Consistent task specification for manipulation systems with general kinematics

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Abstract

Although most part of the literature on manipulation systems deals with systems with as many degrees of freedom as the dimension of their task space, or even with more (redundant manipulators), kinematically defective manipulation systems are often encountered in robotics, in particular when dealing with simple industry–oriented grippers, or when the whole surface of the manipulator limbs is exploited to constrain the manipulated object, as in "whole–arm" manipulation. Kinematically defective systems differ from non–defective and redundant manipulation systems under many regards, some of which have been addressed in the literature. In this paper, we focus on one of the central problems of manipulation, i.e., controlling the manipulator in order to track a desired object trajectory, while guaranteeing that contact forces are controlled so as to comply with contact constraints (friction bounds, etc.) at every instant. We attack this problem by an unified approach that is appropriate for manipulation systems with general kinematics. When dealing with kinematically defective systems, it results that it is not possible to assign arbitrary trajectories of object motions and contact forces. To understand what restrictions position and force reference trajectories should exhibit in order to be feasible by a given system, is the central issue of this work.

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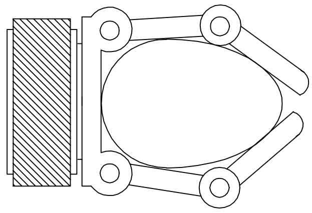

Figure 1: A two–fingered (4 degrees of freedom) gripper with curling fingers grasping an object.

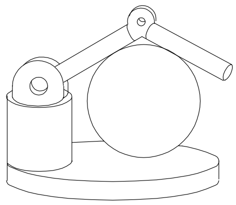

Figure 2: Robust hold of an object by means of an enveloping, "whole–arm" grasp.

1 Introduction

Manipulation systems consisting of one or more actuated linkages cooperatively interacting with one object have been attracting a wide interest in the robotics and mechanics literature at large (Kerr and Roth, 1984), (Mason and Salisbury, 1985), (Trinkle, 1987).

Most part of this literature dealt with systems with as many degrees of freedom (d.o.f's) as the dimension of their task space, or even with more (redundant manipulators). On the other hand, systems occurring for instance when dealing with simple industry–oriented grippers, as illustrated in figure 1; or when the whole surface of the manipulator limbs is exploited to constrain the manipulated object (such as e.g. in tentacle–like arms or in "whole–arm" manipulation, see figure 2), feature limbs with fewer d.o.f.'s than those necessary to achieve arbitrary configurations in their operational space. We call such mechanisms "kinematically defective". In general, kinematic defectivity arises very often when an attempt is made at minimizing the mechanical hardware of the manipulator system for a given task, such as e.g. in (Canny and Goldberg, 1994), (Bicchi et al., 1995b) and (Prattichizzo et al., 1995). Introducing the whole–arm idea, Salisbury (1987) pointed out that kinematic defectivity entails several differences in the treatment of the kinematics and statics. The dynamic analysis, as well as the control of such devices, is also more involved.

A growing attention on kinematically defective systems is witnessed by a number of recent papers. Trinkle et al. (1994) investigated the problem of planning joint motions to reposition and reorient the object within whole–arm grasps. The grasp robustness by kinematically defective devices has been studied in (Zhang et al., 1994) and (Prattichizzo et al., 1995). Howard and Kumar (1995) analyzed the stability of enveloping grasps using a model of the contacts and joints compliances. Bicchi et al. (1995a) studied the rigid–body kinematics of WAM systems and discussed their manipulability. In (Bicchi et al., 1995b) and (Reynaerts and VanBrussel, 1994), the performance of kinematically defective devices is increased by controlling rolling phenomena at contacts.

In a previous work by the authors (Prattichizzo and Bicchi, 1997) the dynamics and the system–theoretic structural properties of general (including kinematically defective and redundant) systems were studied. In particular, in that work, the analysis of pointwise controllability and observability of general manipulation systems was carried out. In the present paper, on the other hand, we are concerned with functional (output trajectory) controllability. We refer the reader to the former paper for details on the derivation and linearization of the dynamic model, and for a few results that are exploited here.

Our focus here is on the problem of tracking a desired trajectory with the manipulated object, while guaranteeing that contact forces are controlled so as to comply with contact constraints at every instant. Notice that such problem is an instance of the well known problem of hybrid position/force control which, for the non–defective case, has been widely studied in literature. For a broad overview of hybrid control the reader is refered to (Siciliano, 1996) and references therein.

When the system is kinematically defective and the desired object trajectory is arbitrarily assigned, the tracking problem is not solvable in general. In the most simple example provided in figure 3, not all trajectories of the object can be controlled in the plane, nor can arbitrary contact forces be applied on the object. Understanding what characteristics required trajectories should have in order to be feasible by a given system is therefore crucial to the design of any planning and control algorithm for these systems. The main result of this paper, stated in Theorem 1, provides a geometric description and an algorithm for evaluating a set of locally feasible trajectories of motions and forces.

The local nature of our results is due to the linearization approach of the dynamics that is used in this work. The use of linearized model dynamics in the analysis of kinematically defective manipulation systems is believed to be a significant advancement with respect to the literature, which is almost solely based on quasi–static models, and in fact provides richer results and better insight. Furthermore, linearized analysis is considered as a fundamental preparatory step towards full nonlinear analysis, which at the moment appears to be too complex to achieve in full generality.

2 Dynamic Model

The starting point of our analysis is the linearized model of the dynamics of general manipulation systems derived in (Prattichizzo, 1995) (see also Prattichizzo and Bicchi, 1997). In this section we summarize some of those results for the reader's convenience.

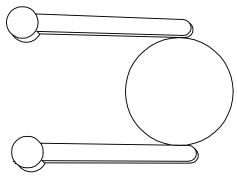

Figure 3: Not all the object trajectories can be controlled by joint inputs.

We denote by $\mathbf{q}$ the vector of manipulator joint positions, τ the vector of joint actuator torques, $(\mathbf{p}, \mathbf{R})$ the position and the orientation of a frame attached to the object, $\dot{\mathbf{u}}$ its generalized velocity, and $\mathbf{w}$ the vector of generalized forces resultant from external forces acting directly on the object. Here and in what follows, we refer the reader to the appendix for details on notation. The Jacobian matrix $\mathbf{J}$ and the grasp (alias "grip" or "wrench") matrix $\mathbf{G}$ of the manipulation system represent the linear maps relating the velocities of the contact points on the links and on the object, with the joint and object velocities, respectively. In building these matrices, the directions of motion of contact points which are relevant to the interaction with the object according to different possible contact models are taken into account, as detailed in the appendix. Finally, the vector $\mathbf{t}$ of all generalized forces exchanged at the contacts between the links and the object is introduced. Note that, in order to model contact interactions properly (in particular for the case of hyperstatic systems), it is necessary to consider visco–elastic effects. This also allows to deal with systems with significant inherent limb/object compliance (Akella and Cutkoski, 1995). The visco–elastic contact model, adopted in this paper, is reported in Appendix.

Manipulation systems may exhibit different kinematic characteristics, which are reflected in the algebraic properties of their Jacobian and grasp matrices. The systematic study of these properties was initiated by Kerr and Roth (1986), although they did not provide a nomenclature for possible interesting cases. For convenience, we will adopt the following definitions for general manipulation systems where $\mathsf{ker}\,(\mathbf{Q})$ denotes the kernel (or right nullspace) of matrix $\mathbf{Q}$:

Definition 1 A manipulation system is said "defective" if $\mathsf{ker}\,(\mathbf{J}^T) \neq \mathbf{0}$; "(motion) indeterminate" if $\mathsf{ker}\,(\mathbf{G}^T) \neq \mathbf{0}$; "redundant" if $\mathsf{ker}\,(\mathbf{J}) \neq \mathbf{0}$; "graspable" if $\mathsf{ker}\,(\mathbf{G}) \neq \mathbf{0}$ and "hyperstatic" if $\mathsf{ker}\,(\mathbf{J}^T) \cap \mathsf{ker}\,(\mathbf{G}) \neq \mathbf{0}$.

