NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1306

Constants and Related Information for Astrodynamic Calculations, 1968

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CALIFORNIA INSTITUTE OF TECHNOLOGY
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SUBJECT: ERRATA

Gentlemen:

Please note the following corrections and modifications to JPL Technical Report 32-1306, "Constants and Related Information for Astrodynamic Calculations, 1968."

1. On Page 18, last paragraph, line 6:
   ± 0.0002 sec/day^2 should be replaced by
   ± 0.0002 sec/day

2. On Page 21, Table 5:
   Headings in columns 3 and 4 should read
   "Latitude, deg" and "Longitude, deg", respectively.

3. On Page 25, Equation (78):
   The subscripts denote positive or negative rotation about the indicated axis through the angle enclosed in the corresponding parentheses.

4. On Page 29, last paragraph, line 6:
   "Ref. 15, pages 111 and 114" should be replaced by
   "Ref. 16, pages 111-115."

5. On Page 29, Table 20:
   Add the following footnotes:

   [b]The recommended AU value is given in Table 17.
   [c]The recommended value of R_{em} is based on the IAU value of $\sin \pi_{☾}$ given in Table 15 and the JPL value of $a_{☾}$ given in Table 17.

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6. On Page 30, Tables 13:

   The entry under the Venus heading, $n^* = 21\ 06631''.3832 + 0''.00009\ 6T$, should be replaced by $n^* = 21\ 06641''.3832 + 0''.000096\ T$.

   The entry under the Mars heading. $L = 293°44'51''.46 + (54^r + 2\ 22117''.33)T + 1''.1184T^2 \ldots$ should be replaced by $L = 293°44'51''.46 + (53^r + 222117''.33)T + 1''.1188T^2 \ldots$

   The entries under the Jupiter heading, $\tilde{\omega} = 12°42'41''.12 + 5800''.79\ T$ and $M = 136°37'44''.88 + 299''.123557\ d$, should be replaced by $\tilde{\omega} = 12°42'41''.13 + 5800''.79\ T$ and $M = 225°13'17''.70 + 299''.123557\ d$, respectively.

   The entry under the Saturn heading, $M = 174°19'45''.64 + 120''.39926\ d$ should be replaced by $M = 175°28'34''.93 + 120''.399263\ d$.

   The entry under the Uranus heading, $M = 73°35'18''.25 + 42''.2131\ d$ should be replaced by $M = 74°19'18''.90 + 42''.213081\ d$.

   The entries under the Neptune heading, $\Omega = 130°41'43''.27 + 3966''.54\ T$, $\tilde{\omega} = 43°45'49''.24 + 3161''.45\ T$, and $M = 41°16'50''.73 + 21''.3092\ d$ should be replaced by $\Omega = 130°40'43''.28 + 3966''.54\ T$, $\tilde{\omega} = 43°44'49''.24 + 3161''.43\ T$, and $M = 41°16'50''.73 + 21''.582952\ d$, respectively.

7. On Page 32, Table 14, second column:

   Across from Mars I, under second column, (a, Au), the values 0.0627 and 0.1570 should be replaced by 0.0627×10^{-3} and 0.1570×10^{-3}, respectively.

8. On Page 35, Table 16, column 1, line 5:

   (units: $Km^3 s^{-3}$) should be replaced by (units: Km^3/sec^2).

9. On Page 35, Table 17, column 4, line 3:

   Entry under Accuracy, 0.6×10^{-7}, should be deleted entirely.

10. On Page 36, Table 18, column 2, line 4:

    The entry 328900.1 ± 0.3 should be replaced by 328900.1 ± 0.4.

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11. On Page 36, Equation (115):

    $r = \ldots$ should be replaced by $\ddot{r} = \ldots$

12. On Page 36, last paragraph, line 9:

    In $C_1 = 1.04 \times 10^8 km^3/kg/sec^2m^2$, remove slash between km^3 and kg.

13. On Page 50, Reference 54:

    Vol. 154 should be replaced by Vol. 158.

14. On Page 51, Reference 65:

    "The Case for the Radius of Venus" should be replaced by "The Case for the Radar Radius of Venus." Vol. 160 should be replaced by Vol. 161.

Very truly yours,

John Kempton, Manager
Publications Section

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TECHNICAL REPORT 32-1306

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Contents

Contents (contd)

Contents (contd)

Tables

Contents (contd)

Figures

Abstract

The principal constants and related information used in space trajectory and navigation calculations are discussed. The values of these constants presently adopted at JPL are described and estimates of their accuracy are provided.

Constants and Related Information for Astrodynamic Calculations, 1968

I. Introduction

The purpose of this report is to provide a current set of principal constants and related information for use in the generation of space trajectories and orbit determination processes. It is important in astrodynamic calculations to use the most accurate set of constants available and to have, wherever practicable, uniform adoption of this set throughout existing software.

The improvement of the values of astronomical constants is a continuing and significant undertaking at JPL and elsewhere; therefore, in some areas it is difficult to pick a definitive set that holds up. Nevertheless, this report reflects the JPL state of knowledge as of early 1968 and is intended to be a definitive source of this information for current use at JPL. It supersedes Ref. 1 and all previous JPL publications regarding astrodynamic constants. For consistency, the authors recommend that this document be used as a standard throughout NASA. It is JPL's intent to update this document occasionally as significant improvements are established.

Section II treats the earth as the observing platform and discusses those constants which arise within this context. This description includes a consideration of the size, shape, and orientation of the earth, timing, station locations, and atmospheric and ionospheric models. The section also includes a discussion of the gravitational potential of the earth.

Sections III and IV treat the constants pertaining to the finite extent of the moon, the planets, and the sun. Section IV also deals briefly with the known characteristics and orbital elements of the principal planets and planetary satellites.

Section V treats those quantities that are most commonly designated as astronomical constants. This includes a discussion of the IAU system of constants and the JPL currently adopted values. Also included are certain physical constants and solar radiation pressure constants.

Section VI discusses the JPL ephemeris system including a description of the Ephemeris Tape System, current ephemerides available for export, and current developmental ephemerides.

It has been tried, in most cases, to provide a measure of the reliability of the constants adopted at JPL. Where the improved values have resulted from unique investigations (e.g., the determination of the mass of Mars from

Mariner IV), a precision measure is readily available from both formal and empirical error analyses performed by the investigators. In these cases, when a precision estimate is quoted, it may be considered as a standard deviation (1-sigma) that reflects the investigator's consideration of the error structure in his observations and in his mathematical model. For constants which are due to the efforts of multiple investigators spread over time and by diverse techniques, providing error measures is not so straightforward. Where precision estimates are given, they may be interpreted as value judgments on JPL's part unless otherwise stated.

II. Earth

A. External Gravity Potential of the Earth

In 1961, the IAU adopted a standard form for the general case of the expression for the earth's external gravity potential (Ref. 2):

$$U = \frac{GE}{r}\left[1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n P_{nm}(\sin\phi') \{C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\}\right] \tag{1}$$

where

r = radius from center of coordinate system

ϕ' = geocentric latitude

λ = geographic longitude

P_{nm} = associated Legendre functions

GE = geocentric gravitational constant

a_e = equatorial radius

Since the coefficients are obtained from satellite orbit observations, the center of coordinates is taken as the dynamical center of mass of the earth, and it can be shown that, in this case, the first-degree ($n=1$) harmonics are zero. It is often convenient to separate the zonal harmonics, $J_n = -C_{no}$, and, hence, the alternate form:

$$U = \frac{GE}{r}\left[1 - \sum_{n=2}^{\infty}\left(\frac{a_e}{r}\right)^n J_n P_n(\sin\phi') + \sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{a_e}{r}\right)^n P_{nm}(\sin\phi') \{C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\}\right] \tag{2}$$

The set of coefficients adopted is selected from solutions by Kaula (Ref. 3), and King-Hele and Cook (Refs. 4 and 5). Values are adopted only for coefficients where there is reasonable agreement among various authors' solutions. In comparing solutions of Kozai (Ref. 6) with those of King-Hele and Cook, it was decided to adopt zonal harmonic values up to J_7. Values of the even harmonics, J_2, J_4, and J_6 are taken from Ref. 4, and are obtained from the secular regression of the node of 14 earth satellites. Values of the odd harmonics are taken from Ref. 5, and are obtained from secular perturbations of the eccentricity of six earth satellites. The adopted values, in units of the sixth decimal, are:

$$10^6 J_2 = 1082.7 \pm 0.1$$
$$10^6 J_3 = -2.56 \pm 0.1$$
$$10^6 J_4 = -1.58 \pm 0.2$$
$$10^6 J_5 = -0.15 \pm 0.2$$
$$10^6 J_6 = 0.59 \pm 0.2$$
$$10^6 J_7 = -0.44 \pm 0.2$$

The quoted accuracies are not strictly those obtained in the solution statistics, but also include the effect of ignoring higher order harmonics, which is the dominant effect on accuracy.

Strictly, the values of the harmonics depend on the adopted values of GE and a_e; however, the significant figures in the above list are well below the number required to reflect probable changes in GE or a_e, and, therefore, may be considered independent constants. It should also be noted that the value of J_2 is equal to that adopted by the IAU in 1964 (Ref. 7).

Values of the tesseral harmonics are selected from those obtained by Kaula (Ref. 3) from Baker–Nunn camera observations of five satellites. The adopted set is taken through degree 4, the maximum degree for which Kaula's solution is complete. Converting from Kaula's normalized coefficients, $\bar{C}_{nm}$ and $\bar{S}_{nm}$, by[1]

$$C_{nm} = \left[\frac{(n-m)!}{(n+m)!}(2n+1)(2-\delta_{mo})\right]^{1/2}\bar{C}_{nm} \tag{3}$$

where

$$\delta_{mo} = 1 \text{ for } m = 0$$
$$\delta_{mo} = 0 \text{ for } m \neq 0$$

[1]Tables of this factor are given on page 185 of Ref. 17.

Table 1. JPL-adopted tesseral harmonics

n	m	C_{nm} $\times 10^6$	S_{nm}	Kaula σ_{nm}	Kaula-Anderle c/s
2	1	0	0		
2	2	1.57	−0.897	±0.01	−0.01 / 0.084
3	1	2.10	0.16	±0.05	−0.22 / −0.13
3	2	0.25	−0.27	±0.03	−0.08 / 0.04
3	3	0.077	0.173	±0.01	−0.04 / −0.053
4	1	−0.58	−0.46	±0.03	−0.12 / 0.08
4	2	0.074	0.16	±0.01	0.014 / 0.01
4	3	0.053	0.004	±0.004	−0.009 / 0.019
4	4	−0.0065	0.0023	±0.002	0.0022/−0.0049

The adopted set is as presented in Table 1.[2] The standard deviations listed by Kaula do not include the effects of omitting higher harmonics. From comparisons with other solutions for tesseral harmonics (e.g., Refs. 8 through 10), it would appear that a more meaningful accuracy is given by the difference between Kaula's results and other recent results, such as Anderle's solution from doppler data (Ref. 9). These differences are given in Table 1.

In Ref. 11, Kaula lists several sets of tesserals up to degree 6 (page 115) and zonals up to degree 14 (page 117), which may be used if degrees higher than those adopted are desired. A complete set of tesserals up to degree 8 is also given in Ref. 8. The coefficients of Guier and Newton are related to the adopted set by:

$$C_{nm}, S_{nm} = \left[\frac{(n-m)!}{(n+m)!}\right]^{1/2} A_{nm}, B_{nm} \tag{4}$$

It should be pointed out, however, that the values of the higher degree coefficients are very uncertain, even as to the sign of the value.

In a recent publication (Ref. 12), Kaula comments that the current best single set from satellite tracking is by Gaposhkin (Ref. 13). He also derives an average set that includes data from gravimetric measurements, which he claims is the best choice. All three sets agree very closely, and it was decided to adopt the more fully documented set in Ref. 3. For comparison, the Gaposhkin values and Kaula's averaged values are presented in Table 2.

[2]Since the earth pole is very nearly a principal axis of inertia, C_{21} and S_{21} are very nearly zero, and are adopted as such. Actually, the values of C_{21}, S_{21}, and all other harmonic coefficients, vary slightly with time because of variations in the pole location (Fig. 2).

Table 2. Alternate tesseral harmonics

n	m	Gaposhkin (Ref. 13) $C_{nm} \times 10^6$	Gaposhkin (Ref. 13) S_{nm}	Kaula Average (Ref. 12) $C_{nm} \times 10^6$	Kaula Average (Ref. 12) S_{nm}
2	2	1.54	−0.871	1.56	−0.878
3	1	2.10	0.29	1.93	0.19
3	2	0.25	−0.18	0.27	−0.26
3	3	0.078	0.226	0.079	0.198
4	1	−0.54	−0.45	−0.53	−0.44
4	2	0.074	0.15	0.067	0.13
4	3	0.051	−0.011	0.055	−0.011
4	4	−0.001	0.0049	−0.001	0.0068

B. Figure of the Earth

The constants of the earth's figure are divided into two categories: primary and derived. Primary constants are those which are directly observed or computed in terms of observations, and are independent from one another. In this report, they are:

(1) a_e, equatorial radius of earth's reference ellipsoid.

(2) GE, geocentric gravitational constant.

(3) J_n, C_{nm}, and S_{nm}, coefficients of the earth's external gravity potential in terms of spherical harmonics (see Subsection II-A).

Derived constants are those which are computed in terms of the primary constants. Although some of these may be observable, the computed values are adopted for consistency. In this report, they are as follows:

(1) f, flattening of the earth's reference ellipsoid.

(2) b, polar radius of the earth's reference ellipsoid.

(3) R_M, mean radius of the earth's reference ellipsoid.

(4) g_e, equatorial surface gravity acceleration.

(5) J_4^*, fourth zonal harmonic of earth's reference ellipsoid.

(6) ω, sidereal rotation rate of the earth.

In addition, useful formulae are presented for several parameters of the earth's figure.

The source for each adopted primary constant is given, along with the source values for the standard deviation.

References are given for the derivation of equations relating the derived constants to the primary constants. Standard deviations of the derived constants are computed in terms of those for the primary constants.

*1. **Definitions.*** The surface of the earth has many definitions. For most geodetic purposes, it is defined as that surface coinciding with mean sea level, and is called the *geoid.* Where land masses occur, the geoid is defined as the surface resulting in imagined narrow ditches connected to the oceans. The mean surface of the free oceans is an equipotential surface resulting from the gravitational and rotational potential:

$$W = U + \frac{1}{2}\,\omega^2 r^2 \cos^2 \phi' \tag{5}$$

The geoid is very closely approximated by an ellipsoid of revolution. It is convenient to define a reference ellipsoid for the earth, and to speak of departures of the geoid from the reference ellipsoid.

An ellipsoid of revolution can be defined in terms of GE, a_e, and f

where

$$f = \frac{a_e - b}{a_e}$$

$$b = \text{polar radius}$$

The gravity constant, GE, is observed from satellites and space probes and is a primary constant. The equatorial radius, a_e, is a primary constant adopted such that the geoid has minimum departures from the reference ellipsoid. The flattening, f, is a derived constant whose value must be such that the potential of the reference ellipsoid agrees with that of the geoid.

The potential of an ellipsoid of revolution may be expressed in terms of even zonal harmonics. In the form of the geoid potential above:

$$U^* = \frac{GE}{r}\left[1 - J_2^*\left(\frac{a_e}{r}\right)^2 P_2(\sin\phi') - J_4^*\left(\frac{a_e}{r}\right)^4 \times P_4(\sin\phi')\cdots\right] + \frac{1}{3}\,\frac{g_e m}{a_e}\,r^2(1 - P_2) \tag{6}$$

where

$$m = \frac{\omega^2 a_e}{g_e} = 0(f)$$

The radius of an ellipsoid is given by

$$r = a_e\left[1 - \left(f + \frac{3}{2}f^2\cdots\right)\sin^2\phi' + \frac{3}{2}f^2\sin^4\phi'\cdots\right] \tag{7}$$

where terms of $0(f^3)$ are dropped. Substituting into U^*, the resulting form is (see Ref. 11):

$$U^* = U_0 + U_2 P_2 + U_4 P_4 + 0(f^3) \tag{8}$$

Since U^* is constant ($= U_0$), the following must be true:

$$U_2 = 0$$

$$U_4 = 0$$

From the first of these, $U_2 = 0$, is obtained the defining equation for the flattening:

$$J_2^* = \frac{2}{3}f - \frac{1}{3}f^2 - \frac{1}{3}m + \frac{1}{2}m^2 + \frac{2}{21}mf + 0(f^3) \tag{9}$$

where J_2^* is taken equal to the observed coefficient in the external gravity field and is called "dynamical form factor," a primary constant (Ref. 7). Solving for f explicitly,

$$f = \frac{3}{2}J_2 + \frac{1}{2}m + \frac{9}{8}J_2^2 + \frac{15}{28}J_2 m - \frac{39}{56}m^2 \tag{10}$$

From $U_4 = 0$ is obtained:

$$J_4^* = -\frac{4}{5}f^2 + \frac{4}{7}mf + 0(f^3) \tag{11}$$

The value of J_4^* is generally different from the observed value of J_4. This difference enters into the computation of geoid heights, discussed below.

An important derived constant is the equatorial gravity, g_e. From the definition:

$$g_e = -\left.\frac{\partial U^*}{\partial r}\right|_{r=a_e} = \frac{GE}{a_e^2}\left(1 + \frac{3}{2}J_2 - \frac{15}{8}J_4^*\right) + g_e m \tag{12}$$

or, finally

$$g_e = \frac{GE}{a_e^2}\left(1 - \mu_a + \frac{3}{2}J_2 - m + \frac{27}{8}J_2^2 - \frac{6}{7}J_2 m + \frac{47}{56}m^2\right) \tag{13}$$

The term μ_a is a small correction introduced to account for the effect of the atmosphere above the surface

$$\mu_a = 0.000001$$

It should be noted that, since m is a function of g_e, Eq. (13) must be solved iteratively. Also, it should be pointed out that m is defined slightly differently in some other works (Refs. 14 and 15), and some caution must be exercised in comparing results.

The mean radius, as defined here, is an average over the area of a sphere

$$R_M = \frac{1}{A}\int_{\text{sphere}} r dA \tag{14}$$

which finally becomes

$$R_M = a_e(1 - f/3 - f^2/5) \tag{15}$$

2. ***Adopted primary constants.*** The adopted values for GE, a_e, and J_2 are given in Section V.

3. ***Derived constants.*** From the defining equations above:

$$f = \frac{1}{298.250} \pm 0.15 \times 10^{-6}$$

$$g_e = 978.0264 \pm 0.002 \text{ cm/s}^2$$

$$10^6 J_4^* = -2.35 \pm 0.003$$

$$b = 6356775 \pm 5 \text{ m}$$

$$R_M = 6371017 \pm 5 \text{ m}$$

The inertial rotation rate of the earth, ω, may be computed from the adopted number of seconds (s) in the tropical year at 1900.0 and the annual rate of precession in right ascension (m), as follows:

$$\omega = \frac{360}{86400} + \frac{360}{s} - \frac{m}{240\, s} \text{ deg/sec} \tag{16}$$

$$s = 31556925.9747 \text{ sec (Ref. 2, page 594, and Ref. 16, page 69)} \tag{17}$$

$$m = 3.07234 \text{ s/tropical year (Ref. 16, page 38)} \tag{18}$$

Then (Ref. 16, page 76),

$$\omega = 0.004178074216 \text{ deg/sec}$$
$$= 15.04106718 \text{ arc sec/sec or deg/h} \tag{19}$$

In the above expression for ω, the three terms are, respectively:

(1) The rate of rotation of earth with respect to mean sun.

(2) The rate of revolution of mean sun with respect to mean equinox.

(3) The motion of the mean equinox along the celestial equator with respect to inertial space.

Although the average length of a day is increasing due to tidal friction at an estimated rate of approximately 1 sec per 100,000 years, this secular increase is too small to be of consequence in this section's calculations for which a value of ω is needed (Refer to Sections II-E and II-F on Time for a discussion of the effects of the irregular variations in the day).

4. ***Useful formulae.*** It is often useful to have expressions for latitude-dependent parameters in terms of series expansions, such as on page 58 of Ref. 16. Most such expressions found in the literature, however, are in terms of the geodetic latitude, ϕ, whereas in astrodynamics, the more fundamental variable is geocentric latitude, ϕ'. Following are some useful formulae, derived in terms of the geocentric latitude, along with the numerical values of the coefficients, as computed from the adopted value of f.

a. Radius on the ellipsoid.

$$r = a_e\left[1 - \frac{f}{2} - \frac{3}{16}f^2 + \frac{f}{2}\cos 2\phi' + \frac{3}{16}f^2 \cos 4\phi'\right]$$

$$= a_e\,[0.99832144 + 0.00167645 \cos 2\phi' + 0.00000211 \cos 4\phi'] \tag{20}$$

b. Normal gravity on surface.[3]

$$g_0 = g_e \left[1 + \left(\frac{5}{2} m - f - \frac{17}{14} mf \right) \sin^2 \phi' + \left(\frac{15}{8} mf - \frac{7}{8} f^2 \right) \sin^2 2\phi' \right]$$
$$= g_e \left[1 + 0.00530244 \sin^2 \phi' + 0.00001196 \sin^2 2\phi' \right] \quad (21)$$

c. Geodetic latitude on surface.

$$\phi - \phi' = \left(f + \frac{f^2}{2} \right) \sin 2\phi' + \frac{f^2}{2} \sin 4\phi'$$
$$= 692''.744 \sin 2\phi' + 1''.159 \sin 4\phi' \quad (22)$$

For a reasonable height, h, above the reference ellipsoid, the geocentric radius is approximately increased by h, and $\phi - \phi'$ remains approximately unchanged (Ref. 16). Exact relationships can be obtained from the relations (Ref. 16):

$$r \cos \phi' = (C + h) \cos \phi \quad (23)$$

$$r \sin \phi' = (S + h) \sin \phi \quad (24)$$

where

$$C = a \left\{ \cos^2 \phi + (1 - f)^2 \sin^2 \phi \right\}^{-1/2}$$
$$S = C (1 - f)^2$$

It should be noted that, in Eqs. (23) and (24), h is height above the reference ellipsoid. To obtain height above mean sea level (height above geoid), h is reduced by the geoid height (height of geoid above reference ellipsoid).

