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Conserved higher spin supercurrents for arbitrary spin massless supermultiplets and higher spin superfield cubic interactions

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ABSTRACT: We give an explicit superspace construction of higher spin conserved supercurrents built out of $4D$, $\mathcal{N} = 1$ massless supermultiplets of arbitrary spin. These supercurrents are gauge invariant and generate a large class of cubic interactions between a massless supermultiplet with superspin $Y_1 = s_1 + 1/2$ and two massless supermultiplets of arbitrary superspin Y_2 . These interactions are possible only for $s_1 \geq 2Y_2$. At the equality, the supercurrent acquires its simplest form and defines the supersymmetric, higher spin extension of the linearized Bel-Robinson tensor.

KEYWORDS: Superspaces, Supersymmetric Effective Theories

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1 Introduction

In previous works [1–6] various conserved, higher spin, supercurrent multiplets of supersymmetric matter theories have been constructed. These were used to generate first order interactions among matter theories and higher spin supermultiplets with the lowest possible number of derivatives. In this note, we go beyond matter theories and consider first order interactions among higher spin supermultiplets.

Finding interactions involving higher spins is non-trivial as one has to satisfy the set of consistency conditions imposed by the gauge invariance of the free higher spin action. For non-supersymmetric theories the search for higher spin interactions is extensive and in many cases interaction terms were successfully constructed for flat spacetime at first order in coupling constant g by using a variety of techniques, such as light-cone approach [7–18], Noether’s procedure [19–23]¹ and BRST [29–36]. Interestingly, most of these results have been obtained by analysing tree level amplitudes of (super)strings [37–39], thus enhancing the connection between string theory and higher spin fields. For (A)dS backgrounds similar results have been obtained [40–44] which eventually led to the fully interacting equations of motion for higher spin fields [45] (see [46] for a review).

Most of these constructions, focus on interaction vertices with the lowest possible number of derivatives, which corresponds to a *minimal coupling* scenario. However we can have *non-minimal* type of interactions which in most cases will lead to lagrangians with higher derivatives. This has been done in [21, 24] using gauge invariant field strengths. An interesting, distinctive aspect of these interactions is their uniqueness up to trivial redefinitions.

In this paper, we generalize these results to supersymmetric theories using the manifest $4D$, $\mathcal{N} = 1$ standard superspace formulation.² Specifically, we give the explicit form of gauge invariant, higher spin, conserved supercurrents for all massless, $4D$ Minkowski,

¹Some of these results were later generalized in [24–27]. For a review see [28].

²We use the conventions of *Superspace* [47].

$\mathcal{N} = 1$ higher spin supermultiplets. A particular example for the supergravity supercurrents of low spins $j \leq 1$ was given in [48]. Our results extends it in two directions: (i) we consider higher spin supermultiplets and (ii) we construct higher spin supercurrents.

Our construction is focused on the description of cubic interactions $Y_1 - Y_2 - Y_2$ between a massless higher spin supermultiplet with arbitrary half-integer superspin $Y_1 = s_1 + 1/2$ (which describes the propagation of massless spins $j = s_1 + 1$ and $j = s_1 + 1/2$) and two massless, higher spin supermultiplets with arbitrary superspin Y_2 . We also assume that this cubic interaction can be written in the form *gauge superfield times supercurrent*. The supercurrent must be quadratic in the Y_2 supermultiplet and the gauge transformation of the Y_1 supermultiplet will impose on it an on-shell conservation equation. Additionally, we demand the supercurrent to be gauge invariant with respect to the gauge transformation of the Y_2 supermultiplet. The Y_2 -gauge invariance fixes the supercurrent to be quadratic in the derivatives of the Y_2 superfield strength and the conservation equation gives a unique solution of this type. Furthermore, we derive a restriction on the allowed values of superspin, $s_1 \geq 2Y_2$, which provides a complete classification for this class of interactions. This is in agreement with the known constraints for the spin values of higher spin interactions of non-supersymmetric theories [16, 21] as well as the Weinberg-Witten theorem [49]. Right at the boundary, when $s_1 = 2Y_2$, the structure of the supercurrent simplifies drastically. Its dependence on the superfield strength becomes algebraic and defines the supersymmetric, higher spin extension of the Bel-Robinson³ tensor [50] (and reference therein).

The organization of the paper is as follows. In section 2, we review the description of free, massless, $4D$, $\mathcal{N} = 1$ higher spin supermultiplets and their corresponding superfield strengths. In section 3, we construct the gauge invariant, higher spin, conserved supercurrent which generates the cubic interactions. In section 4, we include a component discussion for the higher spin currents that can be extracted from the higher spin supercurrent superfield. The last section summarizes our results.

2 Free higher spin supermultiplets and their superfield strengths

The massless, higher spin irreducible representations of the $4D$, $\mathcal{N} = 1$, super-Poincaré were first described in [51] using a component Lagrangian invariant under on-shell supersymmetry. Later a superfield formulation was introduced in [52–54] and further developments can be found in [56–59]. A synopsis of the description of higher spin supermultiplets is the following:

1. The integer superspin $Y = s$ ($s \geq 1$) supermultiplets $(s + 1/2, s)$,⁴ are described by a pair of superfields $\Psi_{\alpha(s)\dot{\alpha}(s-1)}$,⁵ and $V_{\alpha(s-1)\dot{\alpha}(s-1)}$ (is real) with the following zero

³The Bel-Robinson tensor is known in the context of General Relativity and is a spin 4, gauge invariant and conserved tensor constructed out massless spin 2 fields. It is the generalization of the energy-momentum tensor.

