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Adaptive Consensus Output Regulation of A Class of Heterogeneous Nonlinear Systems

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***Abstract*—This paper deals with consensus output regulation of a class of nonlinear systems which consist of network-connected subsystems with unknown parameters. The subsystems may have different dynamics with uncertainties, and be subject to the disturbances generated from an exosystem. Only some subsystems have access to the desired output, which is also formulated as an output of the exosystem, following the standard formulation of output regulation of nonlinear systems. The proposed design makes use of some latest results on Laplacian matrices, and a new design of the internal model, which is based on known functions of with unknown constant parameters. Adaptive backstepping design techniques are integrated with the consensus control design to tackle the nonlinearity and unknown constant parameters in the system, and unknown functions relating to the state of the exosystem. The proposed control inputs and adaptive laws are fully decentralized, and ensure the asymptotic convergence to zero of the regulation errors.**

***Index Terms*—Consensus control, Output regulation, Nonlinear systems, Internal model, Output feedback**

I. Introduction

Consensus control deals with the control design for the same objectives for the subsystems with the same or similar dynamics which are connected together by a network. The research on consensus control has been motivated by applications in formation control of robots and vehicles etc, and has attracted significant attention from the researchers in the control circle in the recent years. One important step in consensus control design is to explore the results on Laplacian matrices in graph theory for connections of the subsystems. For linear systems, typical consensus control designs for homogeneous subsystem dynamics have been presented [1], [2], [3] where the properties of the Laplacian matrices play an important part. There are considerable results published for heterogeneous linear systems and results for nonlinear systems [4], [5], [6], [7], [8], [9], [10], [11]. Compared with the results in the linear systems, the results for nonlinear systems are often more restrictive in the subsystem dynamics and the network connections.

When there are uncertainties in a system, adaptive control strategies would be naturally considered. One challenge in adaptive control design for consensus control is the decentralization of the adaptive laws [12]. In the robust adaptive consensus control shown in [13], [14], the adaptive laws are decentralized, with the influence of the uncertainties of the adjacent subsystems being treated as bounded disturbances, and the resultant consensus control errors are kept bounded instead of the convergence to zero due to the robust adaptive control treatment. Decentralized adaptive laws have been proposed for first-order nonlinear systems in [12]. In this paper, we will deal with adaptive control design for high order nonlinear systems under disturbances generated from an exosystem. All the subsystems are required to track a desired trajectory, which is also a function of the state of the exosystem. Not every subsystem has the access of the desired trajectory. The other subsystems will achieve output regulation through the network connections. Tracking a trajectory and rejecting a disturbance simultaneously are normally formulated as an output regulation problem [15]. A key issue in the solution of output regulation problem is the design of an internal model which generates the designed feedforward input signal. This is a difficulty in consensus output regulation, again due to the requirement of decentralized implementation of the internal models. Our approach to the internal model design is based on the fact the the exosystem states can be factored as the product of a vector of known sinusoidal functions and a constant vector, inspired by the properties of linear exosystems in output regulation [16], [17] and disturbance estimation and rejection [18]. The constant vector depends on the initial state of the exosystem. Adaptive control techniques can then be used to estimate the unknown constant vector. For the consensus control part, we explore the result that the Laplacian matrix of an irreducible network can be make positive definite by adding an positive constant in any of the diagonal elements. With this result, we propose a solution to adaptive output regulation problem using block adaptive backstepping design. Our solution ensures that all the subsystems converge to a common output. An example is included to demonstrate the proposed control design with the simulation results shown.

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II. Problem Statement

In this paper, we consider a set of N subsystems connected by an undirected network. The dynamics of each of subsystems are described by, for $i = 1, \ldots, N$,

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= x_{i3} \\ &\vdots \\ \dot{x}_{in} &= \phi_i^T(x_i)\theta_i + \phi_{0i}(w) + u_i \end{aligned} \qquad (1)$$

where $x_i = [x_{i1}, \ldots, x_{in}]^T \in \mathbb{R}^n$ is the state variable for the ith subsystem with x_{i1} as the output of the subsystem, and $u_i \in \mathbb{R}$ is the input, $\phi_i^T : \mathbb{R}^n \to \mathbb{R}^{p_i}$ is a continuous function, $\theta_i \in \mathbb{R}^{p_i}$ is an unknown constant vector, $\phi_{0i} : \mathbb{R}^n \to \mathbb{R}$ is a polynomial, $w \in \mathbb{R}^s$ is the disturbance generated by an exosystem

$$\dot{w} = Sw \qquad (2)$$

with $S \in \mathbb{R}^{s\times s}$ being a known matrix.