5. *Gravity anomalies and geoid heights.* The "actual" surface of the earth, as represented by the geoid, has small differences from the reference ellipsoid. A complete discussion of these differences is beyond the scope and intent of this document. Two aspects, however, are interesting and fundamental: (1) the difference between the surface gravity on the geoid and that on the ellipsoid; and (2) the height of the geoid above the ellipsoid. These quantities are derived from the disturbing potential, i.e., the difference between the geoid potential and that of the reference ellipsoid:

$$T = W - U^*$$
$$= \frac{GE}{r} \left\{ - J_3 \left(\frac{a_e}{r} \right)^3 P_3 - (J_4 - J_4^*) \left(\frac{a_e}{r} \right)^4 P_4 - \sum_{n=5}^{\infty} J_n \left(\frac{a_e}{r} \right)^n P_n + \sum_{n=2}^{\infty} \sum_{m=2}^{n} \left(\frac{a_e}{r} \right)^n \times P_{nm} \left[C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right] \right\} \quad (25)$$

From Ref. 17, the gravity on the geoid minus the gravity on the ellipsoid, Δg, is:

$$\Delta g = g - g_0$$
$$= - \left(\frac{\partial T}{\partial r} + \frac{2T}{r} \right)$$
$$= \frac{GE}{a_e^2} \left\{ - 2J_3 \left(\frac{a_e}{r} \right)^5 P_3 - 3 (J_4 - J_4^*) \left(\frac{a_e}{r} \right)^6 P_4 - \sum_{n=5}^{\infty} (n-1) J_n \left(\frac{a_e}{r} \right)^{n+2} P_n + \sum_{n=2}^{\infty} \sum_{m=2}^{n} (n-1) \left(\frac{a_e}{r} \right)^{n+2} P_{nm} \left[C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right] \right. \quad (26)$$

To order f^2

$$\Delta g = g_e \Sigma^* (n - 1) P_{nm} \left[C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right] \quad (27)$$

[3]The expression for the normal surface gravity contains a term due to a component normal to the radius, i.e.,

$$g_0^2 = \left(\frac{\partial U^*}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial U^*}{\partial \phi'} \right)^2$$

The expression differs from that found in Jeffrey's book (Ref. 14, page 137) because of the previously mentioned, slightly different definition of m.

where Σ^* denotes the summation obvious from inspection of the above and is carried to as high an order as is consistent with accuracy of order f^2.

If N denotes the height of the geoid above the ellipsoid, then the disturbing potential may be expressed:

$$T = \int g dN \tag{28}$$

To a good approximation

$$N = \frac{T}{g_0} \tag{29}$$

and to $0(f^2)$:

$$N = a_e \Sigma^* P_{nm} [C_{nm} \cos m\lambda + S_{nm} \sin m\lambda] \tag{30}$$

From the above expressions, contours of constant Δg and N may be calculated and plotted on maps. Examples of these plots may be found in Refs. 3, 8, 11, and 14.

C. Orientation of the Axis of Rotation

The direction of the rotational axis of earth is not fixed in space. The action of sun and moon on the equatorial bulge causes a variation in the orientation of the equatorial plane, while the perturbative effects of the planets produce a variation in the orientation of the ecliptic. Once a fundamental inertial reference system is specified, it would be sufficient to tabulate the direction cosines of the rotational axis with respect to the coordinate axes. The problem is not treated this way because of historical and practical reasons.

In practice, the motions of the ecliptic and equator are both explicitly computed as a matter of observational necessity (see Ref. 18 for detailed discussions of these motions). Furthermore, the long-term motions that can be treated as though they are secular (precession) are separated from the short-periodic motions (nutation). The fictitious equator, ecliptic, and equinox defined as being represented by the precessional motions only are called *mean*, while those affected by both precession and nutation are called *true*. Values fixed at the time corresponding to a fundamental reference are values *at the epoch*, while those referring to instantaneous moments are the values *of date*.

1. Precession. The mean equator of date is referred to the mean equator at the epoch t_0 by the equatorial precession elements ζ_0, Z, θ (Fig. 1). Although usually described in different terms, these quantities may be defined as:

$-\zeta_0 =$ right ascension of the mean celestial pole of date, referred to the mean equator and equinox at the epoch.

$180° + Z =$ right ascension of the mean celestial pole at the epoch, referred to the mean equator and equinox of date.

$\theta = 90° -$ declination (i.e., north polar distance) of the mean celestial pole of date, referred to the mean equator at the epoch.

The mean obliquity $\bar{\epsilon}$ is the inclination of the ecliptic on the mean equator, and represents a simple rotation about the x-axis. This quantity also contains a component due to precession.

a. Application. To transform rectangular equatorial coordinates referred to the mean equator and equinox at t_0 to coordinates referred to the mean equator and equinox at time t, the following relation should be used:

$$\mathbf{r}_q = [P]\,\mathbf{r}_0 \tag{31}$$

where $\mathbf{r}_0$ is the position vector referred to mean equator and equinox at the epoch, $\mathbf{r}_q$ is referred to the mean equator and equinox of date, and (Refs. 16 and 18):

$$\begin{aligned}
P_{11} &= \cos\zeta_0 \cos\theta \cos Z - \sin\zeta_0 \sin Z \\
P_{12} &= -\sin\zeta_0 \cos\theta \cos Z - \cos\zeta_0 \sin Z \\
P_{13} &= -\sin\theta \cos Z \\
P_{21} &= \cos\zeta_0 \cos\theta \sin Z + \sin\zeta_0 \cos Z \\
P_{22} &= -\sin\zeta_0 \cos\theta \sin Z + \cos\zeta_0 \cos Z \\
P_{23} &= -\sin\theta \sin Z \\
P_{31} &= \cos\zeta_0 \sin\theta \\
P_{32} &= -\sin\zeta_0 \sin\theta \\
P_{33} &= \cos\theta
\end{aligned} \tag{32}$$

Spherical equatorial coordinates will be transformed by the relations

$$\begin{aligned}
\cos\delta \sin(\alpha - Z) &= \cos\delta_0 \sin(\alpha_0 + \zeta_0) \\
\cos\delta \cos(\alpha - Z) &= \cos\theta \cos\delta_0 \cos(\alpha_0 + \zeta_0) - \sin\theta \sin\delta_0 \\
\sin\delta &= \sin\theta \cos\delta_0 \cos(\alpha_0 + \zeta_0) + \cos\theta \sin\delta_0
\end{aligned} \tag{33}$$

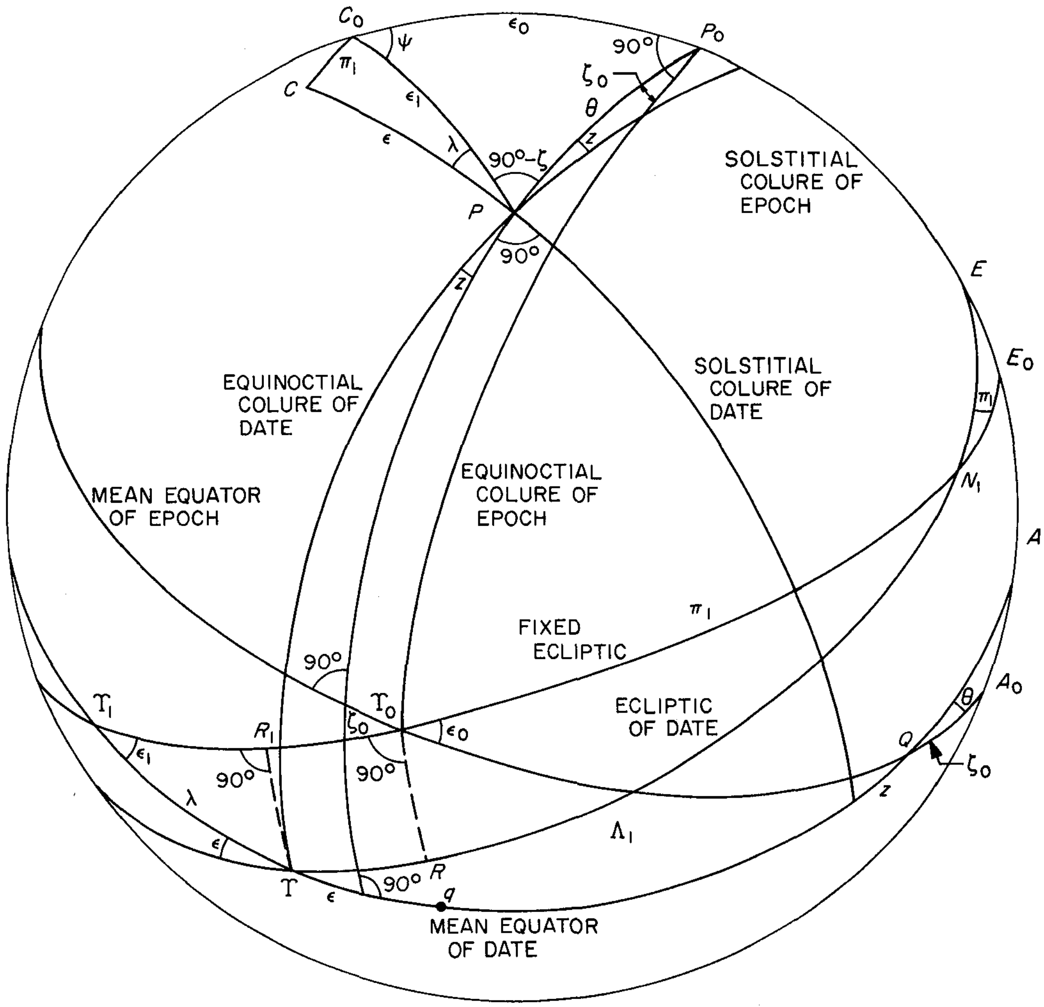

Fig. 1. Precession geometry

It is recommended that precession be applied in this manner. The usual formulae for applying precession directly to the ecliptic coordinates, as well as those popularly used for precessing equatorial coordinates (e.g., Ref. 16, page 38), are first-order approximations derived from the relations given above; use of such approximations can lead to errors of:

$$|\Delta\alpha| \le 0''.2 + 0''.4 \tan^2 \delta_0$$

$$|\Delta\delta| \le 0''.4 \tan \delta_0$$

in the application of precession from the epoch 1950.0 to coordinates of date in 1970.

Mean ecliptic rectangular coordinates of date $\mathbf{r}_c$ may be obtained from mean equatorial coordinates of date by the rotation

$$\mathbf{r}_c = [A(\bar{\epsilon})]\,\mathbf{r}_q \tag{34}$$

where

$$[A(\bar{\epsilon})] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\bar{\epsilon} & \sin\bar{\epsilon} \\ 0 & -\sin\bar{\epsilon} & \cos\bar{\epsilon} \end{bmatrix}$$

The corresponding transformation in spherical coordinates is

$$\begin{Bmatrix} \cos\beta\cos\lambda \\ \cos\beta\sin\lambda \\ \sin\beta \end{Bmatrix} = [A(\bar{\epsilon})] \begin{Bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{Bmatrix} \tag{35}$$

b. Numerical values of precession quantities. The standard expressions for the values of the precession parameters are those of Newcomb, referred to the epoch of 1900 Jan 0.5. In spacecraft trajectory work, however, as in most astronomical research, the standard epoch to which coordinates are referred is that of 1950.0. The expressions given below are evaluations of general relations given by Lieske (Ref. 19). They correspond to Newcomb's expressions referred to the epoch 1950.0, with T denoting the time elapsed from 1950.0 to date, expressed in tropical centuries as follows:

$$\begin{aligned}
\zeta_0 &= 2304''.952\,T + 0''.3022\,T^2 + 0''.0180\,T^3 \\
Z &= 2304.952\,T + 1.0951\,T^2 + 0.0183\,T^3 \\
\theta &= 2004.257\,T - 0.4268\,T^2 - 0.0418\,T^3 \\
\Delta\bar{\epsilon} &= -46.850\,T - 0.0034\,T^2 + 0.0018\,T^3 \\
\bar{\epsilon} &= 23°\,26'\,44''.84 + \Delta\bar{\epsilon}
\end{aligned} \tag{36}$$

2. *Nutation.* Nutation represents the difference between the position of the true celestial pole (rotational axis of earth) and the mean celestial pole. It is entirely composed of the short-period effects due to the action of sun and moon on the figure of the earth, and thus affects only the equatorial plane, not the ecliptic. For this reason, it is most convenient to apply the nutation to ecliptic coordinates, in which the vernal equinox is shifted from its mean position in the mean ecliptic of date to its true position, which is in this same plane. That is, the true ecliptic of date is the mean ecliptic of date also. The true equator of date differs from the mean equator of date by two increments:

$\delta\psi$ = nutation in longitude, which is the true longitude of date of the mean equinox of date

$\delta\epsilon$ = nutation in obliquity

These two quantities, their rates, and their second and fourth modified differences, are tabulated at half-day steps in the JPL Ephemeris Tapes. The values given there are referred to the ecliptic of date and are computed from Woolard's expressions (Ref. 20).

a. Application. A rigorous application of nutation may be obtained for ecliptic coordinates by

$$\mathbf{r}_{te} = [C\,(\delta\psi)]\,\mathbf{r}_c \tag{37}$$

where $\mathbf{r}_{te}$ is the position vector referred to the true equinox and ecliptic of date,
and

$$[C\,(\delta\psi)] = \begin{bmatrix} \cos\delta\psi & -\sin\delta\psi & 0 \\ \sin\delta\psi & \cos\delta\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{38}$$

True equatorial coordinates of date $\mathbf{r}_{tq}$ can be recovered by the transformation

$$\mathbf{r}_{tq} = [A\,(\epsilon)]^T\,\mathbf{r}_{te} \tag{39}$$

where $A(\epsilon)$ is the rotation matrix defined in Eq. (34) of Section (II-C-1a) and

$$\epsilon = \bar{\epsilon} + \delta\epsilon \tag{40}$$

Thus, the entire transformation from mean equatorial coordinates of date to true equatorial coordinates of date is given by

$$\mathbf{r}_{tq} = [A\,(\epsilon)]^T\,[C\,(\delta\psi)]\,[A\,(\bar{\epsilon})]\,\mathbf{r}_q \tag{41}$$

The use of first-order approximate formulae for the application of nutation neglects quantities of the order of one part in 10^8 (0''.002), so their use is inconsistent with double-precision computation.

3. *Uncertainty estimates.* The general precession in longitude is an observationally determined quantity that is related rather directly to the precession elements discussed above. Its conventional value is that of Newcomb for the epoch 1900 Jan 0

$$p = 5025''.64$$

It is now known that this value is too low, but it is retained for reasons of continuity over extended time intervals. The apparent uncertainty is (Ref. 7):

$$e(p) = \begin{Bmatrix} +\,1''.3 \\ -\;\;0 \end{Bmatrix} \tag{42}$$

This upper value of + 1''.3 corresponds to uncertainties in the precession elements of

$$\begin{aligned}
e(\zeta_0) &= 0''.60\,T \\
e(Z) &= +0.60\,T + 0.0004\,T^2 \\
e(\theta) &= +0.52\,T
\end{aligned} \tag{43}$$

where T is measured in tropical centuries from 1950.0.

Other factors contribute to uncertainties in these quantities, but are of much lower significance.

The value of the mean obliquity at some specified epoch is also determined from observation. The conventional epoch for this determination is that of 1900 Jan 0; the uncertainty in that value (Ref. 7) is $\pm 0''.10$. Duncombe (Ref. 21) indicates a correction of $-0.29\ T$ to the rate term of the obliquity. Combining these two, it is estimated the uncertainty in the value of the mean obliquity is

$$e(\bar{\epsilon}) = \pm\ 0''.3 - 0''.3\ T \tag{44}$$

The nutations are computed from series expressions that are derived using Newcomb's theory of the earth and Brown's Lunar Theory. One number is determined from observation, the "constant of nutation" N. It corresponds to the value at 1900 of the coefficient of cos Ω in the series for $\delta\epsilon$. The conventional value of N, along with its estimated uncertainty, is (Ref. 7):

$$N = 9''.210 \pm 0.01 \tag{45}$$

The values of all other coefficients, both in $\delta\epsilon$ and $\delta\psi$, are specified by N and the theory. The Theory assumes a rigid earth, and it has been suggested (Ref. 22) that the coefficient of sin Ω in the series for $\delta\psi$ should also be determined through observation. No estimate can be made of the uncertainty involved in this theoretical defect. Recent corrections to the Lunar Theory will affect the nutations only in the fourth decimal of a second of arc.

In this discussion, only the spatial orientation of the spin axis of earth was of concern. The topics concerning the location of stations on the earth's surface relative to the spin axis are discussed in Sections II-D and II-G.

D. Polar Motion[4]

The earth's axis of figure is not coincident with the axis of rotation; it moves with respect to the latter, causing the so-called polar motion. The precision with which JPL is seeking to evaluate DSN station locations requires consideration of this polar motion and its effects.

1. *Definition of terms and coordinate systems.*

a. Spin axis. This term refers to the axis of the earth's rotation. Free of gravitational and impulsive forces, the spin axis would remain fixed in inertial space. In reality, there are precession and nutation motions (see Section II-C for a discussion of precession and nutation).

b. Polar motion. The polar motion is a motion of the earth's principal moment of inertia axis with respect to its axis of rotation. This manifests itself as a change in the point where the spin axis pierces the earth's crust. It is best to think of the polar motion as a motion or displacement of the earth's crust with respect to the spin axis. The polar motion is not a movement of the spin axis with respect to inertial space.

c. International Polar Motion Service. The International Polar Motion Service (IPMS) has the task of observing and documenting the polar motion. This international organization was initiated in September 1889 with the construction of five observatories, all of which were located at the same north latitude and spaced around the earth. The observatories are located at Mizusawa, Japan (the current administrative center); at Ukiah, Calif.; at Gaithersburg, Md.; at Carloforte, Italy; and at Kitab, U.S.S.R. The IPMS issues yearly reports (Ref. 23) and monthly bulletins containing observations, results, and all other pertinent information and equations, and also assists scientists in obtaining their publications.[5]

d. Polar coordinates. This is the standard system adopted by the IPMS for representing the polar motion. The system consists of a rectangular coordinate grid (X, Y) centered on the origin defined below. The X-Y plane is tangent to the earth at the north pole, with $+X$ along the Greenwich meridian, and $+Y$ along the 90 west-longitude meridian. This coordinate system is used in Fig. 2.

e. Mean pole 1903.0. The mean pole 1903.0[6] refers to a method of data reduction and the resulting origin for the polar coordinates. Reference 23 documents the equations currently used by IPMS to reduce observations. Since other methods may be, and have been, used in this connection, results based on this system are denoted "new system." The origin of polar coordinates is uniquely defined by the new system program.

[4]This section was written by Paul M. Muller, Systems Analysis Research Section, JPL.

[5]Up-to-date data can be obtained by writing to Dr. Shigeru Yumi, Director, IPMS Observatory, Mizusawa, Iwate-Ken, Japan.

[6]Recently named mean pole 1903.0 at the Stresa Symposium, March 1967, but previously known as New System 1900 (i.e., $X_{\text{mean pole 1903.0}} - X_{\text{new system pole 1900}} = 0$, $Y_{\text{mean pole 1903.0}} - Y_{\text{new system pole 1900}} = 0$).

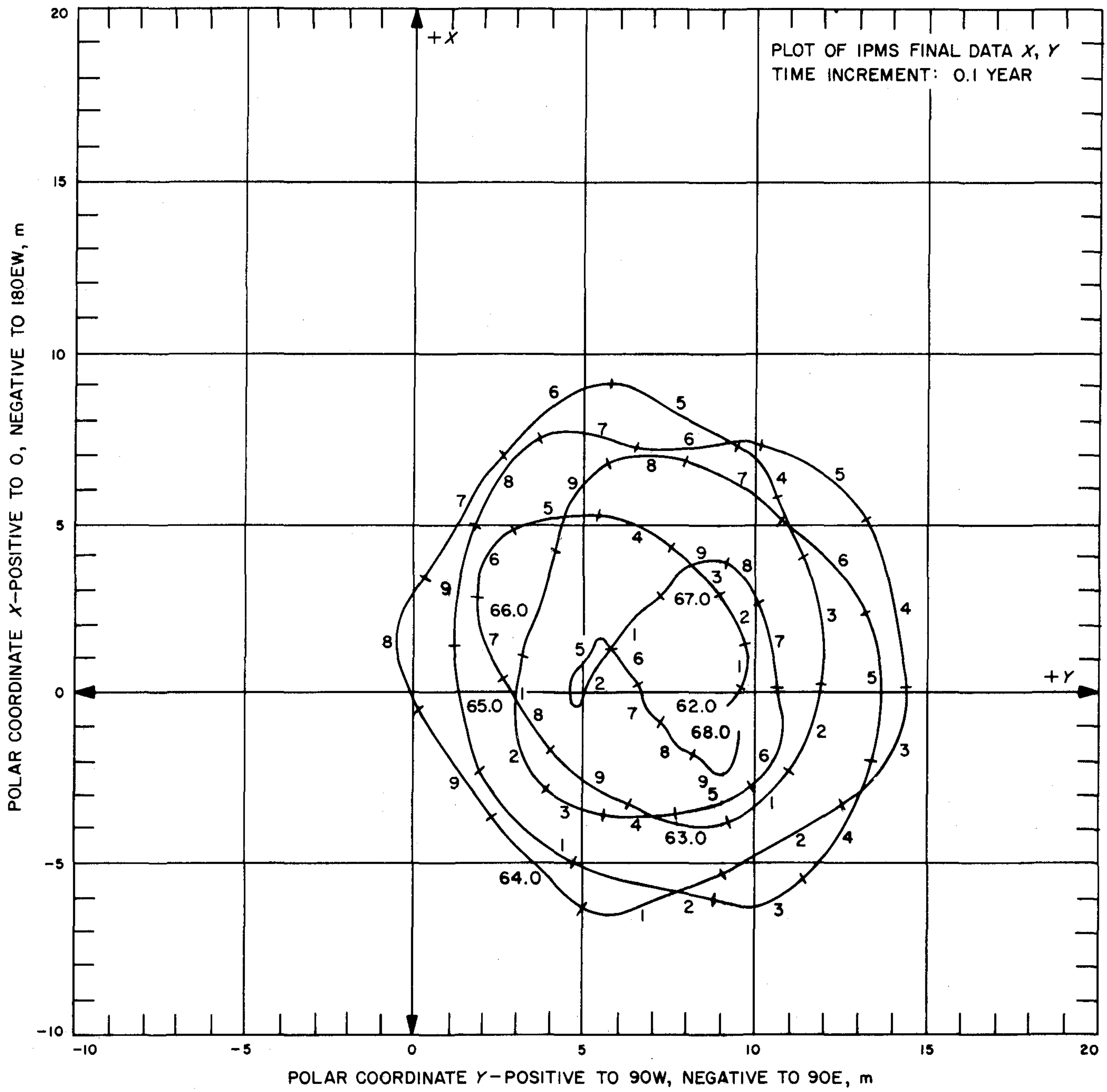

Fig. 2. Polar orbit plot of X vs Y for the period 1962–1968

f. Old system 1900. This system was the program standard of IPMS prior to replacement by the new system in 1955. The origins and results of the two are distinct, and there is no simple relationship between them. To obtain results in a given system, the observations must be processed according to the methods that define it.

g. Local vertical. As defined by astronomers, the local vertical is a line perpendicular to the plane determined by local gravity. Because of gravitational anomalies, the local vertical line does not, in general, pass through the spin axis.

h. Astronomical latitude. This is the complement of the angle between the local vertical and a line through the observatory parallel to the spin axis. This quantity is definitely a function both of the observer's location and of the polar motion. As will be shown, variations in this observed quantity can be related to polar motion.

i. Continental drift and crustal slippage. These motions have effects on station locations. The continental drift is a matter of controversy; however, most observers believe that it is small, bounded by approximately 0.1 m/year. Crustal slippage due to earthquakes and faults has been observed; it amounts to 2 m/50 years (Ref. 24, page 327, and bibliography from same). These small motions will be neglected in the current discussion.

j. Earth-fixed system coordinates. A system of crust-fixed coordinates has been adopted as a reference for this report. Let the pole be at the origin for the mean pole 1903.0. The equator follows from the pole. Choice of the Greenwich meridian is arbitrary, but, once chosen, remains fixed and determines earth-fixed system (EFS) station longitudes and the polar coordinates. This is a system of invariant coordinates for any station anchored to the crust, neglecting crustal slippage. In any comparison of station-location solutions, such as are obtained from orbit determination, time-invariant station coordinates should be used.

k. Instantaneous coordinates. Let the pole be at the location (X,Y) given by the IPMS as the current location. The equator is defined from the pole. Consider a small rotation from EFS to the instantaneous (INS) coordinate system. Rotate through Y first, followed by X, which is an arbitrary choice in small transformations. The INS Greenwich meridian resulting from the change of coordinates still passes through the intersection of the EFS equator and EFS Greenwich meridian. Therefore, the INS Greenwich meridians all pass through this EFS intersection point. Other definitions of the INS Greenwich meridian would be possible; however, they could not be consistent with Eqs. (46) through (48).

l. Longitude, latitude, and r_s. Longitude is measured eastward, from 0 to 360 deg; latitude is measured positive north and negative south of the equator; and r_s is the distance from the spin axis, and is the length of the perpendicular through the station. These may be given in either the EFS or INS system, as required below.