Remark 1 The term "defective" is employed because the row rank of the Jacobian is not full when at least one of the links touching the object possesses less degrees–of–freedom than those necessary to move its contact point in all directions inhibited by the relative contact constraint. Equivalently, in terms of forces, defectivity implies that there exists at least one direction of the contact force $\mathbf{t}$ that does not affect the manipulator joint torques. Defectivity occurs whenever the number t of components of contact forces is

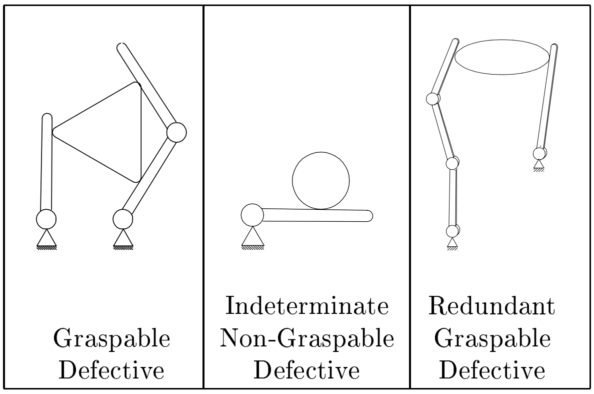

Figure 4: Illustration of Definition 1.

larger than the number q of joints, or when the manipulator is in a singular configuration. The term "motion indeterminate", or "indeterminate" for short, refers to the fact that the object is not completely restrained by contacts, and hence its motion can not be determined quasi–statically (indeterminacy of motion is of course solved when dynamics are taken into account).
The term "redundant" is standard in robotics. Note that here, redundancy of one of the linkages is enough to have redundancy of the whole system, and that redundancy and defectivity may occur in the same mechanism.
The term "graspable" refers to the fact that self–balanced "squeezing" contact forces are possible in this case, so that a multi–finger frictional grasp is possible.
Finally, we use "hyperstatic" for systems where the distribution of contact forces can not be determined by knowledge of joint torques and external forces alone. Such systems have also been termed "indeterminate" with reference to force, but we prefer to avoid this usage here because of possible confusion with motion indeterminacy.

The class of "general manipulation systems" this paper is concerned with is comprised of mechanisms with any number of limbs (open kinematic chains), of joints (prismatic, rotoidal, spherical, etc.) and of contacts (hard and soft finger, complete–constraint, etc.) between a reference member called "object" and links in any position in the limb chains. This includes in particular defective and hyperstatic systems, whose treatment is seldom considered in the literature.

Figure 4 pictorially illustrates such definitions. Recall that in 2D examples the dimension of the contact vector $\mathbf{t}$ is $2n$, being n the number of contacts.

Let ($\mathbf{q} = \mathbf{q}_o, \mathbf{p} = \mathbf{p}_o, \mathbf{R} = \mathbf{R}_o, \dot{\mathbf{q}} = \dot{\mathbf{u}} = \mathbf{0}, \tau = \tau_o, \omega = \omega_o$, and $\mathbf{t} = \mathbf{t}_o$) be a reference equilibrium configuration such that $\tau_o = \mathbf{J}^T \mathbf{t}_o$ and $\mathbf{w}_o = -\mathbf{G}\mathbf{t}_o$. The linear approximation of the manipulation system dynamics is written in a neighborhood of this configuration as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau \tau' + \mathbf{B}_w \mathbf{w}', \tag{1}$$

where state and input vectors are defined as (small) departures from the reference equilibrium configuration: $\mathbf{x} = \left[(\delta\mathbf{q})^T \ (\delta\mathbf{u})^T \ \dot{\mathbf{q}}^T \ \dot{\mathbf{u}}^T\right]^T$, $\tau' = \tau - \mathbf{J}^T\mathbf{t}_o$ and $\mathbf{w}' = \mathbf{w} + \mathbf{G}\mathbf{t}_o$; and system matrices are

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \quad \mathbf{B}_\tau = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}; \quad \mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_o^{-1} \end{bmatrix}, \tag{2}$$

where $\mathbf{M}_h$ and $\mathbf{M}_o$ are the inertia matrix of the manipulator (i.e., the block–diagonal aggregate of the inertia matrices of the limbs) and the inertia matrix of the object, respectively. To simplify notation we will henceforth omit the apices in τ' and $\mathbf{w}'$.

Assuming that variations of the joint torques due to gravity, of the Jacobian and of the grasp matrices are negligible for small displacements $\delta\mathbf{u}$, $\delta\mathbf{q}$, simple expressions are obtained for $\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k$ and $\mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b$, where $\mathbf{M} = \mathrm{diag}(\mathbf{M}_h, \mathbf{M}_o)$, $\mathbf{P}_k = \mathbf{S}^T\mathbf{K}\mathbf{S}$, $\mathbf{P}_b = \mathbf{S}^T\mathbf{B}\mathbf{S}$, and $\mathbf{S} = [\mathbf{J}, \ -\mathbf{G}^T]$.

To our purposes, three possible combinations of states are of interest as outputs, namely object positions, joint positions, and contact forces. The corresponding output matrices are, respectively,

$$\begin{aligned} \mathbf{C}_u &= \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}; \\ \mathbf{C}_q &= \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}; \\ \mathbf{C}_t &= \begin{bmatrix} \mathbf{K}\mathbf{J} & -\mathbf{K}\mathbf{G}^T & \mathbf{B}\mathbf{J} & -\mathbf{B}\mathbf{G}^T \end{bmatrix}, \end{aligned}$$

where the contact force output matrix is evaluated according to the visco–elastic contact model reported in Appendix.

3 Stability and stabilizability

We consider some aspects related to the analysis of the stability of the linearized model of a manipulation and grasping system. The characteristic polynomial of the linearized system is: $\det(s\mathbf{I} - \mathbf{A}) = \det(s^2\mathbf{M} + s\mathbf{P}_b + \mathbf{P}_k)$. While the inertia matrix $\mathbf{M}$ is certainly positive definite, matrices $\mathbf{P}_k$ and $\mathbf{P}_b$ can be positive semidefinite. In fact, if the manipulation system has non-zero mobility μ ($\mu = \dim \ker [\mathbf{J}, \ -\mathbf{G}^T] \neq \mathbf{0}$; see (Bicchi et al. 1995a) and section 4.2), both $\mathbf{P}_k$ and $\mathbf{P}_b$ will have μ eigenvalues in the origin. Accordingly, the eigenvalues of the linearized system will lie in the union of the open left–half–plane and the origin.

