Consensus Control of a Class of Lipschitz Nonlinear Systems with Input Delay

Chunyan Wang, Zongyu Zuo, *Member, IEEE,* Zongli Lin, *Fellow, IEEE,* and Zhengtao Ding, *Senior Member, IEEE*

***Abstract*—This paper deals with the consensus control design for Lipschitz nonlinear multi-agent systems with input delay. The Artstein-Kwon-Pearson reduction method is employed to deal with the input delay and the integral term that remains in the transformed system is analyzed by using Krasovskii functional. Upon exploring certain features of the Laplacian matrix, sufficient conditions for global stability of the consensus control are identified using Lyapunov method in the time domain. The proposed control only uses relative state information of the agents. The effectiveness of the proposed control design is demonstrated through a simulation study.**

***Index Terms*—Consensus control, Chua Circuit, Multi-agent systems, Input delay, Lipschitz nonlinearity.**

I. Introduction

With the recent advances in measurements and communication, dynamic systems, or agents, are often connected together to achieve specified control tasks. In many applications, the cooperative control of such multi-agent systems is to design a distributed control strategy based on local information that enables all agents to achieve an agreement, or consensus. One significant advance in consensus control is to use tools in graph theory, in particular, Laplacian matrices, to characterize the network connection [1].

Consensus control has attracted a significant attention in recent years. Many theoretic and practical issues have been analyzed and reported in the literature, [1]–[5], to cite a few. Many of the results are based on linear system dynamics. Consensus control design for nonlinear systems is more involved than for the linear systems counterpart. The results on consensus control of nonlinear systems are often restricted to local stability [6], or to certain network connection [7], to certain types of nonlinearities [8]. The obstacle to solving consensus control problem for nonlinear systems stems mainly from certain restrictions the nonlinearity imposes on using the information of the individual systems. This problem has attracted significant attention in the control community, as evidenced by recent publications [9]–[16]. Consensus problems of high-order multi-agent systems with Lipschitz nonlinear dynamics under directed or switching graph are studied in [9]–[12]. Synchronization via pinning control, a related but different problem, on general complex dynamical networks are addressed in [13]. The works [14]–[16] address the consensus output regulation problem of nonlinear multi-agent systems.

Delays are inevitable in physical systems due to the time taken for transportation of materials, transmission of signals etc. The importance of addressing delay has been well recognized for a long time (see [17] and the references therein). A wide variety of predictor-based methods such as Smith predictor [18], modified Smith predictor [19], finite spectrum assignment [20], Artstein-Kwon-Pearson reduction method [21]–[22] and the truncated predictor feedback approach [23] are effective for systems with input delays. Among these tools, the Artstein-Kwon-Pearson reduction method is well-known and relatively straightforward for linear systems with any constant input delay. The stabilization problems are reduced to similar problems for ordinary differential equations.

With the internet and other communication tools used in the consensus control of multi-agent systems, time delay often arises. The network communication delay usually occurs in the consensus control input when the inputs depend on the relative state information transmitted via the network. The existing studies [24]–[28] of consensus with time delay mainly focus on linear systems. However, for nonlinear systems, the state transformation can only apply to the linear parts, and the nonlinear functions remain functions of the original state, which leads to extra complexity in the stability analysis. The consensus analysis for nonlinear systems with delay is even more complicated. The difficulty lies in dealing with the nonlinear term in each agent, which remains a function of the original state after the state transformation. Judicious analysis is needed to tackle the influence of the nonlinear terms under the state transformation. In this paper, we systematically investigate the consensus control problem for multi-agent systems with nonlinearity and input delay. A reduction method is adopted to deal with the input delay in the presence of nonlinearity in the agent dynamics. Further rigorous analysis is carried out to ensure that the extra integral terms of the system state associated with nonlinear functions are properly considered by means of Krasovskii functionals. By transforming the Laplacian matrix into the real Jordan form, global stability analysis is put in the framework of Lyapunov functions in real domain. For the control design, only the relative information obtained via the network connection is used, without local

C. Wang and Z. Ding are with Control Systems Centre, School of Electrical and Electronic Engineering, University of Manchester, Sackville Street Building, Manchester M13 9PL, UK (chunyan.wang-2@postgrad.manchester.ac.uk, zhengtao.ding@manchester.ac.uk).

Z. Zuo is with the Seventh Research Division, and Science and Technology on Aircraft Control Laboratory, Beihang University, Beijing 100191, China (zzybobby@buaa.edu.cn).

Z. Lin is with Charles L., Brown Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA 22904-4743, USA (zl5y@virginia.edu).

Copyright (c) 2015 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending an email to pubs-permissions@ieee.org.

feedback control of the agents. A simulation study is carried out to demonstrate the results obtained in the paper.

The remainder of this paper is organized as follows. Some notations and the problem formulation are given in Section II. Section III presents a few preliminary results for the consensus analysis. Section IV presents the main results on the consensus control design and stability analysis. Simulation results are given in Section V. Section VI concludes the paper.

II. Problem Statement

In this paper, we consider control design for a group of N agents, each represented by a nonlinear subsystem that is subject to input delay and Lipschitz nonlinearity,

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t-h) + \phi(x_i), \tag{1}$$

where for agent i, $i = 1, 2, \dots, N$, $x_i \in \mathbb{R}^n$ is the state vector, $u_i \in \mathbb{R}^m$ is the control input vector, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices with (A, B) being controllable, $h > 0$ is input delay, and the initial conditions $x_i(\theta)$, $\theta \in [-h, 0]$, are given and bounded, and $\phi : \mathbb{R}^n \to \mathbb{R}^n$, $\phi(0) = 0$, is a Lipschitz nonlinear function with a Lipschitz constant γ, i.e., for any two constant vectors a, $b \in \mathbb{R}^n$,

$$\|\phi(a) - \phi(b)\| \le \gamma\|a - b\|.$$

The connections among the agents are specified by a directed graph $\mathcal{G}$, which consists of a set of vertices denoted by $\mathcal{V}$ and a set of edges denoted by $\mathcal{E}$. A vertex represents an agent, and each edge represents a connection. Associated with the graph is its adjacency matrix Q, where element q_{ij} denotes the connection between two agents. More specifically, q_{ij} = 1 if there is a connection from agent j to agent i, and q_{ij} = 0 otherwise. The Laplacian matrix $L = l_{ij}$ is commonly defined by

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{N} q_{ik}, & i = j, \\ -q_{ij}, & i \neq j. \end{cases}$$

From the definition of the Laplacian matrix, it is clear that

$$L\mathbf{1} = 0,$$

where

$$\mathbf{1} = [1, 1, \ \cdots, \ 1]^T \in \mathbb{R}^N,$$

which implies that the Laplacian matrix has 0 as an eigenvalue associated with the right eigenvector $\mathbf{1}$.

Assumption 1: The eigenvalue of the Laplacian matrix at 0 is a single eigenvalue.

This assumption is necessary for solving the consensus problem. In terms of the network collections, this condition implies that the network has a spanning tree that connects any two agents in the system.