2. *Observation and determination of polar motion.* The early background of the IPMS effort and the theory behind it are discussed in Refs. 25 and 26. Polar motion is observed indirectly through determination of the variations in latitude of the five IPMS observatories. If the nominal station location of any observatory or DSN station in EFS coordinates is taken, and then its location in the INS system is observed, there will be a difference. This is clearly a coordinate difference, for the station has not moved with respect to the earth. Equations (46) through (48) give INS-EFS coordinate differences for latitude, longitude, and r_s:

$$\Delta\lambda = \lambda_{\mathrm{INS}} - \lambda_{\mathrm{EFS}} = \tan\phi\,(X\sin\lambda + Y\cos\lambda) \qquad (46)$$

$$\Delta\phi = \phi_{\mathrm{INS}} - \phi_{\mathrm{EFS}} = X\cos\lambda - Y\sin\lambda \qquad (47)$$

$$\Delta r_s = -\Delta\phi \cdot \sin\phi \qquad (48)$$

where

$X, Y =$ polar coordinates

$\phi, \lambda =$ station latitude and longitude, respectively, correct to three significant figures (commensurate with X and Y).

It should be noted that $\Delta\lambda$ is zero for stations on the equator. This is a consequence of the definition for the INS Greenwich meridian, which requires that it always pass through a fixed point on the equator. Other requirements could be made; however, first-order changes in the equations would result.

Equation (47) is the fundamental equation used by the IPMS for variation in latitude. The same set of 72 star pairs are observed with vertical zenith telescopes at each observatory. Equation (49) is filled out for an observation from each station, creating five simultaneous equations for each star-pair observed as follows:

$$\phi_{\mathrm{observed}} - \phi_{\mathrm{nominal}} = X\cos\lambda - Y\sin\lambda + Z \qquad (49)$$

The variable Z is the change introduced by any star catalog (or proper-motion) errors in the adopted declinations. With at least three stations observing the same pair, values for X, Y, and Z, may be obtained independently. If the catalog declination errors could not be removed in this fashion, they would swamp-out the very small latitude variations caused by polar motion.

The use of five stations allows for bad weather, and allows possible data-consistency checks. Such checks (Ref. 23) have revealed good agreement. The IPMS values of X and Y are correct to the order of 0.5 m. The method of observing each star pair, including reversal of the instrument between observations, eliminates or reduces such instrumental errors as flexure, axis orientation, etc.

3. Observed nature of the polar motion. Figure 2 is a plot of IPMS data, X versus Y, for the period up to 1968. These so-called polar orbit plots demonstrate the elliptical nature of the motion, and show the period of approximately 405 days. This motion has been represented as the sum of two basic terms, called the annual and Chandler motion (see for example Ref. 27, Fig. 7.4). Their periods differ, and they repeat phase relationships approximately every 6.5 years. This beat phenomenon results in a phase-cancelling in 1961 and in 1967, as manifested by a very small amplitude of motion in the six-month semi-period. The variation in amplitude over one-half revolution ranges from 1 to more than 20 m (compare 1964 with 1967 in Fig. 2, for example). It should be noted that the ellipses are not centered at the

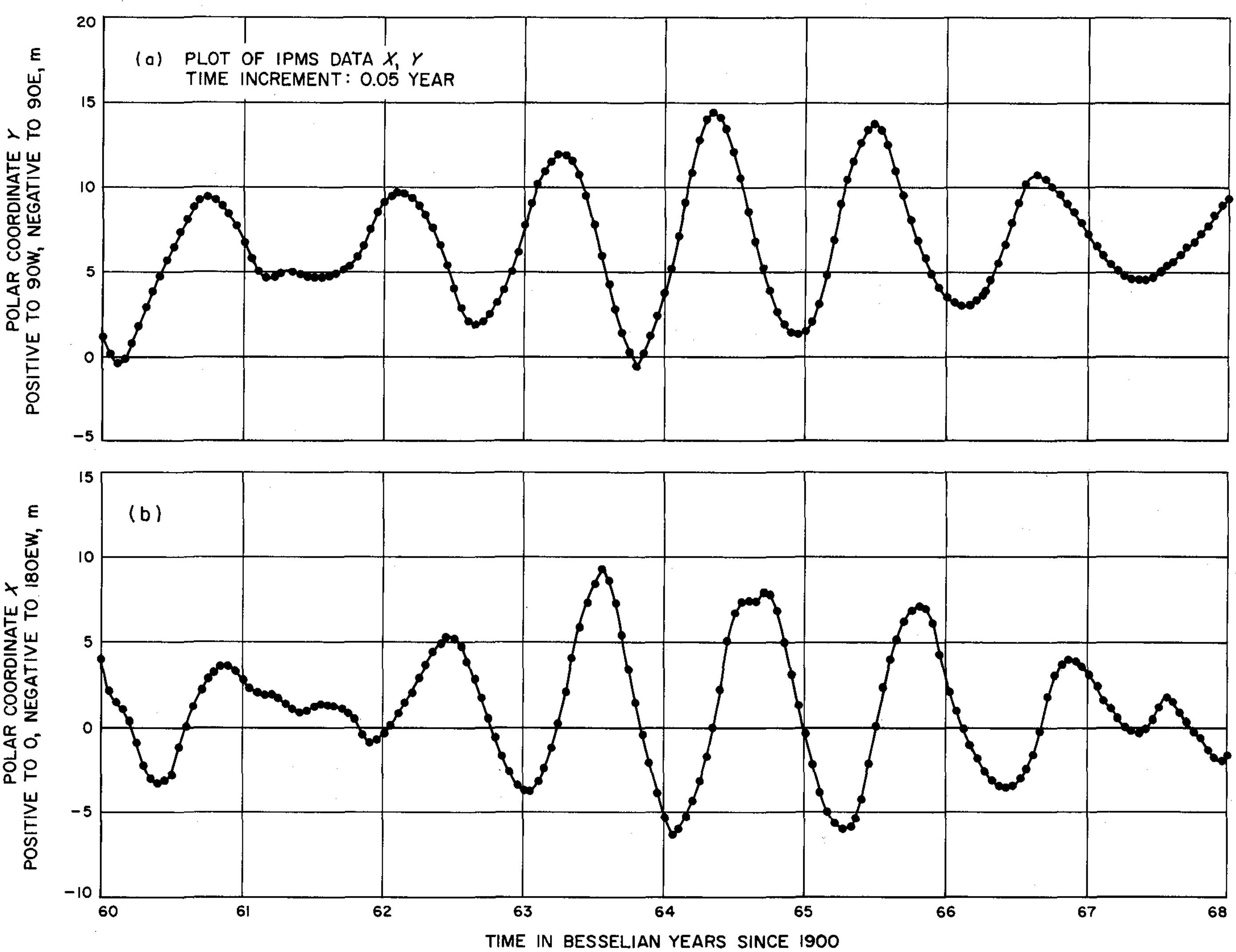

Fig. 3. Polar motion plot of X and Y vs Besselian years 1960–1968

origin. In 1900, they were very nearly so aligned. The change illustrates the secular motion of the pole (Ref. 24) which has covered 10 m in 67 years. Figures 3a and 3b give plots of X and Y alone, versus time since 1960.

*4. **Predictions of the future motion.*** Hattori (Ref. 28) discussed possible modeling of the polar motion. Hattori was successful in empirically fitting trigonometric equations, with approximately 28 terms, to the polar motion over one 6.5-year period (minimum-to-minimum amplitude). The fit was good to within 3 m over the entire interval. He found that the phase and amplitude characteristics of the Chandler term undergo unpredictable changes at each minimum. At this time, there is no reliable prediction service from IPMS or other sources.

E. Time

Time is not physically tangible; it has no unique physical property that permits its laboratory examination. Time is essentially metaphysical; there is no direct way of measuring it, even in principle. Nonetheless, our lives are ordered by it and, more importantly, our physics is also ordered by it. Therefore, time must be measured. The foundations of physical science include the article of faith that a "uniform" time exists that corresponds identically with the variable called time in dynamics. Even Isaac Newton suspected (Ref. 29, page 8) the impossibility of determining "uniform" time, and he commented upon the necessity of distinguishing between this construct of *faith* and the physical measures of time. The failure to reconcile observations with dynamical theories may lead one to amend or discard the laws of dynamics or the means of *determination* of time, but the faith in a "uniform" supertime is untouched.

Any physical measure of time is entirely conventional, established by definition. Most of the physical measures of time represent attempts to approximate "uniform" time more closely than did their predecessors (i.e., their introduction was intended to reduce the discordances between observations and dynamical theory); however, each is nonuniform in some degree. Each of them has some area of continuing utility. These points may serve as a baseline for the following discussions.

*1. **Sidereal time.*** Sidereal time is governed by the rotation of the earth relative to the stars, being defined as the hour angle of the vernal equinox (the ascending node of the sun's geocentric orbit). Thus, an object transits the meridian at a sidereal time equal to its right ascension. Sidereal time referred to the true equinox of date is called apparent sidereal time, while mean sidereal time is referred to the mean equinox of date. Local sidereal time is determined directly from observations in accordance with its definition. Greenwich sidereal time is obtained by adding the west longitude of the local meridian to the local sidereal time.

Because it is referred to the equinox of date, the duration of the sidereal day is not identical with the period of rotation of earth relative to a fixed direction, but is approximately $0\overset{s}{.}0084$ shorter.

*2. **Mean solar time.*** Solar time is governed by the rotation of earth relative to the earth-sun direction. Apparent solar time (referred to the true sun) is not used today, because of its variability of approximately $\pm$ 1%. Instead, mean solar time is defined as 12 h plus the hour angle of the mean equinox of date, minus the right ascension of the "fictitious mean sun." This latter is an imaginary point defined by Newcomb to have a uniform sidereal motion on the mean equator of date, *if time is counted in ephemeris time*. The rate of this fictitious motion is chosen so as to keep it as close as possible to the mean longitude of the true sun. It was originally intended that this definition be identical with 12 h plus the hour angle of the fictitious mean sun; however, the variable rotation of earth and the method of tabulation prevent this correspondence from being exactly satisfied.

The definition of mean solar time does not admit of practical determination in a wholly consistent way for two reasons: (1) the fictitious mean sun cannot be observed, and (2) the definition requires a simultaneous knowledge of ephemeris time (q.v.). In Ref. 16, the section on the calculation of mean solar time does not, in fact, give principles for the practical determination of this quantity, but rather for Universal Time.

*3. **Universal Time.*** Universal Time (UT) is almost identical with mean solar time on the Greenwich meridian. The definition differs from that for Greenwich mean solar time in only one small, but very significant, detail. UT is defined as 12 h plus the Greenwich hour angle of a point moving on the mean equator of date. The expression for the right ascension of this fiducial point is identical with that for the fictitious mean sun, but with time counted in UT. In actual practice, no distinction is made between a unit of UT and a unit of mean solar time. Indeed, UT minus the local longitude is called local mean solar time, ignoring the near-zero difference in the definitions.

The definition of UT cannot be used as a basis for its direct determination, for the fiducial point cannot be observed. However, the definition requires a well-defined and nearly linear relationship with mean sidereal time (Ref. 16). Thus, UT is determined from measures of local mean sidereal time. Such measures are contaminated by variations in the position of the pole of rotation and by variations in the rate of rotation, so that several different levels of UT are defined, corresponding to different levels of approximation to "uniform" time.

The time corresponding to observed local sidereal time, converted to local mean solar time, and augmented by the station west longitude is designated UT0. Because of pole wandering, which causes variations in the meridian, UT0 refers only to the location of the observing station.

Analysis of time determinations and stellar observations from several observatories permits the determination of the true instantaneous geographic location of the pole (see Section II-D). UT0 corrected for the polar motion is designated UT1, which is observatory-independent. The pole can presently be located to an accuracy of $0\overset{''}{.}015$ (Ref. 30).

UT1 is still affected by the variability in the rate of rotation of earth. This variability includes a component, the seasonal inequality, that seems stable enough to be predictable (Ref. 31). When this effect is removed from UT1, the resultant time measure is designated UT2. It is the closest approximate to "uniform" time based primarily on the rotation of earth.

A hybrid, designated UTC, will be discussed under broadcast time.

*4. **Ephemeris time.*** Verification of the nonuniformity of earth's rotation rate implied the unsuitability of earth for time keeping. Ephemeris time was devised as a means of more closely approaching a "uniform" measure of time. If it is recalled that uniformity means consistency with the laws of dynamics, then a uniform measure of time could be defined by means of the dynamical system. The following is excerpted from Ref. 16 (page 69):

> "Ephemeris time is a uniform measure of time depending on the laws of dynamics. It is the independent variable in the gravitational theories of the sun, moon, and planets, and the argument for the fundamental ephemerides in the Ephemeris."

It is imperative to observe the logical flaw in this citation, for it is fundamental to a great deal of confusion over Ephemeris Time (ET). It is implied here that the laws of dynamics, the gravitational theories of all solar system objects, and the tabulated ephemerides of these objects are all consistent with one another. This implication is patently false. Taking the sun as an example, neither Newcomb's *Theory of the Sun* nor his *Tables of the Sun* are completely consistent with either gravitational theory or the observed motion of the sun, nor are they totally consistent with one another.

The formal definition of ET consists of a defined rate and epoch. Both were chosen so as to force ET to be the independent variable in the *Theory of the Sun*; the ephemeris second is the tropical second at 1900 January 0.5 ET, which epoch corresponded by definition to a geometric mean longitude of the sun equal to $279^\circ\,41'\,48\overset{''}{.}04$.

It has been noted that the epoch definition depends on the System of Astronomical Constants, since the geometric position of the sun cannot be observed. If the definition is strictly adhered to, then the value of aberration adopted in 1964 requires a change to the values of ET and corresponding changes to the lunar and planetary theories (Ref. 30). It is suspected that, when action is taken, it will be a redefinition of epoch instead.

There is nothing wrong in principle with defining ET in this way, unless it is assumed that ET, thus defined, is "uniform." In a practical sense, however, the formal definition is an irrelevancy that is inconsistent with the determination of ET.

The determination of ET consistent with its definition would require that observations of the sun be compared with the *Theory of the Sun* under the constraint that admits of no error in the theoretical mean longitude. This is not done in practice because of the difficulty of interpreting solar observations and because of the relative slowness of the solar motion in right ascension. What in fact occurs is that observations of the moon are compared with the Lunar Theory under the constraint that no error exists in the theoretical mean longitude of *that* object. These two procedures would be equivalent only if the independent variables in the two theories bore the same relationship to "uniform" time. They would themselves be uniform if the two theories included no deviations from dynamical theory.

The *de facto* definitions of both rate and epoch are specified by the lunar theory against which the observations are compared. This is implicitly acknowledged in the 1970 Ephemeris (Ref. 32), where, in accordance with an IAU resolution of 1967, the designation ET0 is attached to ET referred to the Improved Brown Lunar Theory and ET1 is attached to ET referred to the revision of that theory incorporating the 1964 System of Astronomical Constants. Another designation will become necessary when the Eckert–Jones theory or the Eckert-corrected Brown theory come into general use. Even with these modifications, however, the Lunar Theory is not adequately consistent with gravitational theory (Refs. 33 and 34), nor is the real lunar motion adequately represented, even by dynamical theory, for any of these measures to be called "uniform."[7]

5. Atomic clock time. All of the previous subsections have dealt with time measures derived from the motion of material objects. In principle, any repetitive process of adequately high reliability could be used to define the unit of time. Such a process is the atomic resonance corresponding to transition between the two hyperfine levels of the ground state of cesium 133. The frequency of this resonance is 9 192 631 770 Hz (per second of ET as determined by Markowitz, Hall, Essen, and Perry, *Phys. Rev. Lett.*, Vol. I, No. 105, 1958), with a stability of 3×10^{-12} (Ref. 30) under laboratory conditions.[8] In October, 1967, the General Conference of Weights and Measures adopted this as the definition of the second in the International System of Units (SI), replacing the ephemeris second, which remains in the IAU System of Astronomical Constants.

Although the rate can be defined in this way, the atomic resonance cannot be used to define epoch, which requires a unique event. The epoch for atomic clock time (A.1) is supplied by definition as follows: At the epoch 1958 January 1, at $0^h0^m0^s$ UT2, A.1 was precisely $0^h0^m0^s$. With rate and epoch thus defined, A.1 is the standard of atomic time used in the U.S. and some other countries. It is not identical with the atomic time scale adopted by the *Bureau International de l'Heure* nor with the National Bureau of Standards.

While A.1 is certainly a closer approximation to "uniform" time than ET1, there are no grounds for calling it uniform. It is known that the transition frequency should be subject to variations due to relativistic effects and to variations in the local magnetic field. Such effects are thought to have been observed; however, the variations are so close to the present threshold of uncertainty that the determinations are not definite.

[7]The quantity inaccurately called ET in the JPL spacecraft tracking and data reduction programs is not, in fact, any of the above measures. It is A.1 plus a constant $32^s\!.25$ (Section II-E-5).

[8]Stability, in this context, means intercomparability between two oscillators operating under identical conditions.

6. Coordinate time. In a general relativistic framework, atomic time kept by an observer is interpreted as the observer's *proper* time. In such a framework, it is necessary to include a transformation from proper time to *coordinate* time, the latter being the independent variable in the differential equations describing the geodesic motion of matter and light. Coordinate time is identified with the ideal of uniform time on which the definition of ephemeris time is based (Ref. 35). Loosely speaking, coordinate time is interpreted as mean atomic time. The transformation from proper time to coordinate time depends on the position and inertial velocity of the observer within the solar system and provides for the instantaneous deviation of proper time from coordinate time. Only the effect of *variations* in the observer's potential and inertial velocity on the proper/coordinate time relationship are of interest since any constant effects are absorbed in the definition of the ephemeris second.

The transformation provided below (Eq. 50) is accurate to approximately one part in 10^{11} and accounts for variations in the orbital motion of the earth and in the vector addition of the heliocentric velocity of the earth and the diurnal motion of the observer. If further precision is required, other observer motions must be considered and it is not advisable to use the transformation provided. If τ is the atomic time and t the coordinate time, the differential relationship is given by (Ref. 36):

$$\begin{aligned}\frac{d\tau}{dt} = 1 &- 3.302 \times 10^{-10}\frac{a}{r}\cos E_\odot \\ &- 1.476 \times 10^{-10}\cos\phi\cos(\mathrm{UT}+\lambda) \\ &+ 0.064 \times 10^{-10}\cos\phi\cos(2\alpha_\odot + \mathrm{UT} + \lambda) \\ &+ \cdots, \end{aligned} \tag{50}$$

where

$E_\odot$ = the eccentric anomaly of the sun

a = semi-major axis of the earth's orbit

r = distance of the sun

UT = universal time in angular measure

ϕ = geocentric latitude of the observer

λ = the longitude of the observer measured eastward from Greenwich. (More precisely, ϕ and λ are the latitude and longitude of the atomic clock)

$\alpha_\odot$ = right ascension of the fictitious mean sun

In converting intervals of coordinate time to observed intervals of atomic time, these terms are significant for the accurate computation of spacecraft range data. At one astronomical unit, these terms in round-trip range produce an effect of approximately 100 m. The integral of this differential expression is needed for time keeping and is given by

$$
\begin{aligned}
t = \tau &+ 32\overset{s}{.}15 + 1.658 \times 10^{-3} [\sin E_{\odot} + 0.0368] \\
&+ 2.03 \times 10^{-6} \cos \phi \ [\sin(\mathrm{UT} + \lambda) - \sin \lambda] \\
&+ \cdots
\end{aligned}
\tag{51}
$$

Here, January 1, 1958, UT = 0 was chosen to be the epoch at which $t_0 - \tau_0 = 32^s15$, $\sin E_0 = -0.0368$. Because of its high frequency nature, the diurnal term is three orders of magnitude smaller than the orbital term in its integrated effect. Other neglected observer motions have integrated effects at the microsecond level.