The connections between the subsystems are specified by an undirected graph $\mathcal{G}$ which consists of a set of vertices denoted by $\mathcal{V}$ and a set of edges denoted by denoted by $\mathcal{E}$. A vertex represents a subsystem, and each edge represents a connection. Associated with the graph, its adjacency matrix A with elements a_{ij} denotes the connections such that $a_{ij} = 1$ if there is a connection from subsystem j to subsystem i, and $a_{ij} = 0$ otherwise. Since the connection is undirected, we have $A = A^T$. Let $D = \text{diag}\{d_i\}$ with $d_i = \sum_{j=1}^N a_{ij}$. The Laplacian matrix is commonly defined as $L = D - A$.

The control objective is to design a control input to ensure that the outputs of all the subsystems converge to the same function, i.e.,

$$\lim_{t\to\infty} (x_{i1} - x_{j1}) = 0 \qquad (3)$$

for $i, j = 1, \ldots, N$, and furthermore, for $i = 1, \ldots, N$

$$\lim_{t\to\infty} (x_{i1} - r(w)) = 0 \qquad (4)$$

where $r : \mathbb{R}^s \to \mathbb{R}$ is a polynomial. Not all the subsystems have the access to the function value of $r(w)$. We use a diagonal matrix B to denote the access to $r(w)$ in the way that if $b_{ii} = 1$, the ith subsystem has access to the value of $r(w)$ for the control design. At least one subsystem has the access. The subsystems which do not have access to the tracking signal rely on the network connections to achieve the consensus tracking.

We make a couple of assumptions about the dynamics of the exosystem and the connections between the subsystems.

Assumption 1. The eigenvalues S are distinct and on the imaginary axis.

Assumption 2. The adjacent matrix A is irreducible.

Remark 1. In the formulation of an output regulation problem, the tracking trajectories and the disturbances are commonly assumed to be functions of the state that is generated by an exosystem. In such a formulation, disturbance rejection and output tracking can be treated together in a unified way. Assumption 1 on the eigenvalues of the exosystem dynamics is common in the formulation of output regulation, as the stable modes in the exosystem do not have an impact asymptotically. From a practical point of view, any periodic signal designed for the tracking trajectory can be approximated by sinusoidal functions with different frequencies, and those sinusoidal functions can be formulated as the state variables of the exosystem under Assumption 1. ◁

Remark 2. The adjacent matrix is irreducible if there exists a connection between any two subsystems. ◁

III. Preliminary Results

Consider the tracking error of a subsystem. Let

$$\bar{x}_{i1} = x_{i1} - r(w)$$

Its derivative is given by

$$\dot{\bar{x}}_{i1} = \frac{d}{dt}\bar{x}_{i1} = x_{i2} - L_{Sw}r(w) := \bar{x}_{i2}$$

where $L_{Sw}r(w)$ denotes the Lie derivative of $r(w)$ along Sw. Similarly, we define

$$\bar{x}_{ij} = \frac{d^{j-1}}{dt^{j-1}}\bar{x}_{i1}$$

for $j = 2, \dots, n$. We can obtain that

$$\dot{\bar{x}}_{in} = \phi_i^T(x_i)\theta_i + u_i - v_i$$

where

$$v_i = -\phi_{0i}(w) + L_{Sw}^{n-1}r(w).$$

In output regulation, we often refer v_i as the desired feedforward term. For the control design to ensure the asymptotic tracking of $r(w)$ and rejection of the disturbance term $\phi_{0i}(w)$, the control input for the subsystem must contain the desired feedforward term. If the variables x_{ij} for $j = 1, \dots, n$, and w are available, the control design given by

$$u_i = \sum_{j=1}^{n} -\lambda_j \bar{x}_{ij} + v_i$$

where λ_i are positive real coefficients such that the polynomial $s^n + \lambda_1 s^{n-1} + \dots + \lambda_n$ is Hurwitz.

