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# Consensus Filters for Sensor Networks and Distributed Sensor Fusion

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***Abstract*— Consensus algorithms for networked dynamic systems provide scalable algorithms for sensor fusion in sensor networks. This paper introduces a distributed filter that allows the nodes of a sensor network to track the average of $n$ sensor measurements using an average consensus based distributed filter called *consensus filter*. This consensus filter plays a crucial role in solving a data fusion problem that allows implementation of a scheme for distributed Kalman filtering in sensor networks. The analysis of the convergence, noise propagation reduction, and ability to track fast signals are provided for consensus filters. As a byproduct, a novel critical phenomenon is found that relates the size of a sensor network to its tracking and sensor fusion capabilities. We characterize this performance limitation as a *tracking uncertainty principle*. This answers a fundamental question regarding how large a sensor network must be for effective sensor fusion. Moreover, *regular networks* emerge as efficient topologies for distributed fusion of noisy information. Though, arbitrary overlay networks can be used. Simulation results are provided that demonstrate the effectiveness of consensus filters for distributed sensor fusion.**

**Keywords:** sensor networks, sensor fusion, consensus problems, distributed Kalman filters, complex networks, networked dynamic systems, graph Laplacians

## I. Introduction

Sensor networks have broad applications in surveillance & monitoring of an environment, collaborative processing of information, and gathering scientific data from spatially distributed sources for environmental modeling and protection [19], [4], [8], [10], [2], [1], [3], [13], [14], [17], [23], [25], [5]. Dealing with sensor networks requires multidisciplinary collaborations among researchers with background in computer science, wireless communication networks, and systems & control science.

A fundamental problem in sensor networks is to solve detection and estimation problems using *scalable algorithms*. This requires development of novel distributed algorithms for estimation and in particularly Kalman filtering that are currently unavailable. In a recent paper, Spanos, Olfati-Saber, and Murray [23] proposed a scalable sensor fusion scheme that requires fusion of sensor measurements combined with local Kalman filtering. The key component of this approach is to develop a distributed algorithm that allows the nodes of a sensor network to track the average of all of their measurements. We refer to this problem as *dynamic average-consensus*. The main contribution of this paper is to develop a distributed low-pass filter that solves this tracking problem via reaching an average-consensus.

Consensus problems [18], [21] for networked dynamic systems have been extensively used by many researchers as part of the solution of more complex problems including collective control of motion of swarms/flocks of mobile agents [15], ultrafast consensus in small-world networks [16], [24], state-dependent graphs [11], random networks [7], and directed networks [12], [20]. More recently, consensus-based information processing has been applied to sensor fusion in sensor networks [23], [22], [25].

This paper generalizes the average-consensus algorithm for $n$ constant values in [18], [21] to the case of $n$ measurements of noisy signals obtained from $n$ sensors in the form of a distributed low-pass filter called the *Consensus Filter*. The role of this consensus filter is to perform distributed fusion of sensor measurements that is necessary for implementation of a scalable Kalman filtering scheme proposed in [23]. We show that consensus filters can be also used independently for distributed sensor fusion.

The outline of the paper is as follows. Section II provides some preliminaries on consensus problems in networked systems and graph Laplacians. In Section III, the main results on design and analysis of distributed consensus filters are presented. Section IV provides detailed simulation results. Finally, concluding remarks are made in Section V.

## II. Consensus Problems in Networked Systems

Let $G = (V, E)$ be a graph with a nonnegative adjacency matrix $\mathcal{A} = [a_{ij}]$ that specifies the interconnection topology of a network of dynamic systems, sensors, or agents. The set of nodes is denoted by $V = \{1, \ldots, n\}$. For complex networks, we refer to $|V|$ and $|E|$ as the *scale* and *size* of the network, respectively. Let $N_i = \{i \in V : a_{ij} \neq 0\}$ denote the set of *neighbors* of node $i$ and $J_i = N_i \cup \{i\}$ denote the set of *inclusive neighbors* of node $i$. A *consensus algorithm* can be expressed in the form of a linear system

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)), \quad x(0) = c \in \mathbb{R}^n. \quad (1)$$