⁴On-shell they describe the propagation of degrees of freedom with helicity $\pm(s + 1/2)$ and $\pm s$.

⁵The notation $\alpha(k)$ is a shorthand for k undotted symmetric indices $\alpha_1\alpha_2 \dots \alpha_k$. The same notation is used for the dotted indices.

order gauge transformations

$$\delta_0 \Psi_{\alpha(s)\dot{\alpha}(s-1)} = -D^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{(s-1)!} \bar{D}_{(\dot{\alpha}_{s-1}} \Lambda_{\alpha(s)\dot{\alpha}(s-2)}), \quad (2.1a)$$

$$\delta_0 V_{\alpha(s-1)\dot{\alpha}(s-1)} = D^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{D}^{\dot{\alpha}_s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)}. \quad (2.1b)$$

Off-shell, this supermultiplet carries $8s^2 + 8s + 4$ bosonic and equal number of fermionic degrees for freedom.⁶

2. The half-integer superspin $Y = s + 1/2$ supermultiplets $(s + 1, s + 1/2)$ have two descriptions. The first is called the transverse formulation ($s \geq 1$) and it uses the pair of superfields $H_{\alpha(s)\dot{\alpha}(s)}$ (is real) and $\chi_{\alpha(s)\dot{\alpha}(s-1)}$ with the following zero order gauge transformations

$$\delta_0 H_{\alpha(s)\dot{\alpha}(s)} = \frac{1}{s!} D_{(\alpha_s} \bar{L}_{\alpha(s-1))\dot{\alpha}(s)} - \frac{1}{s!} \bar{D}_{(\dot{\alpha}_s} L_{\alpha(s)\dot{\alpha}(s-1)}), \quad (2.2a)$$

$$\delta_0 \chi_{\alpha(s)\dot{\alpha}(s-1)} = \bar{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + D^{\alpha_{s+1}} \Lambda_{\alpha(s+1)\dot{\alpha}(s-1)}. \quad (2.2b)$$

This supermultiplet, off-shell describes $8s^2 + 8s + 4$ bosonic and equal fermionic degrees of freedom. The second formulation is called the longitudinal ($s \geq 2$) and it includes the superfields $H_{\alpha(s)\dot{\alpha}(s)}$ (is real) and $\chi_{\alpha(s-1)\dot{\alpha}(s-2)}$ with the following zero order gauge transformations

$$\delta_0 H_{\alpha(s)\dot{\alpha}(s)} = \frac{1}{s!} D_{(\alpha_s} \bar{L}_{\alpha(s-1))\dot{\alpha}(s)} - \frac{1}{s!} \bar{D}_{(\dot{\alpha}_s} L_{\alpha(s)\dot{\alpha}(s-1)}), \quad (2.3a)$$

$$\begin{aligned} \delta_0 \chi_{\alpha(s-1)\dot{\alpha}(s-2)} &= \bar{D}^{\dot{\alpha}_{s-1}} D^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)} + \frac{s-1}{s} D^{\alpha_s} \bar{D}^{\dot{\alpha}_{s-1}} L_{\alpha(s)\dot{\alpha}(s-1)} \\ &+ \frac{1}{(s-2)!} \bar{D}_{(\dot{\alpha}_{s-2}} J_{\alpha(s-1)\dot{\alpha}(s-3)}. \end{aligned} \quad (2.3b)$$

This supermultiplet carries $8s^2 + 4$ off-shell bosonic and equal number of fermionic degrees of freedom. However one can show that it is dual to the first, transverse, formulation [6, 52].

The physical and propagating degrees of freedom of the two massless integer and half-integer superspin theories above are described by corresponding superfield strengths $\mathcal{W}_{\alpha(2s)}$ and $\mathcal{W}_{\alpha(2s+1)}$ respectively. They are defined by:

$$Y = s + 1/2: \quad \mathcal{W}_{\alpha(2s+1)} \sim \bar{D}^2 D_{(\alpha_{2s+1}} \partial_{\alpha_{2s}}^{\dot{\alpha}_s} \partial_{\alpha_{2s-1}}^{\dot{\alpha}_{s-1}} \dots \partial_{\alpha_{s+1}}^{\dot{\alpha}_1} H_{\alpha(s)\dot{\alpha}(s)}) \quad (2.4)$$

$$Y = s: \quad \mathcal{W}_{\alpha(2s)} \sim \bar{D}^2 D_{(\alpha_{2s}} \partial_{\alpha_{2s-1}}^{\dot{\alpha}_{s-1}} \partial_{\alpha_{2s-2}}^{\dot{\alpha}_{s-2}} \dots \partial_{\alpha_{s+1}}^{\dot{\alpha}_1} \Psi_{\alpha(s)\dot{\alpha}(s-1)}) \quad (2.5)$$

These are superfields which respect the gauge symmetries mentioned above and at the component level they include the bosonic and fermionic higher spin field strengths. Moreover, they have two interesting characteristics. Firstly, they are both chiral superfields

$$\bar{D}_{\dot{\beta}} \mathcal{W}_{\alpha(2s+1)} = 0, \quad \bar{D}_{\dot{\beta}} \mathcal{W}_{\alpha(2s)} = 0. \quad (2.6)$$

⁶A detailed counting of the off-shell degrees of freedom can be found in [55–58].

