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Constant and variable loads in failure analyses of composite laminates

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ABSTRACT

A method for assessing the criticality of applied loads in failure analyses of composite laminates is described. The applied load vector is divided into a constant and variable part. The criticality of the combined loading condition is studied with respect to changes in the magnitude of the variable loading. When sets of unrelated loads are applied simultaneously, this approach gives more realistic results than if the resultant load vector as a whole is considered. The use of factors of safety with the method is discussed. A procedure to solve reserve factors and margins of safety based on quadratic failure criteria is outlined. The approach is illustrated with examples.

INTRODUCTION

The main goal of failure analyses is to evaluate the criticality of loads applied to a structure. The criteria for the prediction of failure in composites are typically nonlinear with respect to the applied loading. Instead of the actual values of the failure criteria it is therefore more practical to study how much the applied loading can be increased until failure occurs. The results can be expressed in terms of so-called strength/stress ratios as described, for instance, by Tsai.[1] In the strength/stress ratio approach, the applied loads are considered as a single load vector. Thus, later in the text, this approach is referred to as the resultant load method.

Since loads applied simultaneously to a structure are often due to independent physical phenomena, the assumption that the ratios of the individual load components stay constant is not always valid. An approach to avoid this shortcoming of the resultant load method is presented. The ideas have evolved during the development of ESAComp, a composite analysis and design software[2, 3] in which the approach will be implemented.

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CONSTANT AND VARIABLE LOADS

The approach presented in this paper is based on the partitioning of the applied loads into constant loads (F^c) and variable loads (F^v). Thus, the applied load vector can be expressed as

$$\{\boldsymbol{F}\} = \{\boldsymbol{F}^c\} + \{\boldsymbol{F}^v\} \tag{1}$$

Examples of typical constant and variable loads are mechanical preloads (e.g., static pressure loads, clamp loads) and dynamic loads (e.g., gust loads in aircraft and spacecraft). The preloads are often better predictable and can thus be considered as constant loads when the margin of safety with respect to the increase of the dynamic loads is studied. Typically thermal and moisture loads can also be considered as constant loads. However, the choice of constant and variable loads is not always self-evident. Different combinations of constant and variable loads may have to be studied to assure the adequacy of the design.

Factors of safety

Factors of safety (*FoS*) are parameters which take into account the statistical distribution of material strengths and applied loads so that a specified target reliability for the structure can be achieved. The use of factors of safety in this sense is a standard approach especially in the aerospace field.[4, 5] The factors of safety are also used in design without explicitly considering the statistical aspects.

In mechanical analyses the applied loads are multiplied with appropriate factors of safety to obtain the effective loading:

$$\{\boldsymbol{F}\}_{\text{effective}} = \boldsymbol{FoS}^c\{\boldsymbol{F}^c\} + \boldsymbol{FoS}^v\{\boldsymbol{F}^v\} \tag{2}$$

Different values of factors of safety can be assigned for different types of loads. Since the constant and variable load vectors may include different types of loads, the values of the factors of safety FoS^c and FoS^v could further be defined separately for the load components of $\{F^c\}$ and $\{F^v\}$. For simplicity, this is not shown in Equation (2) or in the rest of this text.

FAILURE PREDICTION

Reserve factors and margins of safety

The linear relation between the applied loading and the loading that leads to failure can be expressed in terms of so-called reserve factors (*RF*). The effective loading defined in Equation (2) is used as the basis for determining the reserve factors. Multiplying the nominal value of the applied load vector with the corresponding reserve factor gives the maximum allowed magnitude

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of the applied load vector. The effective loading is predicted to cause failure if the value of the reserve factor is less or equal to one.

In the constant and variable load approach, the criticality of the combined loading condition is studied with respect to the changes in the magnitude of the variable loading. Thus, the reserve factor *RF* is defined by the expression

$$\{F\}_{\text{failure}} = FoS^{c}\{F^{c}\} + RF\, FoS^{v}\{F^{v}\} \tag{3}$$

In comparison, the reserve factor RF^r of the resultant load method is defined as

$$\{F^{r}\}_{\text{failure}} = RF^{r}\left(FoS^{c}\{F^{c}\} + FoS^{v}\{F^{v}\}\right) \tag{4}$$

Due to the independent nature of the constant and variable loads, the reserve factors RF^c and RF^v which correspond to the constant and variable load vectors applied separately are further defined. The expressions for the corresponding failure loads are

$$\{F^{c}\}_{\text{failure}} = RF^{c} FoS^{c}\{F^{c}\} \tag{5}$$

and

$$\{F^{v}\}_{\text{failure}} = RF^{v} FoS^{v}\{F^{v}\} \tag{6}$$

Values below unity for the reserve factors RF^c and RF^v should normally not be accepted since the removal of the other load vector would lead to failure.