Given a connected network $G$, all the solutions of system (1) converge to an *aligned state* $x^* = (\mu, \ldots, \mu)^T$ with identical elements equal to $\mu = \bar{x}(0) = \frac{1}{n}\sum_i c_i$. This explains why in the term "average-consensus" was first coined in [21] to refer to the distributed algorithm in (1). In a more compact form, system (1) can be expressed as

$$\dot{x} = -Lx, \tag{2}$$

where $L$ is the *Laplacian matrix* [6] of graph $G$ and is defined as

$$L = \Delta - \mathcal{A} \tag{3}$$

where $\Delta = \mathrm{diag}(\mathcal{A}\cdot\mathbf{1})$ is the *degree matrix* of $G$ with diagonal elements $d_i = \sum_j a_{ij}$. Here, $\mathbf{1} = (1,\dots,1)^T \in \mathbb{R}^n$ denotes the vector of ones that is always a right eigenvector of $L$ corresponding to $\lambda_1 = 0$ (i.e. $L\mathbf{1} = 0$). The second smallest eigenvalue $\lambda_2$ of $L$ determines the speed of convergence of the algorithm [16], [18].

## III. Consensus Filters

Consider a sensor network of size $n$ with information flow[1] $G$. Assume each sensor is measuring a signal $s(t)$ that is corrupted by noise $v_i$ that is a zero-mean white Gaussian noise (WGN). Thus, the *sensing model* of the network is

$$u_i(t) = r(t) + v_i(t), \quad i = 1,\dots,n \tag{4}$$

or $u(t) = r(t)\mathbf{1} + v(t)$. Let $R_i$ denote the covariance matrix of $v_i$ for all $i$.

Our *objective* is to design the dynamics of a distributed low-pass filter with state $x = (x_1,\dots,x_n)^T \in \mathbb{R}^n$ that takes $u$ as the input and $y = x$ as the output with the property that asymptotically all nodes of the network reach an $\epsilon$-*consensus* regarding the value of signal $r(t)$ in all time $t$. By $\epsilon$-consensus, we mean there is a ball of radius $\epsilon$ that contains the state of all nodes (i.e. approximate agreement). In most applications, $r(t)$ is a low-to-medium frequency signal and $v(t)$ is a high-frequency noise. Thus, the consensus filter must act as a low-pass filter.

We propose the following *dynamic consensus algorithm*

$$\dot{x}_i(t) = \sum_{j\in N_i} a_{ij}(x_j(t) - x_i(t)) + \sum_{j\in J_i} a_{ij}(u_j(t) - x_i(t)), \tag{5}$$

as a candidate for a distributed low-pass *consensus filter*. The reminder of the paper is devoted to establishing the properties of this distributed filter. Note that the algorithm in (5) only requires communication among neighboring nodes of the network and thus is a distributed algorithm [9].

*Remark* 1. In discrete-time, the dynamic consensus algorithm in (5) can be stated as follows:

$$x_i^+ = x_i + \delta[\sum_{j\in N_i} a_{ij}(x_j - x_i) + \sum_{j\in J_i} a_{ij}(u_j - x_i)], \tag{6}$$

where $x_i$ is the current state of node $i$, $x_i^+$ is the next state, and $\delta$ is the step-size of iterations. We will conduct all of our analysis in continuous-time.

[1]Keep in mind that the *information flow* in a sensor network might (or might not) be the same as the *overlay network* (i.e. communication network).

**Proposition 1.** *The distributed algorithm in* (5) *gives a consensus filter with the following collective dynamics*

$$\dot{x} = -(I_n + \Delta + L)x + (I_n + \mathcal{A})u \tag{7}$$

*that is an LTI system with specification $A = -(I + \Delta + L)$, $B = I_n + \mathcal{A}$, $C = I_n$ and a proper MIMO transfer function.*

*Proof.* First, let us rewrite the system in (5) as

$$\begin{aligned}\dot{x}_i &= \sum_{j\in N_i} a_{ij}(x_j - x_i) + \sum_{j\in J_i} a_{ij}(u_j - u_i + u_i - x_i),\\ &= \sum_{j\in N_i} a_{ij}(x_j - x_i) + \sum_{j\in N_i} a_{ij}(u_j - u_i)\\ &+ |J_i|(u_i - x_i).\end{aligned}$$