Secondly, they have a special index structure. Specifically, they have indices of only one type (the undotted ones) and in both cases the number of indices is $2Y$. Furthermore, on-shell they satisfy the following equations of motions:

$$D^\beta \mathcal{W}_{\beta\alpha(2s)} = 0, \quad D^\beta \mathcal{W}_{\beta\alpha(2s-1)} = 0. \quad (2.7)$$

3 Gauge invariant, conserved, higher spin supercurrent

Now let us consider the cubic interaction $Y_1 - Y_2 - Y_2$ between a massless superspin Y_1 supermultiplet and two massless superspin Y_2 supermultiplets. We select Y_1 to be half-integer $Y_1 = s_1 + 1/2$ for arbitrary positive integer s_1 and Y_2 can be either integer, $Y_2 = s_2$, or half-integer, $Y_2 = s_2 + 1/2$, for arbitrary positive integer s_2 . Moreover, we assume that the first order interaction vertices can be written in a lagrangian form which is generated by a higher spin supercurrent \mathcal{J} and a higher spin supertrace \mathcal{T} as follows:⁷

$$S_I = g \int d^8z \left\{ H^{\alpha(s_1)\dot{\alpha}(s_1)} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} + \chi^{\alpha(s_1)\dot{\alpha}(s_1-1)} \mathcal{T}_{\alpha(s_1)\dot{\alpha}(s_1-1)} \right\}. \quad (3.1)$$

In this case, the supertrace $\mathcal{T}_{\alpha(s_1)\dot{\alpha}(s_1-1)}$ can be ignored because one can find improvement terms to eliminate it. This is related to the fact that a gauge invariant description of supermultiplet Y_2 via the superfield strength has conformal symmetry. Hence the only non-trivial object to focus in the supercurrent $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$.

It is straightforward to show that, due to the gauge transformation (2.2a) the higher spin supercurrent must satisfy on-shell (up to terms that depend on the equation of motion) the following conservation equation

$$\bar{D}^{\dot{\alpha}s_1} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = 0. \quad (3.2)$$

Also due to the reality of $H_{\alpha(s_1)\dot{\alpha}(s_1)}$, $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$ must be real:

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = \bar{\mathcal{J}}_{\alpha(s_1)\dot{\alpha}(s_1)}. \quad (3.3)$$

On top of these requirements, we demand that the higher spin supercurrent is gauge invariant with respect the gauge transformation of the Y_2 supermultiplets. This can be achieved if the supercurrent explicitly depends on the superfield strength $\mathcal{W}_{\alpha(2Y_2)}$ of the Y_2 supermultiplet. Furthermore, due to the cubic nature of the interaction the supercurrent must be quadratic in the Y_2 information. However because of the special index structure of $\mathcal{W}_{\alpha(2Y_2)}$, we conclude that the supercurrent must explicitly depend on both $\mathcal{W}_{\alpha(2Y_2)}$ and $\bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}$. The most general ansatz one can write is:

$$\begin{aligned} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = & \sum_{p=0}^{s_1-2Y_2} \alpha_p \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)} \\ & + \sum_{p=0}^{s_1-2Y_2-1} \beta_p \partial^{(p)} D \mathcal{W}_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-1-p)} \bar{D} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)} \end{aligned} \quad (3.4)$$

⁷Due to the duality between the transverse and longitudinal formulations of the half-integer superspin, we only have to consider one of them. Our choice is the transverse description.

for some coefficients α_p and β_p . We have intentionally not explicitly written the external indices of the spacetime or spinorial derivatives in order to simplify the expression. Also one must not forget the independent symmetrization of all the dotted and undotted indices of the right hand side together with appropriate symmetrization factors that we also omit. The symbol $\partial^{(k)}$ is a replacement for a string of k spacetime derivatives.

An immediate consequence of all the above is that there is a constraint in the Y_1 and Y_2 values. A supercurrent of this type can exist only if:

$$s_1 \geq 2Y_2 \Rightarrow Y_1 \geq 2Y_2 + 1/2. \quad (3.5)$$

In other words, if $Y_2 > s_1/2$ there is not enough room for derivatives in order to have a gauge invariant supercurrent. Hence the conclusion is that if a supersymmetric theory allows the construction of a gauge invariant, conserved (as in (3.2)), real supercurrent of rank s_1 then its spectrum can include massless supermultiplets with superspin $Y > s_1/2$. This is a supersymmetric, higher spin extension of the Weinberg-Witten theorem [49]. Also at the component level the constrain (3.5) is consistent with the restrictions found in [16, 21].

