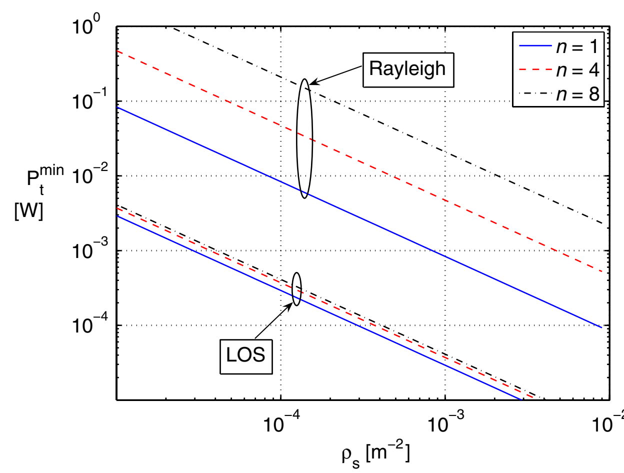

Fig. 3 Minimum transmit power as a function of the node spatial density. Scenarios with strong LOS and with Rayleigh fading, respectively, are compared

A 1-hop-connected network (i.e., the case with $n = 1$) represents a limiting scenario where there are paths connecting pairs of nodes, but the desired quality can be guaranteed only for 1-hop routes (i.e., direct communications). In this situation, the quality of data sent to destinations through routes formed by more than one hop will not meet the desired level. Since it is evident that the minimum transmit power required in a case with $n > 1$ is substantially higher than that in the case with $n = 1$, this confirms that setting the transmit power (or, equivalently, the transmission range) without taking the quality of multi-hop routes into consideration could be misleading. In addition, comparing, for a given value of n, the minimum transmit power in the strong LOS case with that in the Rayleigh fading case, it is clear that multipath fading is a major factor affecting the minimum transmit power required for connectivity. In the presence of multipath fading, a much higher transmit power is required (compared to the case without fading) to achieve network connectivity. In some cases, the difference between the minimum transmit powers required for connectivity in scenarios with and without fading can be as large as 3–4 orders of magnitude. Thus, if the multipath fading can be mitigated, the transmit power required to make the network connected will decrease substantially, and a considerable amount of energy will be saved at the nodes. This, in turn, will prolong the lifetime of the nodes and, consequently, of the network.

The reason why an increase in the number of hops has only a slight impact on the minimum transmit power in the strong LOS case, but has a significant impact on the minimum transmit power in the multipath fading case, can also be justified analytically. Note that the term which is directly affected by n in (27) and (28) is $\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}$. In the case with strong LOS, corresponding to Eq. 27, $\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}$ is an argument of the inverse Q-function and, thus, does not significantly affect the terms in the outermost bracket as n increases. On the contrary, in the case with multipath fading corresponding to (28), the terms in the square bracket are much more sensitive to the value of n.

In Fig. 4, the minimum transmit power for n_{avg}-hop connectivity is shown, as a function of the node spatial density, in a scenario with multipath fading considering various values of σ_{f}^2. Note that $\sqrt{2}\sigma_{\mathrm{f}}$ is the root mean square (RMS) of the received signal power. The BER improves as the value of σ_{f}^2 increases—*provided that* there is coherent detection at the receiving node of each link. As a result, the minimum transmit power required for n_{avg}-hop connectivity decreases as the RMS value increases. In other words, as the severity of fading reduces, the minimum transmit power also decreases. These results further confirm that multipath fading is a key factor affecting the minimum transmit power required for connectivity, and a substantial energy saving at each node can be achieved if the fading can be mitigated.

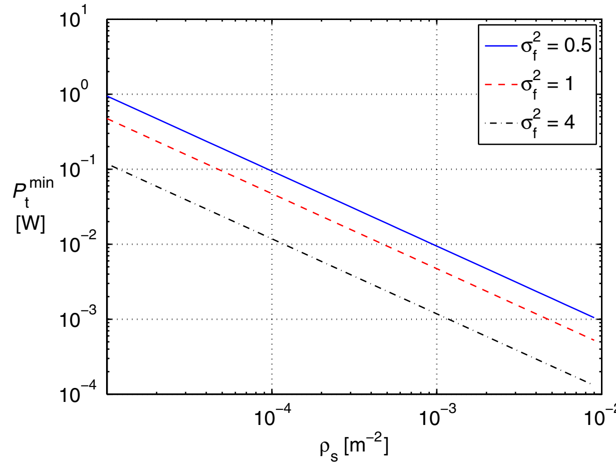

Fig. 4 Minimum transmit power required for n_{avg}-hop connectivity as a function of node spatial density in the scenario with Rayleigh fading. Three values of σ_{f}^2 are considered

6.3 Impact of high-order modulations

A possible extension of the proposed framework consists in evaluating the minimum transmit power required to guarantee connectivity in the presence of high-order modulations. As an illustrative example, we consider MQAM. In [30, 31], it is shown that the link BER for an AWGN (i.e., strong LOS) channel with MQAM and ideal coherent phase detection can be approximated as follows:[6]

$$\overline{\mathrm{BER}}_{\mathrm{L}}^{\mathrm{LOS,MQAM}} \approx a_1 \exp\left(-\frac{a_2}{M-1}\overline{\mathrm{SNR}}_{\mathrm{L}}\right) \qquad (30)$$

where $a_1 = 0.2$ and $a_2 = 1.5$. Expression (30) holds for $M \geq 4$ and $0\,\mathrm{dB} < \overline{\mathrm{SNR}}_{\mathrm{L}} < 30\,\mathrm{dB}$, and the specific expression of the average link SNR depends on the presence/absence of multi-access interference.