Noting that $|J_i| = 1 + d_i$, from the definition of graph Laplacian, we get

$$\begin{aligned}\dot{x} &= -Lx - Lu + (I_n + \Delta)(u - x),\\ &= -(I_n + \Delta + L)x + (I_n + \Delta - L)u\end{aligned}$$

But $\Delta - L = \mathcal{A}$ and therefore $\dot{x} = Ax + Bu, y = Cx$ with matrices that are defined in the question. □

The transfer function of the consensus filter is given by

$$H(s) = [sI_n + (I_n + \Delta + L)]^{-1}(I_n + \mathcal{A}) \tag{8}$$

Applying Geršgorin theorem to matrix $A = I_n + 2\Delta + \mathcal{A}$ guarantees that all poles of $H(s)$ are strictly negative and fall within the interval $[-(1 + d_{min}), -(1 + 3d_{max})]$ with $d_{max} = \max_i d_i$ and $d_{min} = \min_i d_i$. i.e. $1 + d_{min} \le \lambda_i(A) \le (1 + 3d_{max})$ for all $i$. This immediately implies the following stability property of the consensus filter.

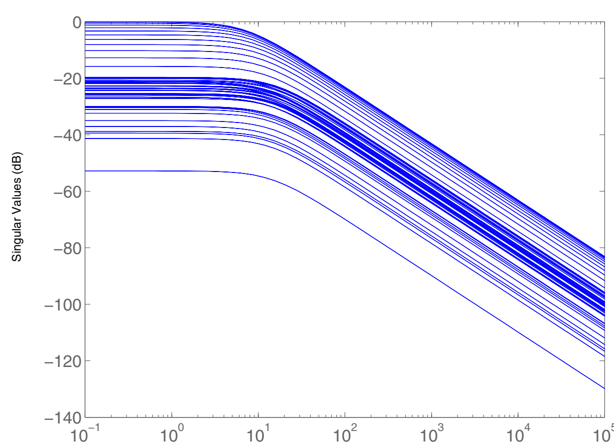



Fig. 1. The singular value plots of the low-pass consensus filter for a regular network.

**Corollary 1.** *The consensus filter in* (8) *is a distributed stable low-pass filter.*

*Proof.* Apparently, all the poles of $H(s)$ are strictly negative and thus the filter is stable. On the other hand, $H(s)$ is a proper MIMO transfer function satisfying $\lim_{s\to\infty} H(s) = 0$ which means it is a low-pass filter. □

Fig. 1 shows the singular value plots of the *low-pass consensus filter* (or CF$_{lp}$) for a regular network with $n = 100$ nodes and degree $k = 6$.

*Remark* 2. The following dynamic consensus algorithm [22]

$$\dot{x} = -Lx + \dot{u}(t)$$

gives a *high-pass consensus filter* (CF$_{hp}$) that is useful for distributed data fusion applications with low-noise data.

It remains to establish that all nodes asymptotically can reach an $\epsilon$-consensus regarding $r(t)$.

**Proposition 2.** *Let $r(t)$ be a signal with a uniformly bounded rate $|\dot{r}(t)| \le \nu$. Then, $x^*(t) = r(t)\mathbf{1}$ is a globally asymptotically $\epsilon$-stable equilibrium of the dynamics of the consensus filter given by*

$$\dot{x} = -Lx - Lu + (I_n + \Delta)(u - x) \tag{9}$$

*with input $u = r(t)\mathbf{1}$ and*

$$\epsilon = \frac{\nu\sqrt{n}(1 + d_{max})\lambda_{max}^{\frac{1}{2}}(A)}{\lambda_{min}^{\frac{5}{2}}(A)} \tag{10}$$