Due to the presence of zero eigenvalues no conclusion can be drawn about the local stability of the full, nonlinear dynamics at the equilibrium configuration. However, the following *restricted stabilizability* lemma holds for systems with $\ker \mathbf{G}^T = \{0\}$ (i.e., excluding indeterminate systems):

Lemma 1 *The dynamics of a determinate manipulation system is made locally asymptotically stable by a constant linear state feedback of joint displacements and rates only, with feedback matrix* $\mathbf{R}' = \begin{bmatrix} \mathbf{R}_q & \mathbf{0} & \mathbf{R}_{\dot{q}} & \mathbf{0} \end{bmatrix}$ *provided that* $\mathbf{R}_q$ *and* $\mathbf{R}_{\dot{q}}$ *are positive definite matrices.*

Proof: Recall that $\det(s\mathbf{I} - (\mathbf{A} - \mathbf{B}_\tau \mathbf{R})) = \det(s^2\mathbf{M} + s\mathbf{P}'_b + \mathbf{P}'_k)$, where $\mathbf{P}'_k = \mathbf{P}_k + \mathrm{diag}(\mathbf{R}_q, \mathbf{0})$ and $\mathbf{P}'_b = \mathbf{P}_b + \mathrm{diag}(\mathbf{R}_{\dot{q}}, \mathbf{0})$. The proof ends by showing that $\mathbf{P}'_k$ and $\mathbf{P}'_b$ are positive definite matrices. Putting $\mathbf{K} = \mathbf{K}^{T/2}\mathbf{K}^{1/2}$, we have that $\mathbf{x}^T\mathbf{P}'_k\mathbf{x} = (\mathbf{K}^{1/2}\mathbf{J}\mathbf{x}_1 - \mathbf{K}^{1/2}\mathbf{G}^T\mathbf{x}_2)^T(\mathbf{K}^{1/2}\mathbf{J}\mathbf{x}_1 - \mathbf{K}^{1/2}\mathbf{G}^T\mathbf{x}_2) + \mathbf{x}_1^T\mathbf{R}_q\mathbf{x}_1 > 0$, and analogously for $\mathbf{P}'_b$. ■

Remark 2 The practical relevance of this lemma is that independent proportional-derivative control at joints is sufficient to stabilize any manipulation system whose motions are quasi–statically determinate, about a reference equilibrium.

4 Functional controllability

As already pointed out, we are interested in the problem of following a desired trajectory with the manipulated object, while guaranteeing that contact forces are controlled so as to comply with contact constraints at every instant. In system theory this problem is known as "functional (output trajectory) controllability". Although functional controllability is generally approached by state–space methods (Sain and Massey, 1969), for linear systems it is most simply studied in terms of input–output representations. A well–known necessary and sufficient condition for the output functional controllability of linear system is reported in the following proposition

Proposition 1 *Let* $\mathbf{Z}(s)$ *be the* $(d \times q)$ *transfer function matrix of a given linear system. A necessary and sufficient condition for the functional (output trajectory) controllability of* d *arbitrary smooth (*C^∞*) outputs by* q *smooth inputs is that the transfer function matrix* $\mathbf{Z}(s)$ *is full row rank over the field of complex numbers.*

Explicitly note that the output functional controllability requires that at least as many inputs are available as there are outputs of concern.

Consider the linearized model (1–2) initially relaxed and fed back by $\mathbf{R}_q$ from joint positions and $\mathbf{R}_{\dot{q}}$ from joint velocities. $\hat{\mathbf{A}}$ and τ will henceforth indicate the dynamic matrix with feedback and the reference input, respectively. Let $\delta\mathbf{u}$ be the system output, in the Laplace domain the input–output representation is

$$\begin{array}{ll}
\mathbf{u}(s) = \mathbf{Z}_{u,\tau}(s)\tau(s) + \mathbf{Z}_{u,w}\mathbf{w}(s), & \text{with} \\
\mathbf{Z}_{u,\tau} = \mathbf{C}_{\mathbf{u}}\ (s\mathbf{I} - \hat{\mathbf{A}})^{-1}\ \mathbf{B}_{\tau} = -\mathcal{D}^{-1}\mathcal{B}^T\mathcal{X}; & \\
\mathbf{Z}_{u,w} = \mathbf{C}_{\mathbf{u}}\ (s\mathbf{I} - \hat{\mathbf{A}})^{-1}\ \mathbf{B}_{\mathbf{w}} = (\mathcal{D} - \mathcal{B}^T\mathcal{A}^{-1}\mathcal{B})^{-1}, & \text{where} \\
\mathcal{A} = s^2\mathbf{M}_h + s(\mathbf{J}^T\mathbf{B}\mathbf{J} + \mathbf{R}_{\dot{q}}) + \mathbf{J}^T\mathbf{K}\mathbf{J} + \mathbf{R}_q; & \\
\mathcal{B} = -s\mathbf{J}^T\mathbf{B}\mathbf{G}^T - \mathbf{J}^T\mathbf{K}\mathbf{G}^T; & \\
\mathcal{D} = s^2\mathbf{M}_o + s\mathbf{G}\mathbf{B}\mathbf{G}^T + \mathbf{G}\mathbf{K}\mathbf{G}^T; & \\
\mathcal{X} = (\mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{B}^T)^{-1}. &
\end{array}$$

Being $\mathbb{R}^d$ the space of object motions, in absence of disturbances $\mathbf{w}$, at least d input torques (joints) are necessary to track arbitrary object trajectories from τ.

Analogous considerations apply when contact forces, $\mathbf{t} \in \mathbb{R}^t$, are considered as outputs,

$$\begin{array}{ll}
\mathbf{t}(s) = \mathbf{Z}_{t,\tau}(s)\tau(s) + \mathbf{Z}_{t,w}\mathbf{w}(s), & \text{with} \\
\mathbf{Z}_{t,\tau} = \mathbf{C}_{\mathbf{t}}\ (s\mathbf{I} - \hat{\mathbf{A}})^{-1}\ \mathbf{B}_{\tau} = (\mathbf{K} + s\mathbf{B})(\mathbf{J}\mathcal{X} - \mathbf{G}^T\mathcal{Z}); & \\
\mathbf{Z}_{t,w} = \mathbf{C}_{\mathbf{t}}\ (s\mathbf{I} - \hat{\mathbf{A}})^{-1}\ \mathbf{B}_{w} = (\mathbf{K} + s\mathbf{B})(\mathbf{J}\mathcal{Y} - \mathbf{G}^T\mathcal{W}), & \text{where} \\
\mathcal{Z} = -\mathcal{D}^{-1}\mathcal{B}^T\mathcal{X}; & \\
\mathcal{W} = (\mathcal{D} - \mathcal{B}^T\mathcal{A}^{-1}\mathcal{B})^{-1}; & \\
\mathcal{Y} = -\mathcal{A}^{-1}\mathcal{B}\mathcal{W}. &
\end{array}$$

Being $\mathbb{R}^t$ the space of contact forces, in absence of disturbances $\mathbf{w}$, at least t input torques are necessary to track arbitrary contact forces.

The relationship between contact forces $\mathbf{t}$ and object motions $\mathbf{u}$ can be written as

$$\mathbf{u}(s) = \mathbf{Z}_{u,t}\mathbf{t}(s) \quad \text{with} \;\; \mathbf{Z}_{u,t} = \mathbf{M}_o^{-1}\mathbf{G}/s^2. \tag{3}$$

Thus the functional controllability of contact forces along with the full row rank of the grasp matrix $\mathbf{G}$ are sufficient conditions to guarantee functional controllability of object trajectories. Conversely, specification of object trajectories $\mathbf{u}_d$ imposes restrictions on possible specification of contact force trajectories $\mathbf{t}_d$, according to $\mathbf{t}_d = \mathbf{Z}^r_{u,t}(s)\mathbf{u}_d + \mathbf{N}_{u,t}\mathbf{y}_t(s)$, where $\mathbf{Z}^r_{u,t}(s)$ and $\mathbf{N}_{u,t}$ are transfer function matrices representing a right–inverse operator and a null–space basis, respectively, such that $\mathbf{Z}_{u,t}\mathbf{Z}^r_{u,t}(s)\mathbf{u}_d(s) = \mathbf{u}_d(s),\ \forall\mathbf{u}(s)$, and $\mathbf{Z}_{u,t}\mathbf{N}_{u,t}(s)\mathbf{y}_t(s) = 0,\ \forall\ \mathbf{y}_t(s)$ which represents the residual freedom in specifying contact forces.