The consensus control problem considered in this paper is to design a control strategy, using the relative state information, to ensure that all agents asymptotically converge to an identical trajectory.

III. Preliminary Results

In this section, we present a few preliminary results which are useful for the consensus analysis later in the paper. We first present an overview of the Artstein-Kwon-Pearson reduction method. Consider an input-delayed system

$$\dot{x}(t) = Ax(t) + Bu(t-h) + \phi(x), \tag{2}$$

with $\phi : \mathbb{R}^n \to \mathbb{R}^n$, $\phi(0) = 0$, being a Lipschitz nonlinear function, and the initial conditions $x(\theta)$, $\theta \in [-h, 0]$, being bounded. Let

$$z(t) = x(t) + \int_t^{t+h} \mathrm{e}^{A(t-\tau)} Bu(\tau - h)\mathrm{d}\tau, \tag{3}$$

Differentiating $z(t)$ against time yields

$$\begin{aligned} \dot{z}(t) &= Ax(t) + \phi(x) + \mathrm{e}^{-Ah}Bu(t) \\ &\quad + A\int_t^{t+h} \mathrm{e}^{A(t-\tau)} Bu(\tau - h)\mathrm{d}\tau \\ &= Az(t) + Du(t) + \phi(x), \end{aligned} \tag{4}$$

where $D = \mathrm{e}^{-Ah}B$. When $\phi(x)$ is absent, system (4) is delay free.

Remark 1: It is straightforward to verify that the controllability of (A, B) and $(A, e^{-Ah}B)$ are equivalent. We need additional conditions in the sequel due to the nonlinearity and the consensus requirement.

We consider a controller

$$u(t) = Kz(t). \tag{5}$$

From (3) and (5), we have

$$\|x(t)\| \le \|z(t)\| + h\left(\max_{-h\le\theta\le 0} \|e^{A\theta}\|\right)\|B\|\|K\|\|z_t(\theta)\|,$$

where $z_t(\theta) := z(t+\theta)$, $-h \le \theta \le 0$. Thus, $x(t) \to 0$ as $z(t) \to 0$. In other words, if the controller (5) stabilizes the transformed system (4), then the original system (2) is also stable with the same controller [21].

Remark 2: With any given bounded initial condition $u(\theta)$, $\theta \in [-h, 0]$, a stable feedback controller (5) implies that $u(t)$ in (3) is locally integrable, which allows for the model reduction as (4).

We next recall a lemma from [10].

Lemma 1: For a Laplacian matrix that satisfies Assumption 1, there exists a similarity transformation T, with its first column being $T_1 = \mathbf{1}$, such that

$$T^{-1}LT = J, \tag{6}$$

with J being a block diagonal matrix in the real Jordan form

$$J = \begin{bmatrix} 0 & & & & & & \\ & J_1 & & & & & \\ & & \ddots & & & & \\ & & & J_p & & & \\ & & & & J_{p+1} & & \\ & & & & & \ddots & \\ & & & & & & J_q \end{bmatrix}, \tag{7}$$

where $J_k \in \mathbb{R}^{n_k}$, $k = 1, 2, \dots, p$, are the Jordan blocks for real eigenvalues $\lambda_k > 0$ with the multiplicity n_k in the form

$$J_k = \begin{bmatrix} \lambda_k & 1 & & & \\ & \lambda_k & 1 & & \\ & & \ddots & \ddots & \\ & & & \lambda_k & 1 \\ & & & & \lambda_k \end{bmatrix},$$

and $J_k \in \mathbb{R}^{2n_k}$, $k = p+1, p+2, \dots, q$, are the Jordan blocks for conjugate eigenvalues $\alpha_k \pm j\beta_k$, $\alpha_k > 0$ and $\beta_k > 0$, with the multiplicity n_k in the form

$$J_k = \begin{bmatrix} \nu(\alpha_k,\beta_k) & I_2 & & & \\ & \nu(\alpha_k,\beta_k) & I_2 & & \\ & & \ddots & \ddots & \\ & & & \nu(\alpha_k,\beta_k) & I_2 \\ & & & & \nu(\alpha_k,\beta_k) \end{bmatrix},$$

with I_2 being the identity matrix in $\mathbb{R}^{2\times 2}$ and

$$\nu(\alpha_k,\beta_k) = \begin{bmatrix} \alpha_i & \beta_i \\ -\beta_i & \alpha_i \end{bmatrix} \in \mathbb{R}^{2\times 2}.$$

We also need the following lemmas from [29].

Lemma 2: For a positive definite matrix P, and a function $x : [a, b] \to \mathbb{R}^n$, with $a, b \in \mathbb{R}$ and $b > a$, the following inequality holds:

$$\begin{aligned} &\left(\int_a^b x^T(\tau)d\tau\right) P \left(\int_a^b x(\tau)d\tau\right) \\ \le&(b-a)\int_a^b x^T(\tau)Px(\tau)d\tau. \end{aligned} \tag{8}$$

Lemma 3: For a positive definite matrix P, the following identity holds

$$\mathrm{e}^{A^T t}P\mathrm{e}^{At} - \mathrm{e}^{\omega t}P = -\mathrm{e}^{\omega t}\int_0^t \mathrm{e}^{-\omega\tau}\mathrm{e}^{A^T\tau}R\mathrm{e}^{A\tau}d\tau, \tag{9}$$

where

$$R = -A^T P - PA + \omega P.$$

Furthermore, if R is positive definite, for $t > 0$,

$$\mathrm{e}^{A^T t}P\mathrm{e}^{At} < \mathrm{e}^{\omega t}P. \tag{10}$$

IV. Consensus Control

For the multi-agent system (1), we use (3) to transform the agent dynamics to

$$\dot{z}_i(t) = Az_i + Du_i(t) + \phi(x_i), \tag{11}$$

where $D = \mathrm{e}^{-Ah}B$.

We propose a control design using the relative state information. The control input takes the structure,

$$u_i = -K\sum_{j=1}^N l_{ij}z_j, \tag{12}$$

where $K \in \mathbb{R}^{m\times n}$ is a constant control gain matrix to be designed later.

The closed-loop system is then described by

$$\dot{z} = (I_N \otimes A - L \otimes DK)z + \Phi(x), \tag{13}$$

where

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix}, \quad \Phi(x) = \begin{bmatrix} \phi(x_1) \\ \phi(x_2) \\ \vdots \\ \phi(x_N) \end{bmatrix},$$

and $\otimes$ denotes the Kronecker product of matrices.

Let us define $r^T \in \mathbb{R}^{1\times N}$ as the left eigenvector of L corresponding to the eigenvalue at 0, that is, $r^T L = 0$. Furthermore, let r be scaled such that $r^T\mathbf{1} = 1$ and let the first row of T^{-1} be $(T^{-1})_1 = r^T$.