*Proof.* Given the input $u = r(t)\mathbf{1}$, the dynamics of the system in (9) reduces to

$$\dot{x} = -Lx + (I_n + \Delta)(r(t)\mathbf{1} - x) \tag{11}$$

with an obvious equilibrium at $x = r(t)\mathbf{1}$ that is an aligned state with elements that are identical to the signal $r(t)$. This is due to the fact that $L\mathbf{1} = 0$. Defining the error variable $\eta = x - r(t)\mathbf{1}$ gives

$$\dot{\eta} = -A\eta + \dot{r}(t)\mathbf{1} \tag{12}$$

where $A = I_n + \Delta + L$ is a positive definite matrix with the property that

$$1 + d_{min} \le \lambda_{min}(A) \le \lambda_{max}(A) \le 1 + 3d_{max}. \tag{13}$$

Let us define the Lyapunov function $\varphi(\eta) = \frac{1}{2}\eta^T A\eta$ for the perturbed linear system in (12). We have

$$\begin{aligned} \dot{\varphi} &= -\|A\eta\|^2 + \dot{r}(t)(\mathbf{1}^T A\eta) \\ &\le -\lambda_{min}^2(A)\|\eta\|^2 + \nu\sqrt{n}(1 + d_{max})\|\eta\|. \end{aligned}$$

This is because

$$\mathbf{1}^T A = \mathbf{1}^T + \mathbf{1}^T\Delta = (1 + d_1, 1 + d_2, \ldots, 1 + d_n),$$

and thus

$$|\mathbf{1}^T A\eta| \le [\sum_i (1 + d_i)^2]^{\frac{1}{2}}\|\eta\| \le \sqrt{n}(1 + d_{max})\|\eta\|.$$

As a result, one obtains

$$\begin{aligned} \dot{\varphi}(\eta) &\le -\left(\lambda_{min}(A)\|\eta\| - \frac{\nu\sqrt{n}(1 + d_{max})}{2\lambda_{min}(A)}\right)^2 \\ &+ \left(\frac{\nu\sqrt{n}(1 + d_{max})}{2\lambda_{min}(A)}\right)^2 \end{aligned}$$

Let $B_\rho$ be a closed ball centered at $\eta = 0$ with radius

$$\rho = \frac{\nu\sqrt{n}(1 + d_{max})}{\lambda_{min}^2(A)} \tag{14}$$

and let $\Omega_c = \{\eta : \varphi(\eta) \le c\}$ be a level-set of the Lyapunov function $\varphi(\eta)$ with $c = \frac{1}{2}\lambda_{max}(A)\rho^2$. Then, $B_\rho$ is contained in $\Omega_c$ because

$$\|\eta\| \le \rho \implies \varphi(\eta) = \frac{1}{2}\eta^T A\eta \le \frac{1}{2}\lambda_{max}(A)\rho^2 = c,$$

and thus $\eta \in \Omega_c$. As a result, any solution of (12) starting in $\mathbb{R}^n \setminus \Omega_c$ satisfies $\dot{\varphi} < 0$. Thus, it enters $\Omega_c$ in some finite time and remains in $\Omega_c$ thereafter (i.e. $\Omega_c$ is an invariant level-set). This guarantees global asymptotic $\epsilon$-stability of $\eta = 0$ with a radius $\epsilon = \rho\lambda_{max}(A)/\lambda_{min}(A)$. To show this, note that

$$\frac{1}{2}\lambda_{min}(A)\|\eta\|^2 \le \varphi(\eta) \le \frac{1}{2}\lambda_{max}(A)\rho^2 \tag{15}$$

Thus, the solutions enter the region

$$\|\eta\| \le \rho\sqrt{\frac{\lambda_{max}(A)}{\lambda_{min}(A)}}$$

which implies the radius of $\epsilon$-stability is

$$\epsilon = \frac{\nu(1 + d_{max})}{\lambda_{min}^2(A)}\sqrt{\frac{n\lambda_{max}(A)}{\lambda_{min}(A)}} \tag{16}$$

Of course, $\epsilon$-stability of $\eta = 0$ implies $\epsilon$-tracking of $r(t)$ by every node of the network (i.e. $\epsilon$-consensus is asymptotically reached). □

The following result describes the occurrence of a *critical phenomenon* in regular complex networks.