We now derive the minimum required transmit power with MQAM in a scenario without multi-access interference. Since the average link SNR has the following expression:

$$\overline{\mathrm{SNR}}_{\mathrm{L}} = \frac{\alpha P_{\mathrm{t}}}{r^2}.$$

Setting $r = r_0$, from (30) it is straightforward to show that the minimum required transmit power to guarantee a link BER equal to $\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}$ has the following expression:

$$P_{\mathrm{t}}^{\mathrm{min,LOS,MAQM}}(n) \approx \frac{(M-1)P_{\mathrm{therm}} r_0^2}{a_2 \alpha} \ln \frac{a_1}{\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}} \qquad (31)$$

where $\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}$ is related to $\mathrm{BER}_{\mathrm{R}}^{\mathrm{req}}$ and n as shown in (24).

[6] In [30, Eq. (17)], it is shown that expression (30) is a tight upper bound for the exact link BER.

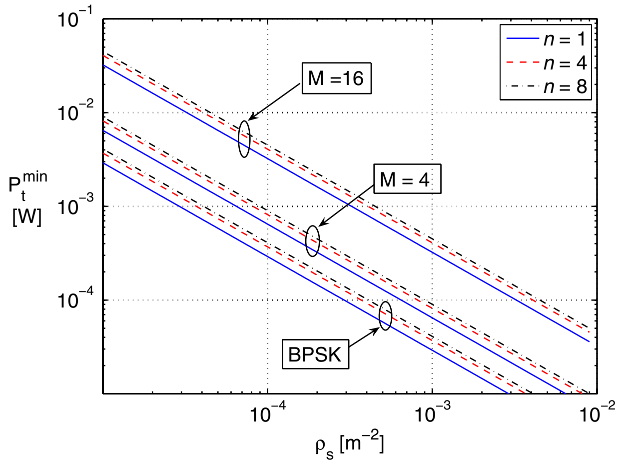

Fig. 5 Minimum transmit power as a function of the node spatial density in the presence of MQAM, considering $M = 4$ and $M = 16$, respectively. For comparison, the minimum transmit power for BPSK is also shown. The strong LOS is assumed in all scenarios

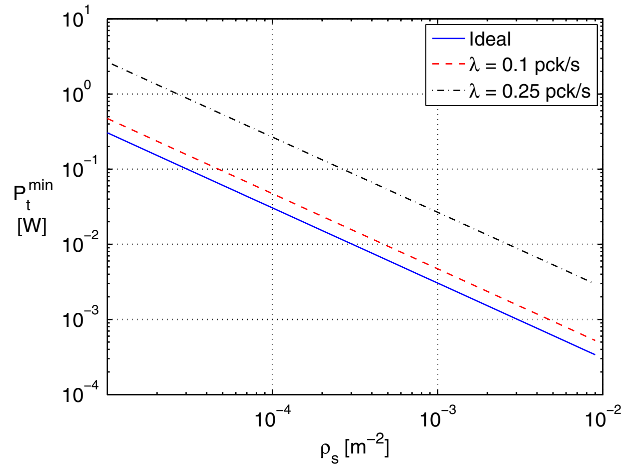

Fig. 6 Minimum transmit power required for n_{avg}-hop connectivity, as a function of the node spatial density, in a scenario with Rayleigh fading. The ideal case without interference (*solid line*) and realistic cases with interference (*dashed* and *dotted lines*) are compared

From (31), one can see that the minimum required transmit power is proportional to the cardinality of the modulation format. This is intuitive: in fact, the higher the modulation cardinality, the lower its robustness against noise. Therefore, in order to meet a desired quality of service (in terms of link BER), the transmit power has to be increased. In Fig. 5, the minimum transmit power required for n-hop connectivity is shown, as a function of the node spatial density, in the cases with $M = 4$ (4-QAM) and $M = 16$ (16-QAM). For comparison, the minimum required transmit power in the BPSK scheme is also shown. The strong LOS is assumed in all scenarios. Three different values for n, as shown in the figure, are considered for each modulation format. As discussed above, increasing the value of M requires a higher transmit power in order to guarantee the same connectivity level. However, as already observed for BPSK, also in the presence of MQAM the impact of n is, for a given value of M, minor.

6.4 Impact of multiple access interference

In Fig. 6, the minimum transmit power for n_{avg}-hop connectivity is shown, as a function of the node spatial density, in a scenario with multipath fading. The value of the minimum transmit power in the ideal (no interference) and in realistic (with interference) cases (considering $\lambda = 0.1$ pck/s and $\lambda = 0.25$ pck/s, respectively) are compared. A higher packet transmission rate corresponds to a higher amount of interference. In general, it can be observed that as the amount of interference increases, the minimum transmit power required for connectivity also increases. However, it is worth mentioning that increasing the transmit power cannot always combat the interference and make the network connected. In fact, there is a limiting value of the packet transmission rate, denoted by λ^*, beyond which the connectivity cannot be achieved *regardless* of the transmit power. This critical packet transmission rate corresponds to the value of λ that makes the BER floor higher than the required link BER quality, i.e., the maximum tolerable link BER. More precisely, in the case with Rayleigh fading, the critical packet transmission rate is such that $\mathrm{BER}_{\mathrm{floor}}^{\mathrm{Ray}} = \mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}$. Substituting $\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}$ for $\mathrm{BER}_{\mathrm{floor}}^{\mathrm{Ray}}$ in (22), using a first order Taylor series expansion centered in zero[7] for the expression of p, as a function of $\lambda L/R_{\mathrm{b}}$, in (15), i.e., $p \approx \frac{\lambda L}{R_{\mathrm{b}}}$, the critical transmission rate in the case with Rayleigh fading can be expressed as

$$\lambda_{\mathrm{Ray}}^* \approx \frac{8R_{\mathrm{b}}\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}}{\mathbb{E}[N_{\mathrm{int}}]\left[\frac{1}{1+r_0^2} + \ln\left(\frac{5r_0^2}{1+r_0^2}\right)\right]L}. \tag{32}$$

If every node transmits at a packet transmission rate higher than λ_{Ray}^*, then network connectivity can never be achieved. The critical packet transmission rate can also be regarded as the maximum traffic load that the network can handle while maintaining its connectivity. Knowing the critical transmission rate helps a network designer determine an appropriate parameter for the MAC protocol. In the case with the simple random access MAC protocol considered in this paper, for example, this corresponds to making sure that the average packet transmission rate of each node is below the critical value, or making sure that the mean time between consecutive packet transmissions is longer than $1/\lambda_{\mathrm{Ray}}^*$.