In kinematically defective devices ($\mathsf{ker}\,(\mathbf{J}^T) \neq \mathbf{0}$, where $\mathbf{J}^T \in \mathbb{R}^{q\times t}$), the dimension t of the space of contact forces is much larger than that of joint displacements, q, thus kinematic defectivity implies a lack of complete functional controllability of contact forces, with the consequence that further restrictions apply to specification of both $\mathbf{u}_d(s)$ and $\mathbf{y}_t(s)$.

In this paper we will focus on the *definition of a new set of outputs that is functionally controllable* and *relevant to the task of manipulation*. In order to do this, the concept of "asymptotic reproducibility" (Brocket and Mesarovich, 1965) is well suited. Asymptotic reproducibility investigates output tracking for a particular class of trajectories, namely those constant in time. In other words, it investigates the possibility of displacing the system from its reference equilibrium configuration to a different nearby equilibrium by means of step inputs. The following definition formalizes the notion of asymptotic reproducibility.

Definition 2 Let $\mathbf{y}(s)/\tau(s) = \mathbf{Z}(s)$ be the transfer matrix of an asymptotically stable system, the subspace of asymptotic reproducibility is defined as the column space of $\mathbf{Z}(0)$. The system output is asymptotically reproducible if the gain matrix $\mathbf{Z}(0)$ is full row rank.

Remark 3 The asymptotic reproducibility of the outputs of an asymptotically stable system is a sufficient condition for the functional reproducibility of the same outputs.

In the sequel, we assume that the manipulation system has no indeterminate modes ($\mathbf{G}$ is f.r.r) and that joint position and rates have been fed back such that all modes of the system are asymptotically stable.

4.1 Contact forces

After some algebraic manipulation, the steady–state gain matrix for contact forces from joint inputs is evaluated as

$$\mathbf{Z}_{t,\tau}(0) = -\mathbf{C_t}\hat{\mathbf{A}}^{-1}\mathbf{B}_\tau = -(\mathbf{I} - \mathbf{G}_{\bar{K}}^{+}\mathbf{G})\mathbf{K}\mathbf{J}, \tag{4}$$

where $\mathbf{G}_{\bar{K}}^{+}$ is the $\bar{\mathbf{K}}$–weighted pseudoinverse of $\mathbf{G}$, and $\bar{\mathbf{K}}^{-1} = \mathbf{K}^{-1} + \mathbf{J}\mathbf{R}_q^{-1}\mathbf{J}^T$ is the equivalent stiffness matrix including the effect of proportional control on joint positions (Cutkosky and Kao, 1989). The subspace

$$\mathcal{F}_{hr} = \mathsf{im}\,(\mathbf{Z}_{t,\tau}(0)) \tag{5}$$

is defined as the subspace of "asymptotically internal forces" and consists of all the contact forces that are reachable at steady–state. Observe that $\mathcal{F}_{hr} \subseteq \mathsf{ker}\,(\mathbf{G})$: these forces are self–balanced and their resultant action on the object dynamics is zero. In robotic grasp literature, forces $\mathbf{t} \in \mathsf{ker}\,(\mathbf{G})$ are customarily defined "internal", and play a fundamental role in grasp contact stability (slippage avoidance).

The importance of controllability of internal forces in grasping was put into evidence in a previous work by Bicchi (1993), where the principle of virtual work was used in a quasi–static approach to describe the subspace of "active" internal forces. Simple calculations show that such subspace coincides with $\mathcal{F}_{hr}$. The example in figure 5 illustrates asymptotically internal contact forces. While the subspace of internal forces ($\mathsf{ker}\,(\mathbf{G})$) is 4–dimensional, only a one–dimensional subspace ($\mathsf{im}\,(\mathbf{E})$) is asymptotically reproducible from joint torques τ.

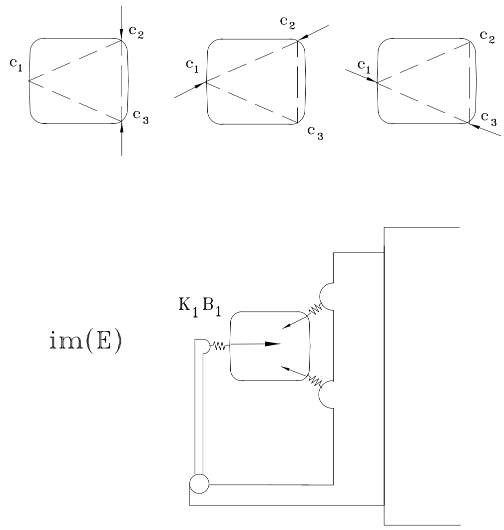

Figure 5: 2D example of asymptotic reproducible contact forces ($\mathcal{F}_{hr} \subset \mathsf{ker}\,(\mathbf{G})$). Stiffness ($\mathbf{K}_1$) and damping ($\mathbf{B}_1$) matrices at contact c_1 are square with dimension 2. For a hard–finger contact model in 2D, the complete stiffness and damping matrices are $\mathbf{K} = \text{diag}(\mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_3)$ and $\mathbf{B} = \text{diag}(\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3)$.

It should be pointed out that, in general, asymptotically reproducible internal forces are internal only at steady–state, and it might not be possible to apply them without a transient phase affecting the equilibrium of the object. Consider for instance the example in figure 5: when a step of torque is applied at the joint to “squeeze” the object, it causes the motion of the object, which recovers a (displaced) equilibrium only after the transient is finished. In other cases, due to symmetries in the mechanism, it might be possible to apply internal forces that remain such during the transients as well. Such “dynamically internal” forces along with a special force/motion nointeracting controller have been investigated in detail by Prattichizzo et al. (1995) and (1996).

4.2 Object motions

The steady–state gain matrix for object motions from joint inputs is evaluated as

$$\mathbf{Z}_{u,\tau}(0) = -\mathbf{C}_{\mathbf{u}}\hat{\mathbf{A}}^{-1}\mathbf{B}_{\tau} = (\mathbf{G}\bar{\mathbf{K}}\mathbf{G}^T)^{-1}\mathbf{G}\bar{\mathbf{K}}\mathbf{J}. \tag{6}$$

The subspace $\mathcal{U}_r = \mathsf{im}\,(\mathbf{Z}_{u,\tau}(0))$ is comprised of all asymptotically reproducible displacements of the object from joint torques. Observe that, in the absence of external disturbances, reference paths built by points in $\mathcal{U}_r$ can be tracked by the object within arbitrary accuracy. In the sequel, it will be shown that every displacement of the object complying with a rigid–body model of the system is asymptotically reproducible.