**Proposition 3.** *Consider a regular network $G$ of degree $k$. Let $r(t)$ be a signal with a finite rate $|\dot{r}| \le \nu$. Then, the dynamics of the consensus filter in the form*

$$\dot{x} = -Lx - Lu + (I + \Delta)(u - x) \tag{17}$$

*satisfies the following properties:*

i) *The mean $\mu(t) = \bar{x}(t)$ of the state of all nodes is the output of a scalar low-pass filter*

$$\dot{\mu} = (k + 1)(\bar{u}(t) - \mu) \tag{18}$$

*with and input $\bar{u}(t) = r(t) + w(t)$ and a zero-mean noise $w(t) = \frac{1}{n}\sum_i v_i(t)$.*

ii) *Assume the network node degree $k = \beta n^\gamma$ is exponentially scale-dependent. Then, there exists a critical exponent $\gamma_c = \frac{1}{2}$ such that for all $\gamma > \gamma_c$ (or networks with more than $O(n^{1.5})$ links), the radius of $\epsilon$-tracking vanishes as the scale $n$ becomes infinity large for any arbitrary $\nu, \beta$ ($\epsilon$ is defined in Proposition 2).*

*Proof.* Part i) follows from the fact that $\mu = \frac{1}{n}(\mathbf{1}^T x)$ and $\mathbf{1}^T L = 0$. Moreover, for regular networks with degree $k$, $I_n + \Delta = (k + 1)I_n$. To show part ii), note that for a regular network with degree $k$, $d_{max} = d_{min} = k = \beta n^\gamma$

and $\lambda_{max}(A) = \lambda_{min}(A) = 1 + k$ (the least conservative upper bound on $\epsilon$ is attained by a regular network). Hence, the expression for $\epsilon$ greatly simplifies as

$$\epsilon = \frac{\nu\sqrt{n}}{1+k} = \frac{\nu\sqrt{n}}{1+\beta n^{\gamma}} \tag{19}$$

Thus, for all $\gamma > \gamma_c = \frac{1}{2}$, $\epsilon_n \to 0$ as $n \to \infty$ regardless of the values of $\beta, \nu < \infty$. In other words, $\epsilon_n$-tracking of $r(t)$ is achieved asymptotically by every node with a vanishing $\epsilon$ for large-scale regular networks of size (i.e. $nk/2$) greater than $O(n^{1.5})$. □

*Remark* 3. The white noise $w(t) = \frac{1}{n}\sum_i v_i(t)$ has a covariance matrix $\frac{1}{n}\bar{R}$ that is $n$ times smaller that the average covariance $\bar{R} = \frac{1}{n}\sum_i R_i$ of all (uncorrelated) $v_i$'s. For a large-scale network, $w(t)$ can possibly become multiple orders of magnitude weaker than all the $v_i$'s.

**Corollary 2.** *(scale-uncertainty principle) A regular complex network with density $\sigma = (2|E|+n)/n^{1.5}$ and tracking uncertainty $\varepsilon = \epsilon/\nu$ that runs the dynamic consensus algorithm in* (5) *satisfies the following uncertainty principle*

$$\textit{(network density)} \times \textit{(tracking uncertainty)} = 1, \tag{20}$$

*or $\sigma \times \varepsilon = 1$.*

*Proof.* The proof follows from (19) and the identity $2|E| := \sum_i d_i = nk$. □

Defining the performance of tracking as $1/\varepsilon$, we get the following trade-off between tracking performance and network density:

$$\text{(network density)} \propto \text{(tracking performance)}.$$

The most common application is to track a signal that has a single, or multiple, sinusoidal components.

**Example 1.** (tracking of sinusoidal signals) Consider the case of a signal $r(t) = asin(\omega t)$ with $a, \omega > 0$ that is being measured by every sensor in a sensor network. This signal could possibly represent the $x$-coordinate of the position of a moving object that goes in circles. The main question of interest is *how large the sensor network must be?* This is important for the purpose of tracking $r(t)$ within a tube of radius $\epsilon \leq \delta a$ (e.g. $\delta = 0.1$).