Based on the vector r, we introduce a state transformation

$$\xi_i = z_i - \sum_{j=1}^N r_j z_j, \tag{14}$$

for $i = 1, 2\dots, N$. Let

$$\xi = [\xi_1^T, \xi_2^T, \cdots, \xi_N^T]^T.$$

We have

$$\begin{aligned} \xi &= z - ((\mathbf{1}r^T)\otimes I_n)z \\ &= (M \otimes I_n)z, \end{aligned}$$

where $M = I_N - \mathbf{1}r^T$. Since $r^T\mathbf{1} = 1$, it can be shown that $M\mathbf{1} = 0$. Therefore the consensus of system (13) is achieved when $\lim_{t\to\infty}\xi(t) = 0$, as $\xi = 0$ implies $z_1 = z_2 = \cdots = z_N$, due to the fact that the null space of M is span($\mathbf{1}$). The dynamics of ξ can then be obtained as

$$\begin{aligned} \dot{\xi} &= (I_N \otimes A - L \otimes DK)z - \mathbf{1}r^T \otimes I_N[I_N \otimes A - L \otimes DK]z \\ &\quad + (M \otimes I_n)\Phi(x) \\ &= (I_N \otimes A - L \otimes DK)\xi + (M \otimes I_n)\Phi(x). \end{aligned} \tag{15}$$

To explore the structure of L, let us introduce another state transformation

$$\eta = (T^{-1} \otimes I_n)\xi. \tag{16}$$

Then we have

$$\dot{\eta} = (I_N \otimes A - J \otimes DK)\eta + \Psi(x), \tag{17}$$

where $\Psi(x) = (T^{-1}M \otimes I_n)\Phi(x)$, and

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_N \end{bmatrix}, \Psi(x) = \begin{bmatrix} \psi_1(x) \\ \psi_2(x) \\ \vdots \\ \psi_N(x) \end{bmatrix},$$

with $\eta_i \in \mathbb{R}^n$ and $\psi_i : \mathbb{R}^{n\times N} \to \mathbb{R}^n$ for $i = 1, 2, \dots, N$. Then from (14) and (16), we have:

$$\begin{aligned} \eta_1 &= \left(r^T \otimes I_n\right)\xi \\ &= \left((r^T M) \otimes I_n\right)z \\ &\equiv 0. \end{aligned}$$

The nonlinear term $\Psi(x)$ in the transformed system dynamic model (17) is expressed as a function of the state x. For the stability analysis, first we need to establish a bound of this nonlinear function in terms of the transformed state η. The following lemma gives a bound of $\Psi(x)$.

Lemma 4: For the nonlinear term $\Psi(x)$ in the transformed system dynamics (17), a bound can be established in terms of the state η as

$$\|\Psi\|^2 \leq \gamma_0^2(\|\eta\|^2 + 4\lambda_\sigma^2(Q)\,\|\delta\|^2), \tag{18}$$

with

$$\gamma_0 = 2\sqrt{2N}\gamma\,\|r\|\,\|T\|_F\,\lambda_\sigma(T^{-1}), \tag{19}$$

$$\delta = -\int_t^{t+h} \mathrm{e}^{A(t-\tau)}BK\eta(\tau-h)\mathrm{d}\tau, \tag{20}$$

where $\lambda_\sigma(\cdot)$ and $\|\cdot\|_F$ denote the maximum singular value and Frobenius norm of a matrix, respectively.

Proof: Based on the state transformations (14) and (16), we have

$$\begin{aligned}\Psi(x) &= \left(T^{-1}\otimes I_n\right)(M\otimes I_n)\,\Phi(x)\\ &= \left(T^{-1}\otimes I_n\right)\mu,\end{aligned}$$

where $\mu = (M\otimes I_n)\Phi(x)$. Then, we have

$$\|\Psi(x)\| \leq \lambda_\sigma(T^{-1})\,\|\mu\|\,, \tag{21}$$

where $\mu = [\mu_1, \mu_2, \ldots, \mu_N]^T$.

Recalling that $M = I_N - \mathbf{1}r^T$, we have

$$\begin{aligned}\mu_i &= \phi(x_i) - \sum_{k=1}^N r_k\phi(x_k)\\ &= \sum_{k=1}^N r_k\left(\phi(x_i)-\phi(x_k)\right).\end{aligned}$$

It then follows that

$$\|\mu_i\| \leq \gamma\sum_{k=1}^N |r_k|\,\|x_i - x_k\| \tag{22}$$

From the state transformation (3), we have

$$\begin{aligned}x_i - x_k &= (z_i-\sigma_i)-(z_k-\sigma_k)\\ &= (z_i - z_k) - (\sigma_i - \sigma_k)\end{aligned}$$

where

$$\sigma_i = \int_t^{t+h}\mathrm{e}^{A(t-\tau)}Bu_i(\tau-h)\mathrm{d}\tau.$$

Then, we have

$$\|\mu_i\| \leq \gamma\sum_{k=1}^N |r_k|\left(\|z_i - z_k\| + \|\sigma_i-\sigma_k\|\right). \tag{23}$$

From $\eta = (T^{-1}\otimes I_n)\xi$, we obtain $\xi = (T\otimes I_n)\eta$, and from the state transformations (14), we have

$$\begin{aligned}z_i - z_k &= \xi_i - \xi_k\\ &= \left((t_i - t_k)\otimes I_n\right)\eta\\ &= \sum_{j=1}^N (t_{ij}-t_{kj})\eta_j,\end{aligned}$$

where t_k denotes the kth row of T. Then, we obtain

$$\|z_i - z_k\| \leq (\|t_i\| + \|t_k\|)\,\|\eta\|\,. \tag{24}$$

We next deal with the derived terms σ_i and σ_k. We have

$$\begin{aligned}\sum_{k=1}^N |r_k|\,\|\sigma_i-\sigma_k\| &\leq \sum_{k=1}^N |r_k|\,\|\sigma_i\| + \sum_{k=1}^N |r_k|\,\|\sigma_k\|\\ &\leq \|r\|\sqrt{N}\,\|\sigma_i\| + \|r\|\,\|\sigma\|\,,\end{aligned} \tag{25}$$

where $\sigma = [\sigma_1^T, \sigma_2^T, \cdots, \sigma_N^T]^T$, and we used the inequality

$$\sum_{i=1}^N |a_i| \leq \sqrt{N}\,\|a\|\,.$$

Then, from (23), (24) and (25), we can obtain that

$$\begin{aligned}\|\mu_i\| &\leq \gamma\sum_{k=1}^N |r_k|\left(\|t_i\|+\|t_k\|\right)\|\eta\|\\ &\quad + \gamma\sqrt{N}\,\|r\|\,\|\sigma_i\| + \gamma\,\|r\|\,\|\sigma\|\\ &\leq \gamma(\|r\|\sqrt{N}\,\|t_i\| + \|r\|\,\|T\|_F)\,\|\eta\|\\ &\quad + \gamma\sqrt{N}\,\|r\|\,\|\sigma_i\| + \gamma\,\|r\|\,\|\sigma\|\\ &= \gamma\,\|r\|\,[(\sqrt{N}\,\|t_i\| + \|T\|_F)\,\|\eta\|\\ &\quad + \sqrt{N}\,\|\sigma_i\| + \|\sigma\|].\end{aligned} \tag{26}$$