7. ***Broadcast time.*** For purposes of practical use by a large variety of users, the various Standard Frequency services disseminate time by radio broadcast. Some users, such as navigators, require that the measure of time available to them approximate UT2 to within approximately 0.1 sec. This led, in 1964, to the introduction of Coordinated Time (UTC) for radio dissemination. The UTC is a hybrid time. The rate is defined relative to atomic clock rate, while the epoch is defined relative to UT2. When, at any time, UTC deviates too far from UT2, either the frequency or the epoch can be offset (adjusted) by international agreement. By convention, frequency offsets are introduced only at the beginning of a year (the most recent was in 1966), while epoch offsets can be introduced at the beginning of any month. The present frequency differs from A.1 by -3×10^{-8}; i.e., the ratio of an A.1 sec divided by a UTC second is $1 - 3 \times 10^{-8}$.

8. ***Julian dates.*** The system of Julian day numbers is, in practical effect, a continuous sequential numbering of days from an epoch so remote that all astronomical events of historical record will be assigned positive Julian dates. The epoch of the Julian cycle is 4713 BC, January 1, Greenwich mean noon, on the Julian calendar,[9] at which time the Julian date was exactly zero.

Two systems of Julian day numbers are in common use: (1) the Julian date (JD), which is measured in days of UT, and (2) the Julian Ephemeris date (JED), measured in days of ET. In either case, the Julian date corresponding to a particular instant of time is specified as the Julian day number followed by the decimal fraction of a day elapsed since 12^h (UT or ET, whichever is appropriate). A table of Julian day numbers is given in Ref. 16, pages 437–439.

At various times in recent years, a proposal has been advanced to adopt a modified JD. Usually this has been defined as JD — 2400000.5 and defended on the grounds of economy of publication, simplification of computer operations, or reduction of transcription labor. The proposal has consistently encountered strong opposition and has never received official sanction. The reason for this is fundamental. The system of Julian dates is the one and only system that is universal and unambiguous in both space and time. No other calendric system has ever been standardized worldwide (Greece still does not have a strictly Gregorian calendar). No other major calendric system has gone unmodified (the Gregorian calendar was shifted by 0.5 days in 1925 in every observatory except Heidelberg, where the shift was effected in 1933). Codification of a modified Julian system would destroy these advantages and create ambiguity where none existed before.

This is not to say that leading digits must always be carried in any use of the Julian date. Indeed, many analysts actually work with some modification of JD during their computations. There can be no objection to this practice, as long as the present system of Julian day numbers remains the primary reference and the computational devices are related unambiguously to it.

9. ***Year.*** The year is defined as the period of revolution of earth in its motion about the sun. Several different years may be defined in conformance with this general statement, depending on the nature of the reference direction with respect to which the motion is measured. Thus, the sidereal year is the period of revolution with respect to the fixed stars, the anomalistic year is the period from one perihelion passage to the next, the eclipse year is the interval between successive passages through the moon's apsidal line, and the Gaussian year is the period associated with Kepler's third law with $a = 1$. In the present context, only four years are of any consequence: Julian, tropical, Besselian, and calendar.

The Julian year (or the Julian century) is used in many of the time-variant relationships of astronomy, both in this document and elsewhere. It was the average of the calendar years in the Julian calendar and has the advantage of an exact decimal fraction representation as follows:

$$\text{Julian year} = 365.25 \text{ days}$$

$$\text{Julian century} = 36525 \text{ days}$$

[9]Julian cycle and Julian date do not refer to Julius Caesar, as in the case of the Julian calendar, but instead honor the father of Josephus Scaliger, author of the system.

The Julian year is an interval only and is not associated with an epoch.

By tradition, it is desirable to have a definition for a year in which the seasons remain fixed, i.e., a year based on the passage of the earth through the equinoxes. The interval of time elapsed between two successive passages of the vernal equinox is called the tropical year and is as follows:

$$\text{Tropical year} = 365.2421988 \text{ days}$$

$$\text{Tropical century} = 36524.21988 \text{ days}$$

It is this interval that calendric systems intend to approximate on the average, so that the seasons do not precess through the calendar. For example, the Gregorian calendar year (present system), averaged over the 400-year cycle, yields 365.2425 days. The tropical year is used only as an interval measure and no epoch is usually associated with it. The epoch for the calendar is specified by the calendric algorithm.

The Besselian year is frequently used for specifying standard and secondary epochs, as well as for some time-variant relations, such as those of Section II-C. The epoch of the Besselian year is the instant of time at which the right ascension of the fictitious mean sun, affected by aberration and referred to the mean equinox of date, is precisely 18^h40^m. This instant, designated by the notation .0 after the year (e.g., 1950.0), always falls near the beginning of the Gregorian calendar year. The length of the Besselian year is almost identical with the tropical year as follows and the difference is usually neglected:

$$\text{Besselian year} = \text{tropical year} - 0\overset{s}{.}148\, T$$

where T is measured in tropical centuries from 1900.0; 1900.0 is the basic epoch, which corresponded to JED 241 5020.31352 = 1900 January 0.81352 ET. The error committed in computing the beginning of a Besselian year by addition of the appropriate number of tropical centuries to the Julian date given above is:

$$+ 0.856 \times 10^{-4}\, T^2 \text{ (days)}$$

This amounts to approximately $1\overset{s}{.}85$ for 1950.0. If extremely high precision is required, one may either compute

$$\text{JED}(1900.0 + \tau) = \text{JED}(1900.0) + (365.2421988 - 0.856 \times 10^{-8}\tau)\tau \qquad (52)$$

where τ is an integer number of years, or one may invert the definition of the right ascension of the fictitious mean sun (Ref. 16, page 73). Rarely will either procedure be justified by necessity.

Every usage of decimals with years implies that Besselian years are intended. The indicated time point should not be confused with January 0.0, January 0.5 or January 1.0, for the error involved can be significant for high-precision work.

F. Timing Polynomials

There are several different times available. These times are referenced to each other by definite offsets and with varying degrees of accuracy. The primary times used in trajectory and orbit determination work are ET, UT1 and UTC, and A.1 (see Section II-E and also Ref. 37). The relationship between the various times has been tabulated by the U.S. Naval Observatory. JPL has fit these tabulated data with polynomials over discrete time periods and interpolates over them. This technique is described in Ref. 38, and the polynomial coefficients from January 1, 1961, to November of 1968 are shown in Table 3. It has been assumed that ET–A.1 is a constant and has the value 32.25[10] sec. Since the epoch for A.1 is January 1958, this is the value of ET–UT1 for 1958 (A.1–UT1 approximately = 0 on this date). Subsequent values of ET–UT1 are assumed to be 32.25[10] + (A.1–UT1) for any date of interest. The U.S. Naval Observatory Atomic Time, A.1, is known with respect to UTC as disseminated by that agency to within microseconds. Atomic time and UTC, as determined by other agencies such as National Bureau of Standards, will, in general, differ from the above. This arises because the atomic scales themselves differ, and because the various transmitting and receiving stations inevitably maintain different time.

For these reasons, A.1 and UTC (U.S. Naval Observatory) have been arbitrarily picked as reference times. Where necessary, the proper corrections are applied as when, for example, published data from National Bureau of Standards are used.

The timing polynomial listings are good to 1 msec. The Double Precision Orbit Determination Program (DPODP) version (Ref. 39) is being computed within 100 μsec, more than adequate in view of uncertainty in ET or UT1. The value of A.1–UT1 is known to ± 0.005 sec after the fact, and to ± 0.0002 sec/day^2 additional when predicting ahead.

[10]The value 32.25 was and is used in JPL's Single Precision Orbit Determination Program (SPODP); the improved estimate, 32.15, is used in JPL's Double Precision Program for the value $\Delta T_{1958.0}$.

Table 3. TPOLY[a]

POLYNOMIAL SUMMARY

ET − UTC = A + B*T[b]   UT1 − ET[b] = C + D*T + E*T*T[b]   ET = A.1 + 32.15[a]

Date[c] From	Date[c] To	JD/1950 (From, To)	A	B	C	D	E
500101	610100	(5000, 4017)	0.29080000E + 02	0.13052167E − 07	−0.29080000E + 02	−0.13020473E − 07	−0.00000000E − 99
610100	610301	(4017, 4077)	0.28429486E + 02	0.14926422E − 07	−0.21298914E + 02	−0.58853102E − 07	0.67453304E − 16
610301	610801	(4077, 4230)	0.28445206E + 02	0.14881933E − 07	0.39503872E + 02	−0.39565606E − 06	0.53355032E − 15
610801	611012	(4230, 4302)	0.28349853E + 02	0.15005795E − 07	−0.14298924E + 03	0.60234415E − 06	−0.83088548E − 15
611012	620101	(4302, 4383)	0.28349753E + 02	0.15005795E − 07	−0.14061474E + 02	−0.89773211E − 07	0.97976158E − 16
620101	620601	(4383, 4534)	0.29111339E + 02	0.12994775E − 07	−0.55893650E + 02	0.13004285E − 06	−0.19077050E − 15
620601	620905	(4534, 4630)	0.29128013E + 02	0.12952030E − 07	−0.50802635E + 02	0.91877394E − 07	−0.12650309E − 15
620905	630101	(4630, 4748)	0.29122413E + 02	0.12966003E − 07	−0.56731408E + 02	0.12927234E − 06	−0.18292754E − 15
630101	630322	(4748, 4828)	0.29119564E + 02	0.12972758E − 07	−0.14166431E + 03	0.53123992E − 06	−0.65811788E − 15
630322	630701	(4828, 4929)	0.29106636E + 02	0.13003815E − 07	0.96463315E + 02	−0.60342523E − 06	0.69349709E − 15
630701	631101	(4929, 5052)	0.29125189E + 02	0.12960409E − 07	−0.20963144E + 03	0.82696476E − 06	−0.97753119E − 15
631101	640101	(5052, 5113)	0.29224045E + 02	0.12962996E − 07	0.49658002E + 02	−0.36078385E − 06	0.38267675E − 15
640101	640401	(5113, 5204)	0.28319556E + 02	0.15010497E − 07	−0.55773549E + 02	0.11642456E − 06	−0.15731393E − 15
640401	640710	(5204, 5304)	0.28435475E + 02	0.14975058E − 07	0.14515182E + 03	−0.77314012E − 06	0.82727231E − 15
640710	640901	(5304, 5357)	0.28440733E + 02	0.14963630E − 07	−0.21128099E + 03	0.77793237E − 06	−0.86013450E − 15
640901	641101	(5357, 5418)	0.28434666E + 02	0.15192921E − 07	−0.23673174E + 03	0.88906376E − 06	−0.98143552E − 15
641101	650101	(5418, 5479)	0.28490095E + 02	0.15074038E − 07	−0.91424256E + 02	0.26298114E − 06	−0.30709080E − 15
650101	650301	(5479, 5538)	0.28563622E + 02	0.15131450E − 07	0.12965656E + 03	−0.67170154E − 06	0.68082041E − 15
650301	650501	(5538, 5599)	0.28696647E + 02	0.15061314E − 07	−0.34236210E + 03	0.13037586E − 05	−0.13860726E − 14
650501	650701	(5599, 5660)	0.28784160E + 02	0.14880915E − 07	0.20918441E + 03	−0.98453356E − 06	0.98736037E − 15
650701	650901	(5660, 5722)	0.28774081E + 02	0.15106046E − 07	−0.33177949E + 03	0.12209437E − 05	−0.12605304E − 14
650901	651101	(5722, 5783)	0.28947903E + 02	0.14956092E − 07	−0.35750054E + 03	0.13229078E − 05	−0.13615332E − 14
651101	660101	(5783, 5844)	0.28940792E + 02	0.14971102E − 07	0.17638474E + 03	−0.81812145E − 06	0.78500217E − 15
660101	660301	(5844, 5903)	0.21348034E + 02	0.30009928E − 07	−0.59641384E + 03	0.22315531E − 05	−0.22236831E − 14
660301	660501	(5903, 5964)	0.21344123E + 02	0.30017464E − 07	−0.19657679E + 03	0.65338382E − 06	−0.66646551E − 15
660501	660701	(5964, 6025)	0.21344100E + 02	0.30017454E − 07	0.29377306E + 03	−0.12478306E − 05	0.11764154E − 14
660701	660901	(6025, 6087)	0.21359804E + 02	0.29987399E − 07	−0.33881691E + 03	0.11721046E − 05	−0.11378846E − 14
660901	661101	(6087, 6148)	0.21367766E + 02	0.29972326E − 07	−0.33493374E + 03	0.11594470E − 05	−0.11278589E − 14
661101	670101	(6148, 6209)	0.21295881E + 02	0.30107599E − 07	0.95276669E + 02	−0.46460702E − 06	0.40484368E − 15
670101	670301	(6209, 6268)	0.21519205E + 02	0.29691716E − 07	−0.36565600E + 03	0.12407315E − 05	−0.11724004E − 14
670301	670501	(6268, 6329)	0.21383454E + 02	0.29942250E − 07	−0.20730884E + 02	−0.29936111E − 07	−0.21609671E − 17
670501	670701	(6329, 6390)	0.21358853E + 02	0.29987391E − 07	0.28709060E + 03	−0.11538954E − 05	0.10238291E − 14
670701	670901	(6390, 6452)	0.21367164E + 02	0.29972330E − 07	−0.32450812E + 03	0.10509295E − 05	−0.96323391E − 15
670901	671101	(6452, 6513)	0.21344894E + 02	0.30012423E − 07	−0.19095332E + 03	0.57462174E − 06	−0.53857933E − 15
671101	680101	(6513, 6574)	0.21342073E + 02	0.30017404E − 07	0.77463410E + 02	−0.37767890E − 06	0.30607119E − 15
680101	680201	(6574, 6605)	0.21487161E + 02	0.29762122E − 07	−0.65159696E + 02	0.12362954E − 06	−0.13444034E − 15
680201	680301	(6605, 6634)	0.21185217E + 02	0.30116296E − 07	−0.60346725E + 02	0.10697501E − 06	−0.12003299E − 15
680301	680501	(6634, 6695)	0.21259206E + 02	0.29987006E − 07	0.60776033E + 01	−0.12226171E − 06	0.77721864E − 16
680501[d]	680701	(6695, 6756)	0.21251736E + 02	0.29999957E − 07	0.41868214E + 03	−0.15455193E − 05	0.13050834E − 14
680701[d]	680901	(6756, 6818)	0.21251745E + 02	0.29999942E − 07	−0.35786413E + 03	0.11064831E − 05	−0.95912393E − 15
680901[d]	681101	(6818, 6879)	0.21251731E + 02	0.29999967E − 07	−0.15360279E + 03	0.41656383E − 06	−0.37656913E − 15
681101[d]	750101	(6879, 9131)	0.21251711E + 02	0.30000000E − 07	−0.21212858E + 02	−0.30000002E − 07	0.18406913E − 23

[a] 32.25 sec was and is used in SPODP; the improved estimate of 32.15 sec is used in DPODP.

[b] T is A.1 sec, but error introduced for T in another system is negligible; epoch from $0^h0^m0^s$ Jan. 1, 1950.

[c] Under "From" and "To," the first two digits on the left represent the year, the two digits in the middle represent the month, and the other two digits the day.

[d] Predictions based on the method of Ref. 38.

Table 4. Deep space station locations[a]

Station	Location	Geocentric					Standard Deviation			References
		Radius, km	ϕ' Latitude, deg	Longitude, deg	$R \cos \phi'$ [b] km	$R \sin \phi'$ [c] km	$R \cos \phi'$ [b] km	Longitude, deg	Relative longitude, deg	
DSS 11 (Pioneer)	Goldstone, Calif.	6372.006	35.20802	243.15070	5206.338	3673.759	0.010	0.00010	0.00006	40[de]
DSS 12 (Echo)	Goldstone, Calif.	6371.990	35.11864	243.19463	5212.050	3665.624	0.010	0.00010	0.00006	40
DSS 13 (Venus)	Goldstone, Calif.	6372.113	35.06652	243.20523	5215.483	3660.952	0.010	0.00010	0.00006	42[de]
DSS 14 (Mars)	Goldstone, Calif.	6371.989	35.24433	243.11059	5203.995	3677.048	0.010	0.00010	0.00006	43[d]
DSS 41	Woomera, Australia	6372.553	−31.21139	136.88759	5450.198	−3302.238	0.010	0.00010	0.00006	40[d]
DSS 42	Canberra, Australia	6371.700	−35.21953	148.98137	5205.350	−3674.628	0.010	0.00010	0.00006	40[d]
DSS 51	Johannesburg, South Africa	6375.528	−25.73945	27.68546	5742.937	−2768.760	0.010	0.00010	0.00006	40[d]
DSS 61	Madrid, Spain	6369.988	40.23854	355.75109	4862.605	4114.829	0.010	0.00010	0.00006	40[d]
DSS 62	Madrid, Spain	6369.993	40.26349	355.63226	4860.816	4116.950	0.030	0.00010	0.00006	40[d]
DSS 72	Ascension Island	6378.239	−7.89991	345.67361	6317.708	−876.644	0.200	0.00200	0.00200	[f]

[a]All locations at the mean pole 1903.0 (or New System 1900 pole, i,e., $\Delta x = \Delta y = 0$ of Fig. 2). The uncertainty in the coordinate along the earth's spin axis is approximately 50 m for all stations when deep space data are used; however, the uncertainty for DSS 11, 12, 13, 14, 41, and 51 is 15 m using Ref. 41. Longitude solutions are a function of a particular ephemeris; there is a problem with the lunar ephemeris epoch, see page 39 and Ref. 40.

[b]Other notation $R_s = U = X_1$.

[c]Other notation $Z = W = X_3$.

[d]The difference between DSS 12 and the particular DSS under consideration from the reference was added to the absolute Longitude of DSS 12, $R \cos \phi'$ and $R \sin \phi'$ are directly from Ref. 40.

[e]Internal communication from B. Bollinger, "First Order Geodetic Survey for DSS 11, 12, and 13," dated October 29, 1964.

[f]Astronomical determination communicated by J. Heller, JPL; uncertainty in $R \sin \phi'$ is 200 m.

G. Station Locations

Listed in Table 4 are the DSS geocentric positions with their associated uncertainties. Listed in Table 5 are some of the AFETR stations. Most of the DSS locations, as noted in the references, were estimated from the various space probes (Refs. 40, 41, and 42). Since deep space tracking data are insensitive to the station coordinate along the earth's axis of rotation, these estimates reflect determinations of distance off the spin axis ($R \cos \phi$) and longitude as indicated by the statistics (i.e., the radius or latitude of the initial land survey being held constant in a particular determination).

The effect of the wandering of the earth's pole of rotation on station locations ranges between 5 and 20 m over a 14-month period (Ref. 23). The uncertainty in the pole's position is approximately 1 m after the fact and approximately 3 m for a 6-month prediction. All station location estimates have been referenced to the mean pole 1903.0.[11]

The conversion to geodetic latitude for proper station elevation angle calculations is:

$$\phi_{\text{Geodetic}} = \phi' + e^2 \cos \phi' \sin \phi' \left[1 + e^2 \cos^2 \phi'\right] \quad (53)$$

where

$$e^2 = 2f - f^2$$

$f = 1/298.25$, nominal flattening (see Section II-B)

$\phi' =$ geocentric latitude in radians

Table 5. AFETR station locations

Stations	Geocentric		
	Radius, km	Latitude, km	Longitude, km
Antigua (91.18)	6376.3	17.036	298.207
Ascension (12.18)	6377.9	−7.921	345.598
Bermuda (MSFN)	6372.1	32.174	295.364
Cape Kennedy (1.16)	6373.3	28.321	279.423
Canary Islands	6373.7	27.733	344.405
Carnarvon	6374.5	−24.751	113.716
Grand Bahama	6373.9	26.482	281.732
Grand Turk	6375.3	21.332	288.868
Pretoria	6375.7	−25.792	28.358

The correction from geocentric to geodetic longitude is assumed to be zero.

H. Atmospheric Model

The model is an empirical expression for the corrections to the tracking station observables. It was formulated by D. Cain of JPL who fit to real atmospheric measurements from the AFETR. Recent comparisons with ray tracing techniques described in Refs. 44 and 45 provide excellent agreement, provided the proper refractivity index is used. The corrections to range and doppler observables are given by:

$$\Delta_r \rho = \left[0.0018958/(\sin \gamma + 0.06483)^{1.4}\right] \frac{N}{340.0} \quad (54)$$

$$\Delta_r \dot{\rho} = \frac{0.0018958}{\tau} \left[\frac{1}{(\sin A + 0.06483)^{1.4}} - \frac{1}{(\sin B + 0.06483)^{1.4}}\right] \frac{N}{340.0} \quad (55)$$

where

$\rho =$ the topocentric range in km

$\dot{\rho} =$ the topocentric range rate in km/sec

$\tau =$ doppler count interval, sec

$N =$ refractivity $\times 10^6$ (nominally 340.0 at sea level)

$$A = \gamma + \frac{\tau \dot{\gamma}}{2}$$

$$B = \gamma - \frac{\tau \dot{\gamma}}{2}$$

$\gamma =$ elevation angle in rad

$$\Delta_r \gamma = 57.2957795 \frac{N}{340.0} b_1 b_2 \qquad \gamma < 0.3 \text{ rad}$$

$$\Delta_r \gamma = 57.2957795\, N \times 10^{-6} \frac{\cos \gamma}{\sin \gamma} \qquad \gamma \geq 0.3 \text{ rad}$$

$$\Delta_r \alpha = \frac{\Delta_r \gamma \cos \phi \sin^2 \alpha}{\cos^2 \gamma \sin \sigma} \qquad \delta < 87 \text{ deg}$$

$$\Delta_r \delta = \frac{(\sin \phi \cos \gamma - \sin \gamma \cos \phi \cos \sigma) \Delta_r \gamma}{\cos \delta} \qquad \delta < 87 \text{ deg}$$

$$\Delta_r \alpha = \Delta_r \delta = 0 \qquad \delta \geq 87 \text{ deg}$$

[11]Also referred to as "New System 1900 Pole" with $\Delta X = \Delta Y = 0$ (see Section II-D on Polar Motion).

where

$$b_1 = 1.0 - (1.216 \times 10^5\, b_3\, \gamma_{\text{rad}}) - (51.0 - 300.0\, \gamma_{\text{rad}})\, (b_3)^{1/2}$$

$$b_2 = [7.0 \times 10^{-4}/(0.0589 + \gamma_{\text{rad}})] - 1.26 \times 10^{-3}$$

$$b_3 = \frac{1}{10^3\,(r - R_e)}$$

α = right ascension

δ = declination

ϕ = latitude

σ = azimuth

r = geocentric radius of probe

The optical hour angle and declination corrections are determined as above; however, $\Delta_r\gamma$ is calculated differently:

$$\Delta_r\gamma = \tan^{-1}\left(\frac{b_4}{\rho - b_5}\right) \qquad (56)$$

where

$$b_4 = \frac{0.00211}{(\gamma_{\text{rad}} + 0.0598)^{2.42}}$$

$$b_5 = \sqrt{b_6^2 - R_e^2 + R_e^2 \sin^2\gamma} - R_e \sin\gamma$$

$$b_6 = R_e + 51.2064$$

where

R_e = equatorial radius of the earth

I. Ionospheric Model

The model is an empirical expression for the corrections to the tracking station observables. It was developed by D. Cain and A. Liu of JPL (Ref. 46 using material from Ref. 47). It should be noted that the dependence on transmitter frequency is the inverse square and that the time of day and elevation angle are direct factors.