The challenge in the consensus output regulation is that w is is not available for the control design, and $r(w)$ is only available to a subset of the subsystems, not to all the subsystems. For the tracking signal $r(w)$, the subsystems that do not have access to $r(w)$ directly can obtain necessary information via the connections with other subsystems when the subsystems are limited to the first order. For high-order subsystems, we need the information of $L_{Sw}^{j-1}r(w)$ for $j = 1, \dots, n$ and $\phi_{0i}(w)$. The following lemma provides a possibility to solve the problem with adaptive control techniques.

Lemma 1: For the state w generated from the exosystem (2), any polynomial of w can be expressed as a product of known time-varying vector and a vector with unknown constant parameters that depend on the initial state $w(0)$, in particular,

$$\begin{aligned} L_{Sw}r(w) &= \psi^T(t)\vartheta_{00}, \\ \phi_{0i}(w) &= \psi_{0i}^T(t)\vartheta_{0i} \text{ for } i = 1, \dots, N \end{aligned}$$

where $\psi(t)$ and $\psi_{0j}(t)$ are known functions, and ϑ_{00} and ϑ_{0i} are unknown constant vectors with their dimensions depending on the exosystem and the nonlinear function $r(w)$ and $\phi_{0i}(w)$. Furthermore, for the function $\psi(t)$, its derivative is given by

$$\frac{d\psi(t)}{dt} = S_\psi \psi(t), \tag{5}$$

where S_ψ is a diagonal matrix, with elements being frequencies of the signals in $\psi(t)$.

Proof: For the exosystem, its state variables are sinusoidal functions with the frequencies depending on the eigenvalues, and the coefficients depending on the initial state. The results shown in Lemma 1 can then established by following similar procedures shown in [17], [18]. Since $\psi(t)$ is a vector of sinusoidal functions, its derivative can be expressed as as in (5). ■

Note that

$$L_{Sw}^j r(w) = \psi^T(t)S_\psi^j \vartheta_{00},$$

for $j = 1, \dots, n-1$, based on the result shown in (5).

We have another useful result for the Laplacian matrix.

Lemma 2: If the adjacency matrix A is irreducible, and the non-negative diagonal matrix B has at least one positive diagonal element, the matrix $(L + B)$ is positive definite.

The proof of this lemma can be found in [19].

Let us denote

$$\chi_j = [x_{1j}, x_{2j}, \dots, x_{Nj}]^T \tag{6}$$

for $j = 1, \dots, n$. and the consensus regulation error

$$e = (L + B)(\chi_1 - \mathbf{1}\ r(w)) \tag{7}$$

where $\mathbf{1} \in \mathbb{R}^N$ is a vector with all the elements 1. Based on the definition of L, we have $L\mathbf{1} = 1$. Since $(L + B)$ is invertible, the control objective (4) is equivalent to $\lim_{t\to\infty} e = 0$. It is worth noting that

$$e = L\chi_1 + B(\chi_1 - \mathbf{1}\ r(w)) \tag{8}$$

which implies that

$$e_i = \sum_{j=1}^{N} a_{ij}(x_{i1} - x_{j1}) + b_{ii}(x_{i1} - r(w)) \tag{9}$$

for $i = 1, \dots, N$. Clearly, e_i is available to the control design for the ith subsystem. For the notational convenience, we denote $Q = L + B$.

IV. Consensus Regulation for 1st-Order Subsystems

Consider the control design for a system with 1st-order subsystems. In this case, we can write the subsystem dynamics as

$$x_{i1} = \phi_i^T(x_{i1})\theta_i + \phi_{0i}(w) + u_i$$

for $i = 1, \dots, N$. Based on the result shown in Lemma 1, we have for, $i = 1, \dots, N$,

$$x_{i1} = \phi_i^T(x_{i1})\theta_i + \psi_{0i}(t)^T\vartheta_{0i} + u_i.$$

Design the control input, for $i = 1, \dots, N$,

$$u_i = -ce_i - \phi_i^T\hat{\theta}_i - \psi_{0i}^T\hat{\vartheta}_{0i} \tag{10}$$

where $\hat{\theta}_i$ and $\hat{\vartheta}_{0i}$ are the estimates of θ_i and ϑ_{0i} respectively, and c is a positive real design constant. The resultant dynamics of x_i are given by