It then follows that

$$\begin{aligned}\|\mu\|^2 &= \sum_{i=1}^N (\|\mu_i\|)^2\\ &\leq 4\gamma^2\,\|r\|^2\sum_{i=1}^N (N\,\|t_i\|^2 + \|T\|_F^2)\,\|\eta\|^2\\ &\quad + 4\gamma^2\,\|r\|^2\sum_{i=1}^N (N\,\|\sigma_i\|^2 + \|\sigma\|^2)\\ &= 8\gamma^2\,\|r\|^2\,N\left[\|T\|_F^2\,\|\eta\|^2 + \|\sigma\|^2\right],\end{aligned} \tag{27}$$

where we have used

$$\sum_{k=1}^N \|t_k\|^2 = \|T\|_F^2\,,$$

and the inequality

$$(a+b+c+d)^2 \leq 4(a^2+b^2+c^2+d^2).$$

Next we need to deal with $\|\sigma\|^2$. From (12), we can get

$$\begin{aligned}\sigma_i &= \int_t^{t+h}\mathrm{e}^{A(t-\tau)}Bu_i(\tau-h)\mathrm{d}\tau\\ &= -\int_t^{t+h}\mathrm{e}^{A(t-\tau)}BK\sum_{j=1}^N l_{ij}z_j(\tau-h)\mathrm{d}\tau.\end{aligned}$$

From the relationship between Q and L, we have

$$\begin{aligned}\sum_{j=1}^N l_{ij}z_j &= \sum_{j=1}^N q_{ij}(z_i - z_j)\\ &= \sum_{j=1}^N q_{ij}\left((t_i - t_j)\otimes I_n\right)\eta\\ &= \sum_{j=1}^N q_{ij}\sum_{l=1}^N (t_{il}-t_{jl})\eta_l.\end{aligned} \tag{28}$$

Here we define δ_l

$$\delta_l = -\int_t^{t+h} \mathrm{e}^{A(t-\tau)} BK\eta_l(\tau - h)\mathrm{d}\tau. \tag{29}$$

Then we can obtain that

$$\sigma_i = \sum_{j=1}^N q_{ij} \sum_{l=1}^N (t_{il} - t_{jl})\delta_l.$$

It then follows that

$$\|\sigma_i\| \le \sum_{j=1}^N q_{ij} \left(\|t_i\| + \|t_j\|\right) \|\delta\| . \tag{30}$$

where $\delta = [\delta_1^T, \delta_2^T, \cdots, \delta_N^T]^T$. With (30), the sum of the $\|\sigma_i\|$ can be obtained

$$\begin{aligned}\sum_{i=1}^N \|\sigma_i\| &\le \|\delta\| \sum_{i=1}^N \sum_{j=1}^N q_{ij}(\|t_i\| + \|t_j\|) \\ &= \|\delta\| \sum_{i=1}^N \sum_{j=1}^N q_{ij} \|t_i\| + \|\delta\| \sum_{i=1}^N \sum_{j=1}^N q_{ij} \|t_j\| \\ &\le \lambda_\sigma(Q) \|T\|_F \|\delta\| + \lambda_\sigma(Q^T) \|T\|_F \|\delta\| \\ &= 2\lambda_\sigma(Q) \|T\|_F \|\delta\| ,\end{aligned} \tag{31}$$

with $\lambda_\sigma(Q) = \lambda_\sigma(Q^T)$. Therefore we have

$$\begin{aligned}\|\sigma\|^2 &= \sum_{i=1}^N (\|\sigma_i\|)^2 \\ &\le \left(\sum_{i=1}^N \|\sigma_i\|\right)^2 \\ &\le 4\lambda_\sigma^2(Q) \|T\|_F^2 \|\delta\|^2 .\end{aligned} \tag{32}$$

Hence, together with (27) and (32), we get

$$\|\mu\|^2 \le 8\gamma^2 \|r\|^2 N \|T\|_F^2 \left(\|\eta\|^2 + 4\lambda_\sigma^2(Q) \|\delta\|^2\right). \tag{33}$$

Finally, we obtain the bound for Ψ as

$$\begin{aligned}\|\Psi\|^2 &\le \lambda_\sigma^2(T^{-1}) \|\mu\|^2 \\ &\le \gamma_0^2 \left(\|\eta\|^2 + 4\lambda_\sigma^2(Q) \|\delta\|^2\right),\end{aligned} \tag{34}$$

with

$$\begin{aligned}\gamma_0 &= 2\sqrt{2N}\gamma \|r\| \|T\|_F \lambda_\sigma(T^{-1}), \\ \delta &= -\int_t^{t+h} \mathrm{e}^{A(t-\tau)} BK\eta(\tau - h)\mathrm{d}\tau.\end{aligned}$$

This completes the proof. ∎

With the control law shown in (12), the control gain matrix K is chosen as

$$K = D^T P, \tag{35}$$

where P is a positive definite matrix. In the remaining part of the paper, we will use Lyapunov-function-based analysis to identify a condition for P to ensure that consensus is achieved by using the control algorithm (12) with the control gain K in (35).

The stability analysis will be carried out in terms of η. As discussed earlier, the consensus control can be guaranteed by showing that η converges to zero, which is sufficed by showing that η_i converges to zero for $i = 2, 3, \ldots, N$, since we have shown that $\eta_1 = 0$.

From the structure of the Laplacian matrix shown in (7), we can see that

$$N_k = 1 + \sum_{j=1}^k n_j,$$

for $k = 1, 2, \ldots, q$. Note that $N_q = N$.

The agent state variables η_i from $i = 2$ to N_p are the state variables which are associated with the Jordan blocks of real eigenvalues, and η_i for $i = N_p+1$ to N are with Jordan blocks of complex eigenvalues.

For the state variables associated with the Jordan blocks J_k of real eigenvalues, i.e., for $k \le p$, we have the dynamics given by

$$\dot{\eta}_i = (A - \lambda_k DD^T P)\eta_i - DD^T P\eta_{i+1} + \psi_i(x),$$

for $i = N_{k-1} + 1,\ N_{k-1} + 2, \cdots, N_k - 1$, and

$$\dot{\eta}_i = (A - \lambda_k DD^T P)\eta_i + \psi_i(x),$$

for $i = N_k$.