The correction in range is given by the expression

$$\Delta_i\rho = C_D\, C_L\, C_S\, C_H \times \left[\left(\frac{B}{39}\right)\left(\frac{2300 \times 10^6}{f_q}\right)^2\left(\frac{N_{\text{MAX}}}{5 \times 10^7}\right)\right]\Delta_0\rho(\gamma) \qquad (57)$$

where

B = scale height, km

f_q = transmitter frequency, Hz

N_{MAX} = maximum electron density, number of electrons/cc

$\Delta_0\rho(\gamma)$ = unmodified range correction, m

C_D = correction factor for diurnal effect

C_L = correction factor for geomagnetic latitude

C_S = correction factor for sunspot activity

C_H = correction factor for scale height variations

The unmodified range correction, which depends only upon the elevation angle γ, is obtained by interpolating between the following values:

γ, deg	$\Delta_0\rho(\gamma)$, m
0	23.434900
10	19.607710
20	14.462294
30	11.128928
40	9.0899870
50	7.8161703
60	7.0007498
70	6.4233956
80	6.0079401
90	6.0079401

An appropriate interpolation method should be utilized.

To calculate the four correction factors, it is first necessary to find the latitude and longitude (ϕ' and λ', respectively) of the sub-400 km point of the signal path. Let R, ϕ, and λ be the coordinates of the tracking station. Let h be an input (nominal value is 400 km) and calculate

$$B = \frac{R}{R + h}\cos\gamma \qquad (58)$$

$$\psi = \tan^{-1}\left(\frac{B}{+\sqrt{1 - B^2}}\right)^{-1} - \gamma \qquad 0 \le \psi \le \pi/2 \qquad (59)$$

Then,

$$\sin\phi' = \sin\phi\cos\psi + \cos\phi\sin\psi\cos\sigma$$

$$-\pi/2 \le \phi' \le \pi/2 \qquad (60)$$

$$\cos\phi' = +(1 - \sin^2\phi')^{1/2} \qquad (61)$$

where

σ = the azimuth angle of the spacecraft at the tracking station

$$\sin \Delta\lambda = \frac{\sin \psi \sin \sigma}{\cos \phi'} \quad (62)$$

$$\cos \Delta\lambda = + (1 - \sin^2 \Delta\lambda)^{1/2} \quad (63)$$

$$\Delta\lambda = \tan^{-1} \frac{\sin \Delta\lambda}{\cos \Delta\lambda} \qquad -\pi/2 \leq \Delta\lambda \leq \pi/2 \quad (64)$$

$$\lambda' = \lambda + \Delta\lambda \text{ (express in deg)} \quad (65)$$

The correction factors are obtained as follows:

$$C_D = 6 + 5 \cos H \quad (66)$$

$$H = \frac{\left(T - \frac{\lambda'}{15} - 15\right)\pi}{12} \quad (67)$$

where

T = current time (UT1) minus time of previous midnight, in hours

$$C_L = 3\left(\frac{1}{2} - \frac{|\phi_M|}{\pi}\right)$$

The geomagnetic latitude, ϕ_M, is computed from:

$$\sin \phi_M = \cos \phi \cos \phi_0 \cos (\lambda' - \lambda_0) + \sin \phi' \sin \phi_0$$

$$-\frac{\pi}{2} \leq \phi_M \leq \frac{\pi}{2} \quad (68)$$

where

λ_0 = longitude of geomagnetic pole = 291 deg

ϕ_0 = latitude of geomagnetic pole = 79 deg

$$C_S = 1 + 0.004\,\mathbf{R}\;|\cos H| \quad (69)$$

where

$\mathbf{R}$ = sunspot index, an integer

$$C_H = \sum_{k=0}^{2} A_k(\gamma)\left(\frac{h_m}{200}\right) \quad (70)$$

where

$A_0(\gamma) = 1.51266 - 0.203970\theta + 0.020460\theta^2$

$A_1(\gamma) = -0.63775 + 0.261150\theta + 0.026720\theta^2$

$A_2(\gamma) = 0.12469 - 0.057180\theta + 0.006260\theta^2$

$\theta = \gamma/10$, deg

h_m = height of maximum electron density, km

The correction in counted doppler is derived by differencing the $\Delta_i\rho$ expression before and after the count time, changing the sign to account for the plasma effects on phase velocity, and dividing by the count time.

III. Moon

A. Gravity Potential

The lunar gravity potential is described by the function, Φ, expressed as a series of spherical harmonics:

$$\Phi = \frac{GM_{☾}}{r}\left\{1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^n P_n^m(\sin \phi) \times (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda)\right\} \quad (71)$$

where

Φ = gravity potential

$GM_{☾}$ = lunar gravity constant

r = radial distance to moon's center of mass

ϕ = latitude referred to moon's true equator

λ = longitude referred to moon's true equator and meridian with zero mean deviation from earth-moon line

R = mean radius of moon (nominal value: 1738.09 km)

$P_n^m(z)$ = associated Legendre polynomial[12,13] of degree n and order m

The coefficients C_{nm} and S_{nm}, referred to as the moon's gravity harmonics, describe the deviation from sphericity. Prior to the launching of the lunar orbiters, lunar libration data provided information on the values of C_{20} and C_{22} (Ref. 49), i.e., the triaxial moon model. Otherwise, none of these gravity harmonics was known. The lunar orbiter tracking data are presently being analyzed by two NASA groups: one at JPL and one at Langley Research Center (LaRC). Since neither group has yet arrived at a final best estimate for the values of the gravity harmonics, a definitive set cannot be included herein.

[12]As defined in Whittaker and Watson, *Modern Analysis*, Fourth Edition, page 323, paragraph 15.5.

[13]In some European books, there is a difference in the definition; e.g., Ref. 48, Russian coefficients have a sign reversal on C_{21}, S_{21}, C_{31}, S_{31}, C_{33}, and S_{33}.

However, the current interim results from both JPL and LaRC are presented as being the best information available to date. It should be noted that the LaRC model includes degree 5 harmonics, whereas the JPL model includes tesserals only through degree 4, but the zonals through degree 8.

Tables 6 (Ref. 50) and Table 7 (Ref. 51) provide the current JPL and LaRC preliminary determinations of the gravity harmonics. In view of their inconsistent nature, JPL recommends no adopted set at this time. Current JPL computer programs use the triaxial model of the moon with coefficients (Ref. 1) given by:

$$J_{20} = 2.0711 \times 10^{-4} \pm 0.05 \times 10^{-4} \tag{72}$$

$$C_{22} = 0.20716 \times 10^{-4} \pm 0.05 \times 10^{-4} \tag{73}$$

$$S_{22} = C_{21} = S_{21} = 0 \tag{74}$$

Table 6. Lunar gravity harmonics derived from *Lunar Orbiters I* through *V* (preliminary JPL estimate)[a, b]

n	m	$C_{nm} \times 10^4$	$\sigma \times 10^4$	$S_{nm} \times 10^4$	$\sigma \times 10^4$
2	0	−1.9076	0.0171		
	1	0.0248	0.0501	0.0260	0.0431
	2	0.0895	0.0406	−0.1091	0.0351
3	0	−0.2058	0.0339		
	1	0.3397	0.0049	0.1046	0.0047
	2	−0.0038	0.0052	0.0305	0.0054
	3	0.0414	0.0110	−0.0311	0.0093
4	0	0.1319	0.0174		
	1	−0.1104	0.0157	0.0786	0.0146
	2	0.0421	0.0084	0.0053	0.0084
	3	0.0231	0.0070	−0.0101	0.0060
	4	−0.0059	0.0018	−0.0020	0.0018
5	0	0.0209	0.0340		
6	0	−0.1364	0.0463		
7	0	0.4030	0.0460		
8	0	−0.2057	0.0424		

[a]The standard deviations (σ) represent the goodness of fit to the data. They are based on the square roots of the diagonal terms in the covariance matrix. In interpreting these deviations, the following must be taken into account: (1) there still remain unknown biases in the data, and (2) the correlations are significant with respect to the data. Thus, it should be anticipated that improved estimates of the gravity coefficients may yield numbers which vary by several σ from the above.

[b]Based on:

$GM_☾ = 4902.78$ km^3/sec^2

$R_☾ = 1738.09$ km.

However, Koziel's reduction of data on physical librations of the moon produces (Ref. 52):

$$J_{20} = 2.054 \times 10^{-4} \tag{75}$$

$$C_{22} = 0.231 \times 10^{-4} \tag{76}$$

$$S_{22} = C_{21} = S_{21} = 0 \tag{77}$$

under the assumption of uniform density. These are considered to be the best values available at this time.

Although lunar orbiter data do sample the lunar potential, the best determination of the central term (i.e., $GM_☾$) is derived from the *Ranger* and *Surveyor* determinations of GE (Ref. 53) and the earth-moon mass ratio obtained from *Mariner V* cruise phase data (Ref. 54 and Footnote 14, see also Section V).

[14]Private communications, J. D. Anderson, results from recent analysis of *Mariner V* cruise and Venus encounter range and doppler tracking data using DE 40, May 7, 1968.

Table 7. Lunar gravity harmonics from *Lunar Orbiters I, III,* and *IV* (preliminary LaRC estimate)[a]

n	m	$C_{nm} \times 10^4$	Standard Deviation $\times 10^4$	$S_{nm} \times 10^4$	Standard Deviation $\times 10^4$
2	0	−2.0596	0.141	—	—
	1	−0.1661	0.051	0.0080	0.039
	2	0.2042	0.029	−0.0342	0.025
3	0	−0.3773	0.180	—	—
	1	0.3012	0.048	0.1762	0.053
	2	0.1294	0.028	−0.0147	0.033
	3	0.0317	0.015	−0.0043	0.018
4	0	0.0798	0.128	—	—
	1	−0.1560	0.036	0.0391	0.028
	2	0.0011	0.010	0.0072	0.013
	3	−0.0082	0.008	−0.0001	0.006
	4	−0.0007	0.003	0.0011	0.003
5	0	−0.5505	0.171	—	—
	1	−0.0385	0.037	0.0829	0.031
	2	+0.0342	0.009	−0.0203	0.008
	3	−0.0071	0.002	−0.0078	0.002
	4	−0.0008	0.001	−0.0013	0.001
	5	−0.0003	0.0002	0.0003	0.0002

[a]Based on:

$GM_☾ = 4902.64$ km^3/sec^2

$R_☾ = 1738.09$.

B. Figure

The figure of the moon is best represented by the work progressing at the Aeronautical Charting and Information Center (ACIC), St. Louis, Missouri, where charts on the scale of 1 to 1,000,000 covering $20 \times 16°$ areas of the lunar surface are available (Ref. 55). Photographs obtained from *Lunar Orbiters I* through *V* will definitely enhance the existing maps and produce better overall continuity, including a complete view of the lunar far side (Ref. 56).

The determination of lunar radii from the dynamical center, or center of mass, to surface points shows reductions from values obtained from ACIC maps. This reduction is attributable to an offset in the assumed ACIC center of figure from the center of mass, as obtained from the JPL lunar ephemeris. Sjogren (Ref. 57) has determined from *Rangers VI–IX* impact data an offset of 2.5 km along the earth-moon line (Δx, offset from center of figure to center of mass) between ACIC maps and the JPL ephemeris.[15] Compton and Wells of LaRC (Ref. 58) using V/H (speed/height) data from *Lunar Orbiter* have also obtained a reduction of the nominal lunar radius of some 2 km with a 1- to 2-km bulge in the central Sinus Medii area. Much more data remains to be processed especially out toward the limb (i.e., Ref. 58 covers $\pm 50°$ longitude). Shapiro, *et al.* (Ref. 58a) of NRL, have also obtained 1- to 2-km decrease in the lunar radius from radar bounce data.

C. Selenographic Coordinates

From the laws of Cassini and the mathematical definitions of the small departures from these laws (called physical librations), a relationship can be stated relating a coordinate system fixed with respect to the moon and the coordinate system aligned with the mean equinox and ecliptic of date. In terms of angular rotations about instantaneous axes, the transformation is (Fig. 4):

$$(\bar{X})_{\text{Selenographic}} = (\text{☾} + \tau - \text{☊} - \sigma + 180°)_{+z}\,(I + \rho)_{-x} \times (\text{☊} + \sigma)_{+z}\,(\bar{X})_{\substack{\text{mean equinox and}\\ \text{ecliptic of date}}} \tag{78}$$

where

☊ = longitude of ascending node of mean lunar orbit on ecliptic, measured from mean equinox of date

☾ = mean longitude of moon, measured from equinox of date along ecliptic to ascending node, then along mean orbit

I = inclination of mean lunar equator to ecliptic

σ = libration in node

ρ = libration in inclination

τ = libration in longitude

The quantities ☊ and ☾ are derived from the lunar theory and are given as polynomials in time (Ref. 16, page 107). Expressions for the physical librations are the result of complex derivations involving the lunar theory and observationally determined values for several constants. Fairly comprehensive descriptions of the theory of physical librations are given in Refs. 57 and 59.

The theory yields expressions which are sine series for τ and $I\sigma$ and cosine series for ρ in terms of fundamental arguments:

$$g, \omega, g', \omega'$$

where

g = mean anomaly of moon

ω = argument of perigee of moon

g' = mean anomaly of sun

ω' = argument of perigee of sun

The coefficients of the series are functions of two quantities:

$$I \text{ and } f$$

[15]Sturms at JPL, using *Ranger* impact time and photographic data, has obtained similar results for the ACIC center of figure: $\Delta x = 3.1 \pm 0.5$, $\Delta y = -0.3 \pm 0.8$, $\Delta z = 0.6 \pm 0.7$.

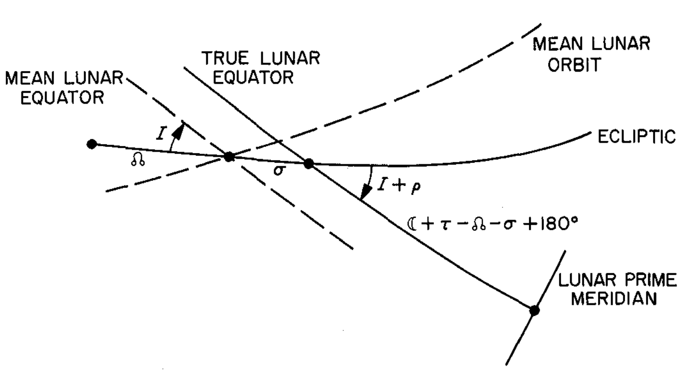

Fig. 4. Libration geometry

where

$$f = \frac{B(C - B)}{A(C - A)}$$

A,B,C = principal moments of inertia of the moon

Koziel (Ref. 57) has performed the latest determinations of the physical libration constants based on heliometer data. The solution, including the free libration in longitude, is:

$$I = 1°32'1'' \pm 7''.1 \tag{79}$$

$$f = 0.633 \pm 0.011 \tag{80}$$

Using Koziel's values in a computer solution for the libration theory, Eckhardt (Refs. 57 and 60) obtains:

$$\tau = \Sigma\, a_i \sin(nl + ml' + pF + qD) \tag{81}$$

$$I\sigma = I\tau + \Sigma b_i \sin(nl + ml' + pF + qD) \tag{82}$$

$$\rho = \Sigma c_i \cos(nl + ml' + pF + qD) \tag{83}$$

where

$$l = g$$

$$l' = g'$$

$$F = g + \omega$$

$$D = g - g' + \omega - \omega'$$

and the coefficients and multipliers are given in Table 8. Terms with coefficients less than 1''.0 are omitted. Considering the uncertainties in the libration constants, it seems satisfactory to adopt a libration model containing only the largest terms in Table 8, e.g., terms greater than 10''. The free libration term in longitude, as determined by Koziel (Ref. 57), has an amplitude greater than 10'' and should be included as follows:

$$A \sin[a + Qt] \tag{84}$$

where

$A = 18''.7 \pm 4''.7$

$a = 334°.3 \pm 15°.7$

t = days from 1800.0

$Q = \left(2.9559 \dfrac{GE}{a_☾^3} \gamma\right)^{1/2}$ [rad/day]

GE = gravity constant for earth

$a_☾$ = perturbed semi-major axis of moon's orbit

$$\gamma = \frac{B - A}{C}$$

The value of f and the moments of inertia can also be inferred from the values of the gravitational harmonics as follows (see particularly Ref. 61):

$$f = \frac{C_{20} + 2C_{22}}{C_{20} - 2C_{22}} \tag{85}$$

The recent determination by Tolson and Gapcynski (Ref. 51), shown in Table 7 for the values of C_{20} and C_{22}, yields

$$f = 0.669 \pm 0.043 \tag{86}$$

On the other hand, the JPL determination (Table 6) provides the rather high value of

$$f = 0.828 \pm 0.06 \tag{87}$$

where the standard deviation indicated is a formal value.

Because of the poor results from the *Lunar Orbiter* determination, Koziel's value should be adopted at this time. However, the gravity potential method should eventually provide the most accurate determination.

The selenographic coordinates described so far are dynamic and are not strictly related to lunar surface features. Large efforts are underway at ACIC and AMS to relate surface features to a selenographic coordinate

Table 8. Series for the physical Librations of the moon

Symbols		Argument Multipliers				Coefficient, arc sec	
		n	m	p	q		
τ	(sin)	0	0	2	−2	(a_i)	1.7
		0	1	0	0		91.7
		1	−1	0	−1		−1.2
		1	0	0	−2		4.2
		1	0	0	−1		−3.5
		1	0	0	0		−16.9
		2	−1	0	−2		1.0
		2	0	−2	0		15.3[a]
		2	0	0	−2		10.0
$I\sigma$	(sin)	0	0	2	−2	(b_i)	−3.2
		0	0	2	0		−10.6
		1	0	−2	0		−23.8
		1	0	0	−2		2.5
		1	0	0	0		−100.7
ρ	(cos)	0	0	2	−2	(c_i)	−3.2
		0	0	2	0		−11.0
		1	0	−2	0		23.9
		1	0	0	−2		−1.9
		1	0	0	0		−98.5

[a]Improved value from private communication with D. H. Eckhardt.

grid. The heliometer analysis of Koziel also requires such a relation, which is given as the coordinates of Mösting A (Ref. 57):

$$\lambda = -5°09'50'' \pm 4''.5$$

$$\beta = -3°10'47'' \pm 4''.4$$

$$h = 932''.98 \pm 0''.19$$

$$= 1738.7 \text{ km} \pm 0.35 \qquad (a_☾ = 384400 \text{ km})$$

Determinations of coordinate grids are strictly done with respect to a system fixed at an assumed center of figure. Therefore, the location of a given feature from, say, an ACIC map, must be corrected by the offset of the center of figure from the center of mass, in order to obtain the coordinates with respect to the dynamical coordinate system defined above.

IV. Planets and the Sun

A. Planetary Figures

The diameters of the planets are directly observed in terms of their apparent equatorial angular semi-diameters. Adopted values are reduced to unit distance (1 AU). The adopted values of semi-diameter at unit distance in the American Ephemeris (AE) are tabulated on page 491 of Ref. 16, and are reproduced in Table 9, along with the conversion to km (1 AU = 149597893 km). The historical authorities for the adopted AE values are listed on page 194 of Ref. 16.

Reference 16 also lists planetary radii in terms of earth radius, for "recent values for the angular semi-diameters," for which sources are unidentified. These are used to determine the values from Ref. 16 in Table 10.

Improved values of the radii of Mercury and Venus are available through radar tracking of the planets and spacecraft (Refs. 62, 63, 64, and 65). The radius values adopted for Mercury and Venus were obtained from an averaging of the results in these references. Based on data from *Mariner IV* (Ref. 66) and an adopted value of the flattening, Cain infers (Ref. 67) an equatorial radius for Mars of 3393.4 ± 4.0 km. The JPL adopted values of planetary radii are given in Table 10.

The flattening of a planet may be observed by either direct observation of the equatorial and polar semi-diameters (geometric flattening), or inferred through the action of the equatorial bulge on its satellites (dynamic flattening). A relationship can be stated between the geometric flattening, f, and the dynamic flattening, in terms of the form factor, J_2, by assuming hydrostatic equilibrium (see Section II-B), as follows:

$$\frac{3}{5} \cdot \frac{C-A}{C} = \frac{3}{2} J_2 = f - \frac{1}{2} m + 0(f^2) \qquad (88)$$

where

$$m = \frac{a_e \omega^2}{g_e}$$

C = polar moment of inertia

A = equatorial moment of inertia

No flattening has been observed by either method for Mercury, Venus, and Pluto. Processing of tracking data from *Mariner V* may eventually yield a value for Venus dynamic flattening. Dicke (Ref. 68) has made a series of measurements to detect an oblateness of the photosphere of the sun in which he quotes a value for f of $(5.0 \pm 0.7) \times 10^{-5}$. However, this result is considered to be provisional at this time. Furthermore, there is some controversy over the issue of inferring from a photospheric

Table 9. Radii based on adopted AE semi-diameters

Body	AE Semi-Diameter at 1 AU, sec of arc	Equatorial Radius, a_e, km
Sun[a]	959.63	695992.
Mercury	3.34	2422.
Venus	8.41	6100.
Mars	4.68	3394.
Jupiter	98.47	71417.
Saturn	83.33	60437.
Uranus	34.28	24862.
Neptune	36.56	26516.
Pluto	10.	7253.

[a]Excluding irradiation.

Table 10. JPL-adopted radii

Body	Equatorial Radius, km	Precision, km	References
Sun	695992.0	—	16
Mercury	2435.0	3	63, 64
Venus	6052.0	3	62, 63, 64 and 65
Mars	3393.4	4	67
Jupiter	71372.0	—	16
Saturn	60401.0	—	16
Uranus	23535.0	—	16
Neptune	22324.0	—	16
Pluto	7016.0	—	16

oblateness a corresponding oblateness for the solar interior and, hence, a derived value for J_2. Consequently, the JPL-adopted value for J_2 of the sun is zero. Furthermore, the adopted value for all harmonic coefficients of the sun and the planets (except the earth and moon) higher than degree 2 is zero. Table 11 is a listing of JPL-adopted geometric flattening and corresponding J_2 and Table 12 is a listing of JPL-adopted dynamic form J_2 and corresponding f.