$$\dot{x}_{i1} = -ce_i + \phi_i^T \tilde{\theta}_i + \psi_{0i}^T \tilde{\vartheta}_{0i} \tag{11}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{\vartheta}_{0i} = \vartheta_{0i} - \hat{\vartheta}_{0i}$. In this paper we will use $\hat{a}$ as an estimate of an unknown vector a, and $\tilde{a} = a - \hat{a}$ to denote the estimation error. The adaptive laws are designed as, for $i = 1, \ldots, N$,

$$\begin{aligned} \dot{\hat{\theta}}_i &= \Gamma_{\theta i} \phi_i(x_i) e_i, & (12) \\ \dot{\hat{\vartheta}}_{0i} &= \Gamma_{0i} \psi_{0i}(t) e_i, & (13) \end{aligned}$$

where $\Gamma_{\theta i}$ and Γ_{0i} are positive definite matrices with proper dimensions. Note that the control inputs shown in (10) and the adaptive laws in (12) and (13) are fully decentralized, as they only use the local information available to the ith subsystem.

Lemma 3: The decentralized control inputs and adaptive laws solve the adaptive consensus regulation problem in the sense that the regulation error e converges to zero asymptotically.

Proof: Denote

$$\begin{aligned} \Phi &= \mathrm{diag}\{\phi_1, \phi_2, \ldots, \phi_N\}, \\ \theta &= [\ \theta_1 \quad \theta_2 \quad \ldots \quad \theta_N\ ]^T, \\ \Psi_0 &= \mathrm{diag}\{\psi_{01}, \psi_{02}, \ldots, \psi_{0N}\}, \\ \vartheta_0 &= [\ \vartheta_{01} \quad \vartheta_{02} \quad \ldots \quad \vartheta_{0N}\ ]^T, \end{aligned}$$

and

$$u = [u_1, \ldots, u_N]^T. \tag{14}$$

The dynamics of the consensus regulation error are obtained as

$$\dot{e} = Q(\Phi^T \theta + \Psi_0^T \vartheta_0 + u) \tag{15}$$

and the control input can then be expressed as

$$u = -ce - \Phi^T \hat{\theta} - \Psi^T \hat{\vartheta}_0 \tag{16}$$

and the adaptive laws as

$$\begin{aligned} \dot{\hat{\theta}} &= \Gamma_\theta \Phi e, & (17) \\ \dot{\hat{\vartheta}}_0 &= \Gamma_0 \Psi e, & (18) \end{aligned}$$

where $\Gamma_\theta = \mathrm{diag}\{\Gamma_{\theta i}\}$ and $\Gamma_0 = \mathrm{diag}\{\Gamma_{0i}\}$.

The resultant regulation error dynamics are given by

$$\dot{e} = Q(-ce + \Phi^T \tilde{\theta} + \Psi_0^T \tilde{\vartheta}_0). \tag{19}$$

Consider a Lyapunov function candidate

$$V = e^T Q^{-1} e + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \tilde{\vartheta}^T \Gamma_0^{-1} \tilde{\vartheta}. \tag{20}$$

From (19), we have

$$\begin{aligned} \dot{V} &= -ce^T e + 2e^T \Phi^T \tilde{\theta} + 2e^T \Psi_0^T \tilde{\vartheta}_0 \\ &\quad -2\dot{\hat{\theta}}^T \Gamma^{-1} \tilde{\theta} - 2\dot{\hat{\vartheta}}^T \Gamma_0^{-1} \tilde{\vartheta} \\ &= -ce^T e. \end{aligned}$$

Following the standard procedures for analysing adaptive control systems, we can conclude the boundedness of all the variables and $\lim_{t\to\infty} e = 0$. ∎

V. Consensus Regulation of High-Order Systems

Using the notations introduced in the earlier sections, we can write the system dynamics as

$$\begin{aligned} \dot{\chi}_1 &= \chi_2 \\ &\vdots \\ \dot{\chi}_{n-1} &= \chi_n \\ \dot{\chi}_n &= \Phi^T \theta + \Psi_0^T \vartheta_0 + u. & (21) \end{aligned}$$

The dynamics of the consensus regulation error e are then obtained as

$$\dot{e} = Q(\chi_2 - \mathbf{1}\psi^T \vartheta_{00}). \tag{22}$$

If the derivatives of $r(w)$ up to nth order are available to the control design, a full-state feedback control design can be easily carried out to ensure the converge to zero of the consensus output regulation error. Due to the unknown derivatives of the tracking errors, adaptive backstepping can be used to carry out the control design. For the backstepping design, we introduce a number of notations,

$$\begin{aligned} z_1 &= e, \\ z_j &= \chi_j - \alpha_{j-1}, \text{ for } j = 2, \ldots, n, \end{aligned}$$

where $\alpha_i \in \mathbb{R}^N$ are stabilising functions obtained in backstepping design. The key point in this adaptive backstepping design is to ensure that the control and adaptive laws can be decentralized.