Now we have to actually check, whether a solution of (3.2) and (3.3) of the form (3.4) exist. The conservation equation (3.2) relates the β_p and α_p coefficients:

$$\beta_p = -i (-1)^{2Y_2} \alpha_{p+1} \frac{p+1}{s_1-p}, \quad p = 0, 1, \dots, s_1 - 2Y_2 - 1 \quad (3.6)$$

whereas the reality of the supercurrent gives the following constraints:

$$\alpha_p = \alpha_{s_1-2Y_2-p}^*, \quad p = 0, 1, \dots, s_1 - 2Y_2, \quad (3.7)$$

$$\beta_p = \beta_{s_1-2Y_2-1-p}^*, \quad p = 0, 1, \dots, s_1 - 2Y_2 - 1. \quad (3.8)$$

The above three constraints fix uniquely the coefficients α_p and β_p

$$\alpha_p = c (i)^{s_1-2Y_2} (-1)^p \frac{\binom{s_1-2Y_2}{p} \binom{s}{p}}{\binom{2Y_2+p}{2Y_2}}, \quad p = 0, 1, \dots, s_1 - 2Y_2, \quad (3.9)$$

$$\beta_p = c (i)^{s_1-2Y_2+1} (-1)^{p+2Y_2} \frac{\binom{s_1-2Y_2}{p} \binom{s}{p}}{\binom{2Y_2+p}{2Y_2}} \frac{s_1 - 2Y_2 - p}{2Y_2 + 1 + p}, \quad p = 0, 1, \dots, s_1 - 2Y_2 - 1 \quad (3.10)$$

up to an overall real constant c . Hence not only is there a solution but it is unique. The higher spin supercurrent we find is

$$\begin{aligned} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = c(i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{\binom{s_1-2Y_2}{p} \binom{s}{p}}{\binom{2Y_2+p}{2Y_2}} \left\{ \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)} \right. \\ \left. + i(-1)^{2Y_2} \frac{s_1 - 2Y_2 - p}{2Y_2 + 1 + p} \partial^{(p)} \mathcal{D} \mathcal{W}_{\alpha(2Y_2)} \partial^{(s_1-2Y_2-1-p)} \bar{\mathcal{D}} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)} \right\}. \end{aligned} \quad (3.11)$$

There are two interesting cases to look at. The first one is at the boundary of (3.5), when $s_1 = 2Y_2$. In this case the supercurrent (3.11) is simplified and takes the following form:

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = c \mathcal{W}_{\alpha(2Y_2)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}. \tag{3.12}$$

This supercurrent is the supersymmetric and higher spin extension of the Bel-Robinson tensor. That means that for $Y_2 = 3/2$ (linearized supergravity) the supercurrent $\mathcal{J}_{\alpha\beta\gamma\dot{\alpha}\dot{\beta}\dot{\gamma}}$ as defined by (3.12) in its components includes the Bel-Robinson tensor of General Relativity (see [50] and references therein). For higher values of Y_2 we get higher spin generalizations of the Bel-Robinson tensor. A supercurrent of this type has been constructed in [60].

The second interesting observation is in the limit $Y_2 = 0$. This case technically is not allowed because it is not a gauge theory but a matter theory, hence there is no zero order transformation and \mathcal{W} is not a superfield strength. However, it is the chiral superfield of the free chiral theory and the supercurrent (3.11) gives the cubic interactions of the higher spin supermultiplet Y_1 with the chiral supermultiplet. Therefore it is allowed to take this limit. In that case the expression (3.11) exactly matches to the higher spin supercurrent of matter theories found in [1, 2].

4 Component structure of the higher spin supercurrent

The components of the higher spin supercurrent (3.11) will include bosonic and fermionic gauge invariant, conserved higher spin currents. It will be useful to extract the expressions for these currents and compare with known results for non-supersymmetric theories. For the projection of the superfield to its components, we will follow [2, 56, 57]. Due to the conservation equation (3.2), on-shell the higher spin supercurrent $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$ has only three independent components⁸ (2 bosonic and one fermionic)

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}^{(0,0)} = \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} \Big|_{\substack{\theta=0 \\ \bar{\theta}=0}}, \quad \mathcal{J}_{\alpha(s_1+1)\dot{\alpha}(s_1)}^{(1,0)} = \frac{1}{(s_1+1)!} D_{(\alpha_{s_1+1}} \mathcal{J}_{\alpha(s_1))\dot{\alpha}(s_1)} \Big|_{\substack{\theta=0 \\ \bar{\theta}=0}}, \tag{4.1}$$

$$\mathcal{J}_{\alpha(s_1+1)\dot{\alpha}(s_1+1)}^{(1,1)} = -\frac{1}{2(s_1+1)!(s_1+1)!} [D_{(\alpha_{s_1+1}}, \bar{D}_{\dot{\alpha}(s_1+1)}] \mathcal{J}_{\alpha(s_1))\dot{\alpha}(s_1)} \Big|_{\substack{\theta=0 \\ \bar{\theta}=0}}.$$

The rest of them either vanish or are derivatives of the components above. It is straight forward to show, using (3.3) and (3.2), that these components satisfy the traditional spacetime conservation equations

$$\partial^{\alpha_{s_1}\dot{\alpha}_{s_1}} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}^{(0,0)} = 0, \quad \partial^{\alpha_{s_1+1}\dot{\alpha}_{s_1}} \mathcal{J}_{\alpha(s_1+1)\dot{\alpha}(s_1)}^{(1,0)} = 0, \quad \partial^{\alpha_{s_1+1}\dot{\alpha}_{s_1+1}} \mathcal{J}_{\alpha(s_1+1)\dot{\alpha}(s_1+1)}^{(1,1)} = 0 \tag{4.2}$$