For the state variables associated with the Jordan blocks J_k, i.e., for $k > p$, corresponding to complex eigenvalues, we consider the dynamics of the state variables in pairs. For notational convenience, let

$$\begin{aligned}i_1(j) &= N_{k-1} + 2j - 1 \\ i_2(j) &= N_{k-1} + 2j\end{aligned}$$

for $j = 1, 2, \ldots, n_k/2$. The dynamics of η_{i_1} and η_{i_2} for $j = 1, 2, \ldots, n_k/2 - 1$ are expressed by

$$\begin{aligned}\dot{\eta}_{i_1} &= (A - \alpha_k DD^T P)\eta_{i_1} - \beta_k DD^T P\eta_{i_2} - DD^T P\eta_{i_1+2} + \psi_{i_1}, \\ \dot{\eta}_{i_2} &= (A - \alpha_k DD^T P)\eta_{i_2} + \beta_k DD^T P\eta_{i_1} - DD^T P\eta_{i_2+2} + \psi_{i_2}.\end{aligned}$$

For $j = n_k/2$, we have

$$\begin{aligned}\dot{\eta}_{i_1} &= (A - \alpha_k DD^T P)\eta_{i_1} - \beta_k DD^T P\eta_{i_2} + \psi_{i_1}, \\ \dot{\eta}_{i_2} &= (A - \alpha_k DD^T P)\eta_{i_2} + \beta_k DD^T P\eta_{i_1} + \psi_{i_2}.\end{aligned}$$

Let

$$V_i = \eta_i^T P\eta_i, \tag{36}$$

for $i = 2, 3 \ldots, N$. Let

$$V_0 = \sum_{i=2}^N \eta_i^T P\eta_i. \tag{37}$$

For the convenience of presentation, we borrow the following results for V_0 from [10].

Lemma 5: For a network-connected dynamic system (1) with the transformed state η, $\dot{V}_0$ has following bounds specified in one of the following two cases:

1) If the eigenvalues of the Laplacian matrix L are distinct, i.e., $n_k = 1$ for $k = 1, 2, \ldots, q$, $\dot{V}_0$ satisfies

$$\begin{aligned}\dot{V}_0 \le& \sum_{i=2}^N \eta_i^T \left(A^T P + PA - 2\alpha PDD^T P + \kappa PP\right)\eta_i \\ &+ \frac{1}{\kappa}\|\Psi\|^2,\end{aligned} \tag{38}$$

with κ being any positive real number and

$$\alpha = \min\{\lambda_1, \lambda_2, \ldots, \lambda_p, \alpha_{p+1}, \alpha_{p+2}, \ldots, \alpha_q\}.$$

2) If the Laplacian matrix L has multiple eigenvalues, i.e., $n_k > 1$ for any $k \in \{1, 2, \cdots, q\}$, $\dot{V}_0$ satisfies

$$\dot{V}_0 \leq \sum_{i=2}^{N} \eta_i^T \left(A^T P + PA - 2(\alpha-1)PDD^T P + \kappa PP\right)\eta_i + \frac{1}{\kappa}\|\Psi\|^2, \tag{39}$$

with κ being any positive real number.

Using Lemmas 4 and 5, we easily obtain

$$\dot{V}_0 \leq \sum_{i=2}^{N} \eta_i^T \left(A^T P + PA - 2\alpha PDD^T P + \kappa PP + \frac{\gamma_0^2}{\kappa} I_n\right)\eta_i + \frac{4\gamma_0^2}{\kappa}\lambda_\sigma^2(Q)\Delta, \tag{40}$$

for Case 1) with $\Delta = \delta^T\delta$, and

$$\dot{V}_0 \leq \sum_{i=2}^{N} \eta_i^T \left(A^T P + PA - 2(\alpha-1)PDD^T P + \kappa PP + \frac{\gamma_0^2}{\kappa} I_n\right)\eta_i + \frac{4\gamma_0^2}{\kappa}\lambda_\sigma^2(Q)\Delta, \tag{41}$$

for Case 2). Here we have used $\|\eta\|^2 = \sum_{i=2}^N \|\eta_i\|^2$.

The remaining analysis is to explore the bound of Δ. With δ_l in (29) and Lemma 2, we have

$$\begin{aligned}\Delta_i =& \int_t^{t+h} \eta_i^T(\tau-h)K^T B^T \mathrm{e}^{A^T(t-\tau)}\mathrm{d}\tau \\ &\times \int_t^{t+h} \mathrm{e}^{A(t-\tau)} BK\eta_i(\tau-h)\mathrm{d}\tau \\ \leq& \, h\int_t^{t+h} \eta_i^T(\tau-h)PDD^T \mathrm{e}^{A^T h}\mathrm{e}^{A^T(t-\tau)} \\ &\times \mathrm{e}^{A(t-\tau)}\mathrm{e}^{Ah}DD^T P\eta_i(\tau-h)\mathrm{d}\tau.\end{aligned}$$

In view of Lemma 3 with $P = I_n$, provided that

$$R = -A^T - A + \omega_1 I_n > 0, \tag{42}$$

we have

$$\mathrm{e}^{A^T t}\mathrm{e}^{At} < \mathrm{e}^{\omega_1 t} I_n,$$

and

$$\begin{aligned}\Delta_i \leq& \, h\int_t^{t+h} \mathrm{e}^{\omega_1(t-\tau)}\eta_i^T(\tau-h)PDD^T e^{A^T h} \\ &\times \mathrm{e}^{Ah}DD^T P\eta_i(\tau-h)\mathrm{d}\tau \\ \leq& \, \rho^2 h\mathrm{e}^{\omega_1 h}\int_t^{t+h} \mathrm{e}^{\omega_1(t-\tau)}\eta_i^T(\tau-h)\eta_i(\tau-h)\mathrm{d}\tau \\ \leq& \, \rho^2 h\mathrm{e}^{2\omega_1 h}\int_t^{t+h} \eta_i^T(\tau-h)\eta_i(\tau-h)\mathrm{d}\tau,\end{aligned}$$

where ρ is a positive real number satisfying

$$\rho^2 I_n \geq PDD^T DD^T P. \tag{43}$$

Then the summation of Δ_i can be obtained as

$$\Delta = \sum_{i=2}^{N}\Delta_i \leq \sum_{i=2}^{N}\rho^2 h\mathrm{e}^{2\omega_1 h}\int_t^{t+h}\eta_i^T(\tau-h)\eta_i(\tau-h)\mathrm{d}\tau. \tag{44}$$

For the integral term Δ shown in (44), we consider the following Krasovskii functional

$$W_i = \int_t^{t+h} \mathrm{e}^{\tau-t}\eta_i^T(\tau-h)\eta_i(\tau-h)\mathrm{d}\tau + \int_t^{t+h}\eta_i^T(\tau-2h)\eta_i(\tau-2h)\mathrm{d}\tau.$$

A direct evaluation gives that

$$\begin{aligned}\dot{W}_i =& -\int_t^{t+h}\mathrm{e}^{\tau-t}\eta_i^T(\tau-h)\eta_i(\tau-h)\mathrm{d}\tau \\ &- \eta_i(t-2h)^T\eta_i(t-2h) + \mathrm{e}^h\eta_i^T(t)\eta_i(t) \\ \leq& -\int_t^{t+h}\eta_i^T(\tau-h)\eta_i(\tau-h)\mathrm{d}\tau + \mathrm{e}^h\eta_i^T(t)\eta_i(t).\end{aligned}$$

With $W_0 = \sum_{i=2}^N W_i$, we have

$$\begin{aligned}\dot{W}_0 =& \sum_{i=2}^{N}\dot{W}_i \\ \leq& -\sum_{i=2}^{N}\int_t^{t+h}\eta_i^T(\tau-h)\eta_i(\tau-h)\mathrm{d}\tau \\ &+ \sum_{i=2}^{N}\mathrm{e}^h\eta_i^T(t)\eta_i(t).\end{aligned} \tag{45}$$

