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Consistent 2-D Phase Unwrapping Guided by a Quality Map

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Abstract -- The problem of 2-D phase unwrapping arises when a spatially varying quantity is measured modulo some period. One needs to reconstruct a smooth unwrapped phase, consistent with the original data, by adding a multiple of the period to each sample. Smoothness typically cannot be enforced over all of the scene, due to noise and localized jumps. An unwrapping algorithm may form a mask within which phase discontinuities are allowed. In interferometry a quality map is available, indicating the reliability of the measurements. In this case, the mask should be contained as much as possible in areas of low quality. This paper presents an algorithm for phase unwrapping in which the mask design is guided by the quality map. The mask is grown from the residues (as defined by Goldstein et al.) into areas where the quality is below a threshold. A connected component of the mask stops growing when its residue charge becomes balanced. The threshold is raised as necessary to allow growth. This stage terminates when all components are balanced. The mask is then thinned by removing points that are not needed to cover the residues correctly. The unwrapped phase is found by simple 1-D unwrapping along paths that avoid the mask. We present an example solution found by the algorithm and discuss possible modifications.

INTRODUCTION

The problem of 2-D phase unwrapping arises when a spatially varying quantity is measured modulo some period. Such measurements can be thought of as phase angles, hence the term "phase unwrapping." The task is to make a reconstruction of the unwrapped phase, removing the periodicity of the measurement. It is assumed that the phase difference between adjacent pixels is almost always a small fraction of a cycle. Hence, along a typical path, the unwrapping can be done by wrapping the differences between successive samples (adding cycles as necessary to make the differences less than one-half cycle in magnitude) and accumulating. The resulting unwrapped phase is consistent with the original data (different at each pixel by an integer number of cycles). This method can fail due to regions in which the assumption of smoothness fails to hold. These can cause the unwrapped phase found at one pixel, starting out from another, to depend on the choice of path. Equivalently, the sum of wrapped phase differences around a closed path may be a nonzero number of cycles. The choice of path must be restricted to avoid this condition.

A systematic criterion for path restriction has been found by Goldstein et al. [1]. They observe that unwrap path dependency is caused by pointlike sources called "residues." A residue is a loop of four neighboring pixels for which the clockwise sum of wrapped phase differences around the loop is not zero, but plus or minus one cycle; the residue is said to have charge +1 or -1. The sum of wrapped phase differences around any closed path is equal, in cycles, to the sum of the charges of the enclosed residues. If all closed paths are constrained to enclose zero net residue charge, then the phase difference sum around any path is zero, and the unwrapped phase is defined uniquely. The constraint can be imposed by forming the residues into clusters, each with zero charge sum, and requiring that the path enclose either all or none of the residues in each cluster. If there is a mask of forbidden pixels such that the residues within each cluster are connected by mask pixels, then any path made up entirely of unmasked pixels is guaranteed to be suitable for unwrapping. (The edge of the image can serve as a residue charge source in this method: an unbalanced cluster is allowed if the mask connects it to the edge.) The problem of unwrapping is thus reduced to that of finding a good clustering of residues and a mask that enforces it.

The residues in each cluster should be as close together as possible, so that no long links are needed in the mask. The algorithm of [1] starts a cluster with a single residue, then adds nearby residues found by a local search. The cluster stops growing when it becomes balanced or reaches the edge. The growing process then starts again with another unpaired residue. The algorithm terminates when all residues are in clusters. There is no global search for the best set of clusters. Depending on the order in which the residues are accessed, a residue may be left without unclustered near neighbors, making a link to a faraway residue necessary. Such links can create erroneous jumps in the unwrapped phase. Recently developed algorithms [2,3] include an optimization stage that interchanges the residues among clusters to reduce the length of the links.

An interferogram is formed by complex correlation of neighborhoods of image pixels [4]. The phase of the correlation forms the data to be unwrapped, while its magnitude indicates the quality of the phase data, ranging

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from 0 (poor) to 1 (perfect). This *quality map* can help to guide phase unwrapping: the mask should be confined as much as possible to pixels of low quality, as well as covering as few pixels as possible.