⁸The labels (0, 0), (1, 0) and (1, 1) correspond to the position of the components in the θ and $\bar{\theta}$ expansion of the supercurrent.

hence they correspond to gauge invariant, conserved, higher spin currents. Their explicit expressions are:

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}^{(0,0)} \sim (i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{\binom{s_1-2Y_2}{p} \binom{s}{p}}{\binom{2Y_2+p}{2Y_2}} \left\{ \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)}^{(0,0)} \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}^{(0,0)} \right. \quad (4.3a)$$

$$\left. + i(-1)^{2Y_2} \frac{s_1-2Y_2-p}{2Y_2+1+p} \partial^{(p)} \mathcal{W}_{\alpha(2Y_2+1)}^{(1,0)} \partial^{(s_1-2Y_2-1-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2+1)}^{(0,1)} \right\},$$

$$\mathcal{J}_{\alpha(s_1+1)\dot{\alpha}(s_1)}^{(1,0)} \sim (i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{\binom{s_1-2Y_2}{p} \binom{s}{p}}{\binom{2Y_2+p}{2Y_2}} \frac{s_1+1}{2Y_2+1+p} \partial^{(p)} \mathcal{W}_{\alpha(2Y_2+1)}^{(1,0)} \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}^{(0,0)}, \quad (4.3b)$$

$$\mathcal{J}_{\alpha(s_1+1)\dot{\alpha}(s_1+1)}^{(1,1)} \sim (i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{\binom{s_1-2Y_2}{p} \binom{s}{p}}{\binom{2Y_2+p}{2Y_2}} \left\{ i \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)}^{(0,0)} \partial^{(s_1+1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}^{(0,0)} \right. \quad (4.3c)$$

$$- i \frac{2s_1-2Y_2+1-p}{2Y_2+1+p} \partial^{(p+1)} \mathcal{W}_{\alpha(2Y_2)}^{(0,0)} \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}^{(0,0)}$$

$$+ (-1)^{2Y_2} \frac{s_1+2Y_2+2+p}{2Y_2+1+p} \partial^{(p)} \mathcal{W}_{\alpha(2Y_2+1)}^{(1,0)} \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2+1)}^{(0,1)}$$

$$\left. - (-1)^{2Y_2} \frac{s_1-2Y_2-p}{2Y_2+1+p} \partial^{(p+1)} \mathcal{W}_{\alpha(2Y_2+1)}^{(1,0)} \partial^{(s_1-2Y_2-1-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2+1)}^{(0,1)} \right\}.$$

The components $\mathcal{W}_{\alpha(2Y_2)}^{(0,0)}$ and $\mathcal{W}_{\alpha(2Y_2+1)}^{(1,0)}$ are the only non-trivial components of the superfield strength $\mathcal{W}_{\alpha(2Y_2)}$

$$\mathcal{W}_{\alpha(2Y_2)}^{(0,0)} = \mathcal{W}_{\alpha(2Y_2)} \Big|_{\theta=0}, \quad \mathcal{W}_{\alpha(2Y_2+1)}^{(1,0)} = \frac{1}{(2Y_2+1)!} D_{(\alpha 2Y_2+1)} \mathcal{W}_{\alpha(2Y_2)} \Big|_{\substack{\theta=0 \\ \bar{\theta}=0}} \quad (4.4)$$

and are the higher spin field strengths for spins $j = Y_2$ and $j = Y_2 + 1/2$ respectively. These (4.3a) higher spin currents are consistent with the results of [21, 24].

5 Discussion

We consider cubic interactions $Y_1 - Y_2 - Y_2$ among massless supermultiplets of superspin Y_1 and Y_2 , where Y_1 is half integer $Y_1 = s_1 + 1/2$ for arbitrary positive integer s_1 and Y_2 has an arbitrary non-negative integer or half-integer value. Specifically, we consider the class of these interactions that are generated by a higher spin supercurrent $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$ which respects the gauge symmetry of the Y_2 supermultiplet. We find that:

- i.* There is a unique gauge invariant, conserved, real, higher spin supercurrent $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$ (3.11).
- ii.* The existence of this supercurrent puts constraints (3.5) on the allowed Y_2 superspin values [$s_1 \geq 2Y_2$]. This is a supersymmetric and higher spin generalization of the Weinberg-Witten theorem and also is consistent with various spin restrictions coming from the consideration of non-supersymmetric, higher spin, cubic interactions.

- iii. For the special case of $s_1 = 2Y_2$ the higher spin supercurrent simplifies a lot (3.12) and agrees with the results of [60]. This supercurrent provides a supersymmetric, higher spin extension of the linearized Bel-Robinson tensor known in the context of General Relativity.
- iv. Another special limit is $Y_2 = 0$ where we recover previously known results for the higher spin supercurrent of the free, massless chiral theory.
- v. The component structure includes two bosonic and one fermionic gauge invariant, conserved, higher spin currents (4.3a) which depend on the derivatives of higher spin field strengths for spins $j = Y_2$ and $j = Y_2 + 1/2$.