[7] The first order Taylor series expansion is motivated by the fact that in typical operative conditions it holds that $p \ll 1$, i.e., $\frac{\lambda L}{R_{\mathrm{b}}} \ll 1$.

Similarly, substituting $\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}$ for $\mathrm{BER}_{\mathrm{floor}}^{\mathrm{LOS}}$ in (18), the critical packet transmission rate in a scenario with strong LOS communications can be expressed as

$$\lambda_{\mathrm{LOS}}^{*} \approx \frac{8R_{\mathrm{b}}\mathrm{BER}_{\mathrm{L}}^{\mathrm{req}}}{\mathbb{E}[N_{\mathrm{int}}]L}. \quad (33)$$

6.5 Impact of data-rate

In Fig. 7, the minimum transmit power for n_{avg}-hop connectivity is shown, as a function of the data-rate, in a scenario with multipath fading. It can be observed that there is an *optimal* data-rate at which the minimum transmit power required for connectivity is minimized. This can be explained as follows. At low data-rates, it takes a long time to transmit a packet. Consequently, the vulnerable interval for which the packet transmission will be interfered is large, and the minimum transmit power required to keep the network connected must be high in order to combat the interference. As the data-rate increases, the vulnerable interval becomes shorter, and the probability that the packet transmission will be interfered becomes lower. Thus, the minimum transmit power decreases as the data-rate increases. However, it must be noted that even though the interference is reduced, the thermal noise power increases as the the data-rate increases. Hence, the minimum transmit power starts increasing again for data-rate values beyond the optimal data-rate. This suggests that the data-rate also plays an important role in the design of wireless ad hoc and sensor networks—that is, for a given node spatial density, if the data-rate is carefully chosen, the transmit power can be minimized, prolonging the network lifetime.

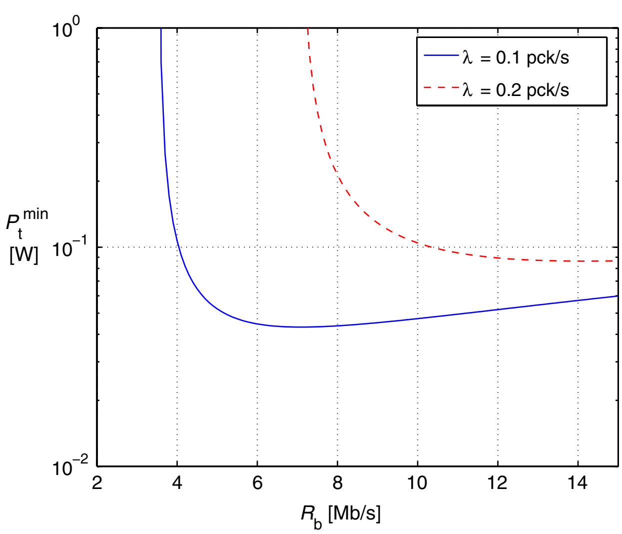

Fig. 7 Minimum transmit power required for n_{avg}-hop connectivity as a function of the data-rate in a scenario with Rayleigh fading. The considered node spatial density is $\rho_{\mathrm{s}} = 10^{-4}\mathrm{m}^{-2}$

In addition, it can be observed that there is a critical data-rate, below which the connectivity cannot be sustained regardless of the transmit power. This is the data-rate at which the minimum transmit power becomes infinitely large and corresponds to the situation where the corresponding route BER floor becomes higher than the maximum tolerable route BER. Our results also show that the critical data-rate increases as the packet transmission rate increases.

6.6 Tradeoff between transmit power and delay

The minimum transmit power for network connectivity presented in this paper is computed under the assumption that there is no packet retransmission in any link of the route. In some data networking applications (such as delay tolerant networks), where delay is not a major concern, packet retransmission may also be allowed. Allowing retransmission can decrease the minimum transmit power required to keep the network connected because retransmission improves the packet error rate (PER). In other words, with retransmission, the desired PER or BER can be achieved with lower transmit power.

We now compare the minimum transmit power predicted by our model, in the case where retransmissions are allowed, with that predicted by the graph-theoretic approach. Recall that in a graph-theoretic approach, a critical transmission range r_0 for network connectivity is estimated, and the minimum transmit power for connectivity is determined by finding the lowest transmit power that makes the SNR at the critical transmission range r_0 higher than the required threshold. It is typically assumed that a node can receive a packet "properly" if the received signal power at the critical transmission range r_0 is higher than a specified receiver sensitivity [6]. However, important channel impairments such as interference and multipath fading are not taken into account for determining the minimum transmit power. In addition, it is assumed that if a particular transmit power value is sufficient to meet the quality requirement on a single link, then there is no problem for the entire route (i.e., packets can still be delivered to the final destination with an acceptable quality). This tacitly underlines either of the following assumptions: (i) a packet can be retransmitted as many times as needed to guarantee the acceptable BER or PER quality; or (ii) a powerful error correction code is employed. Both of these two strategies increase the transmission delay.

Consequently, one may argue that, depending on the types of applications, if a sufficiently large number of retransmissions is allowed, the minimum transmit power determined by a graph-theoretic approach still allows a packet to be sent across an n-hop route with the desired quality (e.g., packets received at the destination still satisfy the desired BER). However, we argue that even though retransmission is allowed, using only a graph model, *without taking into account interference and fading, still underestimates the minimum transmit power required for connectivity*. In the following, we will compare the minimum transmit power predicted by the graph-theoretic approach and the minimum transmit power obtained from our approach when the same number of retransmissions per link is considered (i.e., comparing them at the same transmission delay).