The least-error-path method, due to Roth [5], uses the quality map to choose unwrap paths directly. It assumes that a path that goes entirely through high-quality pixels is suitable for unwrapping. Regions of very high quality are seeded with initial unwrap values. An unwrap solution is then grown from each seed point by summing the wrapped phase differences, starting with the highest-quality pixels and working downward. The growing process leaves the image covered by smooth locally unwrapped patches. These are joined by successively locating the highest-quality pixel pair on the boundary between two patches, then adding cycles to minimize the unwrapped phase difference at that pair. The algorithm performs no explicit residue detection or matching, so it has no direct control on the location of discontinuities.

We now present an algorithm to compute an unwrapped phase solution, consistent with the data, in which the phase discontinuities are confined to an explicitly computed mask, chosen to lie as much as possible within regions of low quality. The algorithm can be thought of as the reverse of the least-error-path method: instead of growing unwrap paths through high-quality regions, one grows the mask through regions of low quality, starting from the residues. The algorithm keeps track of the residue charge of each connected component of the mask, terminating when all components are balanced. The unwrapped phase is then computed by path following, avoiding the mask.

IMPLEMENTATION

We are given, at each location (i,j), the phase $\phi_{ij} \in [0, 2\pi)$ and quality $\rho_{ij} \in [0, 1]$. Introducing the phase wrapping operator

$$W(\phi) \equiv ((\phi + \pi) \bmod 2\pi) - \pi, \qquad (1)$$

the residue map is computed as follows:

$$r_{ij} = \frac{1}{2\pi}[W(\phi_{i+1,j} - \phi_{ij}) + W(\phi_{i+1,j+1} - \phi_{i+1,j}) - W(\phi_{i,j+1} - \phi_{ij}) - W(\phi_{i+1,j+1} - \phi_{i,j+1})]. \qquad (2)$$

The r_{ij} can have value 0, +1 or -1. Locations where r_{ij} is nonzero are called positive or negative residues according to their sign.

The inputs to the mask-design algorithm are the residue map and the quality map. The phase data are used only to find the residues. The pixels adjoining residues are the seed points for the mask. To confine the mask to regions of low-quality data, an outer loop allows it to grow only into eligible pixels (seed points and pixels adjoining the mask) whose quality lies below a threshold. The threshold is initialized, then incremented when mask growth becomes impossible. The initial threshold and increment size are the only adjustable parameters in the algorithm. The loop terminates when all mask components have zero residue charge (so that further growth is unnecessary).

Within the outer loop, growing is done by an inner loop which makes repeated passes through the image, adding to the mask eligible pixels whose quality is below the threshold. Once a pixel is added, its neighbors become eligible. To reduce unnecessary growth, uncharged components are not extended. If a pixel adjoins several components, adding it to the mask merges them and combines their residue charges. The residue-charge table and component labels are updated to reflect the merger. To simplify recordkeeping, the newly merged component is not allowed to grow further until the next pass. A component that reaches the edge is marked as uncharged. The inner loop terminates when a pass through the image adds no pixels to the mask (so that further growth is impossible without raising the quality threshold). In summary, the mask-growing stage of the algorithm acts as follows:

```
Replace quality map with its 3x3 minimum
Compute residues
Declare pixels surrounding residues to be seed points
Initialize threshold
repeat
  repeat
    Declare all components unmerged
    for (each pixel in image)
      if (quality < threshold and pixel is not in mask
       and [pixel is a seed point
             or pixel adjoins a charged component]
       and pixel adjoins no merged component) then
        Add pixel to mask
        if (pixel adjoins several components)
          merge all adjoined components
        if (pixel's component adjoins edge)
          set charge to zero
      end
    end
  until (no pixels added)
  Increment threshold
until (all components are uncharged)
Add all neighbors of residues to mask
```

Mask growing typically puts many pixels in the mask that are not needed to cover the residues correctly. A thinning stage is used to remove them. This starts by identifying and labeling the connected components of *unmasked* pixels, then masking out all but the largest component. It then makes repeated passes through the mask, each time removing pixels that adjoin the unmasked region. A pixel is removed if: 1) it does not adjoin a residue; 2) its quality is above the initial threshold used in growing; and 3) removing it does not change the connectivity of the mask. The connectivity is tested by examining the 3x3 neighborhood of the candidate pixel. Removal is forbidden if it would leave disconnected mask pixels within the neighborhood. The thinning stage ends when a pass finds no removable pixels.