Rigid–body kinematics are of particular interest in the control of manipulation systems. Since they do not involve visco–elastic deformations of bodies, they can be regarded

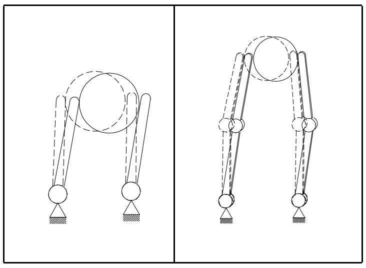

Figure 6: Representative motions for the subspace $\mathsf{im}\,\left[\mathbf{\Gamma}_{qc}^T\ \mathbf{\Gamma}_{uc}^T\right]^T$.

as low–energy motions. In a sense, they represent the natural way to change the object posture. Rigid–body kinematics have been studied in a quasi–static setting (Bicchi et al. 1995a) and in terms of unobservable subspaces in (Prattichizzo and Bicchi, 1997). In both cases rigid kinematics were described by a matrix $\mathbf{\Gamma}$ whose columns form a basis for $\mathsf{ker}\left[\mathbf{J}\ \ -\mathbf{G}^T\right]$. On the assumption that the system is not indeterminate,

$$\mathbf{\Gamma} = \mathsf{ker}\left[\begin{array}{cc} \mathbf{J} & -\mathbf{G}^T \end{array}\right] = \left[\begin{array}{cc} \mathbf{\Gamma}_r & \mathbf{\Gamma}_{qc} \\ \mathbf{0} & \mathbf{\Gamma}_{uc} \end{array}\right], \tag{7}$$

where $\mathbf{\Gamma}_r$ is a basis matrix of the subspace of redundant motions $\mathsf{ker}\,(\mathbf{J})$, and $\mathbf{\Gamma}_{qc}$ and $\mathbf{\Gamma}_{uc}$ are conformal partitions of a complementary basis matrix. The image spaces of $\mathbf{\Gamma}_{qc}$ and $\mathbf{\Gamma}_{uc}$ consist of coordinated rigid–body motions of the mechanism, for the links and the object parts, respectively. Figure 6 illustrates such subspaces for two simple devices.

It can be shown that rigid–body coordinate motions of the object are asymptotically reproducible from joint torques,

$$\mathsf{im}\,(\mathbf{\Gamma}_{uc}) \in \mathsf{im}\,((\mathbf{G}\bar{\mathbf{K}}\mathbf{G}^T)^{-1}\mathbf{G}\bar{\mathbf{K}}\mathbf{J}) = \mathcal{U}_r. \tag{8}$$

Observe that rigid–body motions are not the only asymptotically reproducible object motions; $\mathcal{U}_r$ also contains motions due to deformations of elastic elements in the model, as for instance, horizontal motions of the object in the device of figure 5.

4.3 Functional controllability of contact forces and object motions

In manipulation systems with general kinematics, it is possible that not all the object motions and contact forces result functionally controllable by joint torques. According to (6) and (8) and to Remark 3, desired object trajectories $\mathbf{u}_{des}$ can be executed if they remain within the subspace $\mathcal{U}_r$ and analogously, according to (4) and (5), arbitrary contact force trajectories $\mathbf{t}_{des}$ can be executed if they evolve within the subspace $\mathcal{F}_{hr}$. Obviously we are considering the case of object and contact forces trajectories disjointly controlled.

However in manipulation, due to the presence of unilateral, conic contact constraints, task specifications can not be given disjointly in terms of either object positions or contact forces. Therefore conditions $\mathbf{u}_{des} \in \mathcal{U}_r$ and $\mathbf{t}_{des} \in \mathcal{F}_{hr}$ are only necessary, but no longer sufficient, for *joint* functional controllability of object motions and contact forces. Moreover, specifications of jointly functionally controllable object motions and contact forces may not exhaust the control capabilities of the system.

Our goal is therefore to define a set of outputs for a general manipulation systems that is guaranteed to be feasible, that fully exploits the control inputs and that is convenient for the specification of the tasks. The first requirement implies that the new outputs are functionally controllable; the second that the input–output system is square and the third that the new outputs incorporate the typical priorities of a manipulation task with its priorities:

a) object trajectories that can be accommodated for by the mechanism;

b) contact forces that can be steered so as to avoid violation of contact constraints;

c) reconfiguration of limbs in presence of redundancy.

The following theorem proposes a functionally controllable and task–oriented set of outputs for general manipulation systems

Theorem 1 *In the hypothesis that* $\mathsf{ker}\,(\mathbf{G}^T) = \mathbf{0}$*, consider the linearized dynamics described by the triple* $(\mathbf{A}, \mathbf{B}_\tau, \mathbf{C})$*, where* $\mathbf{A}$ *and* $\mathbf{B}_\tau$ *are as in Section 2, and the output matrix* $\mathbf{C}$ *is defined as*

$$\mathbf{C} = \begin{bmatrix} \mathbf{\Gamma}_{uc}^{+}\mathbf{C}_{\mathbf{u}} \\ \mathbf{E}^{+}\mathbf{C}_{\mathbf{t}} \\ \mathbf{\Gamma}_{r}^{+}\mathbf{C}_{\mathbf{q}} \end{bmatrix}, \tag{9}$$

where $\mathbf{\Gamma}_r$ *and* $\mathbf{\Gamma}_{uc}$ *have been defined in (7), and* $\mathbf{E}$ *is a basis matrix for* $\mathcal{F}_{hr}$*. Then, for any constant linear state feedback* $\mathbf{R} = [\mathbf{R}_q\ \mathbf{R}_u\ \mathbf{R}_{\dot{q}}\ \mathbf{R}_{\dot{u}}]$ *such that* $\mathbf{A} - \mathbf{B}_\tau\mathbf{R}$ *is asymptotically stable, the system* $(\mathbf{A} - \mathbf{B}_\tau\mathbf{R},\ \mathbf{B}_\tau,\ \mathbf{C})$ *is square and functionally controllable.*

Proof: Notice that the existence of such feedback matrix $\mathbf{R}$ is guaranteed by Lemma 1.

a) The system is square if the number of rows of $\mathbf{C}$ is equal to the input space dimension, q. In formulas, denoting by $\#(\mathbf{N})$ the number of colums of matrix $\mathbf{N}$, the system is square if $\#(\mathbf{\Gamma}_{uc}) + \#(\mathbf{E}) + \#(\mathbf{\Gamma}_r) = q$. Since $\mathbf{\Gamma}_r$, $\mathbf{\Gamma}_{uc}$, and $\mathbf{E}$ are full column rank by definition, from (7) we have $\#(\Gamma_{uc}) + \#(\Gamma_r) = \dim(\ker[\mathbf{J} \quad -\mathbf{G}^T]) - \dim(\ker(\mathbf{G}^T))$. Observing that $\mathsf{ker}\,(\mathbf{I} - \mathbf{G}_K^R\mathbf{G}) = \mathsf{im}\,(\mathbf{K}\mathbf{G}^T)$, from (4) we obtain

$$\begin{aligned} &\#(\mathbf{E}) = \#(\mathbf{J}) - \dim(\mathsf{ker}\,(\mathbf{J})) - \dim(\mathsf{im}\,(\mathbf{J}) \cap \mathsf{im}\,(\mathbf{G}^T)) = \\ &= q - \dim(\mathsf{ker}\,(\mathbf{J})) - \Big[\dim(\mathsf{ker}\,([\mathbf{J} \quad -\mathbf{G}^T])) \\ &-\dim(\mathsf{ker}\,(\mathbf{J})) - \dim(\mathsf{ker}\,(\mathbf{G}^T))\Big] = q - \#(\Gamma_r) - \#(\Gamma_{uc}); \end{aligned}$$

b) To prove output functional controllability of the system $(\mathbf{A} - \mathbf{B}_\tau\mathbf{R},\ \mathbf{B}_\tau,\ \mathbf{C})$, it will be shown that $\mathbf{Z}_C(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A} + \mathbf{B}_\tau\mathbf{R})^{-1}\mathbf{B}_\tau$ has rank q over the complex field.

According to Remark 3 it suffices to show that the steady–state gain matrix $\mathbf{Z}_C(0)$ is full row rank.

$$\mathbf{Z}_C(0) = \begin{bmatrix} \mathbf{\Gamma}_{uc}^{+}(\mathbf{GKG}^T)^{-1}\mathbf{GKJ} \\ \mathbf{E}^{+}(\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ} \\ \mathbf{\Gamma}_r^{+} \end{bmatrix} \Xi,$$

where $\Xi = -\left(\mathbf{J}^T\mathbf{KJ} + \mathbf{R}_q - (\mathbf{J}^T\mathbf{KG}^T - \mathbf{R}_u)(\mathbf{GKG}^T)^{-1}\mathbf{GKJ}\right)^{-1}$.