For comparison, we consider a scenario where the number of nodes is $N = 100$, the node spatial density is $\rho_s = 10^{-4}\mathrm{m}^{-2}$, the connectivity probability is $q = 0.999$, the packet size is $L = 1000$ bits, and the data rate is 10 Mb/s. In this scenario, the critical transmission range is $r_0 = 191.42$ m, and the average number of hops is $n_{\mathrm{avg}} = 4$ hops. The minimum transmit power predicted by the graph-theoretic model corresponds to that required in the ideal scenario (with strong LOS communications and without interference and multipath fading) for achieving 1-hop connectivity in our model. Assuming that the required BER quality is 10^{-3}, the corresponding minimum transmit power in this case is $P_t = 0.3$ mW. With this transmit power, the desired route BER quality of 10^{-3} for an n_{avg}-hop route can be achieved if, on average, 3 retransmissions per link are allowed. To make a fair comparison, we use our model to compute the minimum transmit power in realistic scenarios with interference and multipath fading, assuming that 3 retransmissions per link are allowed. In each scenario, the packet transmission rate is $\lambda = 0.1$ pck/s. The considered values of σ_f^2 are 1, 4, and 6.25, respectively. The minimum transmit power required in each scenario is shown in Table 2.

In general, it can be observed that although packet retransmission is considered, the minimum transmit power predicted by our model is still substantially higher than that predicted by the graph-theoretic approach. This is due to the fact that our model takes realistic channel impairments such as interference and multipath fading into consideration, whereas these factors are ignored in the conventional graph-theoretic approach. The difference between the minimum transmit powers predicted by the two approaches depend on the severity of interference and fading. As observed from Table 2, the difference between the minimum transmit power predicted by the graph-theoretic approach and that predicted by our approach increases as the severity of fading increases (i.e., as the value of σ_f^2 decreases).

Table 2 Comparison between the minimum transmit power required for connectivity predicted by a graph-theoretic approach and a communication-theoretic approach with different levels of multipath fading

Scenario	$P_t^{\min}$ (mW)
Graph-theoretic approach	0.3
Multipath fading with $\sigma_f^2 = 1$	7.1
Multipath fading with $\sigma_f^2 = 4$	1.8
Multipath fading with $\sigma_f^2 = 6.25$	1.1

6.7 Experiments on realistic network scenarios

To gain more insights and to further support the main conclusion that connectivity of an ad hoc wireless network should be analyzed from a communication-theoretic viewpoint, we perform simulations of a realistic Zigbee (with the bottom two layers adhering to the IEEE 802.15.4 standard) wireless network [32] using NS-2 [33]. In our simulation scenario, there are N nodes uniformly distributed in a 50 m $\times$ 50 m area. The network is a peer-to-peer wireless sensor network using a non-slotted carrier sense multiple access with collision avoidance (CSMA/CA) MAC protocol without the use of acknowledgment messages. The transmission and carrier sensing ranges of all nodes are set to 100 m, the packet size is 64 bytes, the packet header is 13 bytes, and the data rate is 250 kbps. In our simulation scenarios, we randomly select $\lceil N/3 \rceil$ multi-hop paths with a priori selected number of hops.

In Fig. 8(a), (b), the packet error rate and the delay, respectively, are shown as functions of the number of hops in the routes. The number of nodes N is set to either 20 (the corresponding node spatial density ρ_S is $8 \times 10^{-3}\mathrm{m}^{-2}$) or 40 ($\rho_S = 1.6 \times 10^{-2}\mathrm{m}^{-2}$). For each number of nodes, two values for the packet generation rate λ are considered: 2 and 4 pck/s. It can be observed that as the number of hops and/or the packet generation rate increase (i.e., the level of interference increases), the packet error rate becomes higher. On the other hand, the delay seems to depend marginally on the packet generation rate, but it increases for increasing number of nodes. Note that the networks, in the simulated scenarios, have connectivity. Therefore, from a graph-theoretic viewpoint, any pairs of nodes should be able to communicate. If, however, instead of single-hop communications the routing strategy leads to the selection of multi-hop routes, then the packet error rate becomes intolerable as the number of hops in the routes increases. In other words, although there is a multi-hop path connecting a pair of nodes, the communication between the two nodes becomes ineffective. Clearly, this further supports the main point of the paper.

7 Discussion

While the numerical results in Sect. 6 are based on the specific MAC protocol chosen in this paper, it is important

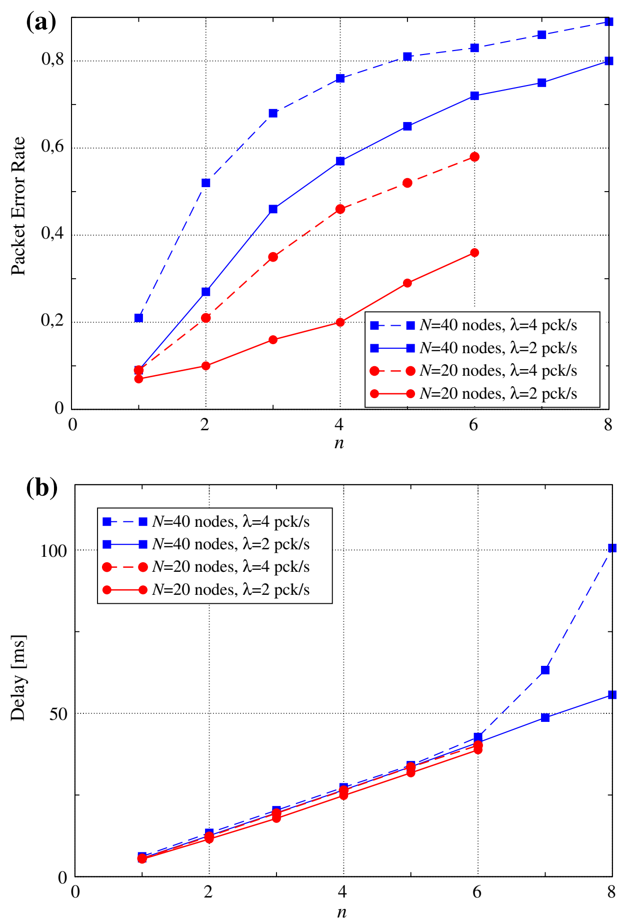

Fig. 8 Performance, in terms of **a** packet error rate and **b** delay, as a function of number of hops. The results shown are obtained through NS-2-based simulations

to understand that the key conclusion of this paper will hold irrespective of the MAC protocol used. In other words, connectivity of wireless ad hoc and sensor networks using any MAC protocol can be determined more accurately by the communication-theoretic framework presented in this paper as opposed to the graph-theoretic approaches used by several previous studies. Of course, while the core of the communication-theoretic framework will remain the same, each MAC protocol calls for a different interference analysis which must be carefully performed to determine the exact impact of interference on connectivity.