Table 11. JPL-adopted geometric flattening and corresponding J_2

Planet	Observed f	Reference	Computed J_2
Mars	0.0105	69, page 25	0.0055
Jupiter	0.0667	16, page 139	0.0161
Saturn	0.105	16, page 139	0.022
Uranus	0.0625	16, page 491	0.023
Neptune	None	—	—

Table 12. JPL-adopted dynamic form J_2 and corresponding f

Planet	Observed J_2	Reference	Computed f
Mars	0.00197	67, 69	0.00525
Jupiter	0.0296	16, page 328	0.0878
Saturn	0.027	16, page 328	0.115
Uranus	None	—	—
Neptune	0.0035	16, page 390	0.0177

B. Planetary Rotations

The rotation of the planets and the sun can be obtained from observations of surface features, action on planetary satellite orbits, and radar spectra. A complete specification of the rotation includes:

(1) Adopted values of the right ascension and declination of the pole at some reference epoch, $\alpha_0(t_0)$, $\delta_0(t_0)$.

(2) The rates of right ascension and declination of the pole, $\dot{\alpha}$, $\dot{\delta}$.

(3) An adopted value of the hour angle of the vernal equinox measured from the prime meridian at some reference epoch, $V_0(t'_0)$.

(4) The sidereal period of rotation, P.

Item 1 is conventionally given with respect to the mean earth equator and equinox of date. The convention of this document is to take the north pole in the direction of the angular momentum vector, which for Venus is in the south ecliptic hemisphere.

Item 2 then is the sum of the precessions of the earth's equator on the ecliptic and the planet's equator on its orbit.

Item 3 is generally computed from the adopted value of the central meridian at some reference epoch. Complete specifications are adopted for the sun, Venus, Mars, and Jupiter. The rotation period is given for all planets. The form of the specifications is:

$$\alpha = \alpha_0 + \dot{\alpha}(t-t_0) \tag{89}$$

$$\delta = \delta_0 + \dot{\delta}(t-t_0) \tag{90}$$

$$V = V_0 + \frac{2\pi}{P}(t - t'_0) \tag{91}$$

In the following expressions, t refers to Besselian date (see Section II-E-9) and JD to Julian Ephemeris date.

1. Sun. The rotational data for the sun is as follows (Ref. 16, page 307)[16]:

$$\alpha = 285^{\circ}.82 + 0.001993\,(t - 1850.0) \tag{92}$$

$$\delta = 63^{\circ}.62 + 0.001518\,(t - 1850.0) \tag{93}$$

$$V = 180^{\circ} + 14.18439716\,(\mathrm{JD} - 2398220.0) \tag{94}$$

$$P = 25.38\ \mathrm{d} \tag{95}$$

2. Mercury. The rotational data for Mercury is as follows (Refs. 70, 71, 72, and 73):

$$P = 58.67 \pm 0.03\ \mathrm{d} \tag{92}$$

The ⅔ synchronism of Mercury's rotation with its orbital motion was discovered in 1965 by Pettengill and Dyce (Ref. 72), but the precision obtainable with radar technique is considerably poorer than is possible, at least in principle, from optical data. The adopted figure is based on a recent re-examination of *Pic du Midi* data (Ref. 73).

3. Venus. The rotational data for Venus is as follows (Refs. 74 and 75):

$$\alpha = 98^{\circ} - 0.0015551\,(t - 1964.5) \tag{93}$$

$$\delta = -69^{\circ} - 0.0007748\,(t - 1964.5) \tag{94}$$

$$V = 238.75 + 1.483924\,(\mathrm{JD} - 2438566.5) \tag{95}$$

$$P = 242.6\ \mathrm{d} \tag{96}$$

[16]The adopted rotational period is that associated with the mean synodic rate (see Ref. 16, pages 307 and 489).

It should be noted that, for Venus, V is measured from the vernal equinox (i.e., the ascending node of the orbit on the equator) in the direction of rotation to the prime meridian. Because of Venus' retrograde rotation, the obliquity of the orbit is greater than 90°, and the longitude of the central meridian, LCM, on 2438566.5 is 40°, taken positive in the direction of rotation (or −40° in the AE convention, which is positive west, to be used in the equation $V - A_E = LCM$.). This is opposite to the selections shown in Refs. 74 and 75.

4. Mars. The rotational data for Mars is as follows (Refs. 76 and 77):

$$\alpha = 316^\circ\!.55 + 0.006751\,(t - 1905.0) \tag{97}$$

$$\delta = 52^\circ\!.85 + 0.003480\,(t - 1905.0) \tag{98}$$

$$V = 149^\circ\!.475 + 350.891962\,(\mathrm{JD} - 2418322.0) \tag{99}$$

$$P = 24^{\mathrm{h}}\,37^{\mathrm{m}}\,22^{\mathrm{s}}\!.6689 \tag{100}$$

It should be noted that the rates $\dot{\alpha}, \dot{\delta}$ are computed as in Ref. 77; the values of $\dot{\alpha}, \dot{\delta}$ and, therefore, $V_0(t'_0)$, differ from those adopted in AE 1968, but are consistent with the values to be adopted beginning with AE 1971.[17]

5. Jupiter. The rotational data for Jupiter is as follows (Ref. 16, page 338; Ref. 76):

$$\alpha = 268^\circ\!.0035 + 0.00103\,(t - 1910.0) \tag{101}$$

$$\delta = 64^\circ\!.5596 - 0.00017\,(t - 1910.0) \tag{102}$$

$$V = 281^\circ\!.001 + 877.90\,(\mathrm{JD} - 2414120.0) \tag{103}$$

$$P = 9^{\mathrm{h}}50^{\mathrm{m}}30^{\mathrm{s}}\!.003 \tag{104}$$

It should be noted that System I for the equatorial region is adopted.

6. Saturn. The rotational data for Saturn is as follows (Ref. 16, pages 365, 491):

$$\alpha = 38^\circ\!.6159 + 0.00011802\,(\mathrm{JD} - 2435000.5) \tag{105}$$

$$\delta = 83^\circ\!.3308 + 0.00001182\,(\mathrm{JD} - 2435000.5) \tag{106}$$

$$P = 10^{\mathrm{h}}\,14^{\mathrm{m}} \tag{107}$$

[17]Private communication, R. L. Duncombe, April 17, 1968.

7. Uranus The rotational data for Uranus is as follows (Ref. 16, pages 387, 491):

$$\alpha = 76^\circ\!.051 + 0.0142\,(t - 1900.0) \tag{108}$$

$$\delta = 14^\circ\!.855 + 0.0013\,(t - 1900.0) \tag{109}$$

$$P = 10^{\mathrm{h}}\,49^{\mathrm{m}} \tag{110}$$

8. Neptune. The rotational data for Neptune is as follows (Ref. 16, pages 391, 491):

$$\alpha = 295^\circ\!.153 + 0.008364\,(t - 1900.0) \tag{111}$$

$$\delta = 41^\circ\!.348 + 0.002367\,(t - 1900.0) \tag{112}$$

$$P = 14^{\mathrm{h}} \tag{113}$$

9. Pluto. The rotation period for Pluto is as follows (Ref. 16, page 491):

$$P = 6.39\ \mathrm{d} \tag{114}$$

For those planets having defined poles, additional angles describing the orientation of orbit and equator can be obtained as in Section 11-G of Ref. 16. Mean orbital elements are taken from Section 4-D of the same reference.

C. Planetary Orbits

For mission design and other qualitative purposes, a convenient and moderately accurate representation of the planetary motions is often very useful. With this in mind, the mean elements of the major planets are presented in Table 13; the sources for this material are given in Ref. 15, pages 111 and 114. Osculating elements for JED 2433280.5 presented in Table 23 are based on planetary theories very similar to those from which Table 13 is derived. The mean Keplerian elements in Table 13 are referred to the mean equinox and ecliptic of date for all planets except Pluto. For elements of Pluto, the use of the osculating elements in Table 23 is recommended. The epoch is 1900 January 0.5 ET or, equivalently, JED 2415020.0. The time interval from the epoch is denoted by T when measured in Julian centuries of 36525 ephemeris days, by $D = 3.6525\ T$ when measured in units of 10000 ephemeris days, and by $d = 10000\ D = 36525\ T$ when measured in ephemeris days.

Table 13. Mean ecliptic elements[a]

Mercury	
$a = 0.38709\,86$ $i = 7^\circ\,00'\,10''.37 + 6''.699\,T - 0''.066\,T^2$ $\Omega = 47^\circ\,08'\,45''.40 + 4266''.75\,T + 0''.626\,T^2$ $\tilde{\omega} = 75^\circ\,53'\,58''.91 + 5599''.76\,T + 1''.061\,T^2$ $n^* = 53\,81016''.3093 - 0''.00049\,5\,T$	$e = 0.20561\,421 + 0.00002\,046\,T - 0.00000\,0030\,T^2$ $M = 102^\circ\,16'\,45''.77 + (415^r + 2\,61055''.04)T + 0''.024\,T^2$ † $= 102^\circ.27938\,1 + 4^\circ.09233\,44364\,d + 0^\circ.00000\,050\,D^2$ $L = 178^\circ\,10'\,44''.68 + (415^r + 2\,66654''.80)T + 1''.084\,T^2$ $= 178^\circ.17907\,8 + 4^\circ.09237\,70233\,d + 0^\circ.00002\,26\,D^2$
Venus	
$a = 0.72333\,16$ $i = 3^\circ\,23'\,37''.07 + 3''.621\,T - 0''.0035\,T^2$ $\Omega = 75^\circ\,46'\,46''.73 + 3239''.46\,T + 1''.476\,T^2$ $\tilde{\omega} = 130^\circ\,09'\,49''.8 + 5068''.93\,T - 3''.515\,T^2$ $n^* = 21\,06631''.3832 + 0''.00009\,6\,T$	$e = 0.00682\,069 - 0.00004\,774\,T + 0.00000\,0091\,T^2$ $M = 212^\circ\,36'\,11''.59 + (162^r + 7\,12093''.95)T + 4''.6298\,T^2$ $= 212^\circ.60321\,9 + 1^\circ.60213\,01540\,d + 0^\circ.00009\,6400\,D^2$ $L = 342^\circ\,46'\,01''.39 + (162^r + 7\,17162''.88)T + 1''.1148\,T^2$ $= 342^\circ.76705\,3 + 1^\circ.60216\,87039\,d + 0^\circ.00002\,3212\,D^2$
Earth (Barycenter)	
$a = 1.00000\,023$ $L = 99^\circ\,41'\,48''.04 + 1296\,02768''.13\,T + 1''.089\,T^2$ $= 99^\circ\,69667\,8 + 0^\circ.98564\,73354\,d + 0^\circ.00002\,267\,D^2$ $\tilde{\omega} = 101^\circ\,13'\,15''.00 + 6189''.03\,T + 1''.63\,T^2 + 0''.012\,T^3$ $= 101^\circ.22083\,3 + 0^\circ.00004\,70684\,d + 0^\circ.00003\,39\,D^2$ $+ 0^\circ.00000\,007\,D^3$ $M = 358^\circ\,28'\,33''.04 + 1295\,96579''.10\,T - 0''.54\,T^2$ $- 0''.012\,T^3$	$= 358^\circ.47584\,5 + 0^\circ.98560\,02670\,d - 0^\circ.00001\,12\,D^2$ $- 0^\circ.00000\,007\,D^3$ $e = 0.01675\,104 - 0.00004\,180\,T - 0.00000\,0126\,T^2$ $= 0.01675\,104 - 0.00001\,1444\,D - 0.00000\,00094\,D^2$ $\bar{\epsilon} = 23^\circ\,27'\,08''.26 - 46''.845\,T - 0''.0059\,T^2 + 0''.00181\,T^3$ $= 23^\circ.45229\,4 - 0^\circ.00356\,26\,D - 0^\circ.00000\,0123\,D^2$ $+ 0^\circ.00000\,00103\,D^3$ $\pi = 0''.4711 - 0''.0007\,T = 0^\circ.00013\,086 - 0^\circ.00000\,0053\,D$ $\Pi = 173^\circ\,57'.06 + 54'.77\,T = 173^\circ\,9510 - 0^\circ.2499\,D$
Mars	
$a = 1.52369\,15$ $i = 1^\circ\,51'\,01''.20 - 2''.430\,T + 0''.0454\,T^2$ $\Omega = 48^\circ\,47'\,11''.19 + 2775''.57\,T - 0''.005\,T^2 - 0''.0192\,T^3$ $\tilde{\omega} = 334^\circ\,13'\,05''.53 + 6626''.73\,T + 0''.4675\,T^2 - 0''.0043\,T^3$ $n^* = 6\,89050''.9262 + 0''.00016\,9\,T$ $e = 0.09331\,290 + 0.00009\,2064\,T - 0.00000\,0077\,T^2$	$M = 319^\circ\,31'\,45''.93 + (53^r + 2\,15490''.60)T + 0''.6509\,T^2$ $+ 0''.0043\,T^3$ $= 319^\circ.52942\,5 + 0^\circ.52402\,07666\,d + 0^\circ.00001\,3553\,D^2$ $+ 0^\circ.00000\,0025\,D^3$ $L = 293^\circ\,44'\,51''.46 + (54^r + 2\,22117''.33)T + 1''.1184\,T^2$ $= 293^\circ.74762\,8 + 0^\circ.52407\,11638\,d + 0^\circ.00002\,3287\,D^2$
Jupiter	
$n^* = 109256''.6481$ $a = 5.202803$ $i = 1^\circ\,18'\,31''.30 - 20''.00\,T$ $\Omega = 99^\circ\,26'\,16''.30 + 3639''.50\,T$	$\tilde{\omega} = 12^\circ\,42'\,41''.12 + 5800''.79\,T$ $e = 0.0483376 + 0.00016302\,T$ $M = 136^\circ\,37'\,44''.88 + 299''.123557\,d$

[a]Definitions of symbols:

n^* = the sidereal mean motion in a Julian year

a = the perturbed semi-major axis of the orbit; n, the mean daily motion, and a are related by $n^2a^3 = k^2\,(1 + m)$, where k is the Gaussian gravitational constant and m is the mass of the planet expressed in terms of the sun's mass

i = the inclination of the orbit to the ecliptic

Ω = the longitude of the ascending node of the orbit on the ecliptic, measured from the equinox

$\tilde{\omega}$ = the longitude of perihelion, measured from the equinox along the ecliptic to the node, and then along the orbit from node to perihelion, i.e., $\tilde{\omega} = \Omega + \omega$, where ω is the argument of perihelion

e = the eccentricity of the orbit

M = the mean anomaly, defined by the relation $M = n$ (time in days since perihelion passage); this is related to L, the mean longitude, by the relation $L = M + \tilde{\omega}$

$\bar{\epsilon}$ = mean obliquity of the ecliptic

π = annual rate of rotation of the ecliptic

Π = longitude of axis of rotation of the ecliptic

† the superscript r denotes revolutions.

Table 13 (contd)

Saturn	
$n^* = 43996''.18875$ $a = 9.538843$ $i = 2°\,29'\,33''.30 - 16''.00\,T$ $\Omega = 112°\,47'\,00''.84 + 3143''.43\,T$	$\tilde{\omega} = 91°\,05'\,19''.72 + 7053''.97\,T$ $e = 0.0558900 - 0.00034705\,T$ $M = 174°\,19'\,45''.64 + 120''.39926\,d$
Uranus	
$n^* = 15426''.33375$ $a = (19.182281 - 0.00057008\,T)$ $i = 00°\,46'\,21''.80 + 2''.00\,T$ $\Omega = 73°\,29'\,23''.65 + 1838''.25\,T$	$\tilde{\omega} = 169°\,02'\,11''.22 + 5846''.80\,T$ $e = 0.0470463 + 0.00027204\,T$ $M = 73°\,35'\,18''.25 + 42''.2131\,d$
Neptune	
$n^* = 7864''.563$ $a = (30.057053 + 0.001210166\,T)$ $i = 1°\,46'\,45''.30 - 33''.00\,T$ $\Omega = 130°\,41'\,43''.27 + 3966''.54\,T$	$\tilde{\omega} = 43°\,45'\,49''.24 + 3161''.45\,T$ $e = 0.00852849 + 0.00007701\,T$ $M = 41°\,16'\,50''.73 + 21''.3092\,d$

D. Planetary Satellites

Each of the major planets, from earth to Neptune, possesses one or more natural satellites. Earth's own satellite, the moon, is treated in Section III of this document. Therefore, this section will be restricted to a brief discussion of the satellites of the other planets.

The most notable thing about the planetary satellites is the degree to which knowledge of them is incomplete and inhomogeneous. Masses, even very approximate ones, are known for less than half of these satellites, physical dimensions for even fewer, compositions are quite unknown. It is doubtful that an entirely satisfactory orbital theory exists for any one of them, and the theories that do exist are frequently difficult to compare with one another on a rational basis. Numerical integration of the equations of motion are available for only a few objects. Therefore, it would require a major effort of doubtful reliability to provide a consistent set of data, such as osculating elements at some common epoch, for all of these objects. Accordingly, Table 14 lists conventional values of the semi-axis, sidereal period, eccentricity and inclination, the mass and radius (if known), the author of the most reliable published orbit, the availability of ephemerides, and the investigators known to be currently interested in improving the orbits.

Many of the satellites are subjected to very strong perturbations arising from solar attraction, planetary oblateness, or mutual resonances. Therefore, the orbital data are given primarily for informational purposes rather than computational. As an extreme example, Jupiter VIII varies from 0.3 to 0.66 in osculating eccentricity in a relatively short span of time. The orbits for the Uranian satellites are extremely uncertain. As may be seen from their radii, the Galilean satellites (Jupiter I through IV) are large enough to be significant for close encounter missions to Jupiter. Two of them are suspected of having atmospheres.

Table 14. Orbital and physical data of planetary satellites[a]

Planet	a, AU	P, days	e	i, deg	Mass	Radius, km	Orbit	Ephemeris	Known Current Investigators
Mars I (Phobos)	0.0627	0.319	0.0170	0.95 A			Struve	A1	W
II (Deimos)	0.1570	1.262	0.0031	1.73 A			Struve	A1	W
Jupiter I (Io)	0.00282	1.769	0	0 B	26(3)	1670	de Sitter	A5	Ma, Me
II (Europa)	0.00449	3.551	0.0003	0 B	40(3)	1460	de Sitter	A5	Ma, Me
III (Ganymede)	0.00716	7.154	0.0015	0 B	123(2)	2550	de Sitter	A5	Ma, Me
IV (Callisto)	0.01259	16.689	0.0075	0 B	20(3)	2360	de Sitter	A5	Ma, Me
V	0.00121	0.498	0.0030	0.4 B			van Woerkom	A4	
VI	0.07672	250.6	0.1580	28.4 C			Bobone	A3	B, H, Mu
VII	0.07846	259.6	0.2072	27.8 C			Bobone	A3	B, H, Mu
VIII	0.1572	737.	0.40	147. C			Grosch	I2	B, H, K
IX	0.1585	758.	0.275	157. C			Nicholson	I2	B, H
X	0.07705	253.	0.1405	28.4 C			Wilson	I2	B, H, Mu
XI	0.15083	692.	0.2068	163.4 C			Herget	I2	B, H
XII	0.14177	631.	0.1687	146.7 C			Herrick	I2	B, H
Saturn I (Mimas)	0.00124	0.942	0.0201	1.52 B	15(6)		Struve	A1	
II (Enceladus)	0.00159	1.370	0.0044	0.03 B	8(6)		Struve	A1	
III (Tethys)	0.00197	1.887	0	1.10 B	87(4)		Struve	A1	
IV (Dione)	0.00252	2.737	0.0022	0.03 B	55(4)		Struve	A1	
V (Rhea)	0.00352	4.517	0.0010	0.34 B	25(4)	700	Struve	A1	
VI (Titan)	0.00817	15.95	0.0291	0.33 B	4150	2440	Struve	A1	
VII (Hyperion)	0.00989	21.28	0.1042	0.43 B	5(6)		Struve	A1, A3	
VIII (Iapetus)	0.02380	79.33	0.0283	18.4 D	5(5)		Struve	A1, A3	
IX (Phoebe)	0.08658	550.4	0.1632	173.9 D			Zadunaisky	A3	
X	Undetermined								
Uranus I (Ariel)	0.00128	2.520	0	0 B			Struve, Newcomb	A1	
II (Umbriel)	0.00179	4.144	0	0 B			Struve, Newcomb	A1	
III (Titania)	0.00293	8.706	0	0 B			Struve	A1	
IV (Oberon)	0.00392	13.46	0	0 B			Struve	A1	
V (Miranda)	0.00085	1.414	0	0 B					
Neptune I (Triton)	0.00236	5.877	0	159.9 B	750	2000	Alden	A1	
II (Nereid)	0.03718	360.	0.7493	27.8 E			V. Biesbroeck		K

[a]Inclination with respect to:
- A = Laplacian plane
- B = planetary equator
- C = planetary orbit
- D = ecliptic
- E = earth equator

Mass:
- Number in parenthesis is power of 10, e.g., 2(2) = 2 × 10^2
- Mass unit is inverse mass referred to planet

Ephemeris source:
- A = American Ephemeris and Nautical Almanac
- I = IAU circulars when available

Ephemeris type:
- 1 = apparent distance and position angle
- 2 = α, δ
- 3 = $\Delta\alpha$, $\Delta\delta$
- 4 = elongation times
- 5 = geocentric phenomena

Known current investigators:
- B = A. Bec, Bureau des Longitudes, Paris
- H = P. Herget, Cincinnati Observatory
- K = J. Kovalevsky, Bureau des Longitudes, Paris
- Ma = B. G. Marsden, Smithsonian Astrophysical Observatory
- Me = S. F. Mello, São Paulo
- Mu = J. D. Mulholland, JPL
- W = G. A. Wilkins, Royal Greenwich Observatory

V. Astronomical Constants

A. The International Astronomical Union System

In many fields of Astronomy, it is necessary to compare observations made over wide spans of time. So as to have a common basis for comparison, it is essential that star catalogs and ephemerides be based on a fixed and consistent set of astronomical constants. Consequently, there is a reluctance to introduce new determinations of constants into theories of celestial mechanics, ephemerides, or star catalogs. On the other hand, in space trajectory and navigation applications, it is important that the most accurate set of constants available be used. This has resulted in a dual system in a sense: (1) the set of nearly self-consistent constants used by the astronomical community not readily subject to change, and (2) the set of constants used in space technology which experiences almost continual updating.