The list of the supercurrents we find refer only to non-minimal couplings (higher number of derivatives) and that is due to the explicit dependence of the supercurrent on the superfield strengths. Therefore, in spite of this list being infinite (for arbitrary values of Y_1 and Y_2 that respect (3.5)), it is incomplete. This is simply understood from the fact that ordinary conserved currents such as the energy momentum tensor and their higher spin extensions [61] are not included in the class of gauge invariant currents. It would be interesting to search for non gauge invariant higher spin supercurrents that generate minimal coupling (least number of derivatives) among higher spins and give gauge invariant conserved charges.

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References

- [1] S.M. Kuzenko, R. Manvelyan and S. Theisen, *Off-shell superconformal higher spin multiplets in four dimensions*, *JHEP* **07** (2017) 034 [[arXiv:1701.00682](https://arxiv.org/abs/1701.00682)] [[INSPIRE](#)].
- [2] I.L. Buchbinder, S.J. Gates and K. Koutrolikos, *Higher Spin Superfield interactions with the Chiral Supermultiplet: Conserved Supercurrents and Cubic Vertices*, *Universe* **4** (2018) 6 [[arXiv:1708.06262](https://arxiv.org/abs/1708.06262)] [[INSPIRE](#)].
- [3] J. Hutomo and S.M. Kuzenko, *Non-conformal higher spin supercurrents*, *Phys. Lett. B* **778** (2018) 242 [[arXiv:1710.10837](https://arxiv.org/abs/1710.10837)] [[INSPIRE](#)].
- [4] J. Hutomo and S.M. Kuzenko, *The massless integer superspin multiplets revisited*, *JHEP* **02** (2018) 137 [[arXiv:1711.11364](https://arxiv.org/abs/1711.11364)] [[INSPIRE](#)].

- [5] K. Koutrolikos, P. Kočí and R. von Unge, *Higher Spin Superfield interactions with Complex linear Supermultiplet: Conserved Supercurrents and Cubic Vertices*, *JHEP* **03** (2018) 119 [[arXiv:1712.05150](#)] [[INSPIRE](#)].
- [6] I.L. Buchbinder, S.J. Gates and K. Koutrolikos, *Interaction of supersymmetric nonlinear σ -models with external higher spin superfields via higher spin supercurrents*, *JHEP* **05** (2018) 204 [[arXiv:1804.08539](#)] [[INSPIRE](#)].
- [7] A.K.H. Bengtsson, I. Bengtsson and L. Brink, *Cubic Interaction Terms for Arbitrary Spin*, *Nucl. Phys.* **B 227** (1983) 31 [[INSPIRE](#)].
- [8] A.K.H. Bengtsson, I. Bengtsson and L. Brink, *Cubic Interaction Terms for Arbitrarily Extended Supermultiplets*, *Nucl. Phys.* **B 227** (1983) 41 [[INSPIRE](#)].
- [9] A.K.H. Bengtsson, I. Bengtsson and N. Linden, *Interacting Higher Spin Gauge Fields on the Light Front*, *Class. Quant. Grav.* **4** (1987) 1333 [[INSPIRE](#)].
- [10] E.S. Fradkin and R.R. Metsaev, *A Cubic interaction of totally symmetric massless representations of the Lorentz group in arbitrary dimensions*, *Class. Quant. Grav.* **8** (1991) L89 [[INSPIRE](#)].
- [11] R.R. Metsaev, *Note on the cubic interaction of massless representations of the Poincaré group in $D = 5$ space-time*, *Class. Quant. Grav.* **10** (1993) L39 [[INSPIRE](#)].
- [12] R.R. Metsaev, *Cubic interaction vertices of totally symmetric and mixed symmetry massless representations of the Poincaré group in $D = 6$ space-time*, *Phys. Lett.* **B 309** (1993) 39 [[INSPIRE](#)].
- [13] R.R. Metsaev, *Generating function for cubic interaction vertices of higher spin fields in any dimension*, *Mod. Phys. Lett.* **A 8** (1993) 2413 [[INSPIRE](#)].
- [14] E.S. Fradkin and R.R. Metsaev, *Cubic scattering amplitudes for all massless representations of the Poincaré group in any space-time dimension*, *Phys. Rev.* **D 52** (1995) 4660 [[INSPIRE](#)].
- [15] R.R. Metsaev, *Cubic interaction vertices of massive and massless higher spin fields*, *Nucl. Phys.* **B 759** (2006) 147 [[hep-th/0512342](#)] [[INSPIRE](#)].
- [16] R.R. Metsaev, *Cubic interaction vertices for fermionic and bosonic arbitrary spin fields*, *Nucl. Phys.* **B 859** (2012) 13 [[arXiv:0712.3526](#)] [[INSPIRE](#)].
- [17] D. Ponomarev and E.D. Skvortsov, *Light-Front Higher-Spin Theories in Flat Space*, *J. Phys.* **A 50** (2017) 095401 [[arXiv:1609.04655](#)] [[INSPIRE](#)].
- [18] E.D. Skvortsov, T. Tran and M. Tsulaia, *Quantum Chiral Higher Spin Gravity*, *Phys. Rev. Lett.* **121** (2018) 031601 [[arXiv:1805.00048](#)] [[INSPIRE](#)].
- [19] F.A. Berends, G.J.H. Burgers and H. Van Dam, *On spin three selfinteractions*, *Z. Phys.* **C 24** (1984) 247 [[INSPIRE](#)].
- [20] F.A. Berends, G.J.H. Burgers and H. van Dam, *On the Theoretical Problems in Constructing Interactions Involving Higher Spin Massless Particles*, *Nucl. Phys.* **B 260** (1985) 295 [[INSPIRE](#)].
- [21] F.A. Berends, G.J.H. Burgers and H. van Dam, *Explicit Construction of Conserved Currents for Massless Fields of Arbitrary Spin*, *Nucl. Phys.* **B 271** (1986) 429 [[INSPIRE](#)].
- [22] S. Deser and Z. Yang, *Inconsistency of Spin 4–Spin 2 Gauge Field Couplings*, *Class. Quant. Grav.* **7** (1990) 1491 [[INSPIRE](#)].
- [23] R. Manvelyan, K. Mkrtchyan and W. Rühl, *Off-shell construction of some trilinear higher spin gauge field interactions*, *Nucl. Phys.* **B 826** (2010) 1 [[arXiv:0903.0243](#)] [[INSPIRE](#)].