Margins of safety (*MoS*) are alternatives for reserve factors to describe the relative margin between the applied loading and the allowed loading. The simple relation between margins of safety and reserve factors is

$$MoS = RF - 1 \tag{7}$$

Margins of safety are often expressed as percentages. Thus, a reserve factor of 1.25 corresponds to a 25% margin of safety. A negative margin of safety indicates how much the loading has to be reduced to obtain an acceptable load level.

Although this paper only touches linear laminate failure analyses, the definitions of reserve factors and margins of safety given above are not dependent on the theoretical basis of the failure analyses performed.

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Computation of reserve factors

To determine the reserve factors of a composite laminate, the reserve factors of the individual plies have to be computed first. The solution is given here for the so-called quadratic failure criteria. Closed form solutions can also be written for the maximum stress and strain criteria, and for criteria such as the Puck criterion where the maximum stress and quadratic conditions are combined.

The general form of the quadratic criteria in stress space is

$$\sum_{i,j=1}^{6} F_{ij}\sigma_i\sigma_j + \sum_{i=1}^{6} F_i\sigma_i = 1 \tag{8}$$

In the classical lamination theory, plane stress state is assumed. Thus, only the terms corresponding to the indexes 1, 2, and 6 need to be considered. For a linear analysis, the ply stresses corresponding to the failure loading in Equation (3) can be written in the form

$$\{\sigma\}_{\mathbf{failure}} = \{\sigma^c\} + \boldsymbol{RF}\{\sigma^v\} \tag{9}$$

where $\{\sigma^c\}$ and $\{\sigma^v\}$ are the stresses caused by the load vectors $FoS^c\{F^c\}$ and $FoS^v\{F^v\}$, respectively. Substituting the expression for the ply stresses in Equation (8) and rearranging the terms yields

$$\begin{aligned} \boldsymbol{RF}^2 \sum_{i,j=1}^{6} F_{ij}\sigma_i^v\sigma_j^v + \boldsymbol{RF}\left[\sum_{i,j=1}^{6} F_{ij}(\sigma_i^c\sigma_j^v + \sigma_i^v\sigma_j^c) + \sum_{i=1}^{6} F_i\sigma_i^v\right] \\ + \sum_{i,j=1}^{6} F_{ij}\sigma_i^c\sigma_j^c + \sum_{i=1}^{6} F_i\sigma_i^c - 1 = 0 \end{aligned} \tag{10}$$

When the end of the effective constant load vector is within the failure envelope, the second order equation gives a positive and a negative root, from which *RF* is the positive root. If the end of the effective constant load vector is outside of the failure envelope, both roots are positive, negative, or imaginary. All these special cases are generally unacceptable, and the value of *RF* can thus be left undefined to indicate failure. However, if the effective resultant vector is within the envelope, the larger of the two positive roots can be selected bearing in mind that the effective constant load vector applied alone is critical (see Load case 2B later in the text). Reserve factors RF^v can be found with the same procedure by setting the constant loads equal to zero, and RF^c by further replacing σ^v's with σ^c's in Equation (10).

In laminate first ply failure (FPF) analyses, the laminate reserve factor is taken as the minimum of the ply reserve factors. Failure of any individual ply indicates failure of the laminate as well. In last ply failure (LPF) analyses, ply properties of individual plies are replaced with degraded properties as failure occurs. Thus, consecutive FPF type analyses are performed until the final failure stage of the laminate is reached and the laminate reserve factor can be determined.

EXAMPLE FAILURE ANALYSES

The significance of considering constant and variable loads in laminate failure analyses is studied through examples in which FPF analyses based on the classical lamination theory are performed for a symmetric graphite/epoxy laminate. The laminate structure and the ply properties corresponding to a typical unidirectional T800/epoxy ply are given in Table 1.