In addition, our study shows that, for given data-rate and desired route BER quality, there exists an *optimal data-rate* which minimizes the minimum transmit power required for connectivity. This suggests that the data-rate also plays an important role in the design of wireless ad hoc and sensor networks. In other words, for a given node spatial density, if the data-rate is carefully chosen, the transmit power can be minimized, prolonging the network lifetime.

Moreover, our results show that multipath fading has a significant impact on the minimum transmit power required for network connectivity. If the multipath fading can be mitigated, the minimum transmit power required for connectivity will substantially decrease. This observation suggests that a key property which should be possessed by a MAC protocol designed for ad hoc wireless networks is the ability to mitigate multipath fading. Therefore, code division multiple access (CDMA) becomes an attractive option [34–36], since it can cope with multipath fading through the use of RAKE receivers [37].

Finally, our model has not taken other important physical effects such as shadow fading into consideration yet. The shadow fading will cause the transmission range to be random [38]. To incorporate the shadow fading into our model, one would need to model the received signal stochastically. This is an interesting extension to our current model. However, we believe that the effects of random shadow fading will further support the main conclusion presented in this paper. That is, a graph-theoretic approach alone is not sufficient in analyzing the connectivity of an ad hoc wireless network. To analyze the network connectivity more accurately, one would need to consider a more complete approach such as our communication-theoretic model.

8 Related work

Selection of the "optimal" transmit power and the range assignment problem in wireless ad hoc and sensor networks have been studied in the past [39]. In earlier works [40–42], an optimal transmission range, which maximizes a one-hop forward progress toward the direction of the destination, is investigated. However, the transmission range which makes the network connected is not the focus of these studies. The transmission range required for network connectivity has been later explored in [1–10], where a Poisson point process is used to model the distribution of nodes in an ad hoc wireless network. In addition, in most of these works it is usually assumed that nodes use a *common* transmit power level. In [1], network connectivity is studied in the context of broadcast percolation. A stochastic model for broadcast percolation is proposed, and the effects of node spatial density and transmission range on the number of broadcast cycles are investigated. However, an analytical expression for the critical transmission range for network connectivity is not given. In [2, 3], network connectivity is studied as a covering problem, and analytical expressions for the minimum transmission range required for network connectivity are derived. The connectivity probability of a *one-dimensional* ad hoc network is studied in [4, 5]. In [6], an expression for the critical transmission range that makes the network connected is derived for a two-dimensional network. The critical range assignment problem for higher dimension networks is analyzed in [7, 8]. In [9], a critical transmission range is studied on the basis of a Bernoulli model, where nodes are active with a certain probability. The critical transmission range for network connectivity, in a scenario where nodes move according to the random waypoint mobility model, is studied in [10].

There are also studies where the number of neighbors is used as an indicator of network connectivity [11, 43, 44]. In these works, it is typically suggested that the transmit power should be adjusted so that each node has the required number of neighbors.

Although the transmit power and the transmission range for connectivity have been studied extensively, this has been mainly done *on the basis of a graph-theoretic notion of connectivity*. More specifically, physical layer characteristics of wireless channels, such as *multiple access interference and multipath fading, are not taken into account* in existing works.

9 Conclusions

In this paper, it is shown that connectivity of wireless ad hoc and sensor networks can be more accurately determined via a communication-theoretic approach as opposed to the conventional graph-theoretic approaches that are prevalent in the literature. While the foregoing conclusion is valid for any MAC protocol, the discrepancy between the predictions of the former and the latter approaches becomes large when the network has to operate under strong multipath fading and multiple access interference conditions. In particular, our results show that in wireless ad hoc and sensor networks operating under strong multipath fading and multiple access interference conditions, the graph theoretic-approaches may grossly underestimate the transmit power requirements for full connectivity. This, in turn, suggests the use of the communication-theoretic approach outlined in this paper as a serious alternative to graph-theoretic approaches for determining the connectivity requirements of wireless ad hoc networks in realistic scenarios.

Acknowledgment We would like to thank Stefano Busanelli (University of Parma, Italy) for helping us in obtaining the NS-2-based simulation results presented in Sect. 6.

Appendix 1: Derivation of the BER Floor for the Strong LOS Case

Consider the link between the transmitter (node Tx) and the receiver (node Rx) shown in Fig. 9. For simplicity, we will

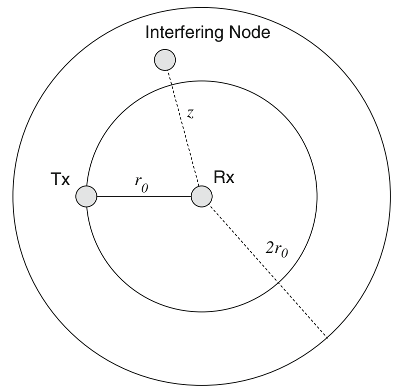

Fig. 9 A scenario where there is only one potential interfering node in the effective interference region

first consider the scenario where there is only one interfering node within the effective interference region, which is the circular area of radius $2r_0$ around the receiver. We will also assume that $r_0 \geq 1$, which will be true for all the scenarios considered in this paper. The amplitude of the observable signal can generally be written as

$$S_r = S_{sig} + S_i + W_{therm}$$

where S_{sig} is the amplitude of the signal transmitted by node Tx, S_i is the amplitude of the signal from the interfering node, and W_{therm} is the amplitude of the additive white Gaussian noise (AWGN) with variance P_{therm}.