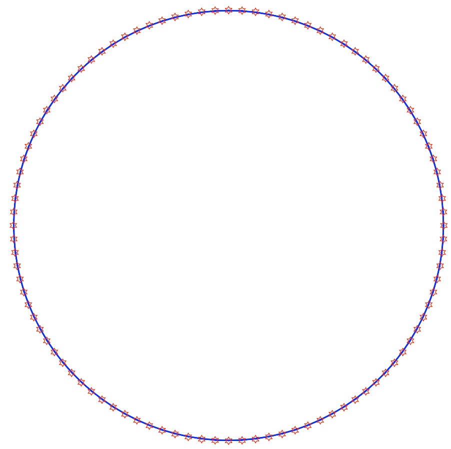
Notice that $\nu = a\omega$ and therefore the tracking uncertainty satisfies To guarantee $\epsilon \leq \delta a$, we must have $\varepsilon = \epsilon/\nu \leq \delta/\omega$. Using the uncertainty principle, $\sigma \times \varepsilon = 1$ and thus $\omega \leq \delta \times \sigma$.

For a network with $n = 1000$ nodes and weighted degree $k = \beta n^{\gamma}$ with $\beta = 10, \gamma = 0.6 > \gamma_c$ (all weights of the graph are in $\{0, \beta\}$), we get $k = 631$ and $\omega \leq 2$ (rad/sec) for $\epsilon = 0.1a$ accuracy. This is a relatively conservative bound and in practice the network is capable of tracking much faster signals with only 100 nodes. Finding a less conservative uncertainty principle is a real challenge.

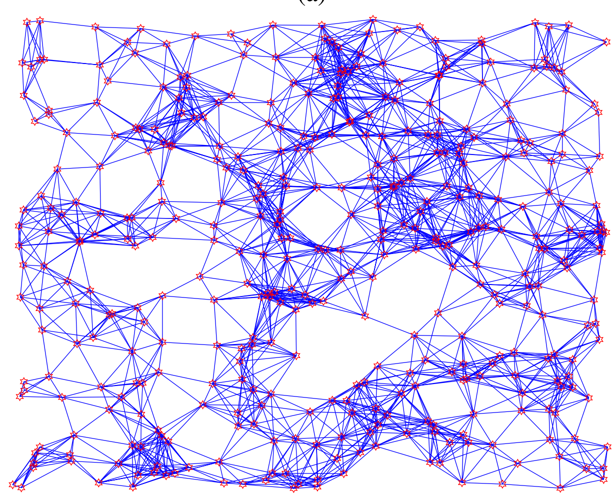
One cannot arbitrarily increase $\beta$ because based on the low-pass filter with state $\mu$, this is equivalent to using a high-gain observer for $\bar{u}$ that amplifies noise.

## IV. Simulation Results

In this section, we present simulation results for sensor networks with two type of topologies: a) a regular network of degree $k = 6$ and b) a random network obtained as a spatially induced graph from $n = 400$ points with coordinates $\{q_i\}_{i\in V}$ that are distributed uniformly at random in an $n \times n$ square region with a set of neighbors $N_i = \{q_j : \|q_i - q_j\| < \rho_0\}$ and a radio range of $\rho_0 = 2\sqrt{n}$. These networks are shown in Fig 2. Networks (a) and (b), shown in Fig. 2, have an average-degree of 6 and 7.1, respectively. Apparently, the random network is irregular.



(a)



(b)

Fig. 2. Sensor network topologies: a) a regular network with $n = 100$ and degree $k = 6$ and b) a random network with $n = 400$ and 2833 links.

We use the following three test signals

$$\begin{aligned} r_1(t) &= \sin(2t); \\ r_2(t) &= \sin(t) + \sin(2t+3) + \sin(5t+4), \\ r_3(t) &= \sin(10t). \end{aligned}$$

For $r_1$ and $r_2$, we set the covariance matrix to $R_i = 0.3$ for all nodes and for $r_3$, $R_i = 0.6$ for all $i$.

Fig. 3 demonstrates sensor fusion using a low-pass consensus filter with a regular network topology for sensor measurements $r_1(t) + v_i(t)$ obtained from $n = 100$ nodes. The fused measurements Fig. 3 (b) have a covariance that is almost 100 times smaller than the covariance of the sensor data.