- [24] O.A. Gelfond, E.D. Skvortsov and M.A. Vasiliev, *Higher spin conformal currents in Minkowski space*, *Theor. Math. Phys.* **154** (2008) 294 [[hep-th/0601106](#)] [[INSPIRE](#)].
- [25] X. Bekaert, N. Boulanger and S. Leclercq, *Strong obstruction of the Berends-Burgers-van Dam spin-3 vertex*, *J. Phys. A* **43** (2010) 185401 [[arXiv:1002.0289](#)] [[INSPIRE](#)].
- [26] R. Manvelyan, K. Mkrtchyan and W. Rühl, *Direct Construction of A Cubic Selfinteraction for Higher Spin gauge Fields*, *Nucl. Phys. B* **844** (2011) 348 [[arXiv:1002.1358](#)] [[INSPIRE](#)].
- [27] R. Manvelyan, K. Mkrtchyan and W. Rühl, *General trilinear interaction for arbitrary even higher spin gauge fields*, *Nucl. Phys. B* **836** (2010) 204 [[arXiv:1003.2877](#)] [[INSPIRE](#)].
- [28] X. Bekaert, N. Boulanger and P. Sundell, *How higher-spin gravity surpasses the spin two barrier: no-go theorems versus yes-go examples*, *Rev. Mod. Phys.* **84** (2012) 987 [[arXiv:1007.0435](#)] [[INSPIRE](#)].
- [29] I.G. Koh and S. Ouvry, *Interacting Gauge Fields of Any Spin and Symmetry*, *Phys. Lett. B* **179** (1986) 115 [Erratum *ibid.* **B 183** (1987) 434] [[INSPIRE](#)].
- [30] A.K.H. Bengtsson, *BRST approach to interacting higher spin gauge fields*, *Class. Quant. Grav.* **5** (1988) 437 [[INSPIRE](#)].
- [31] L. Cappiello, M. Knecht, S. Ouvry and J. Stern, *BRST Construction of Interacting Gauge Theories of Higher Spin Fields*, *Annals Phys.* **193** (1989) 10 [[INSPIRE](#)].
- [32] G. Bonelli, *On the tensionless limit of bosonic strings, infinite symmetries and higher spins*, *Nucl. Phys. B* **669** (2003) 159 [[hep-th/0305155](#)] [[INSPIRE](#)].
- [33] I.L. Buchbinder, A. Fotopoulos, A.C. Petkou and M. Tsulaia, *Constructing the cubic interaction vertex of higher spin gauge fields*, *Phys. Rev. D* **74** (2006) 105018 [[hep-th/0609082](#)] [[INSPIRE](#)].
- [34] A. Fotopoulos and M. Tsulaia, *Interacting higher spins and the high energy limit of the bosonic string*, *Phys. Rev. D* **76** (2007) 025014 [[arXiv:0705.2939](#)] [[INSPIRE](#)].
- [35] A. Fotopoulos, N. Irges, A.C. Petkou and M. Tsulaia, *Higher-Spin Gauge Fields Interacting with Scalars: The Lagrangian Cubic Vertex*, *JHEP* **10** (2007) 021 [[arXiv:0708.1399](#)] [[INSPIRE](#)].
- [36] A. Fotopoulos and M. Tsulaia, *Gauge Invariant Lagrangians for Free and Interacting Higher Spin Fields. A Review of the BRST formulation*, *Int. J. Mod. Phys. A* **24** (2009) 1 [[arXiv:0805.1346](#)] [[INSPIRE](#)].
- [37] A. Sagnotti and M. Taronna, *String Lessons for Higher-Spin Interactions*, *Nucl. Phys. B* **842** (2011) 299 [[arXiv:1006.5242](#)] [[INSPIRE](#)].
- [38] D. Polyakov, *Interactions of Massless Higher Spin Fields From String Theory*, *Phys. Rev. D* **82** (2010) 066005 [[arXiv:0910.5338](#)] [[INSPIRE](#)].
- [39] D. Polyakov, *Gravitational Couplings of Higher Spins from String Theory*, *Int. J. Mod. Phys. A* **25** (2010) 4623 [[arXiv:1005.5512](#)] [[INSPIRE](#)].
- [40] E.S. Fradkin and M.A. Vasiliev, *On the Gravitational Interaction of Massless Higher Spin Fields*, *Phys. Lett. B* **189** (1987) 89 [[INSPIRE](#)].
- [41] E.S. Fradkin and M.A. Vasiliev, *Cubic Interaction in Extended Theories of Massless Higher Spin Fields*, *Nucl. Phys. B* **291** (1987) 141 [[INSPIRE](#)].