Table 1: Laminate structure and ply properties

Laminate lay-up		$(2(+45/-45) / 2(0/0/90) / 0)_s$				
Laminate thickness		$22 \cdot 0.20$ mm = 4.40 mm				
Ply engineering constants		Ply failure stresses				
E_1	155.0 GPa	X_t	2000 MPa	X_c	1500 MPa	
E_2	8.5 GPa	Y_t	40 MPa	Y_c	220 MPa	
G_{12}	5.5 GPa	S	80 MPa			
ν_{12}	0.30					
Ply thermal expansion coefficients		α_1	$-0.50 \cdot 10^{-6}$/°C			
		α_2	$30 \cdot 10^{-6}$/°C			

For the Tsai-Hill failure criterion that is used in the analyses, the nonzero coefficients in Equation (10) are

$$F_{11} = \frac{1}{X^2} \qquad F_{22} = \frac{1}{Y^2} \qquad F_{12} = -\frac{1}{2X^2} \qquad F_{66} = \frac{1}{S^2} \tag{11}$$

Tensile or compressive failure stresses X and Y are selected according to the ply stress state. In the constant and variable load approach, however, the ply stresses corresponding to failure are not explicitly known. Thus, the ply reserve factors have to be solved first from Equation (10) for the four possible combinations of the failure stresses X_t, X_c, Y_t, and Y_c. From the four values obtained, the one for which the stress state corresponds to the failure stresses is selected.

The three load cases used in the study are given in Table 2. The temperature difference from the stress-free environment of the laminate is constant through the laminate. Table 3 shows the values of laminate reserve factors for the given load cases and factors of safety. Besides of the reserve factors RF, RF^r, RF^c, and RF^v, the reserve factor RF^{v+c} is computed based on a reversed assumption on constant and variable loads: the variable load vector stays constant while the magnitude of the constant load vector is increased.

Table 2: Load cases

Load case	ΔT (°C)	N_x (kN/m)	N_y (kN/m)	N_{xy} (kN/m)
1 constant	-50	–	–	–
variable	–	-1000	–	500
2 constant	-50	–	–	500
variable	–	-1000	–	–
3 constant	–	-1000	900	–
variable	–	–	-1500	–

Table 3: Laminate reserve factors

Load case	FoS^c	FoS^v	RF	RF^r	RF^c	RF^v	RF^{v+c}
1A	1.0	1.0	1.58	1.41	3.29	1.88	2.21
1B	1.3	1.5	<u>0.97</u>	<u>0.98</u>	2.53	1.25	<u>0.92</u>
2A	1.0	1.0	3.07	1.41	1.19	3.63	1.37
2B	1.3	1.5	1.76	1.09	<u>0.92</u>	2.42	1.09
3	1.0	1.0	2.04	4.06	1.05	1.34	2.43

The effect of constant and variable loads can be illustrated with failure envelopes constructed in the coordinate system defined by the two load vectors. The load components in Load cases 1 and 2 are the same, but the partitioning into constant and variable loads is different. Therefore, the shapes of the envelopes in Figures 1a and 1b are fully different. The envelope for Load case 3 (Figure 3c) is a 'classical' failure envelope, the axes of which represent two load components. Since these are the only load components applied in Load case 3, the constant and variable load vectors can be shown in the plane of the envelope.

In Load cases 2A and 2B, the values of the reserve factor RF are considerably larger than those of RF^r. Thus, there is much more latitude for the increase of the variable loads than could be presumed based on the resultant load method. This information may be important since the variable loads are often of dynamic nature and thus prone to changes during the design process. Load case 3 demonstrates that the resultant load method is not always