The dynamics of z_1 can be obtained as

$$\dot{z}_1 = Q(z_2 + \alpha_1 - \mathbf{1}\psi^T \vartheta_{00}). \tag{23}$$

Design α_1 as

$$\alpha_1 = -c_1 z_1 + \Psi^T \hat{\vartheta} \tag{24}$$

where c_1 is a positive constant, $\Psi = I_N \otimes \psi$ and $\hat{\vartheta}$ is an estimate of $\mathbf{1} \otimes \vartheta_{00}$ with $\otimes$ denoting the Kronecker product of matrices. It is noted that there are N copies of estimates of ϑ_{00} in $\hat{\vartheta}$, and indeed, we can denote $\hat{\vartheta} = [\hat{\vartheta}_1^T, \ldots, \hat{\vartheta}_N^T]^T$. The multiple copies $\hat{\vartheta}_i$ are for the decentralized implementation of the adaptive laws. For the notational convenience, we denote $\vartheta = \mathbf{1} \otimes \vartheta_{00}$. The resultant dynamics of z are obtained as

$$\dot{z}_1 = Q(-c_1 z_1 + z_2 - \Psi^T \tilde{\vartheta}). \tag{25}$$

The adaptive law for $\hat{\vartheta}$ is designed through tuning functions to avoid multiple estimations of ϑ in adaptive backstepping [20]. The tuning functions for subsystems in the first step, τ_{1i}, are designed as

$$\tau_{1i} = -z_{1i} \Gamma_i \psi \tag{26}$$

for $i = 1, \ldots, N$, where Γ_i is a positive definite matrix with a proper dimension. We put the tuning functions for subsystems together to have

$$\tau_1 = -\Gamma \Psi z_1 \tag{27}$$

where $\Gamma = \mathrm{diag}\{\Gamma_i\}$. In the subsequent design, we will use c_j and τ_j for $j = 2, \ldots, n$ to denote positive design parameters and tuning functions respectively.

In the second step, the dynamics of z_2 are obtained as

$$\begin{aligned} \dot{z}_2 &= \chi_3 + c_1 \dot{e} - \Psi^T S_\Psi \hat{\vartheta} - \Psi^T \dot{\hat{\vartheta}} \\ &= z_3 + \alpha_2 + c_1 Q(\chi_2 - \Psi^T \vartheta) \\ &\quad -\Psi^T S_\Psi \hat{\vartheta} - \Psi^T \dot{\hat{\vartheta}} & (28) \end{aligned}$$

where $S_\Psi = I_N \otimes S_\psi$. We design α_2 as

$$\begin{aligned} \alpha_2 &= -z_1 - c_2 z_2 - c_1 Q(\chi_2 - \Psi^T \hat{\vartheta}) \\ &\quad + \Psi^T S_\Psi \hat{\vartheta} + \Psi^T \tau_2. \end{aligned} \quad (29)$$

The resultant dynamics of z_2 are obtained as

$$\dot{z}_2 = -z_1 - c_2 z_2 + z_3 - c_1 Q \Psi^T \tilde{\vartheta} - \Psi^T (\dot{\hat{\vartheta}} - \tau_2). \quad (30)$$

The tuning function τ_2 is designed as

$$\tau_2 = -\Gamma(\Psi z_1 + c_1 \Psi Q z_2) \quad (31)$$

In the subsequent step, we have, for $j = 3, \ldots, n-1$, the dynamics of z_j are obtained as

$$\begin{aligned} \dot{z}_j &= \chi_{j+1} - \frac{\partial \alpha_{j-1}}{\partial e} Q(\chi_2 - \Psi^T \vartheta) \\ &\quad - \sum_{k=2}^{j-1} \frac{\partial \alpha_{j-1}}{\partial \chi_k} \chi_{k+1} - \frac{\partial \alpha_{j-1}}{\partial \hat{\vartheta}} \dot{\hat{\vartheta}} \\ &\quad - \frac{\partial \alpha_{j-1}}{\partial \Psi} S_\Psi \Psi. \end{aligned} \quad (32)$$