Assuming free space propagation loss, the received signal power observed at the receiver can be written as

$$P_r = \frac{\alpha P_t}{r_0^2}$$

where $\alpha = \frac{G_t G_r c^2}{(4\pi)^2 f_c^2}$, G_t and G_r are the transmitter and the receiver antenna gains, f_c is the carrier frequency, c is the speed of light, P_t is the transmit power, and r_0 is the distance between the transmitter and the receiver. Assuming BPSK modulation, the amplitude of the signal transmitted by node Tx observed at the receiver can then be written as

$$S_{sig} = \begin{array}{ll} \sqrt{\alpha E_b}/r_0 & \text{if } +1 \text{ is transmitted} \\ -\sqrt{\alpha E_b}/r_0 & \text{if } -1 \text{ is transmitted} \end{array}$$

where $\sqrt{E_b} \triangleq \sqrt{P_t/R_b}$ is the transmit bit energy and R_b is the data-rate. Let z be the distance between the interfering node and the receiver. Similarly, the amplitude of the interfering signal can be written as

$$S_i = \begin{cases} \sqrt{\alpha E_b}/z & \text{if } +1 \text{is transmitted} \\ -\sqrt{\alpha E_b}/z & \text{if } -1 \text{is transmitted} \\ 0 & \text{if the interfering node does not transmit.} \end{cases} \quad (34)$$

Assuming a simple random access MAC protocol with Poisson transmission, the probability that the interfering node will interfere with an ongoing transmission can be given as

$$p = 1 - e^{-\frac{\lambda L}{R_b}} \tag{35}$$

where λ is the packet transmission rate in pck/s and L is the number of bits in a packet. From (34) and (35), the probability mass function (PMF) of the amplitude of the interfering signal can then be given as

$$P\{S_i\} = \begin{cases} \frac{1}{2}p & \text{if} \quad S_i = \sqrt{\alpha E_b}/z \\ \frac{1}{2}p & \text{if} \quad S_i = -\sqrt{\alpha E_b}/z \\ 1-p & \text{if} \quad S_i = 0. \end{cases}$$

Since we are considering a binary modulation, due to symmetry we can assume that node Tx transmits "+1" (i.e., $S_{sig} = \sqrt{\alpha E_b}/r_0$). Given that there is only one potential interfering node, the bit error probability can be expressed as follows [28]:

$$\begin{aligned} &P\{\text{bit error}|1 \text{ interfering node at distance } z\} \\ &\quad = P\{S_r < 0 | S_i = \sqrt{\alpha E_b}/z\} P\{S_i = \sqrt{\alpha E_b}/z\} \\ &\qquad + P\{S_r < 0 | S_i = -\sqrt{\alpha E_b}/z\} P\{S_i = -\sqrt{\alpha E_b}/z\} \\ &\qquad + P\{S_r < 0 | S_i = 0\} P\{S_i = 0\} \\ &\quad = \frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} + \frac{1}{z}\right)\right) + \frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} - \frac{1}{z}\right)\right) \\ &\qquad + (1-p) Q\left(\frac{\sqrt{\alpha E_b}}{\sigma r_0}\right) \end{aligned} \tag{36}$$

where $\sigma = \sqrt{FkT_0/2}$, F is the noise figure, $k = 1.38 \times 10^{-23}$J/K, $T_0 = 300$ K is the room temperature, and $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \mathrm{d}u$ is the standard Q-function.

As observed from the simulation results, at a particular node spatial density, if the transmit power is high enough, the link BER converges to a BER floor. Thus, to derive the BER floor, we take the limit, as E_b approaches ∞, of the conditional link BER in (36), obtaining:

$$\begin{aligned} &\lim_{E_b \to \infty} P\{\text{bit error}|1 \text{ interfering node at distance } z\} \\ &\quad = \lim_{E_b \to \infty} \left[\frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} + \frac{1}{z}\right)\right) + \frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} - \frac{1}{z}\right)\right) \right. \\ &\qquad \left. + (1-p) Q\left(\frac{\sqrt{\alpha E_b}}{\sigma r_0}\right)\right] \\ &\quad = \lim_{E_b \to \infty} \frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} - \frac{1}{z}\right)\right) \end{aligned} \tag{37}$$

where in the last passage we have used the fact that $Q(+\infty) = 0$. Note that the argument of the Q-function in the last line in (37) can either be positive or negative, depending on the value of z. To find the total bit error probability, we have to consider all possible values of z. With a straightforward algebra, it can be shown that the PDF of z is

$$f_Z(z) = \frac{z}{2r_0^2} \quad 0 \le z \le 2r_0.$$

Given that there is only one potential interfering node, the link BER floor can then be written as[8]

$$\begin{aligned} &\{\text{BER}_{\text{floor}}^{\text{LOS}} | 1 \text{ interfering node}\} \\ &= \int \lim_{E_b \to \infty} P\{\text{bit error}|1 \text{interf. node at distance } z\} f_Z(z) \mathrm{d}z \\ &= \int_0^1 \lim_{E_b \to \infty} \frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} - 1\right)\right) \frac{z}{2r_0^2} \mathrm{d}z \\ &\quad + \int_1^{2r0} \lim_{E_b \to \infty} \frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} - \frac{1}{z}\right)\right) \frac{z}{2r_0^2} \mathrm{d}z \\ &= \int_0^1 \frac{p}{2} Q(-\infty) \frac{z}{2r_0^2} \mathrm{d}z + \int_1^{r_0} \frac{p}{2} Q(-\infty) \frac{z}{2r_0^2} \mathrm{d}z \\ &\quad + \int_{r_0}^{2r_0} \frac{p}{2} Q(+\infty) \frac{z}{2r_0^2} \mathrm{d}z = \frac{p}{8}. \end{aligned}$$