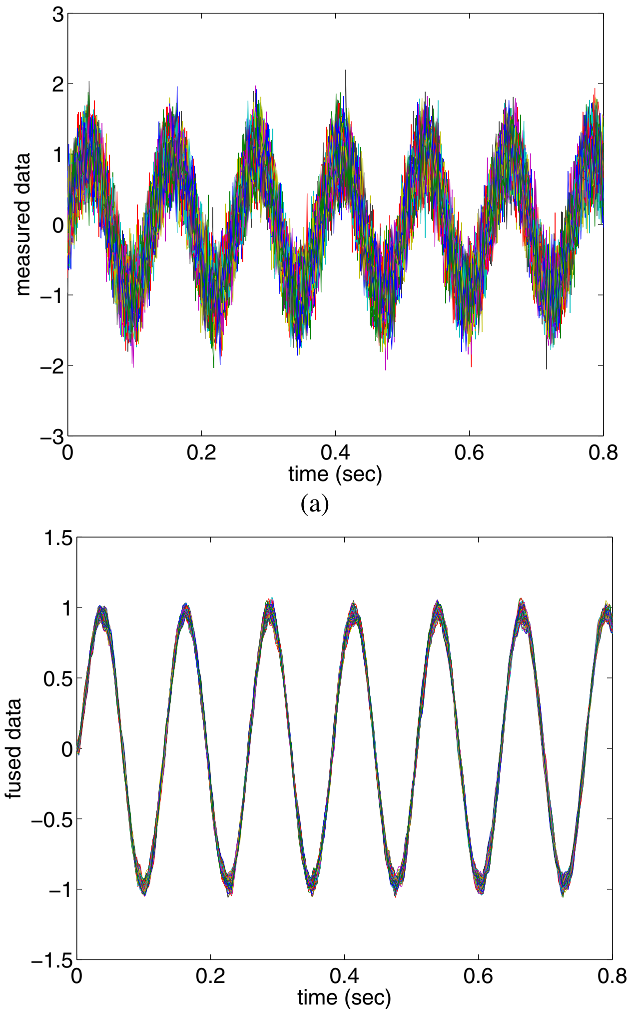



Fig. 3. a) sensor measurements $r_1(t) + v_i(t)$ and b) fused sensor data via a low-pass consensus filter in a regular network.

Similarly, Fig. 4 demonstrates sensor fusion using a distributed low-pass consensus filter for sensor data $r_2(t) + v_i(t)$ obtained from $n = 100$ nodes. Again, the network topology is regular. All nodes are apparently capable of tracking $r_3(t)$ within a radius of uncertainty that is determined by $|\dot{r}_3$ and the noise covariance $R_i$.

Now, to demonstrate tracking capabilities of larger networks, we consider tracking $r_3(t)$ that is 5 times faster than $r_1(t)$ using a consensus filter in a network with random topology. The results of the sensor fusion are shown in Fig. fig:measurements3.