We design α_j as

$$\begin{aligned} \alpha_j &= -z_{j-1} - c_j z_j + \frac{\partial \alpha_{j-1}}{\partial e} Q(\chi_2 - \Psi^T \hat{\vartheta}_j) \\ &\quad + \sum_{k=2}^{j-1} \frac{\partial \alpha_{j-1}}{\partial \chi_k} \chi_{k+1} + \frac{\partial \alpha_{j-1}}{\partial \hat{\vartheta}} \tau_j + \frac{\partial \alpha_{j-1}}{\partial \Psi} S_\Psi \Psi \\ &\quad + \sum_{k=2}^{j-1} \frac{\partial \alpha_{j-1}}{\partial e} Q \Psi^T \Gamma \left(\frac{\partial \alpha_{k-1}}{\partial \hat{\vartheta}} \right)^T z_k. \end{aligned} \quad (33)$$

The resultant dynamics of z_j are obtained as

$$\begin{aligned} \dot{z}_j &= -z_{j-1} - c_j z_j + z_{j+1} \\ &\quad + \frac{\partial \alpha_{j-1}}{\partial e} Q \Psi^T \tilde{\vartheta} - \frac{\partial \alpha_{j-1}}{\partial \hat{\vartheta}} (\dot{\hat{\vartheta}} - \tau_j) \\ &\quad + \sum_{k=2}^{j-1} \frac{\partial \alpha_{j-1}}{\partial e} Q \Psi^T \Gamma \left(\frac{\partial \alpha_{k-1}}{\partial \hat{\vartheta}} \right)^T z_k. \end{aligned} \quad (34)$$

The tuning function τ_j is designed as

$$\tau_j = \Gamma \Psi Q \sum_{k=1}^{j} \left(\frac{\partial \alpha_{k-1}}{\partial e} \right)^T z_k \quad (35)$$

where we set $\frac{\partial \alpha_0}{\partial e} = -Q^{-1}$ for the notational convenience.

In the final step, we have

$$\begin{aligned} \dot{z}_n &= u + \Phi^T \theta + \Psi_0^T \vartheta_0 - \frac{\partial \alpha_{n-1}}{\partial e} Q(\chi_2 - \Psi^T \vartheta) \\ &\quad - \sum_{k=2}^{j-1} \frac{\partial \alpha_{n-1}}{\partial \chi_k} \chi_{k+1} - \frac{\partial \alpha_{n-1}}{\partial \hat{\vartheta}} \tau_n - \frac{\partial \alpha_{n-1}}{\partial \Psi} S_\Psi \Psi. \end{aligned} \quad (36)$$

We design u as

$$\begin{aligned} u &= -z_{n-1} - c_n z_n + \frac{\partial \alpha_{n-1}}{\partial e} Q(\chi_2 - \Psi^T \hat{\vartheta}_j) \\ &\quad + \sum_{k=2}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \chi_k} \chi_{k+1} + \sum_{k=2}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\vartheta}_k} \dot{\hat{\vartheta}}_k \\ &\quad + \frac{\partial \alpha_{n-1}}{\partial \Psi} S_\Psi \Psi + \sum_{k=2}^{n-1} \frac{\partial \alpha_{n-1}}{\partial e} Q \Psi^T \Gamma \left(\frac{\partial \alpha_{k-1}}{\partial \hat{\vartheta}} \right)^T z_k \\ &\quad - \Phi^T \hat{\theta} - \Psi_0^T \hat{\vartheta}_0. \end{aligned} \quad (37)$$

The resultant dynamics of z_n are obtained as

$$\begin{aligned} \dot{z}_n &= -z_{n-1} - c_n z_n + \frac{\partial \alpha_{n-1}}{\partial e} Q \Psi^T \tilde{\vartheta} \\ &\quad + \sum_{k=2}^{n-1} \frac{\partial \alpha_{n-1}}{\partial e} Q \Psi^T \Gamma \left(\frac{\partial \alpha_{k-1}}{\partial \hat{\vartheta}} \right)^T z_k \\ &\quad + \Phi^T \tilde{\theta} + \Psi_0^T \tilde{\vartheta}_0. \end{aligned} \quad (38)$$