Now, let us consider the case where there are two potential interfering nodes within the effective interference region, as shown in Fig. 10. Let the interfering node 1 be at distance z_1 from to the receiver and let the interfering node 2 be at distance z_2 from the receiver. Following the same approach as in the case with a single interfering node, the conditional link BER floor, given z_1 and z_2, can be written as

$$\begin{aligned} &\lim_{E_b \to \infty} P\{\text{bit error}|2 \text{ interf. nodes at distances } z_1 \text{ and } z_2\} \\ &= \lim_{E_b \to \infty} \left[\left(\frac{p}{2}\right)^2 \sum_{i,j=\pm 1} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} + \frac{i}{z_1} + \frac{j}{z_2}\right)\right) \right. \\ &\quad + \frac{p}{2}(1-p) \sum_{i=\pm 1} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} + \frac{i}{z_1}\right)\right) \\ &\quad + \frac{p}{2}(1-p) \sum_{j=\pm 1} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0} + \frac{j}{z_2}\right)\right) \\ &\quad \left. + (1-p)^2 Q\left(\frac{\sqrt{\alpha E_b}}{\sigma r_0}\right)\right]. \end{aligned}$$

For small values of p (as in operative conditions with the used MAC protocol), the terms with coefficient p^2 can be neglected, and the conditional link BER floor can be approximated as

[8] For $z < 1$, we assume that the interference power is αP_t and not $\alpha P_t/z^2$; otherwise, the interference power will be amplified as opposed to be attenuated.

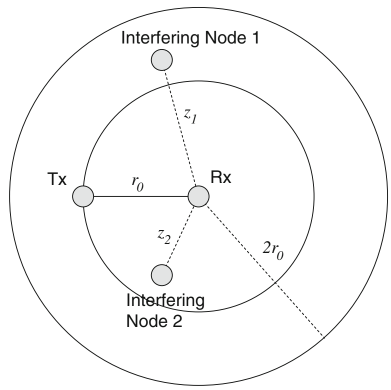

Fig. 10 A scenario where there are two potential interfering nodes in the effective interference region

$$\lim_{E_b\to\infty} P\{\text{bit error}|2\text{ interf. nodes at distances } z_1 \text{ and } z_2\}$$

$$\approx \lim_{E_b\to\infty} \frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0}-\frac{1}{z_1}\right)\right) + \lim_{E_b\to\infty} \frac{p}{2} Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{1}{r_0}-\frac{1}{z_2}\right)\right). \quad (38)$$

To find the average BER floor with respect to all possible positions of the pairs of interfering nodes, one can integrate the expression in (38) by weighing it with the joint PDF of z_1 and z_2. With straightforward algebra, the link BER floor, given that there are two potential interfering nodes, can be approximated as

$$\{\text{BER}_{\text{floor}}^{\text{LOS}}|2\text{ interfering nodes}\} \approx 2\times\frac{p}{8}.$$

Similarly, for the scenarios with m interfering nodes, the BER floor can be approximated as

$$\{\text{BER}_{\text{floor}}^{\text{LOS}}|m\text{ interfering nodes}\} \approx m\times\frac{p}{8}.$$

While the observed derivation leads to the computation of the BER floor for a given number of interferers, in reality, however, the number of potential interfering nodes in the effective interference region is random. Let Y be a random variable denoting the number of nodes in the effective interference region, and let M be the number of *interfering* nodes. Excluding the transmitter and the receiver, the number of potential interfering nodes is $M = Y - 2$ (given that $Y \geq 2$), and the PMF of M is

$$P\{M=m\} = \frac{1}{1-e^{-\rho_s\nu}-\rho_s\nu e^{-\rho_s\nu}} \frac{(-\rho_s\nu)^{(m+2)}}{(m+2)!} e^{-\rho_s\nu}$$
$$m = 0, 1, 2,$$

where $\nu = \pi(2r_0)^2$ is the area of the effective interference region. Finally, the overall average BER floor becomes

$$\text{BER}_{\text{floor}}^{\text{LOS}} \approx \sum_{m=0}^{\infty}\frac{p}{8} m P\{M=m\} = \frac{p}{8}\mathbb{E}[M] = \frac{p}{8}\left(\frac{\rho_s\nu + 2 + (\rho_s\nu - 2)e^{\rho_s\nu}}{e^{\rho_s\nu}-\rho_s\nu-1}\right). \quad (39)$$

The link BER floor given in (39) can be further simplified. Since $P\{Y=0\}$ and $P\{Y=1\}$ are typically very small with the network sizes of interest, $\mathbb{E}[M]$ is very close to $\mathbb{E}[Y]-2$, where $\mathbb{E}[Y] = \rho_s\nu$. It can be shown that $\mathbb{E}[M]$ and $\mathbb{E}[Y]-2$ are almost identical [28]. Thus, the BER floor can be approximated as

$$\text{BER}_{\text{floor}}^{\text{LOS}} \approx \frac{p}{8}(\rho_s\nu - 2). \quad (40)$$

The validity of the approximate expression (40) has been verified through Monte Carlo simulations [28].

Appendix 2: Derivation of the BER floor in the Rayleigh fading case

In this appendix, we focus on the case with multipath fading. Consider the scenario with only one interfering node in the effective interference region, as shown in Fig. 9. With multipath fading, the amplitude of the observed signal can generally be written as

$$S_r = X_s S_{sig} + X_i S_i + W_{therm}$$

where X_s and X_i are Rayleigh distributed random variables. The bit error probability given that $X_s = x_s$, $X_i = x_i$, and the interfering node is at distance z relative to the receiver can be written as

$$\begin{aligned} &P\{\text{bit error}|X_s=x_s, X_i=x_i, Z=z\} \\ &= P\{S_r<0|S_i=\sqrt{\alpha E_b}/z\}P\{S_i=\sqrt{\alpha E_b}/z\} \\ &\quad + P\{S_r<0|S_i=-\sqrt{\alpha E_b}/z\}P\{S_i=-\sqrt{\alpha E_b}/z\} \\ &\quad + P\{S_r<0|S_i=0\}P\{S_i=0\} \\ &= \frac{p}{2}Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{x_s}{r_0}+\frac{x_i}{z}\right)\right) + \frac{p}{2}Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{x_s}{r_0}-\frac{x_i}{z}\right)\right) \\ &\quad + (1-p)Q\left(\frac{x_s\sqrt{\alpha E_b}}{\sigma r_0}\right). \end{aligned} \quad (41)$$