From remark 3, the full row rank of $\mathbf{Z}_C(0)$ is a sufficient condition for functional controllability, that can be shown by proving that $\mathsf{ker}\,(\mathbf{Z}_C(0)^T) = \mathbf{0}$. Transposing $\mathbf{Z}_C(\mathbf{0})$, we get that

$$\mathsf{ker}\,(\mathbf{Z}_C(0)^T) = \mathsf{ker} \begin{bmatrix} \mathbf{\Gamma}_{uc}^{+}(\mathbf{GKG}^T)^{-1}\mathbf{GKJ} \\ \mathbf{E}^{+}(\mathbf{I} - \mathbf{G}_K^{+}\mathbf{G})\mathbf{KJ} \\ \mathbf{\Gamma}_r^{+} \end{bmatrix}^T.$$

Observe that each row block of the matrix on the right–hand side of equation above is full column rank, in fact

i: $\mathsf{im}\,(\mathbf{\Gamma}_{uc}) \subseteq \mathsf{im}\,((\mathbf{GKG}^T)^{-1}\mathbf{GKJ})$, directly from (8);

ii: $\mathbf{E}$ is a basis for $\mathsf{im}\,((\mathbf{I} - \mathbf{G}_K^{+}\mathbf{G})\mathbf{KJ})$ (cf. (4));

iii: $\mathbf{\Gamma}_r$ is a basis matrix for $\mathsf{ker}\,(\mathbf{J})$; Hence, to prove that $\mathsf{ker}\,(\mathbf{Z}_C(0)^T) = \mathbf{0}$ it suffices to show that the raw spaces of the three blocks are also mutually linearly independent;

iv: The columns of the third block span $\mathsf{ker}\,(\mathbf{J})$, while the span of the columns of the first two blocks lies within $\mathsf{im}\,(\mathbf{J}^T)$;

v: $\mathsf{im}\,(\mathbf{G}_K^{+}\mathbf{\Gamma}_{uc})$ and $\mathsf{im}\,(\mathbf{I}-\mathbf{G}_K^{+}\mathbf{G})\mathbf{KE})$ are disjoint, then so are the spans of the columns of the first and second blocks. ■

Notice that the task–oriented priority order in the choice of outputs is reflected in their top–down ordering. In fact, the first group of outputs are coordinates for the subspace of rigid–body displacements of the manipulated object (in the basis $\mathbf{\Gamma}_{uc}$); similarly the second group of outputs for the subspace $\mathcal{F}_{hr}$ of active internal contact forces (in the basis $\mathbf{E}$), and the third group for the subspace of redundant degrees–of–freedom (in the basis $\mathbf{\Gamma}_r$). As a result of Theorem 1, all of these three subspaces are functionally controllable, and so is their direct sum. The chosen outputs provide a basis of the set of all functionally controllable outputs, that exactly corresponds to the task specifications introduced above and exhaust the control capabilities of the manipulation system.

5 Examples

Theorem 1 has been applied to the simple four 2D devices reported in figure 7. For the sake of simplicity and without loss of generality, it is assumed at first that in each example the manipulated object is a disk of unit radius, mass, and barycentral moment of inertia and that link masses with their distributions are such that the inertia matrix of the manipulator $\mathbf{M}_h(\cdot)$ is equal to the identity matrix. Moreover links are assumed to

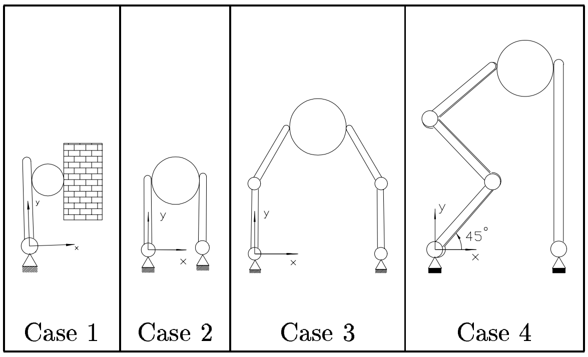

Figure 7: Four simple 2D manipulators.

have unit length except for the link of the second limb in case 4 (its length is $3\cos(\pi/4)$). The length of a link involving contact with the object is meant to be measured between the joint axis and the contact point. Contacts are always assumed to be of hard–finger type. Finally matrices $\mathbf{K}$, $\mathbf{B}$, $\mathbf{R}_q$ and $\mathbf{R}_{\dot{q}}$ are assumed to be normalized to the identity matrix. A more realistic case is reported later on in this paper.

Case 1: $\mathbf{\Gamma} = \mathbf{0}$ and $\mathbf{E} = [1\ 0\ -1\ 0]^T$. Being matrix $\mathbf{\Gamma}$ null, there are neither redundant motions for the manipulator nor rigid–body coordinate motions for the objects. The device can only apply force trajectories lying on $\mathsf{im}\,(\mathbf{E})$. According to Theorem 1, the output matrix $\mathbf{C}$ has one row, namely $\mathbf{C} = [1\ |0\ 0\ 0\ |1\ |0\ 0\ 0\ ]$.

Case 2: In this case the manipulator has 2 joints, and at most two outputs can be functionally controllable. These can be specified according to the proposed method as one rigid–body coordinate motion of the object in the horizontal direction and one controllable internal forces:

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{qc} \\ \Gamma_{uc} \end{bmatrix} = [-1\ -1\ |1\ 0\ 0]^T; \quad \mathbf{E} = [0.7\ 0\ -0.7\ 0]^T.$$

Then the output matrix results

$$\mathbf{C} = \left[\begin{array}{cc|ccc|cc|ccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & -1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{array}\right].$$

Case 3: The angle between the links is 30deg.

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{qc} \\ \Gamma_{uc} \end{bmatrix} = \left[\begin{array}{ccc} 0 & -6.2 & 0 \\ -6.8 & 6.5 & 0 \\ -2.7 & -2.7 & -1 \\ -1 & -1 & 2.2 \\ \hline -7.6 & 7 & 0 \\ 1 & 1.2 & 1 \\ -3.4 & 1 & 1 \end{array}\right]; \quad \mathbf{E} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}.$$

Being $\mathbf{\Gamma}_{uc}$ full row rank, it follows that the device can execute arbitrary object trajectories in $\mathbb{R}^3$ (locally approximating $\mathbb{R}^2 \times S^1$) along with arbitrary internal contact forces trajectories ($\mathsf{im}\,(\mathbf{E}) = \mathsf{ker}\,(\mathbf{G})$).

$$\mathbf{C} = \left[\begin{array}{cccc|ccc|cccc|ccc} 0 & 0 & 0 & 0 & 0 & .2 & -.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .1 & .2 & -.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.1 & .5 & .5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2.2 & 1 & -2.2 & -1 & 0 & 0 & 0 & 2.2 & 1 & -2.2 & -1 & 0 & 0 & 0 \end{array}\right],$$

Case 4: The manipulator is redundant and the angle between the consecutive links is equal to ± 90deg. Regarding the output organization, Theorem 1 suggests to use two input degrees-of freedom to control rigid–body coordinate motion, one for internal contact forces ($\mathsf{im}\,(\mathbf{E})$) and the last one for redundancy. In fact

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{qc} & \Gamma_{qc} \\ 0 & \Gamma_{uc} \end{bmatrix} = \left[\begin{array}{c|cc} -1 & 1 & -1 \\ 1 & -1 & -4.6 \\ 1 & 2 & 3.6 \\ 0 & 1 & -2.9 \\ \hline 0 & -2 & 6.6 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{array}\right]; \quad \mathbf{E} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix},$$

$$\mathbf{C} = \left[\begin{array}{cccc|ccc|cccc|ccc} 0 & 0 & 0 & 0 & 1 & -1.3 & 2.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.5 & 1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 3 & 2 & 1 & -3 & 0 & 0 & 0 & 3 & 2 & 1 & -3 & 0 & 0 & 0 \\ \hline 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Case 2 (revisited): We illustrate now an application of the results of this paper to the construction of a decoupling prefilter for the manipulation system of case 2. Steady–state input/output decoupling is simply obtained as

$$\tau(s) = \mathbf{Z}_C(0)^{-1}\nu(s)$$

where $\mathbf{Z}_C(s) = \mathbf{C}(s\mathbf{I} - \hat{\mathbf{A}})^{-1}\mathbf{B}$.