The official organ for establishing the astronomical constants is the International Astronomical Union (IAU). In 1963, the directors of the principal national and international ephemerides, together with other experts, met in Paris to consider the system of astronomical constants (Ref. 78). This was the fourth such meeting; the first one occurred in 1896. The primary motivation for this meeting was due to improvements in certain constants resulting from early space applications, such as *Mariner 2* and planetary radar bounce measurements. A working group was organized at this meeting to establish a new system of astronomical constants. The report of this working group was approved during the 12th General Assembly of the IAU in 1964, and is included in the proceedings (Ref. 7). The Working Group defined three categories of constants as defining constants, primary constants, and derived constants.

The number of ephemeris seconds in 1 tropical year and the Gaussian constant are the two defining constants. These constants define the units of time and length used in Celestial Mechanics and are not subject to improvement.

The choice of constants designated as primary is based mainly on the direct nature of their determination, although, in some cases, the designation of primary or derived is rather arbitrary. The designated primary constants form an independent set. With the list of auxiliary constants and factors, all derived constants may be directly determined from these defining and primary constants.

The IAU system of constants is presented in Table 15, and the set of accompanying notes are presented in the Appendix. The constants and notes have been taken directly from the report of the Working Group (Refs. 7 and 76). These accompanying notes include explanatory material, differential correction factors relating derived quantities to primary quantities, and error bound estimates on the primary constants. (For a more extensive discussion of this system see Refs. 7 and 22.)

B. The Adopted JPL System

During the past few years, dramatic improvement in astronomical constants has been achieved through the use of spacecraft radio tracking data and planetary radar bounce measurements. This is emphasized by the fact that certain constants in the IAU system of 1964 are out of date and of insufficient accuracy for astrodynamic purposes. For this reason, it became necessary to modify the IAU system to a system suitable for space trajectory and navigation purposes; this modified system is called the adopted JPL System of Astronomical Constants (Tables 16 and 17).

With one exception, JPL's choice of primary and derived constants is the same as the IAU system. The exception is JPL's choice of the light time of 1 AU, τ_A, as a primary constant and the astronomical unit in kilometers as a derived constant. The rationale for this choice is because of the precision with which τ_A may be determined from planetary radar bounce time delay measurements. The current estimated accuracy of τ_A is 15 μsec, or approximately 0.03 ppm. The accuracy of the astronomical unit A, expressed in metric units such as kilometers, depends on the measured value of the speed of light through the relation $A = c\tau_A$. The accuracy of c is approximately 0.3 km/sec or 1 ppm. Thus, not only is τ_A the quantity that is directly measured, but it is also one of the most precisely determined constants in astronomy.

A further consideration for JPL's choice of τ_A as a primary constant is the superfluous nature of the meter as a unit of distance in deep space trajectory and navigation applications. The light-second (the distance traveled in a gravity-free field by a photon in 1 ephemeris sec) is the fundamental distance unit in deep space applications. With this choice of unit, the velocity of light in metric units is explicitly absent in the observable equations for range and doppler measurements (see Ref. 79, for example). The range observable become a time-delay observable and the doppler observable becomes the

Table 15. IAU system of constants

Constant	Value
Defining constants	
1 Number of ephemeris seconds in 1 tropical year (1900)	$s = 31556925.9747$
2 Gaussian gravitational constant, defining the AU	$k = 0.01720209895$
IAU primary constants	
3 Measure of 1 AU in m	$A = 149600 \times 10^6$
4 Velocity of light in m/sec	$c = 299792.5 \times 10^3$
5 Equatorial radius for earth in m	$a_e = 6378160$
6 Dynamical form-factor for earth	$J_2 = 0.0010827$
7 Geocentric gravitational constant (units: m^3s^{-2})	$GE = 398603 \times 10^9$
8 Ratio of the masses of the moon and earth	$\mu = 1/81.30$
9 Sidereal mean motion of moon in rad/sec (1900)	$n_{\text{☾}}{}^* = 2.661699489 \times 10^{-6}$
10 General precession in longitude per tropical century (1900)	$p = 5025''.64$
11 Obliquity of the ecliptic (1900)	$\epsilon = 23°27'08''.26$
12 Constant of nutation (1900)	$N = 9''.210$
Auxiliary constants and factors	
k/86400, for use when the unit of time is 1 sec	$k' = 1.990983675 \times 10^{-7}$
Number of seconds of arc in 1 rad	$= 206264.806$
Factor for constant of aberration (note 15)	$F_1 = 1.000142$
Factor for mean distance of the moon (note 20)	$F_2 = 0.999093142$
Factor for parallactic inequality (note 23)	$F_3 = 49853''.2$

Constant	Value
IAU derived constants	
13 Solar parallax	$\arcsin(a_e/A) = \pi_{\odot} = 8''.79405 \; (8''.794)$
14 Light time for unit distance	$A/c = \tau_A = 499^{s}.012 = 1^{s}/0.00200396$
15 Constant of aberration	$F_1 k' \tau_A = \kappa = 20''.4958 \; (20''.496)$
16 Flattening factor for earth	$f = 0.0033529 = 1/298.25$
17 Heliocentric gravitational constant (units: m^3s^{-2})	$A^3k'^2 = GS = 132718 \times 10^{15}$
18 Ratio of masses of sun and earth	$(GS)/(GE) = S/E = 332958$
19 Ratio of masses of sun and earth + moon	$S/E(1+\mu) = 328912$
20 Perturbed mean distance of moon, in meters	$F_2(GE(1+\mu)/n_{\text{☾}}^{*2})^{1/3} = a_{\text{☾}} = 384400 \times 10^3$
21 Constant of sine parallax for moon	$a_e/a_{\text{☾}} = \sin \pi_{\text{☾}} = 3422''.451$
22 Constant of lunar inequality	$\frac{\mu}{1+\mu}\frac{a_{\text{☾}}}{A} = L = 6''.43987 \; (6''.440)$
23 Constant of parallactic inequality	$F_3 \frac{1-\mu}{1+\mu}\frac{a_{\text{☾}}}{A} = P_{\text{☾}} = 124''.986$

IAU system of planetary masses

Planet	Reciprocal mass
Mercury	6000000
Venus	408000
Earth + moon	329390
Mars	3093500
Jupiter	1047.355
Saturn	3501.6
Uranus	22869
Neptune	19314
Pluto	360000

change in phase time delay per unit time. The only necessity for c in metric units is in the matching of launch trajectory and ballistic trajectory initial conditions (also, in the application of midcourse maneuvers); launch trajectories are in metric units, since the thrust of the launch vehicle is calibrated in metric units. However, the error in this matching process because of an error in c is negligible compared to execution errors in the guidance system. All of the constants appearing in space applications involving distance have been determined in terms of light-seconds more accurately than in any other unit. The gravitational constant (GM) of a planet, for example, is determined in units of (light-sec)^3/sec^2. It follows that space trajectories and observables described in terms of units of light-seconds and seconds, along with the required constants expressed in the same units, form a self-

Table 16. JPL-adopted astronomical constants–primary constants

Constant	Symbol	Value	Accuracy	Source
Light time for 1 AU in A.1 sec[a]	τ_A	499.004788	0.000015	Refs. 62, 63, and 64
Speed of light in km/sec	c	299792.5	0.3	IAU
Equatorial radius for earth in km	a_e	6378.160	0.005	IAU, Ref. 3
Dynamical form-factor for earth	J_2	0.0010827		IAU, Ref. 4
Geocentric gravitational constant (units: $km^3 s^{-3}$)	GE	398601.2	0.4	Ref. 81
Ratio of the masses of the earth and moon	μ^{-1}	81.3010	0.001	Ref. 84 and b
Sidereal mean motion of moon in rad/sec (1900)	$n^*_{☾}$	$2.661699489 \times 10^{-6}$	0.5×10^{-15}	IAU
General precession in longitude per tropical century (1900)	p	5025″.64	1″.0	IAU
Obliquity of the ecliptic (1900)	ϵ	23°27′08″.26	0″.1	IAU
Constant of nutation (1900)	N	9″.210	0″.01	IAU

[a]The length of an A.1 second and an ET second are considered to be the same in this report.
[b]Private communications, J. D. Anderson, results from recent analysis of *Mariner V* cruise and Venus encounter range and doppler tracking data using DE 40, May 7, 1968.

Table 17. JPL-adopted astronomical constants–derived constants

Constant	Symbol	Value	Accuracy
Solar parallax	$\pi_\odot$	8″.79417	
Measure of 1 AU in km	$A = c\tau_A$	149597893.0	5.0[a]
Constant of aberration	$\kappa = F_1 k' \tau_A$	20″.4955	0.6×10^{-7}
Flattening factor for earth	f (see Section II-A for definition)	$\begin{cases} = 0.0033529 \\ = 1/298.25 \end{cases}$	5×10^{-7}
Heliocentric gravitational constant (km^3/sec^2)	$GS = A^3 k'^2$	132712499×10^3	15×10^3[a]
Ratio of masses of sun and earth	S/E	332945.6	0.4
Ratio of masses of sun and earth plus moon	$S/E(1+\mu)$	328900.1	0.4
Perturbed mean distance of moon in km	$F_2\,[GE(1+\mu) n^{*2}_{☾}]^{1/3} = a_{☾}$	384399.3	0.2
Constant of sine parallax for moon	$a_e / a_{☾} = \sin \pi_{☾}$	3422″.457	0″.004
Constant of lunar inequality	$\frac{\mu}{1+\mu} \frac{a_{☾}}{A} = L$	6″.43987	0″.00008[a]
Constant of parallactic inequality	$F_3 \frac{1-\mu}{1+\mu} \frac{a_{☾}}{A} = P_{☾}$	124″.9873	0″.0001[a]
Lunar gravitational constant (km^3/sec^2)	$GM = \mu\, GE$	4902.78	0.06

[a]Assumes that c has exactly the IAU value of 299792.5 km/sec.

consistent system; the metric value of c is not required to completely describe this dynamic process. At JPL, for conceptual convenience, space applications quantities are often expressed in terms of kilometers and seconds, such as GE or the AU, and probable errors for these quantities are quoted assuming that c has exactly the IAU value of 299792.5 km/sec. It should be remembered, however, that this is for conceptual convenience only.

The moon-to-earth mass ratio μ has been retained as a primary constant, although there is some argument to do otherwise. The planetary probes measure the lunar inequality directly (Ref. 80), while the lunar probes measure directly and separately the GM values of the earth and the moon. It appears that, at present, the planetary probes give a slightly more accurate determination of μ.

The JPL choice of planetary masses (Table 18) differs significantly from the IAU system, which, of all the IAU constants, are the most seriously out of date. Even the IAU noted the need for a revision (see item 24 in the Appendix). The accuracies of the mass values of Venus, the earth, the moon, and Mars have improved by two or three orders of magnitude through the use of spacecraft measurements (Refs. 54, 80, 81, 82, and 83). These refined mass values are needed for accurate radio guidance and have been adopted at JPL. After tampering

Table 18. JPL system of planetary masses

Body	Reciprocal mass	Reference
Sun	1	
Mercury	5983000 ± 25000	a
Venus	408522 ± 3	88, b
Earth and moon	328900.1 ± 0.3	81
Mars	3098700 ± 100	c
Jupiter	1047.3908 ± 0.0074	84, 85
Saturn	3499.2 ± 0.4	84
Uranus	22930 ± 6	84
Neptune	19260 ± 100	d
Pluto	1812000 ± 40000	e

[a]Private communication, W. A. Melbourne, D. A. O'Handley, and R. E. Reed, results from analysis of Venus radar observations, Aug. 15, 1968.

[b]Private communication, J. D. Anderson, results from recent analysis of *Mariner V* cruise and Venus encounter range and doppler tracking data using DE 40, May 7, 1968.

[c]Private communication, G. W. Null, results from reprocessing *Mariner IV* Mars encounter tracking data using DE 40, May 28, 1968.

[d]This value is a weighted mean based on the sources in Ref. 84 and a recent determination using Triton. (See Gill, J. R., and Gault, B. L., "A new determination of the orbit of Triton, role of Neptune's equator, and mass of Neptune," Astron. papers, vol. 21, part I, in press).

[e]Private communication from R. L. Duncombe, Aug. 29, 1968. See also, Duncombe, R. L., Klepczynski, W. J., and Seidelmann, P. K., "The mass of Pluto," Science, in press.

with the inner planet IAU mass values, it was decided to simultaneously incorporate a more recent set of values for the outer planets that are, in the cases of Jupiter and Saturn, significantly improved (Refs. 84, 85, and 86).

Because the spacecraft measurements provide direct determinations of GM in (light-sec)^3/sec^2, for those objects for which spacecraft measurements are involved, the reciprocal mass is computed from the ratio GM/GS, where GS is the derived heliocentric gravitational constant in (light-sec)^3/sec^2.

It should be recognized that JPL's choice of planetary mass values introduces a small inconsistency with respect to the centennial rate terms of the expressions for the precession constants given in Section II (Eq. 36) which are based on the IAU mass system. This inconsistency may be removed through the differential correction procedure provided in Ref. 19. However, it has long been recognized that the IAU constant of general precession in longitude per tropical century, p, is in error by at least $0\overset{''}{.}8$ per century. This error produces, in turn, corresponding errors of similar size in the centennial rate terms of ζ_0, Z, and θ that are two orders of magnitude larger than the inconsistencies introduced by JPL's new planetary mass values. Similarly, the inconsistency caused in the obliquity is an order of magnitude lower than the uncertainty noted earlier for that quantity. There is a good argument in the astronomical community for not changing the value of p for the sake of preserving consistency among star catalogs. It is safe to say that this subject area involving precessional constants and possible changes in value of these constants, from the point of view of JPL, has an unsatisfactory status at this writing and needs considerable review.

Finally, there are several important relationships among the astronomical constants in addition to the IAU-derived constants (see for example, Ref. 87) which should be explored for self-consistency with these JPL values in hand. This task is beyond the scope of this report.

In addition to the auxiliary constants and factors listed under the IAU system, the constants listed in Table 19 are provided.

Table 19. Auxiliary constants

Constant	Symbol	Value
Universal gravitational constant (Ref. 89)	G	6.673×10^{-23} km^{3}sec^{-2}g^{-1}
Pi	π	3.1415926536 ...
Base for natural logarithms	e	2.718281828 ...
Feet to meter[a]	—	0.3048 (exactly)
Statute mile to kilometer[a]	—	1.609344 (exactly)
New international nautical mile to kilometer	—	1.852 (exactly)
Radians to degrees	—	57.29577951 ...

[a]Based on the new international inch = 2.54 cm, exactly.

C. Solar Pressure

Solar pressure acceleration on a spacecraft is presently modeled with the following vector expression:

$$\mathbf{r} = \frac{C_1 A_p}{m r^2_{sp}} \left[\mathbf{G} + \mathbf{G}' (< EPS) + \Delta\mathbf{G} \right] \qquad (115)$$

where

$$C_1 = \frac{J A_e^2}{c}$$

J = solar flux constant = 2.00 ±0.01 g cal/min cm^2 (determined by Johnson, 1954, see Ref. 90). From this value of J, it follows that $C_1 = 1.04 \times 10^8$ km^3/kg/sec^{2}m^2. Sun spot cycles of 11 years and solar flares may affect this value by 2%

A_p = the sun-lit area of the spacecraft projected onto a plane normal to the sun-spacecraft line, m^2

$m =$ instantaneous mass of probe, kg

$\mathbf{G} =$ effective area which is a function of the size, shape, orientation, and reflective characteristics of the parts of the spacecraft exposed to sunlight divided by A_p. Reference 82 (pp. 8–10, 187–188) gives detailed example.

$$\mathbf{G}' = \frac{d\,\mathbf{G}}{d \measuredangle EPS}$$

$\measuredangle EPS =$ earth-probe-sun angle, radians; to account for antenna position

$\Delta\mathbf{G} =$ increments on $\mathbf{G}$ input as a function of time; to compensate for moving parts on the spacecraft or degradation of the surface reflectivity as a function of time

$A_e =$ mean distance between the earth and sun

$c =$ velocity of light

$r_{sp} =$ sun-probe distance

VI. JPL Ephemeris Tape System

A. Description

Predictions of the motion of celestial bodies can be presented in either of two forms: (1) as general but complicated formulas, with time as argument, from which position at any epoch can be computed, or (2) as tables listing positions at discrete, pre-specified epochs, from which positions at other than tabular epochs can be obtained by interpolation. These tables are called ephemerides. Predictions computed by a special perturbation method can be presented only in ephemeris form.

Because of the many current investigations into the motion of the moon and planets, no set of predicted motions now available can be considered final. Accordingly, the procedures, computer programs, and tape archives of ephemeris data that were used in preparing the new ephemerides have been collected into an operational system that will permit the issuing of updated ephemerides whenever a new issue is desirable and feasible. This system is called the JPL Ephemeris Tape System. The tapes issued under this system are called JPL Ephemeris Tapes. These tapes may be used directly by digital computer programs that require predictions of lunar and planetary positions and velocities.

Input to the system consists of predictions of the positions of the moon and planets obtained by a suitable means and recorded in simple format onto magnetic tapes. These tapes are preserved permanently in archives designated as the source tape library.

The ephemeris data available to users are recorded on a sequence of JPL Ephemeris Tapes. Each set of ephemeris tapes in the sequence consists of three tapes which collectively cover the years 1950 through 2000 with overlaps between tapes, and which carry tabulations of rectangular coordinates and velocity components of the moon and the nine planets with respect to the mean equator and equinox of 1950.0, plus nutations in longitude and obliquity, and their rates, plus modified second and fourth differences of these quantities to facilitate interpolation. The format of the ephemeris tapes is defined in Refs. 91 and 92.

The aim of the JPL Ephemeris Tape System is to maintain ephemeris tapes containing the most accurate predictions of lunar and planetary motion available. The system was designed to prevent degradation of the accuracy of source data during the data processing. The principal provisions for ensuring the accuracy of the data include the following:

(1) Positions and velocities are carried as double-precision floating point numbers (i.e., to approximately 16 decimal places). Thus, predictions more accurate than those now available can be assimilated into the Ephemeris Tape System.

(2) Intervals of tabulation were chosen so that the use of Everett's interpolation formula retaining second and fourth modified differences yields sufficient accuracy.

(3) Formal procedures for checking each step of the processing of ephemeris data have been instituted to ensure that the published ephemerides are free from human and mechanical error.

The data in any set of three ephemeris tapes are complete over the years 1950 through 2000—in fact, they are generally more complete than any particular problem would require. Generation of a new set of JPL Ephemeris tapes is only a question of data processing; no changes in the system are required. Revision of ephemeris tapes is simplified by maintaining tape libraries and system data-processing programs. Only a few weeks are required for the generation of a new set of ephemeris tapes, including full checking and documentation. Careful documentation of these tapes is required so that questions concerning the origin, accuracy, and format of data can be answered quickly and correctly. This documentation is achieved by a system of internal memoranda by which the contents of each tape are identified. This identification is repeated, in part, on each tape.

A clear distinction must be maintained between the version of the JPL Ephemeris Tapes actually available for external distribution (the Export Ephemeris) at a given point in time and other ephemerides generated under the Ephemeris Tape System and used for special purposes within JPL. This latter category includes obsolete export conversions, intermediate steps in the production of export versions, special-purpose experimental ephemerides, and ephemerides that represent further development beyond the present Export Ephemeris, but which have not been approved for export purposes. An ephemeris is not granted export status until extensive use with real data has demonstrated it to be clearly superior to the current Export Ephemeris, nor until adequate documentation is available. This means that an ephemeris may have been in limited use for several months before being made available to outside users. Any user to whom the highest attainable accuracy is important should keep informed as to the identification of the most current JPL Export Ephemeris available.

The items distributed to users, which are the only parts of the JPL Ephemeris Tape System of direct interest to them, are collected into a JPL Ephemeris Computer Package, which contains the following items:

(1) The ephemeris interpolation subroutine.

(2) A particular set of three JPL Ephemeris Tapes that carries the tabulation of data (the Export Ephemeris).

(3) The corresponding tape document, in the form of a JPL technical memorandum or technical report, that describes the data of item 2 in detail.

The first item is not changed by revision or replacement of the last two. The sets of ephemeris tapes, and the three tapes in each set (item 2), are given distinct alphanumeric names, which are repeated in the title of the relevant tape document in a BCD label on each tape to permit positive identification. The tape document (item 3), is assembled from the internal memoranda that identify the various library tapes used for the development of the corresponding set of JPL Ephemeris Tapes.

The following sections describe the presently available Export Ephemeris, referred to as Development Ephemeris No. 19 (DE 19), as well as the status of the ephemeris improvement effort.

B. Lunar Ephemeris

1. Export Ephemeris. The current Export Ephemeris, DE 19, contains JPL Lunar Ephemeris No. 4 (LE 4). The rectangular coordinates of the moon are significantly different from those contained in the original Export Ephemeris released in 1964 (Ref. 93). The changes that have been made are fully discussed in Ref. 94. The data were obtained as discussed below. The basic data consisted of the evaluation of the Improved Brown Lunar Theory reported in Ref. 93, which gave the parallax, semidiameter, and longitude and latitude referred to the mean ecliptic and equinox of date, partially corrected for aberration. These data have been extended in accuracy by the application of transformation corrections (Ref. 95). A one-term correction in longitude, suggested by Dr. W. J. Eckert as being indicated by a new solution to the lunar problem (Ref. 96), has been added. The removal of aberration, rendering the ephemeris strictly geometric, has been performed by means of analytically derived relations (Ref. 97), eliminating the need for numerical iteration procedures used previously. In addition, corrections were derived for introducing the new earth-moon mass ratio, based on *Mariner II* data and adopted by the IAU in 1964. These corrections have been applied, as has the new value of the constant of sine parallax which was adopted at the same time. Thus, the ephemeris is consistent with the current IAU astronomical constants (Ref. 7), excepting the earth oblateness. Subsequent to these modifications, the data were converted to rectangular coordinates, referred to the mean equator and equinox 1950.0, and expressed in earth radii.

The computations of the transformation corrections were checked against comparison computations provided by Dr. Eckert and by the U. S. Nautical Almanac Office. The other computations were compared with extensive Nautical Almanac Office computations and with detailed hand computations for one epoch.

Lunar velocity components were obtained by high-order numerical differentiation.

Three error sources are known to exist in these lunar data: (1) spacecraft trajectory data indicate that IAU values for the gravitational constants of earth and moon still require significant adjustment; (2) an archaic value of J_2 for earth is embedded in the ephemeris; and (3) the theory suffers from significant truncation error in the planetary terms.