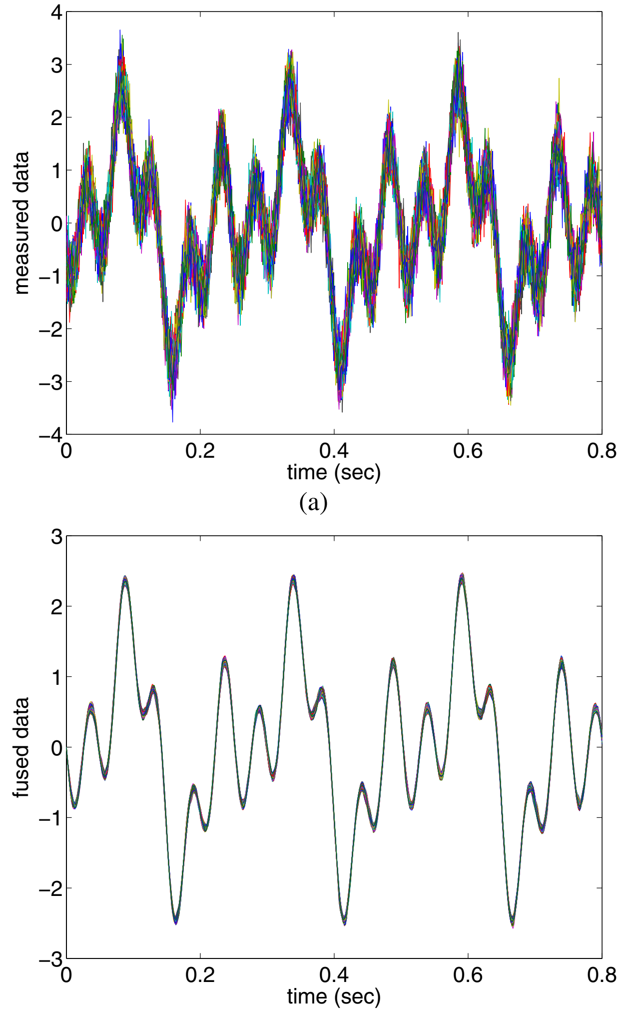
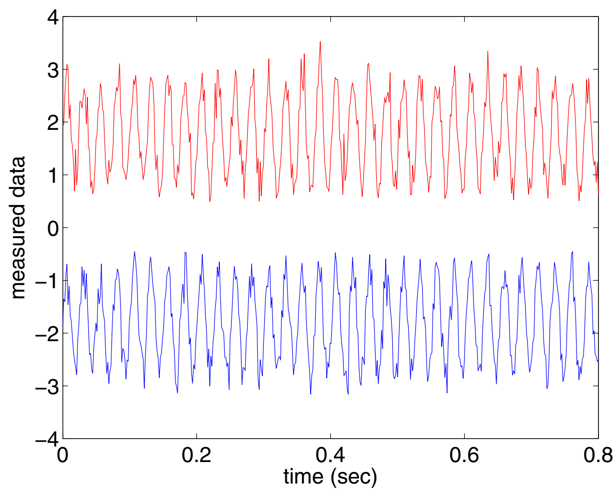


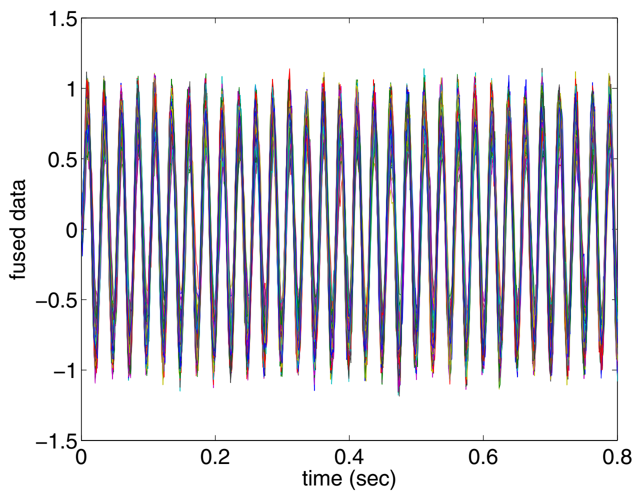

Fig. 4. a) sensor measurements $r_2(t) + v_i(t)$ and b) fused sensor data via a low-pass consensus filter with a regular network topology.

## V. Conclusions

We introduced consensus filters as a tool for distributed sensor fusion in sensor networks. The consensus filter is a dynamic version of average-consensus algorithm that has been extensively used for sensor fusion as well as other applications that involves networked dynamic systems and collaborative decision making. It was mentioned that based on a new scalable Kalman filtering scheme, a crucial part of the solution is to estimate the average of $n$ signals in a distributed way. It was shown that consensus filters effectively solve this dynamic average-consensus problem. This distributed filter acts as a low-pass filter induced by the information flow in the sensor network. In addition, $\epsilon$-tracking properties of consensus filters for sensor fusion was analyzed in details. The byproduct of this analysis was a novel type critical phenomenon in complex networks that relates the size of the sensor network to its capability to track relatively fast signals. This limitation was characterized as a tracking uncertainty principle. Simulations




(a)




(b)

Fig. 5. (a) upper and lower envelops $(\max_i u_i(t), \min_i u_i(t))$ of sensor measurements $r_3(t) + v_i(t)$, and b) fused measurements (i.e. states $x_i$) after consensus filtering in a sensor network with randomly distributed nodes.

results for large regular and random sensor network were presented.

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