Let

$$V = V_0 + \rho^2 h\mathrm{e}^{2\omega_1 h}\frac{4\gamma_0^2}{\kappa}\lambda_\sigma^2(Q)W_0. \tag{46}$$

From (40), (41), (44) and (45), we obtain that

$$\dot{V} \leq \eta^T(t)\left(I_N \otimes H\right)\eta(t), \tag{47}$$

where

$$H := A^T P + PA - 2\alpha PDD^T P + \kappa PP + \frac{\gamma_0^2}{\kappa}\left(1 + \lambda_\sigma^2(Q)\rho^2 h\mathrm{e}^{(2\omega_1+1)h}\right)I_n, \tag{48}$$

for Case 1), and

$$H := A^T P + PA - 2(\alpha-1)PDD^T P + \kappa PP + \frac{\gamma_0^2}{\kappa}\left(1 + \lambda_\sigma^2(Q)\rho^2 h\mathrm{e}^{(2\omega_1+1)h}\right)I_n, \tag{49}$$

for Case 2).

The above expressions can be used for consensus analysis of network-connected systems with Lipschitz nonlinearity and input delay. The following theorem summarizes the results.

Theorem 1: For an input-delayed multi-agent system (1) with the associated Laplacian matrix that satisfies Assumption 1, the consensus control problem can be solved by the control algorithm (12) with the control gain $K = D^T P$ specified in one of the following two cases:

1) If the eigenvalues of the Laplacian matrix L are distinct, the consensus is achieved if the following conditions are satisfied for $W = P^{-1}$ and $\rho > 0$, $\omega_1 \geq 0$,

$$(A - \frac{1}{2}\omega_1 I_n)^T + (A - \frac{1}{2}\omega_1 I_n) < 0, \tag{50}$$

$$\rho W \geq DD^T, \tag{51}$$

$$\begin{bmatrix} WA^T + AW - 2\alpha DD^T + \kappa I_n & W \\ W & \dfrac{-\kappa I_n}{\gamma_0^2(1 + 4h_0\rho^2)} \end{bmatrix} < 0, \tag{52}$$

where κ is any positive real number and $h_0 = \lambda_\sigma^2(Q)h\mathrm{e}^{(2\omega_1+1)h}$.

2) If the Laplacian matrix L has multiple eigenvalues, the consensus is achieved if the conditions (50), (51) and the following condition are satisfied for $W = P^{-1}$ and $\rho > 0$, $\omega_1 \geq 0$,

$$\begin{bmatrix} WA^T + AW - 2(\alpha - 1)DD^T + \kappa I_n & W \\ W & \dfrac{-\kappa I_n}{\gamma_0^2(1 + 4h_0\rho^2)} \end{bmatrix} < 0, \tag{53}$$

where κ is any positive real number and $h_0 = \lambda_\sigma^2(Q)h\mathrm{e}^{(2\omega_1+1)h}$.

Proof: When the eigenvalues are distinct, from the analysis in this section, we know that the feedback law (12) will stabilize η if the conditions (42), (43) and $H < 0$ in (48) are satisfied. Indeed, it is easy to see the conditions (42) and (43) are equivalent to the conditions specified in (50) and (51). From (48), it can be obtained that $H < 0$ is equivalent to

$$\begin{aligned} &P^{-1}A^T + AP^{-1} - 2\alpha DD^T + \kappa I_n \\ &\quad + \frac{\gamma_0^2}{\kappa}(1 + 4h_0\rho^2)P^{-1}P^{-1} < 0, \end{aligned} \tag{54}$$

which is further equivalent to (52). Hence we conclude that η converges to zero asymptotically.

When the Laplacian matrix has multiple eigenvalues, the feedback law (12) will stabilize η if the conditions (42), (43) and $H < 0$ in (49) are satisfied. Following the similar procedure as Case 1), we can show that, under the conditions (50), (51) and (53), η converges to zero asymptotically. The proof is completed. ■

Remark 3: The conditions shown in (50) to (53) can be checked by standard LMI routines for a set of fixed values ρ and ω_1. The iterative methods developed in [31] for single linear system may also be applied here.

V. Simulation

In this section, we will illustrate in some details the proposed consensus control design through a circuit example. The system under consideration is a connection of four agents (i.e. $N = 4$) as shown in Figure 1, each of which is described by a second-order dynamic model as

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = f(v_i) + u_i(t - h), \end{cases} \tag{55}$$

where $p_i = [p_{ix}, p_{iy}, p_{iz}]^T \in \mathbb{R}^3$ denotes the position vector of agent i, $v_i = [v_{ix}, v_{iy}, v_{iz}]^T \in \mathbb{R}^3$ the velocity vector, $f(v_i) \in \mathbb{R}^3$ the intrinsic dynamics of agent i, governed by the chaotic Chua circuit [30]

$$f(v_i) = \begin{bmatrix} -0.59v_{ix} + v_{iy} - 0.17(|v_{ix} + 1| - |v_{ix} - 1|) \\ v_{ix} - v_{iy} + v_{iz} \\ -v_{iy} - 5v_{iz} \end{bmatrix}.$$

Let $x_i = [p_i^T, v_i^T]^T \in \mathbb{R}^6$. The dynamic equation (55) of each agent can be re-arranged as the state space model (2) with

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.59 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and $\phi(x_i) = [0, 0, 0, -0.17(|v_{ix} + 1| - |v_{ix} - 1|), 0, 0]^T$. The adjacency matrix is given by

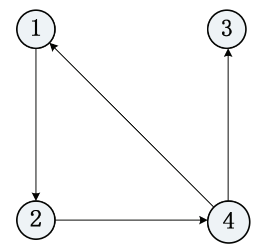

Fig. 1. Communication topology.

$$Q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

and the resultant Laplacian matrix is obtained as

$$L = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

The eigenvalues of L are $\{0, 1, 3/2 \pm j\sqrt{3}/2\}$, and therefore Assumption 1 is satisfied. Furthermore, the eigenvalues are distinct. We obtain that

$$J = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & -\frac{\sqrt{3}}{2} & \frac{3}{2} \end{bmatrix},$$

with the matrices

$$T = \begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & 0 & -1 & 0 \\ 1 & -2 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix},$$

and $r^T = [1/3, 1/3, 0, 1/3]^T$.