The adaptive laws for $\hat{\theta}$, $\hat{\vartheta}$ and $\hat{\vartheta}_0$ are designed as

$$\dot{\hat{\theta}} = \Gamma_\theta \Phi z_n, \quad (39)$$

$$\dot{\hat{\vartheta}} = \tau_n := \Gamma \Psi Q \sum_{k=1}^{n} \left(\frac{\partial \alpha_{k-1}}{\partial e} \right)^T z_k, \quad (40)$$

$$\dot{\hat{\vartheta}}_0 = \Gamma_0 \Psi z_n. \quad (41)$$

For the proposed adaptive backstepping, we have the following theorem.

Theorem 4: For the network-connected nonlinear system (1) under Assumptions 1 and 2, the adaptive controller that consists of the control input (37) and the adaptive laws (39), (40), and (41) ensures the solution of the consensus output regulation problem in the sense that $\lim_{t\to\infty} e(t) = 0$. Furthermore, the proposed control input and adaptive laws are decentralized.

Proof: Consider a Lyapunov function candidate

$$\begin{aligned} V &= z_1^T Q^{-1} z_1 + \sum_{i=2}^{N} z_j^T z_j + \tilde{\theta}^T \Gamma_\theta^{-1} \tilde{\theta} \\ &\quad + \tilde{\vartheta}^T \Gamma^{-1} \tilde{\vartheta} + \tilde{\vartheta}_0^T \Gamma_0^{-1} \tilde{\vartheta}_0. \end{aligned}$$

From the dynamics of z_j for $j = 1 \ldots, n$ shown in (25), (30), (34) and (38), and the adaptive laws (39), (40), and (41), it can be obtained that

$$\dot{V} = -\sum_{j=1}^{n} c_j z_j^T z_j.$$

Hence from the standard adaptive control analysis, we can conclude that all the variables are bounded and we have $\lim_{t\to\infty} z_j(t) = 0$, for $j = 1, \ldots, n$, which implies $\lim_{t\to\infty} e(t) = 0$, as $e = z_1$.

The decentralized implementation can be shown by the fact that all the matrices involved in the adaptive backstepping are diagonal or block diagonal matrices, expect the matrix Q. The variables involved with Q can be obtained using the network connections between neighbourhood subsystems. Therefore, the control input and adaptive laws are decentralized. ■

VI. Example

Consider a system with 5 subsystems described by

$$\begin{aligned} &\dot{x}_{11} = x_{12}, && \dot{x}_{12} = x_{11} x_{12} \theta_1 + w_2 + u_1, \\ &\dot{x}_{21} = x_{22}, && \dot{x}_{22} = x_2^2 \theta_2 + w_2 + u_2, \\ &\dot{x}_{31} = x_{32}, && \dot{x}_{32} = x_{31}^2 \theta_3 + w_2 + u_3, \\ &\dot{x}_{41} = x_{42}, && \dot{x}_{42} = x_{41} x_{42} \theta_4 + w_1^3 + u_4, \\ &\dot{x}_{51} = x_{52}, && \dot{x}_{52} = x_{52}^2 \theta_5 + w_1^3 + u_5 \end{aligned} \quad (42)$$

with

$$\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

where ω is a known positive constant.

The desired trajectory $r(w) = w_1$. The adjacency matrix is given by

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and the B matrix is given by $B = \text{diag}\{0\ 1\ 0\ 0\ 0\}$ and therefore

$$Q = \begin{bmatrix} 3 & 1 & 1 & 0 & 1 \\ 1 & 4 & 1 & 1 & 0 \\ 1 & 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 3 & 1 \\ 1 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

The system (42) is in the format of (1), and it can be checked that the system satisfies Assumptions 1 and 2. For $r = w_1$, we have $r = \psi^T \vartheta_{00}$ with $\psi = [\sin(\omega t),\ \cos(\omega t)]^T$ and $\vartheta_{00} \in \mathbb{R}^2$ being an unknown constant vector. Note that the unknown constant vector only depends on the initial condition $w(0)$. Similarly for $\phi_{01}(w) = \phi_{02}(w) = \phi_{03}(w) = w_2$, we have $\psi_{01} = \psi_{02} = \psi_{03} = [\sin(\omega t),\ \cos(\omega t)]^T$ and ϑ_{0i} for $i = 1, 2, 3$ are unknown constant vectors in $\mathbb{R}^2$. It can be shown that for $\phi_{04} = \phi_{05} = w_1^3$, we have $\psi_{04} = \psi_{05} = [\sin(\omega t),\ \cos(\omega t),\ \sin(3\omega t),\ \cos(3\omega t)]^T$ and ϑ_{04} and ϑ_{05} are unknown constant vectors in $\mathbb{R}^4$.