With the mask prepared, the unwrapped phase is computed by path-following. In the absence of a reference point whose wrap count is known, we can start with an arbitrary point, giving it a wrap count of 0.

EXAMPLE

Figs. 1 and 2 display a portion of a synthetic-aperture radar interferogram computed from an ERS-1 image pair taken over Ft. Irwin, California. Fig. 1 contains the phase data, and Fig. 2 contains the quality map. Occurrences of layover appear as discontinuities in the phase data, with corresponding low regions in the quality map. The initial quality threshold was 32 (on a scale of 0 to 255) and the increment was 2. The mask is presented in Fig. 3. The algorithm has clearly succeeded in finding a mask that covers a small number of pixels and allows only localized discontinuities. The unwrapped phase is displayed in Fig. 4. It has been inverted, and a tilt has been removed, to make terrain features more visible. Some of the masked regions appear as jumps in the unwrapped phase.

CONCLUSIONS AND FUTURE WORK

We have presented an algorithm for phase unwrapping, for data that include a quality map. It works by growing a mask, preferentially through low-quality regions, so as to enforce a residue-balance condition, allowing unwrapping by path following in the unmasked region. The method is intended to find a small, compact mask, largely confined to low-quality regions. Computations on actual interferograms indicate that this goal is achieved in most cases.

The algorithm produces inappropriate results on some data sets. Its worst failures happen when groups of residues are not joined by low-quality paths, so that the growing stage connects them, not to each other, but to the edge. The thinning stage leaves bridges to the edge, across which the unwrapped phase makes erroneous jumps. This problem points out how the performance of the algorithm depends on the accuracy of the quality map. An alternative version of the algorithm, in which the mask is grown through regions where the wrapped phase differences are large, often works well in cases where the quality map is missing or inaccurate.

A more robust algorithm would make short connections as needed to achieve residue balance, overriding the quality threshold. This might be done by combining the mask-growing method with minimum-distance residue matching as in [2,3]. An initial stage of mask growing would form a small number of mask components, some of them charged. The distances between components would then be computed and a minimum-distance pairing found. Links enforcing this pairing would then be added to the mask.

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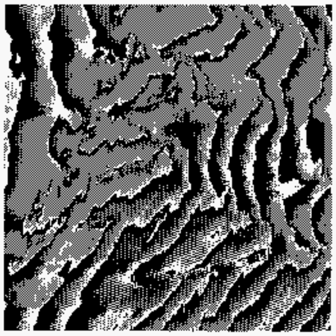

Fig. 1: Phase data

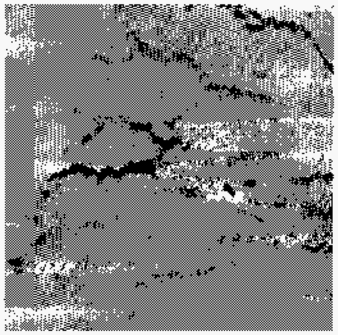

Fig. 2: Quality map

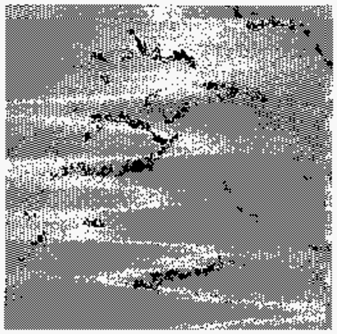

Fig. 3: Mask

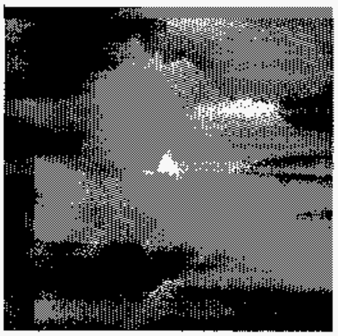

Fig. 4: Unwrapped phase

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