The nonlinear function $\phi(x_i)$ in each agent dynamics is globally Lipschitz with a Lipschitz constant $\gamma = 0.34$, which gives $\gamma_0 = 3.7391$ by (19). Based on the Laplacian matrix

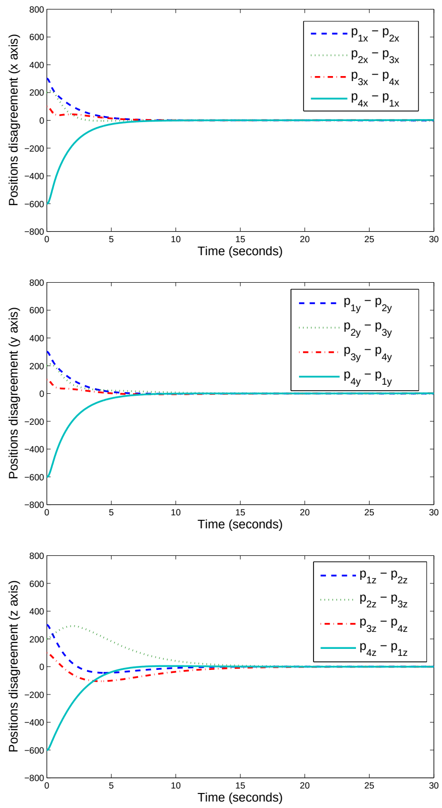

Fig. 2. The positions disagreement of 4 agents: $h = 0.03$s.

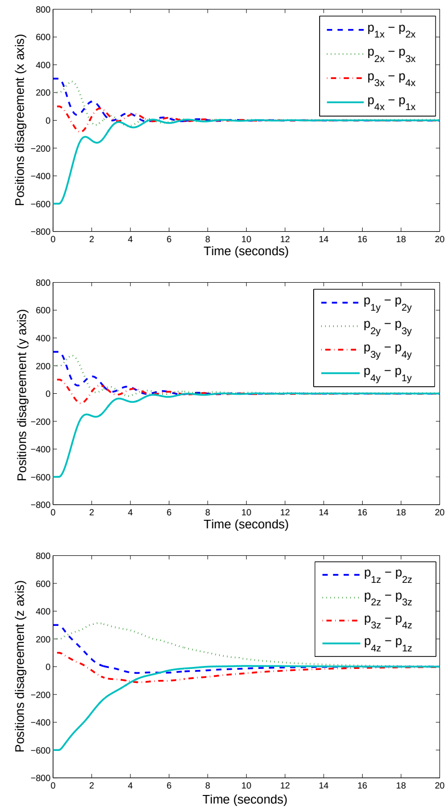

Fig. 3. The positions disagreement of 4 agents: $h = 0.3$s.

L, we have $\alpha = 1$. In simulation, the input delay is set as $h = 0.03$s. A positive definite matrix P can be obtained with $\kappa = 0.01$, $\omega_1 = 1.5$ and $\rho = 2$, as

$$P = \begin{bmatrix} 5.03 & -0.53 & 0.18 & 2.58 & 0.29 & 0.08 \\ -0.53 & 5.37 & 0.43 & 0.28 & 2.39 & 0.47 \\ 0.18 & 0.43 & 7.75 & -0.08 & -0.38 & 1.58 \\ 2.58 & 0.28 & -0.08 & 2.65 & 0.93 & 0.17 \\ 0.29 & 2.39 & -0.38 & 0.93 & 2.17 & 0.25 \\ 0.08 & 0.47 & 1.58 & 0.17 & 0.25 & 0.92 \end{bmatrix},$$

to satisfy the conditions of Theorem 1. Consequently, the control gain is obtained as

$$K = \begin{bmatrix} -2.19 & -0.12 & -0.01 & -2.46 & -0.74 & -0.15 \\ -0.13 & -2.10 & 0.30 & -0.75 & -2.08 & -0.32 \\ -0.09 & -0.43 & -1.64 & -0.18 & -0.18 & -1.27 \end{bmatrix}.$$

Simulation study has been carried out with the results shown in Figure 2 for the positions state disagreement of each agent. Clearly the conditions specified in Theorem 1 are sufficient for the control gain to achieve consensus control for the multi-agent systems. The same control gain has also been used for different values of input delay. The results shown in Figure 3 indicate that the conditions could be conservative in the control gain design for a given input delay and Lipschitz nonlinear function. Indeed, extensive simulation shows that the same control gain can possibly achieve consensus control for the system with a much larger delay and Lipschitz constant.

VI. Conclusion

This paper has investigated the impacts of nonlinearity and input delay in consensus control. This input delay may represent some delays in the network communication. Sufficient conditions are derived for the multi-agent systems to guarantee the global consensus using Lyapunov-Krasovskii method in the time domain. The significance of this research is to provide a feasible method to deal with consensus control of a class of Lipschitz nonlinear multi-agent systems with input delay which includes some common circuits such as Chua circuits.

Acknowledgment

The authors would like to thank the support of the National Natural Science Foundation of China (Nos. 61273105 and 61203022), 111 Project (B08015), the US National Science Foundation (CMMI-1462171), and the China Scholarship Council (CSC).

References

[1] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[2] W. Ren, "On consensus algorithms for double-integrator dynamics," *IEEE Trans. Autom. Control*, vol. 53, no. 6, pp. 1503–1509, 2008.

[3] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multi-agent systems and synchronization of complex networks: a unified viewpoint," *IEEE Trans. Circuits Syst. I*, vol. 57, no. 1, pp. 213–224, 2010.

[4] Z. Guan, Y. Wu, and G. Feng, "Consensus analysis based on impulsive systems in multiagent networks," *IEEE Trans. Circuits Syst. I*, vol. 59, no. 1, pp. 170–178, 2012.