Through such prefilter, object positions and contact forces are simply commanded by controls $\nu_1(s)$ and $\nu_2(s)$, respectively. Such prefiltering will facilitate the design of the two independent position and force control loops (the design of the closed–loop controllers is out of the scope of this paper).

A realistic simulation is presented where parameters for the second example above are as follows. An object with different visco elastic parameters at the two contacts is considered: $\mathbf{K} = \mathrm{diag}(\mathbf{K}_1, \mathbf{K}_2), \mathbf{K}_1 = \mathrm{diag}(200N/m, 200N/m)$, $\mathbf{K}_2 = 0.5\mathbf{K}_1$, $\mathbf{B} = \mathrm{diag}(\mathbf{B}_1, \mathbf{B}_2)$, $\mathbf{B}_1 = \mathrm{diag}(66Ns/m, 66Ns/m)$, $\mathbf{B}_2 = 0.5\mathbf{B}_1$; the uniformly distributed link (object) mass and the link length (object radius) are $\mathbf{m}_l = 0.3kg$ ($\mathbf{m}_o = 0.25kg$), $l = 0.3m$ ($R = 0.15m$), respectively. The joint position and velocity feedback are set to $\mathbf{R}_q = \mathrm{diag}(10, 10)$ and $\mathbf{R}_{\dot{q}} = \mathrm{diag}(1, 1)$. Grasp and Jacobian matrices are

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ R\sin\theta & -R\cos\theta & -R\sin\theta & R\cos\theta \end{bmatrix}; \quad \mathbf{J} = \begin{bmatrix} -l\cos q_1 & 0 \\ -l\sin q_1 & 0 \\ 0 & -l\cos q_2 \\ 0 & -l\sin q_2 \end{bmatrix}.$$

The contact point is assumed fixed at a distance l from the joints on the links. Reachable internal forces $\mathsf{im}\,(\mathbf{E})$ and rigid–body object motion $\mathsf{im}\,(\Gamma_{uc})$ do not change with respect to the previous analysis of Case 2. The transfer function from the joint inputs to the outputs (specified as in Theorem 1) is evaluated at the reference configuration of fig. 7 (with $q_1(0) = q_2(0) = 0$) as

$$\mathbf{Z}_C(s) = \frac{1}{d(s)} \begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix}$$

where

$$\begin{aligned}
d(s) &= s^10 + 2E03s^9 + 1.4E06s^8 + 4.2E08s^7 + 4.9E10s^6 + 1.6E12s^5 + 2.5E13s^4 + 2E14s^3 + 7.6E14s^2 + 1.5E15s \\
n_{11}(s) &= -3.2E03s^7 - 4.4E06s^6 - 1.6E09s^5 - 1.6E11s^4 - 2.1E12s^3 - 1E13s^2 - 2.3E13s - 1.9E13; \\
n_{21}(s) &= -5.7E02s^10 - 9.3E05s^8 - 4.7E08s^7 - 8.1E10s^6 - 2.8E12s^5 - 4.4E13s^4 - 3.2E14s^3 - 1.1E15s^2 - 2E15s - \\
n_{12}(s) &= -1.6e + 003s^7 - 2.4e + 006s^6 - 1e + 009s^5 - 1.4e + 011s^4 - 1.7e + 012s^3 - 8.2e + 012s^2 - 1.8e + 013s - 1. \\
n_{22}(s) &= 2.9E02s^9 + 5.8E05s^8 + 3.7E08s^7 + 7.7E10s^6 + 2.7E12s^5 + 4.3E13s^4 + 3.1E14s^3 + 1.1E15s^2 + 2E15s + 1.
\end{aligned}$$

The slowest time constant of the linearized system is 0.33sec. A step of 0.5N is commanded to the input ν_2 corresponding to internal forces. The input ν_1 corresponding to the horizontal displacement of the object's center is a sinusoid of period 9sec., with amplitude 30cm, started 3 seconds after the step in ν_2. System responses are reported in fig. 8. Note that the outputs of the linear approximation closely follows their references, in spite of the persistency of the variation of inputs. More interestingly, a good behaviour is also retained by the real system, in particuar with regard to position control, notwithstanding the important nonlinearities encountered in the manipulator joints' excursion which is larger than 90 degrees.

6 Conclusions

This paper analyzes the problem of controlling motions of objects manipulated by general mechanisms. Our main result consists in the suggestion of an organization of the output vector, which results functional controllable, exhaust the control capabilities and incorporates the constraints as well as the task requirements for the manipulation system. The approximate linearization method employed to study the problem renders our result valid only locally around an equilibrium point. The problem of generalizing this to the full nonlinear model is an interesting, albeit probably difficult, problem, especially in connection with the inclusion of rolling (nonholonomic) phenomena in the model.

Acknowledgements

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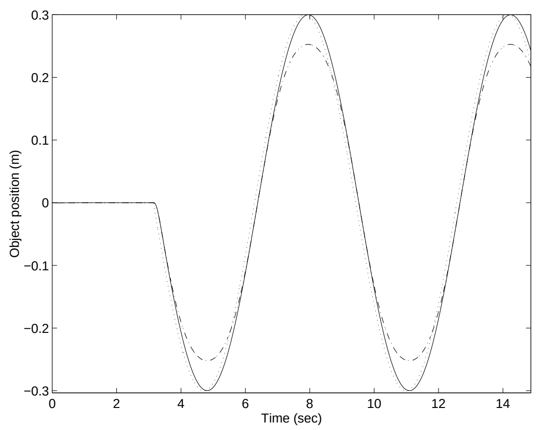
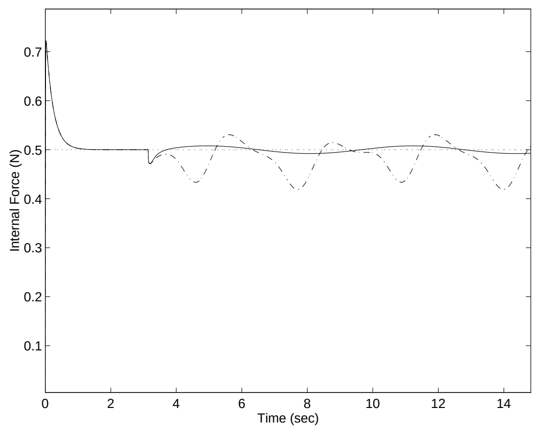
Appendix

The quantities introduced in text are defined as follows. Let $s = 2, d = 3$ for 2D mechanisms, and $s = 3, d = 6$ for 3D ones. Let also q be the number of actuated joints, n the number of contacts, and set

$$\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_q]^T; \quad \dot{\mathbf{q}} \in \mathbb{R}^q;$$



a)



b)

Figure 8: a) Variation of the object horizontal position for the linearized (continuous line) and the nonlinear dynamics (dash–dotted line), and b) intensity of internal force for the linearized (continuous line) and the nonlinear dynamics (dash–dotted line), corresponding to reference inputs (dotted lines).

$$
\begin{aligned}
\tau &= [\tau_1, \tau_2, \ldots, \tau_q]^T; \quad \tau \in \mathbb{R}^q; \\
\dot{\mathbf{u}} &= [\mathbf{v}^T, \omega^T]^T; \quad \dot{\mathbf{u}} \in \mathbb{R}^d; \\
\mathbf{w} &= [\mathbf{f}^T, \mathbf{m}^T]^T; \quad \mathbf{w} \in \mathbb{R}^d,
\end{aligned}
$$

where $\mathbf{v}$ (ω) is the linear (angular) velocity of object and $\mathbf{f}$ ($\mathbf{m}$) is the force (moment) on the object.