At any node spatial density, provided that the transmit power is large enough, the link BER converges to a BER floor [28]. Thus, to derive the BER floor, we take the limit, as E_b approaches ∞, of the conditional link BER in (41), obtaining

$$\lim_{E_b\to\infty} P\{\text{bit error}|X_s=x_s, X_i=x_i, Z=z\}$$

$$= \lim_{E_b\to\infty}\left[\frac{p}{2}Q\left(\frac{\sqrt{\alpha E_b}}{\sigma}\left(\frac{x_s}{r_0}+\frac{x_i}{z}\right)\right)\right.$$

$$+\frac{p}{2}Q\left(\frac{\sqrt{\alpha E_{\mathrm{b}}}}{\sigma}\left(\frac{x_{\mathrm{s}}}{r_0}-\frac{x_i}{z}\right)\right)+(1-p)Q\left(\frac{x_{\mathrm{s}}\sqrt{\alpha E_{\mathrm{b}}}}{\sigma r_0}\right)\Bigg]$$
$$=\lim_{E_{\mathrm{b}}\to\infty}\frac{p}{2}Q\left(\frac{\sqrt{\alpha E_{\mathrm{b}}}}{\sigma}\left(\frac{x_{\mathrm{s}}}{r_0}-\frac{x_i}{z}\right)\right) \tag{42}$$

where in the last passage we have used the fact that $Q(+\infty)=0$. Note that the argument of the Q-function in the last line in (42) can either be positive or negative, depending on the values of x_s, x_i, and z. The argument of the Q-function will be negative if $\frac{x_{\mathrm{s}}}{r_0}-\frac{x_i}{z}<0$, or equivalently $x_i > x_{\mathrm{s}}z/r_0$. Thus, the probability can now be written as

$$\lim_{E_{\mathrm{b}}\to\infty} P\{\text{bit error}|Z=z\}$$
$$=\int\int\lim_{E_{\mathrm{b}}\to\infty}\frac{p}{2}Q\left(\frac{\sqrt{\alpha E_{\mathrm{b}}}}{\sigma}\left(\frac{x_{\mathrm{s}}}{r_0}-\frac{x_i}{z}\right)\right)\frac{x_{\mathrm{s}}}{\sigma_{\mathrm{f}}^2}\cdot e^{-x_{\mathrm{s}}^2/2\sigma_{\mathrm{f}}^2}\frac{x_{\mathrm{i}}}{\sigma_{\mathrm{f}}^2}e^{-x_i^2/2\sigma_{\mathrm{f}}^2}\mathrm{d}x_i\mathrm{d}x_{\mathrm{s}}$$
$$=\frac{p}{2}\left(1-\int_0^\infty\int_0^{x_{\mathrm{s}}z/r_0}\frac{x_{\mathrm{s}}}{\sigma_{\mathrm{f}}^2}e^{-x_{\mathrm{s}}^2/2\sigma_{\mathrm{f}}^2}\frac{x_{\mathrm{i}}}{\sigma_{\mathrm{f}}^2}e^{-x_i^2/2\sigma_{\mathrm{f}}^2}\mathrm{d}x_i\mathrm{d}x_{\mathrm{s}}\right)$$
$$=\frac{p}{2}\left(1-\int_0^\infty\frac{x_{\mathrm{s}}}{\sigma_{\mathrm{f}}^2}e^{-x_{\mathrm{s}}^2/2\sigma_{\mathrm{f}}^2}\int_0^{x_{\mathrm{s}}z/r_0}\frac{x_{\mathrm{i}}}{\sigma_{\mathrm{f}}^2}e^{-x_i^2/2\sigma_{\mathrm{f}}^2}\mathrm{d}x_i\mathrm{d}x_{\mathrm{s}}\right)$$
$$=\frac{p}{2}\left(1-\int_0^\infty\frac{x_{\mathrm{s}}}{\sigma_{\mathrm{f}}^2}e^{-x_{\mathrm{s}}^2/2\sigma_{\mathrm{f}}^2}\left[1-e^{-\frac{x_{\mathrm{s}}^2z^2}{2r_0^2\sigma_{\mathrm{f}}^2}}\right]\mathrm{d}x_{\mathrm{s}}\right)$$
$$=\frac{p}{2}\left(1-\frac{z^2}{z^2+r_0^2}\right).$$

Integrating over all possible values of z, one gets

$$\{\mathrm{BER}_{\mathrm{floor}}^{\mathrm{Ray}}|1\text{ interfering node}\}$$
$$=\int\lim_{E_{\mathrm{b}}\to\infty}P\{\text{bit error}|Z=z\}f_Z(z)\mathrm{d}z$$
$$=\int_0^1\frac{p}{2}\left(1-\frac{1}{1+r_0^2}\right)\mathrm{d}z+\int_1^{2r_0}\frac{p}{2}\left(1-\frac{z^2}{z^2+r_0^2}\right)\mathrm{d}z$$
$$=\frac{p}{8(1+r_0^2)}+\frac{p}{8}\ln\left(\frac{5r_0^2}{1+r_0^2}\right)$$
$$=\frac{p}{8}\left[\frac{1}{1+r_0^2}+\ln\left(\frac{5r_0^2}{1+r_0^2}\right)\right].$$

Following the same approach as considered in the case with single interfering node, the BER floor given that there are two interfering nodes and small values of p can be approximated as [28]

$$\{\mathrm{BER}_{\mathrm{floor}}^{\mathrm{Ray}}|2\text{ interfering nodes}\}\approx 2\times\frac{p}{8}\left[\frac{1}{1+r_0^2}+\ln\left(\frac{5r_0^2}{1+r_0^2}\right)\right]$$

and the overall BER floor (without conditioning on the number of interfering nodes) can be approximated as

$$\mathrm{BER}_{\mathrm{floor}}^{\mathrm{Ray}}\approx(\rho_{\mathrm{s}}v-2)\frac{p}{8}\left[\frac{1}{1+r_0^2}+\ln\left(\frac{5r_0^2}{1+r_0^2}\right)\right].$$

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