[5] Z. Ding, "Consensus disturbance rejection with disturbance observers," *IEEE Trans. Ind. Electron.*, vol. 62, no. 9, pp. 5829–5837, 2015.
[6] W. Yu, J. Cao, G. Chen, J. Lu, J. Han, and W. Wei, "Local synchronization of a complex network model," *IEEE Trans. Syst. Man Cybern. B*, vol. 39, no. 1, pp. 230–241, 2009.
[7] W. Yu, G. Chen, M. Cao, and J. Kurths, "Second-order consensus for multi-agent systems with directed topologies and nonlinear dynamics," *IEEE Trans. Syst. Man Cybern. B*, vol. 40, no. 3, pp. 881–891, 2010.
[8] Q. Song, J. Cao, and W. Yu, "Second-order leader-following consensus of nonlinear multi-agent systems via pinning control," *Syst. Control Lett.*, vol. 59, no. 9, pp. 553–562, 2010.
[9] Z. Li, X. Liu, M. Fu, and L. Xie, "Global H_∞ consensus of multi-agent systems with Lipschitz non-linear dynamics," *IET Control Theory Appl.*, vol. 6, no. 13, pp. 2041–2048, 2012.
[10] Z. Ding, "Consensus control of a class of Lipschitz nonlinear systems," *Int. J. Control*, vol. 87, no. 11, pp. 2372–2382, 2014.
[11] Z. Li, W. Ren, X. Liu, and M. Fu, "Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols," *IEEE Trans. Autom. Control*, vol. 58, no. 7, pp. 1786–1791, 2013.
[12] G. Wen, Z. Duan, G. Chen, and W. Yu, "Consensus tracking of multi-agent systems with Lipschitz-type node dynamics and switching topologies," *IEEE Trans. Circuits Syst. I*, vol. 61, no. 2, pp. 499–510, 2014.
[13] W. Yu, G. Chen, J. Lu, and J. Kurths, "Synchronization via pinning control on general complex networks," *SIAM J. Contr. Optim.*, vol. 51, no. 2, pp. 1395–1416, 2013.
[14] Z. Ding, "Consensus output regulation of a class of heterogeneous nonlinear systems," *IEEE Trans. Autom. Control*, vol. 58, no. 10, pp. 2648–2653, 2013.
[15] Y. Su and J. Huang, "Cooperative adaptive output regulation for a class of nonlinear uncertain multi-agent systems with unknown leader," *Syst. Control Lett.*, vol. 62, no. 6, pp. 461–467, 2013.
[16] Z. Ding, "Adaptive consensus output regulation of a class of nonlinear systems with unknown high-frequency gain," *Automatica*, vol. 51, no. 1, pp. 348–355, 2015.
[17] J. Richard, "Time-delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, no. 10, pp. 1667–1694, 2003.
[18] O. Smith, "A controller to overcome dead time," *ISA*, vol. 6, no. 2, pp. 28–33, 1959.
[19] Z. Palmor, "Modified predictors," in *The Control Handbook*, W. Levine, Ed: CRC Press, 1996, Section 10.9.
[20] A. Manitius and A. Olbrot, "Finite spectrum assignment problem for systems with delays," *IEEE Trans. Autom. Control*, vol. 24, no. 2, pp. 541–552, 1979.
[21] W. Kwon and A. Pearson, "Feedback stabilization of linear systems with delayed control," *IEEE Trans. Autom. Control*, vol. 25, no. 2, pp. 266–269, 1980.
[22] Z. Artstein, "Linear systems with delayed controls: a reduction," *IEEE Trans. Autom. Control*, vol. 27, no. 4, pp. 869–879, 1982.
[23] S. Y. Yoon and Z. Lin, "Truncated predictor feedback control for exponentially unstable linear systems with time-varying input delay," *Syst. Control Lett.*, vol. 62, no. 10, pp. 837–844, 2013.
[24] L. Moreau, "Stability of multi-agent systems with time-dependent communication links," *IEEE Trans. Autom. Control*, vol. 50, no. 2, pp. 169–182, 2005.
[25] Z. Qu, J. Wang, A. Richard, and J. Martin, "Cooperative control design and stability analysis for multi-agent systems with communication delays," in *Proc. 2006 IEEE Int. Conf. Robotics and Automation*, 2006, pp. 970–975.
[26] Y. Sun and L. Wang, "Consensus of multi-agent systems in directed networks with non-uniform time-varying delays," *IEEE Trans. Autom. Control*, vol. 54, no. 7, pp. 1607–1613, 2009.
[27] Z. Meng, W. Ren, Y. Cao, and Z. You, "Leaderless and leader-following consensus with communication and input delays under a directed network topology," *IEEE Trans. Syst. Man Cybern. B*, vol. 41, no. 1, pp. 75–88, 2011.
[28] B. Zhou and Z. Lin, "Consensus of high-order multi-agent systems with large input and communication delays," *Automatica*, vol. 50, no. 2, pp. 452–464, 2014.
[29] Z. Ding and Z. Lin, "Truncated state prediction for control of Lipschitz nonlinear systems with input delay," in *Proc. 53rd IEEE Conf. Decision Control*, 2014, pp. 1966–1971.
[30] X. Wang, G. Zhong, K. Tang, K. Man, and Z. Liu, "Generating chaos in Chua's circuit via time-delay feedback," *IEEE Trans. Circuits Syst. I*, vol. 48, no. 9, pp. 1151–1156, 2001.
[31] S. Y. Yoon, P. Anantachaisilp, and Z. Lin, "An LMI approach to the control of exponentially unstable systems with input time delay," in *Proc. 52nd IEEE Conf. Decision Control*, 2013, pp. 312–317.

Chunyan Wang received his B.Eng. degree in automatic control from Dezhou University, Shandong, China, in 2006, his Master of Engineering degree in control theory and control engineering from Soochow University, Jiangsu, China, in 2009, and his M.Sc degree in Electrical and Electronic Engineering from the University of Greenwich, UK, in 2012. He is now a Ph.D. candidate in control engineering with the School of Electrical and Electronic Engineering at the University of Manchester. His research interests include consensus control, robot control, and time-delay system.

Zongyu Zuo received his B.Eng. degree in automatic control from Central South University, Hunan, China, in 2005, and Ph.D. degree in control theory and applications from Beihang University (BUAA), Beijing, China, in 2011. He was an academic visitor at the School of Electrical and Electronic Engineering, University of Manchester from September 2014 to September 2015 and held an inviting associate professorship at Mechanical Engineering and Computer Science, UMR CNRS 8201, Université de Valenciennes et du Hainaut-Cambrésis in October 2015. He is currently an assistant professor at the School of Automation Science and Electrical Engineering, Beihang University. His research interests are in the fields of nonlinear system control, control of UAVs, and coordination of multi-agent system.

Zongli Lin is the Ferman W. Perry Professor of Electrical and Computer Engineering at University of Virginia. He received his B.S. degree in mathematics and computer science from Xiamen University, Xiamen, China, in 1983, his Master of Engineering degree in automatic control from Chinese Academy of Space Technology, Beijing, China, in 1989, and his Ph.D. degree in electrical and computer engineering from Washington State University, Pullman, Washington, in 1994. His current research interests include nonlinear control, robust control, and control applications. He was an Associate Editor of *IEEE Transactions on Automatic Control* (2001-2003), *IEEE/ASME Transactions on Mechatronics* (2006-2009) and *IEEE Control Systems Magazine* (2005-2012). He was an elected member of the Board of Governors of the IEEE Control Systems Society (2008-2010). He has served on the operating committees several conferences and will be the program chair of the 2018 American Control Conference. He currently chairs the IEEE Control Systems Society Technical Committee on Nonlinear Systems and Control and serves on the editorial boards of several journals and book series, including *Automatica*, *Systems & Control Letters*, *Science China Information Sciences*, and Springer/Birkhauser book series *Control Engineering*. He is a Fellow of IEEE, IFAC, and AAAS.

Zhengtao Ding received the B.Eng. degree from Tsinghua University, Beijing, China, and the M.Sc. degree in systems and control and the Ph.D. degree in control systems from the University of Manchester Institute of Science and Technology, Manchester, U.K. After working as a Lecturer with Ngee Ann Polytechnic, Singapore, for ten years, in 2003, he joined The University of Manchester, Manchester, U.K., where he is currently a Senior Lecturer of control engineering with the School of Electrical and Electronic Engineering. He is the author of the book *Nonlinear and Adaptive Control Systems* (IET, 2013) and a number of journal papers. His research interests include nonlinear and adaptive control theory and their applications. Dr. Ding serves as an Associate Editor for the *IEEE Transactions on Automatic Control*, *Transactions of the Institute of Measurement and Control*, *Control Theory and Technology*, *Mathematical Problems in Engineering*, *Unmanned Systems*, and *the International Journal of Automation and Computing*.