The control design follows the procedures proposed in the previous section. The control input and the adaptive laws are designed as, for $i = 1, \dots, 5$,

$$\begin{aligned} u_i &= -z_{1i} - c_2 z_{2i} - c_1 Q_{(i)}(x_{i2} - \psi^T \hat{\vartheta}_i) \\ &\quad + \psi^T S_\psi \hat{\vartheta}_i + \psi^T \dot{\hat{\vartheta}}_i - \phi_i \hat{\theta}_i - \psi_{0i}^T \hat{\vartheta}_{0i} \end{aligned} \tag{43}$$

and

$$\dot{\hat{\theta}}_i = \Gamma_{\theta i} \phi_i z_{2i}, \tag{44}$$

$$\dot{\hat{\vartheta}}_i = \Gamma_i(-\psi z_{1i} - c_1 \psi Q_{(i)} z_2), \tag{45}$$

$$\dot{\hat{\vartheta}}_{0i} = \Gamma_{0i} \Psi z_{2i}, \tag{46}$$

where $S_\psi = \text{diag}\{\omega,\ \omega\}$ and $Q_{(i)}$ denotes the ith row of Q. It can be seen that the control inputs and the adaptive laws are decentralized.

The simulation study has been carried out, and the results for the subsystem outputs are shown in Figure 1 and the control inputs in Figure 2. For the simulation results shown in Figures 1 and 2, we used $\omega = 1$, $c_1 = c_2 = 5$, $\Gamma_{\theta i} = 10000I$, $\Gamma_i = I$ and $\Gamma_{0i} = 10I$ for $i = 1, \dots, 5$.

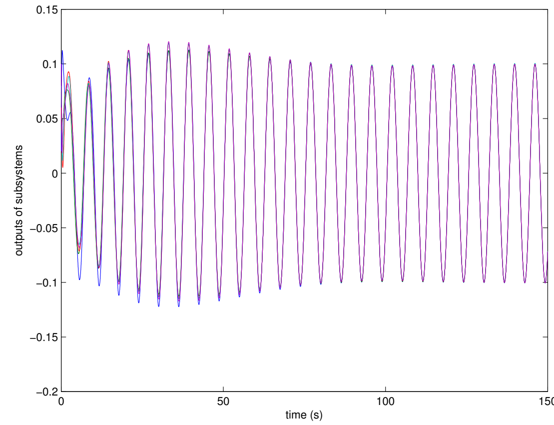

Fig. 1. Subsystem outputs

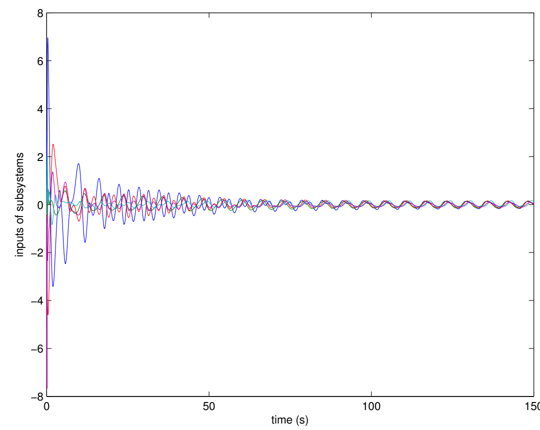

Fig. 2. Subsystem inputs

VII. Conclusions

We have proposed a control design based on block adaptive backstepping design for consensus output regulation of a class heterogeneous nonlinear systems with uncertainties. The success of the proposed design depends on the exploitation of the properties of linear exosystems for nonlinear dynamic systems and the property of Laplacian matrices. Block adaptive backstepping has been used to deal with the high relative degrees in subsystems. The proposed control inputs and adaptive laws are decentralized and they are implemented in each subsystems locally. An example has been used to demonstrate the proposed design, with good simulation results.

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