Denoting by $\mathbf{c}_i$ the position of the i–th contact point and by $\mathbf{p}$ the object center of mass, let

$$
\begin{aligned}
\tilde{\mathbf{G}} &= \left[\begin{array}{ccc|ccc} \mathbf{I}_s & \cdots & \mathbf{I}_s & & \mathbf{O}_{c\times ns} & \\ \mathbf{S}(\mathbf{c_1}-\mathbf{p}) & \cdots & \mathbf{S}(\mathbf{c_n}-\mathbf{p}) & \mathbf{I}_s & \cdots & \mathbf{I}_s \end{array}\right]; \quad \mathbf{G} \in \mathbb{R}^{d\times nd} \\
\tilde{\mathbf{J}}^T &= \left[\begin{array}{ccc|ccc} \mathbf{D}_{1,1} & \cdots & \mathbf{D}_{n,1} & \mathbf{L}_{1,1} & \cdots & \mathbf{L}_{n,1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{D}_{1,r} & \cdots & \mathbf{D}_{n,r} & \mathbf{L}_{1,r} & \cdots & \mathbf{L}_{n,r} \end{array}\right]; \quad \mathbf{J} \in \mathbb{R}^{nd\times q};
\end{aligned}
$$

where

$$
\mathbf{S}(\mathbf{c_i}) = \begin{bmatrix} 0 & -c_{i,y} & c_{i,z} \\ c_{i,y} & 0 & -c_{i,x} \\ -c_{i,z} & c_{i,x} & 0 \end{bmatrix}, \quad for \quad s = 3;
$$

$$
\mathbf{S}(\mathbf{c_i}) = \begin{bmatrix} -c_{i,y} & c_{i,x} \end{bmatrix}, \quad for \quad s = 2;
$$

blocks $\mathbf{D}_{i,j}$ and $\mathbf{L}_{i,j}$ are defined as

$$
\mathbf{D}_{i,j} = \begin{cases} [0\ 0\ 0] & \text{if the } i\text{-th contact force does} \\ & \text{not affect the } j\text{-th joint;} \\ \mathbf{z}_j^T & \text{for prismatic } j\text{-th joint;} \\ {\mathbf{z}'_j}^T\ \mathbf{S}(\mathbf{c_i} - \mathbf{o_j}) & \text{for rotational } i\text{-th joint;} \end{cases}
$$

$$
\mathbf{L}_{i,j} = \begin{cases} [0\ 0\ 0] & \text{if the } i\text{-th contact force does} \\ & \text{not affect the } j\text{-th joint;} \\ [0\ 0\ 0] & \text{for prismatic } j\text{-th joint;} \\ {\mathbf{z}'_j}^T & \text{for rotational } j\text{-th joint;} \end{cases} ;
$$

where $\mathbf{o}_j$ and $\mathbf{z}_j$ are the center and z-axis unit vector of the Denavit-Hartenberg frames associated with the j-th joint while $\mathbf{z}'_j = \mathbf{z}_j$ if $s = 3$ and $\mathbf{z}'_j = 1$ if $s = 2$.

The column space of matrices $\tilde{\mathbf{G}}^T$ and $\tilde{\mathbf{J}}$ represent linear and angular velocities (in all directions) of frames attached to all contact points as a function of object and joint velocities respectively.

Rigid–body contact constraints of different types (Mason and Salisbury, 1985) can be written as

$$
\mathbf{H}(\tilde{\mathbf{J}}\dot{\mathbf{q}} - \tilde{\mathbf{G}}^T\dot{\mathbf{u}}) \stackrel{def}{=} \mathbf{J}\dot{\mathbf{q}} - \mathbf{G}^T\dot{\mathbf{u}} = 0
$$

where the selection matrix $\mathbf{H}$ is built according to different contact models as reported in table 1.

Contact Type	Force Selector FS_i	Moment Selector MS_i
Point Contact w/o Friction	$\mathbf{z}_i^T$	$\mathbf{0}_{1\times(d-s)}$
Point Contact w/h Friction (Hard–Finger)	$\mathbf{I}_s$	$\mathbf{0}_{1\times(d-s)}$
Line Contact w/o Friction	$\mathbf{z}_i^T$	$(S(\mathbf{z}_i)\mathbf{x}_i)^T$
3D Line Contact w/h Friction	$\mathbf{z}_i^T$	$\begin{bmatrix} (S(\mathbf{z}_i)\mathbf{x}_i)^T \\ \mathbf{z}_i^T \end{bmatrix}$
3D Planar Contact w/o Friction	$\mathbf{z}_i^T$	$\begin{bmatrix} \mathbf{x}_i^T \\ \mathbf{y}_i^T \end{bmatrix}$
Planar Contact w/h Friction (Complete–Constraint)	$\mathbf{I}_s$	$\mathbf{I}_{d-s}$
3D Soft Finger	$\mathbf{I}_3$	$\mathbf{z}_i^T$

Table 1: Selectors for different contact types used to build the selection matrix $\mathbf{H}$. Vector $\mathbf{z}_i$ is the unit surface normal at the i–th contact while $\mathbf{x}_i$ and $\mathbf{y}_i$ are two unit vectors defining the line and plane of contact.

The overall contact selection matrix $\mathbf{H}$ is obtained by removing the zero rows from matrix

$$\hat{\mathbf{H}} = \mathrm{diag}(FS_1, \ldots, FS_n, MS_1, \ldots, MS_n).$$

In this paper, the rigid–body model is replaced by a visco–elastic contact model, taking contact forces explicitly into account. Consider a dn–dimensional vector $\tilde{\mathbf{t}}$ of all forces and torques at contacts defined as

$$\tilde{\mathbf{t}} = \left[\mathbf{f}_1^T,\ \ldots,\ \mathbf{f}_n^T,\ \mathbf{m}_1^T,\ \ldots,\ \mathbf{m}_n^T\right]$$

and the t–dimensional vector $\mathbf{t} = \mathbf{H}\tilde{\mathbf{t}}$, whereby only the non–zero components of contact forces and torques are listed. Changes in contact forces are related to displacements of object and links by stiffness and damping matrices $\mathbf{K}$ and $\mathbf{B}$, respectively, as

$$\delta\mathbf{t} = \mathbf{K}(\mathbf{J}\delta\mathbf{q} - \mathbf{G}^T\delta\mathbf{u}) + \mathbf{B}(\mathbf{J}\dot{\mathbf{q}} - \mathbf{G}^T\dot{\mathbf{u}}).$$

By physical reasons, $\mathbf{K}$ and $\mathbf{B}$ are block–diagonal positive definite symmetric matrices.

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