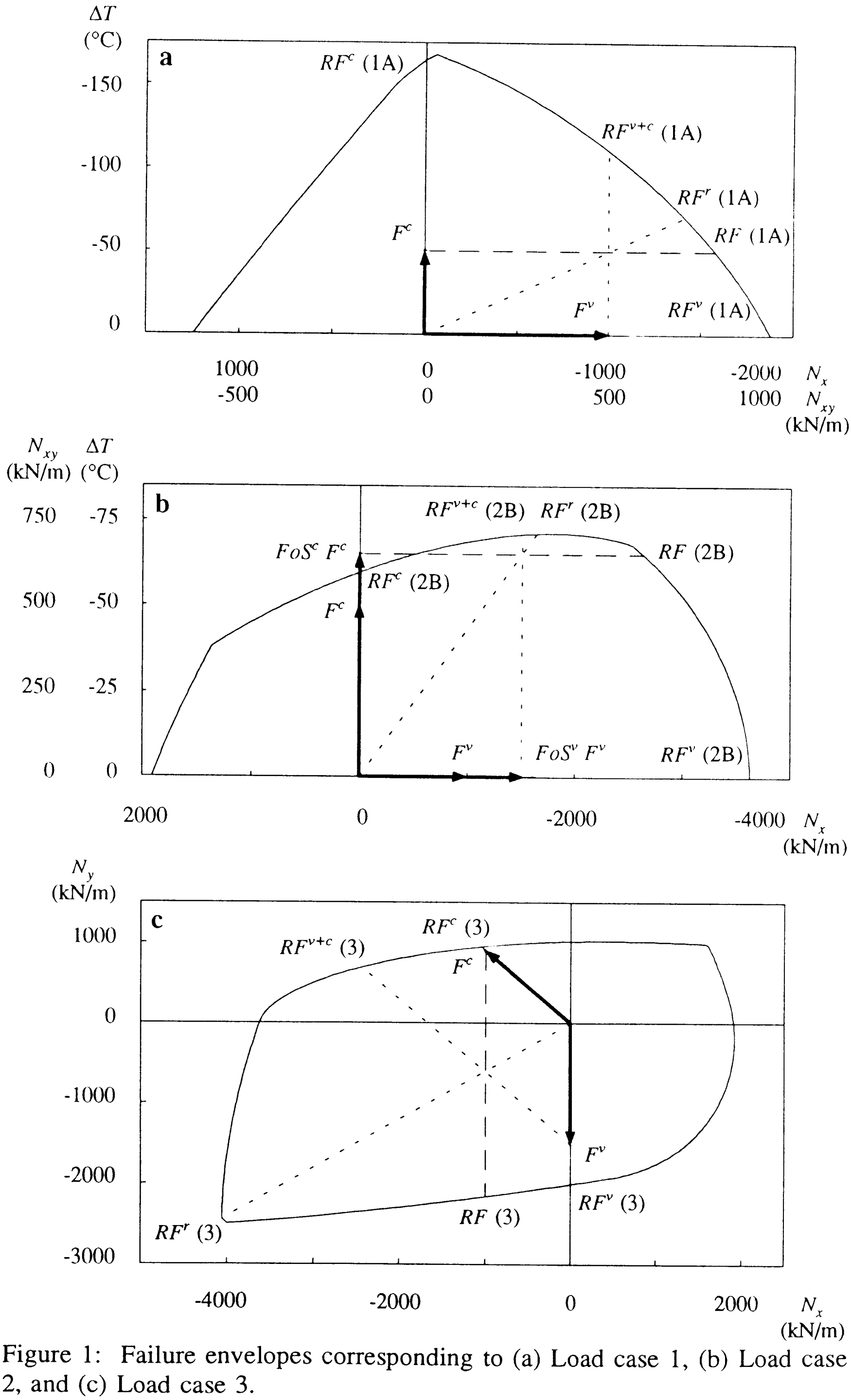

Figure 1: Failure envelopes corresponding to (a) Load case 1, (b) Load case 2, and (c) Load case 3.

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conservative. RF^r indicates over 300% margin of safety, whereas the margin predicted with the constant and variable load approach is approximately 100%.

The importance of considering the reserve factors RF^c and RF^v is also demonstrated by the example load cases. In Load case 2B, the effective constant loading causes failure when applied alone. Still, the combined loading is not critical. For the constant and variable load vectors of Load case 3, the corresponding margins of safety are only 5% and 34%.

CONCLUSIONS

In the computation of reserve factors and margins of safety, the constant and variable load approach leads to results that may be quite different from the results based on resultant loads. The former allows better evaluation of the possible increase of the variable loads. It has been shown that this may result in more stringent limits than those determined with the standard resultant load method, which is not always conservative. The reserve factors RF^c and RF^v may reveal that the effective constant or variable loading applied alone would cause failure. These situations could easily be overlooked with the resultant load approach.

Laminate stress-strain states corresponding to failure obtained with the constant and variable load approach are different from those computed with the resultant load approach. With careful selection of the load vectors, the probable failure mechanisms and critical plies of laminates can also be predicted better. In the design process, laminate structures can therefore be improved with respect to the most likely failure mechanisms.

The ESAComp software will apply the approach presented in this paper to laminate FPF and LPF analyses. To better illustrate the effect of the applied loads, the types of failure envelopes shown in Figure 1 can be constructed with the software. In the future, the approach will further be utilized in nonlinear failure analyses.

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