- [42] M.A. Vasiliev, *Cubic interactions of bosonic higher spin gauge fields in AdS_5* , *Nucl. Phys. B* **616** (2001) 106 [Erratum *ibid.* **B 652** (2003) 407] [[hep-th/0106200](#)] [[INSPIRE](#)].
- [43] K.B. Alkalaev and M.A. Vasiliev, *$N = 1$ supersymmetric theory of higher spin gauge fields in AdS_5 at the cubic level*, *Nucl. Phys. B* **655** (2003) 57 [[hep-th/0206068](#)] [[INSPIRE](#)].
- [44] M.A. Vasiliev, *Cubic Vertices for Symmetric Higher-Spin Gauge Fields in $(A)dS_d$* , *Nucl. Phys. B* **862** (2012) 341 [[arXiv:1108.5921](#)] [[INSPIRE](#)].
- [45] M.A. Vasiliev, *Nonlinear equations for symmetric massless higher spin fields in $(A)dS_d$* , *Phys. Lett. B* **567** (2003) 139 [[hep-th/0304049](#)] [[INSPIRE](#)].
- [46] V.E. Didenko and E.D. Skvortsov, *Elements of Vasiliev theory*, [arXiv:1401.2975](#) [[INSPIRE](#)].
- [47] S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, *Superspace Or One Thousand and One Lessons in Supersymmetry*, *Front. Phys.* **58** (1983) 1 [[hep-th/0108200](#)] [[INSPIRE](#)].
- [48] I.L. Buchbinder and S.M. Kuzenko, *Ideas and methods of supersymmetry and supergravity: Or a walk through superspace*, IOP, Bristol U.K. (1998) [[INSPIRE](#)].
- [49] S. Weinberg and E. Witten, *Limits on Massless Particles*, *Phys. Lett. B* **96** (1980) 59 [[INSPIRE](#)].
- [50] C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation*, W.H. Freeman, San Francisco U.S.A. (1973) [ISBN: 9780716703440].
- [51] T. Curtright, *Massless Field Supermultiplets With Arbitrary Spin*, *Phys. Lett. B* **85** (1979) 219 [[INSPIRE](#)].
- [52] S.M. Kuzenko, A.G. Sibiryakov and V.V. Postnikov, *Massless gauge superfields of higher half integer superspins*, *JETP Lett.* **57** (1993) 534 [*Pisma Zh. Eksp. Teor. Fiz.* **57** (1993) 521] [[INSPIRE](#)].
- [53] S.M. Kuzenko and A.G. Sibiryakov, *Massless gauge superfields of higher integer superspins*, *JETP Lett.* **57** (1993) 539 [*Pisma Zh. Eksp. Teor. Fiz.* **57** (1993) 526] [[INSPIRE](#)].
- [54] S.M. Kuzenko and A.G. Sibiryakov, *Free massless higher superspin superfields on the anti-de Sitter superspace*, *Phys. Atom. Nucl.* **57** (1994) 1257 [*Yad. Fiz.* **57** (1994) 1326] [[arXiv:1112.4612](#)] [[INSPIRE](#)].
- [55] S.J. Gates Jr., S.M. Kuzenko and A.G. Sibiryakov, *$N = 2$ supersymmetry of higher superspin massless theories*, *Phys. Lett. B* **412** (1997) 59 [[hep-th/9609141](#)] [[INSPIRE](#)].
- [56] S.J. Gates Jr. and K. Koutrolikos, *On 4D, $\mathcal{N} = 1$ massless gauge superfields of arbitrary superhelicity*, *JHEP* **06** (2014) 098 [[arXiv:1310.7385](#)] [[INSPIRE](#)].
- [57] S.J. Gates Jr. and K. Koutrolikos, *On 4D, $N = 1$ Massless Gauge Superfields of Higher Superspin: Half-Odd-Integer Case*, [arXiv:1310.7386](#) [[INSPIRE](#)].
- [58] S.J. Gates and K. Koutrolikos, *From Diophantus to Supergravity and massless higher spin multiplets*, *JHEP* **11** (2017) 063 [[arXiv:1707.00194](#)] [[INSPIRE](#)].
- [59] I.L. Buchbinder and K. Koutrolikos, *BRST Analysis of the Supersymmetric Higher Spin Field Models*, *JHEP* **12** (2015) 106 [[arXiv:1510.06569](#)] [[INSPIRE](#)].
- [60] P.S. Howe, K.S. Stelle and P.K. Townsend, *Superactions*, *Nucl. Phys. B* **191** (1981) 445 [[INSPIRE](#)].
- [61] P.A. Smirnov and M.A. Vasiliev, *Gauge-noninvariant higher-spin currents in four-dimensional Minkowski space*, *Theor. Math. Phys.* **181** (2014) 1509 [[arXiv:1312.6638](#)] [[INSPIRE](#)].