It is possible for the user to make a first-order correction to compensate for the first of these error sources. Kepler's third law, as it applies to the lunar motion, is[18]

$$n^2 a_{☾}^3 = F_2^3 \, GE(1 + \mu) \tag{116}$$

The ephemeris, however, is given in "earth radii," related to the mean distance through the constant of sine parallax

$$R_{em} = a_{☾} \sin \Pi_{☾} \tag{117}$$

These two relations (Eqs. 116 and 117) are satisfied by the IAU System of Astronomical Constants. If better values of GE and μ are known, the major part of their effect on the lunar ephemeris can be accounted for by holding the values of n and $\sin \Pi_{☾}$ fixed and computing the value of R_{em} corresponding to the new system. Thus, R_{em} no longer represents the true earth radius, but is only a scaling factor for the lunar ephemeris. Table 20 lists the currently recommended constants for use with the lunar ephemeris, as well as their IAU values. The effect of the recommended R_{em} is to move the moon approximately 0.6 km closer to earth than is the case with the IAU system.

Table 20. Constants appropriate for the lunar ephemeris[a]

Constant	IAU Value	Recommended for use with LE 4 (DE 19)
Earth gravitational constant, GE	398603.0	398601.2 km^3/sec^2
Earth-moon mass ratio, μ^{-1}	81.30	81.3010
Scale factor, R_{em}	6378.160	6378.1492 km

[a]These values supersede those given in Refs. 92 and 94.

The Brown Lunar Theory was constructed on the basis of a value of 0.00111157 for J_2 of the earth. The modern value of 0.0010827 is not represented in LE 4. The errors due to this cause have maximum values of approximately 0.4 km in both longitude and latitude, with periods of about 19 years for longitude and 1 month for latitude. No observable error in radial distance arises from this cause.

The Lunar Theory consists of algebraic expressions for the geocentric spherical coordinates of the moon. These expressions, Fourier series whose parameters are explicit functions of time, were obtained by formal integration of the equations of motion, suitably expanded in series form. The author of such a theory must exercise judgement in determining the precision to which the expressions are carried. If the Theory contains omissions of terms that are significantly large, then the Theory does not predict positions that are strictly in accordance with the law of gravitation. The Brown Lunar Theory suffers from such omissions; it is no slur upon Brown to admit this, for he would have had to be a remarkable visionary to foresee the demands that technology would make on his theory half a century after its completion. Clemence (Ref. 98) has estimated that the errors due to neglected planetary perturbation terms might exceed 0.2 km in longitude and latitude. This estimate has recently been verified, both computationally and observationally (Refs. 33 and 34). The maximum range error from this cause exceeds 0.5 km.

These findings indicate that, at an exceptional time when all error sources experienced maximum values, the geocentric position predicted by LE 4 could be in error by 1.2 km. A realistic estimate of the $1 - \sigma$ uncertainty is 0.4 km in geocentric position. The range, which now seems to be known better than the individual rectangular coordinates, has maximum error and $1 - \sigma$ uncertainty of approximately 0.6 and 0.2 km, respectively.[19]

In addition to these known error sources, the ephemeris is doubtless contaminated in other ways. There is an ambiguity in the construction of the Improved Brown Lunar Theory[20] associated with a monthly latitude fluctuation of 75-m amplitude. It is suspected that another error source may exist in the fitting of observations to the Lunar Theory (Ref. 99), which would have significant effects on longitude and latitude, but not range. This study is not yet complete, nor the suspicion verified; therefore, it is not possible to estimate its implications for the JPL ephemeris.

2. ***Recent lunar ephemeris development.*** At present, the best long-range lunar ephemeris generated at JPL is LE 6. Theoretical correction terms were applied to LE 4 to incorporate the modern value of J_2. Therefore, this error source in LE 4 has been eliminated. Although LE 6 has not yet been tested against observational data, it may be expected that the maximum error in geocentric position at an exceptional time has been reduced to

[18]F_2 is a constant arising in the derivation of the Lunar Theory. Its value is 0.99909 3419 75298.

[19]These figures, based on more recent information, supersede those given in Ref. 92, page 2.

[20]Private communication, T. C. Van Flandern, April 30, 1968.

approximately 0.8 km, with a $1-\sigma$ value of perhaps 0.3 km. The range is unaffected by this modification. LE 6, which is fully documented in Ref. 100, has been incorporated into DE 43, whose planetary data are discussed below.

An effort is now underway to construct a long-range lunar ephemeris by numerical integration; however, this work has not yet been brought to a satisfactory conclusion (Ref. 33).

C. The Planetary Ephemerides

1. Export Ephemeris. The planetary ephemerides included in DE 19 are single-body numerical integrations of the differential equations of planetary motion, with epoch values chosen so as to obtain a Gaussian least-squares fit to some standard source positions for each body. The differential equations contain a relativity term based on the Schartzschild metric; those for the earth-moon barycenter consider earth and moon as separate bodies. The program used for these computations was a modified version of that described in Ref. 101. After each new integration fit, plots were made of the residuals, and the new fitted ephemeris was automatically merged into a new ephemeris system to provide the positions of the perturbing planets from which the perturbing attractions on the next planet to be fitted were calculated. In each case, the numerical integration used a predictor-corrector second-sum method with fourteenth differences of the accelerations retained. A ½-day integration step size was used for Mercury; a 2-day step size was used for Venus and the earth-moon barycenter and a 4-day step size was used for Mars and the outer planets.

Values of the sun/planet mass ratios that were used are those in the current IAU system (Ref. 7, see also Section V-A). Equations of condition for radial distance, latitude, and longitude factored by the cosine of the latitude were formed at 2-day intervals for Mercury, and at 4-day intervals for Venus, the earth-moon barycenter, Mars, and the outer planets. These equations were accumulated into normal equations and, at the completion of integration, were solved for differential corrections to the ecliptic Keplerian elements a, e, I, Ω, ω, and M_0, osculating at the epoch (JED 243 3280.5). In particular, the quantities $\Delta M_0 + \Delta r$, Δp, Δq, $e\Delta r$, $\Delta a/a$, and Δe obtained as a solution to the normal equations were solved for Δa, Δe, ΔI, $\Delta\Omega$, $\Delta\tilde{\omega}$, and ΔM_0, where $\Delta\omega$ was then obtained from $\Delta\tilde{\omega}$ by subtraction of $\Delta\Omega$. These corrections were then added to the osculating elements at epoch, giving corrected osculating elements and a corresponding new direction cosine matrix PQR. The corrected osculating elements and corresponding new direction cosine matrix PQR were then used to obtain new position-velocity coordinates at epoch. The integration was then started again with the new $x_0 y_0 z_0$ and $\dot{x}_0 \dot{y}_0 \dot{z}_0$. This process was repeated until no further significant reduction in the sum of the squares of residuals could be made.

The integration was performed in the mean equatorial system of 1950.0. The value assigned to the obliquity to transform between the equatorial and ecliptic system was $\epsilon = 0.409\ 206\ 194\ 142\ 9905$ rad.

The source positions to which the integrations were fitted are summarized in Table 21. The statistical summary given in Ref. 92 indicates that the residuals (source—DE 19) are smaller than the discordances between the source data and observations. Therefore, the error bounds on DE 19 will be, in fact, the error bounds on the source data. Table 22 estimates the uncertainties

Table 21. Source positions for DE 19

Planet	Source	References
Mercury	Newcomb theory with Clemence corrections	102
Venus	Newcomb theory (no corrections)	103
Earth-moon	Newcomb theory (no corrections)	104
Mars	Clemence theory	105, 106
Jupiter	SSEC integration with Clemence corrections	107, 108
Saturn	SSEC integration with Clemence corrections	107, 108
Uranus	SSEC integration with Clemence corrections	107, 108
Neptune	SSEC integration with Clemence corrections	107, 108
Pluto	SSEC integration with Clemence corrections	107, 108

Table 22. Uncertainty estimates for the planetary positions in DE 19

Planet	Uncertainty estimate, km	Basis for estimate
Mercury	200–300	Refs. 102, 64, radar observations and JPL ephemeris development
Venus	300–500	Refs. 21, 64, radar observations and JPL ephemeris development
Earth-moon	150–300	Refs. 21, 64, and JPL ephemeris development
Mars	200–400	Refs. 105, 106, 67, radar observations and JPL ephemeris development
Jupiter	2000	Refs. 107, 108
Saturn	2000	Refs. 107, 108
Uranus	3000	Refs. 107, 108
Neptune	6000	Refs. 107, 108
Pluto	2×10^6	Ref. 109

Table 23. Coordinates and osculating elements for DE 19 at JED 2433280.5

Equatorial rectangular coordinates	Ecliptic elements
Mercury	
x = 0.3439 2628 2595 0512D 00 AU	a = 0.3870 9767 6143 4659D 00 AU
y = 0.4561 2086 7296 2349D — 01 AU	n = 0.7142 5062 8941 1173D — 01 rad/day
z = −0.1092 5237 7672 6325D — 01 AU	e = 0.2056 2139 2696 1776D 00
$\dot{x}$ = −0.8466 3252 4718 1976D — 02 AU/day	I = 0.1222 3790 4101 3027D 00 rad
$\dot{y}$ = 0.2561 4777 7129 3402D — 01 AU/day	Ω = 0.8331 9221 2035 1764D 00 rad
$\dot{z}$ = 0.1458 6762 7826 5551D — 01 AU/day	ω = 0.5051 7122 8386 4736D 00 rad
	M_0 = 0.5416 5385 1667 6136D 01 rad
Venus	
x = 0.1429 7052 9471 8901D 00 AU	a = 0.7233 3519 4140 1882D 00 AU
y = 0.6470 0406 9944 5343D 00 AU	n = 0.2796 2279 9327 9817D — 01 rad/day
z = 0.2824 8152 5073 4543D 00 AU	e = 0.6810 9366 0405 9934D — 02
$\dot{x}$ = −0.1989 3793 1396 6423D — 01 AU/day	I = 0.5923 9147 6374 4428D — 01 rad
$\dot{y}$ = 0.3113 2345 4293 3853D — 02 AU/day	Ω = 0.1330 5193 1376 6087D 01 rad
$\dot{z}$ = 0.2659 4964 8941 4391D — 02 AU/day	ω = 0.9562 7719 8582 3581D 00 rad
	M_0 = 0.5378 2134 9887 7672D 01 rad
Earth-moon barycenter	
x = −0.1363 6133 0110 0869D 00 AU	a = 0.1000 0078 7236 0544D 01 AU
y = 0.8933 9775 6948 9210D 00 AU	n = 0.1720 1921 9320 3717D — 01 rad/day
z = 0.3874 5843 1572 1408D 00 AU	e = 0.1675 0114 1626 6023D — 01
$\dot{x}$ = −0.1732 0015 4902 0189D — 01 AU/day	I = 0.3019 5435 1067 9661D — 05 rad
$\dot{y}$ = −0.2244 2676 7908 6503D — 02 AU/day	Ω = 0.5866 9905 6919 3158D 01 rad
$\dot{z}$ = −0.9733 4185 6786 7504D — 03 AU/day	ω = 0.2198 0752 5929 5807D 01 rad
	M_0 = 0.6213 6054 9876 8146D 01 rad
Mars	
x = −0.1369 8312 5671 4650D 01 AU	a = 0.1523 7494 5726 3805D 01 AU
y = 0.8431 3716 1771 0496D 00 AU	n = 0.9145 5781 6237 8094D — 02 rad/day
z = 0.4238 3609 4908 1033D 00 AU	e = 0.9326 0810 2118 8317D — 01
$\dot{x}$ = −0.7384 5892 9362 9874D — 02 AU/day	I = 0.3228 8222 5779 1756D — 01 rad
$\dot{y}$ = −0.9477 3446 0757 8137D — 02 AU/day	Ω = 0.8581 8453 5514 8757D 00 rad
$\dot{z}$ = −0.4151 6538 4773 5353D — 02 AU/day	ω = 0.4991 5092 0674 7051D 01 rad
	M_0 = 0.2938 8519 6401 7474D 01 rad
Jupiter	
x = 0.3349 3566 4461 7624D 01 AU	a = 0.5202 6553 8209 9465D 01 AU
y = −0.3473 7659 9386 1919D 01 AU	n = 0.1450 2774 0969 2968D — 02 rad/day
z = −0.1572 1612 0308 8752D 01 AU	e = 0.4891 0755 7695 3513D — 01
$\dot{x}$ = 0.5585 6549 6410 6917D — 02 AU/day	I = 0.2281 0075 4232 1975D — 01 rad
$\dot{y}$ = 0.4962 2486 5918 9390D — 02 AU/day	Ω = 0.1744 6738 0279 8027D 01 rad
$\dot{z}$ = 0.1992 2660 7644 6181D — 02 AU/day	ω = 0.4777 3851 4538 7904D 01 rad
	M_0 = 0.5279 6468 2494 4107D 01 rad
Saturn	
x = −0.8972 4944 5632 1996D 01 AU	a = 0.9522 6887 3876 7056D 01 AU
y = 0.2279 7122 9499 0131D 01 AU	n = 0.5854 6921 3211 8279D — 03 rad/day
z = 0.1330 3683 2124 4373D 01 AU	e = 0.5349 2699 3422 3929D — 01
$\dot{x}$ = −0.1858 2755 1290 4143D — 02 AU/day	I = 0.4345 2611 3823 3223D — 01 rad
$\dot{y}$ = −0.4983 8513 7862 8318D — 02 AU/day	Ω = 0.1976 5298 1653 3096D 01 rad
$\dot{z}$ = −0.1980 2491 2780 6463D — 02 AU/day	ω = 0.5883 0963 5241 1480D 01 rad
	M_0 = 0.1180 0954 0981 9556D 01 rad

Table 23 (contd)

Equatorial rectangular coordinates	Ecliptic elements
Uranus	
x = −0.1002 9132 6963 7378D 01 AU	a = 0.1916 3718 8923 4575D 02 AU
y = 0.1732 3491 9404 7683D 02 AU	n = 0.2050 5541 8127 4017D − 03 rad/day
z = 0.7604 8360 5654 0355D 01 AU	e = 0.4620 9970 2713 5565D − 01
$\dot{x}$ = −0.3955 2571 5130 4471D − 02 AU/day	I = 0.1347 3849 5921 9508D − 01 rad
$\dot{y}$ = −0.3759 0074 7195 8835D − 03 AU/day	Ω = 0.1287 3633 3315 9055D 01 rad
$\dot{z}$ = −0.1088 4370 6981 7871D − 03 AU/day	ω = 0.1703 6263 2661 5070D 01 rad
	M_0 = 0.5005 8317 3611 7874D 01 rad
Neptune	
x = −0.2919 4267 7427 1409D 02 AU	a = 0.3006 8940 4016 5808D 02 AU
y = −0.7719 2336 0132 2025D 01 AU	n = 0.1043 3151 1142 8949D − 03 rad/day
z = −0.2427 2492 8084 7945D 01 AU	e = 0.7942 6832 7095 8694D − 02
$\dot{x}$ = 0.8207 8983 1005 6175D − 03 AU/day	I = 0.3098 3362 6924 2442D − 01 rad
$\dot{y}$ = −0.2772 0922 4647 0526D − 02 AU/day	Ω = 0.2291 3580 3446 3485D 01 rad
$\dot{z}$ = −0.1156 1171 1628 3100D − 02 AU/day	ω = 0.4587 6871 3081 3981D 01 rad
	M_0 = 0.2809 7734 6432 2865D 01 rad
Pluto	
x = −0.2623 1756 1603 1849D 02 AU	a = 0.3937 3641 3530 0176D 02 AU
y = 0.2056 1529 7781 2375D 02 AU	n = 0.6962 6357 0829 8997D − 04 rad/day
z = 0.1444 3690 2947 7839D 02 AU	e = 0.2488 0330 5362 3924D 00
$\dot{x}$ = −0.1315 7218 4319 3501D − 02 AU/day	I = 0.2996 7069 7085 9694D 00 rad
$\dot{y}$ = −0.2619 8366 1853 7051D − 02 AU/day	Ω = 0.1914 3375 5010 2258D 01 rad
$\dot{z}$ = −0.4270 5125 4357 5085D − 03 AU/day	ω = 0.1995 5817 5268 9690D 01 rad
	M_0 = 0.5265 3021 0022 1278D 01 rad

in the planetary positions; however, it should be mentioned that these are quite subjective estimates. Some indication is given of the informational inputs to these estimates. The effect of the uncertainty in the astronomical units is *not* included. The estimates for the outer planets are based on the residuals of normal points, rather than discrete observations.

The data presented in Table 23 are the DE 19 values at the epoch of the JPL Ephemeris Tape System, JED 243 3280.5, of the equatorial rectangular coordinates and velocities and ecliptic elements osculating at the epoch. They are referred to the mean equinox, equator, and ecliptic of 1950.0 (JED 2433282.423).

2. *Recent planetary ephemeris development.* The fundamental procedures under which the JPL Ephemeris Tape System is used to generate ephemerides has recently undergone drastic revision as a result of the continuing evolution of the system. The Planetary Orbit Determination (PLOD) program has been replaced by the Solar System Data Processing System (SSDPS), which performs a simultaneous integration of all the major planets, differentially correcting any subset to fit radar and/or optical observations. This program, using the planetary mass values adopted in Ref. 84, has been applied to the 1949–1967 USNO meridian circle data for all planets except Pluto and to all available 1964–1967 time-delay radar bounce data for Mercury, Venus, and Mars. The resulting ephemeris, originally DE 40 (Ref. 64) but now combined with LE 6 as DE 43, has much improved positional data for the inner planets. The change is not very striking in the optical residuals, because of the inherent precision level of optical measures, but the range residuals are drastically reduced (Ref. 64). The deviations are reduced nearly two orders of magnitude for Mercury and one order for Venus and Mars. Figures 5 through 8 show the Mercury and Venus residuals with DE 35 and DE 43, before and after the correction. DE 35 is comparable to DE 19 for all bodies except Venus and earth-moon barycenter, which had received a previous correction.

The DE 43 positions for the planets Jupiter through Uranus are not likely to be significantly better than those of DE 19 even though the residuals for the restricted data set in DE 43 are smaller. This situation arises from the circumstance that DE 19 is ultimately related to 150 years of optical data, compared with 17 years for DE 43. No radar data, which would supply strength to the solution, exist for these objects. DE 43 positions of Neptune are decidedly better than those of DE 19 because of a secular drift in the latter.

The Pluto ephemeris in DE 43 is integrated from starting conditions obtained by fitting the new orbit of Pluto determined by Cohen, Hubbard, and Oesterwinter (Ref. 109); it has not been corrected because the cited work seems both exhaustive and definitive. The positional uncertainty is probably no greater than 5×10^4 km.

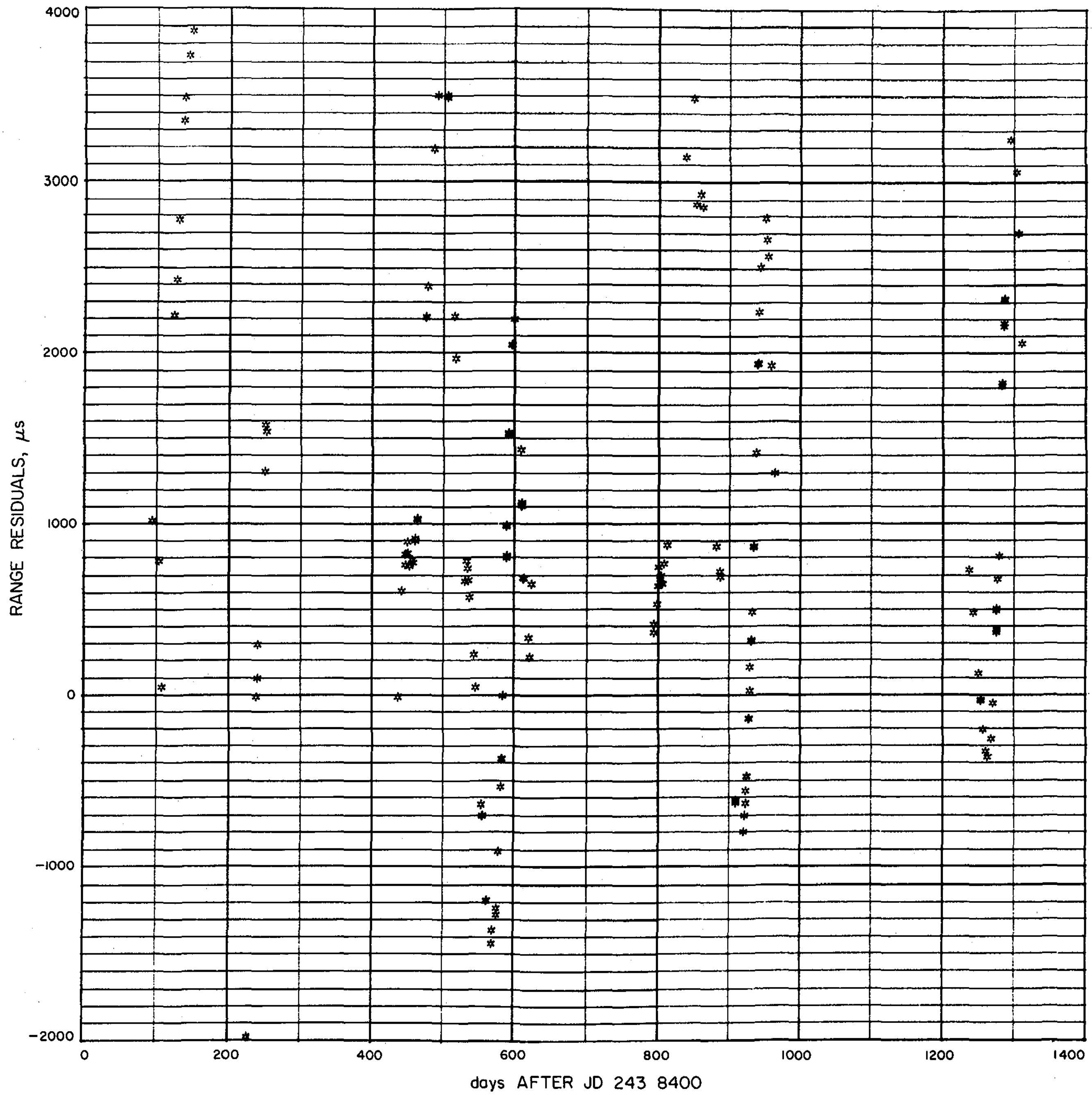

Fig. 5. Mercury radar range residuals with DE 35

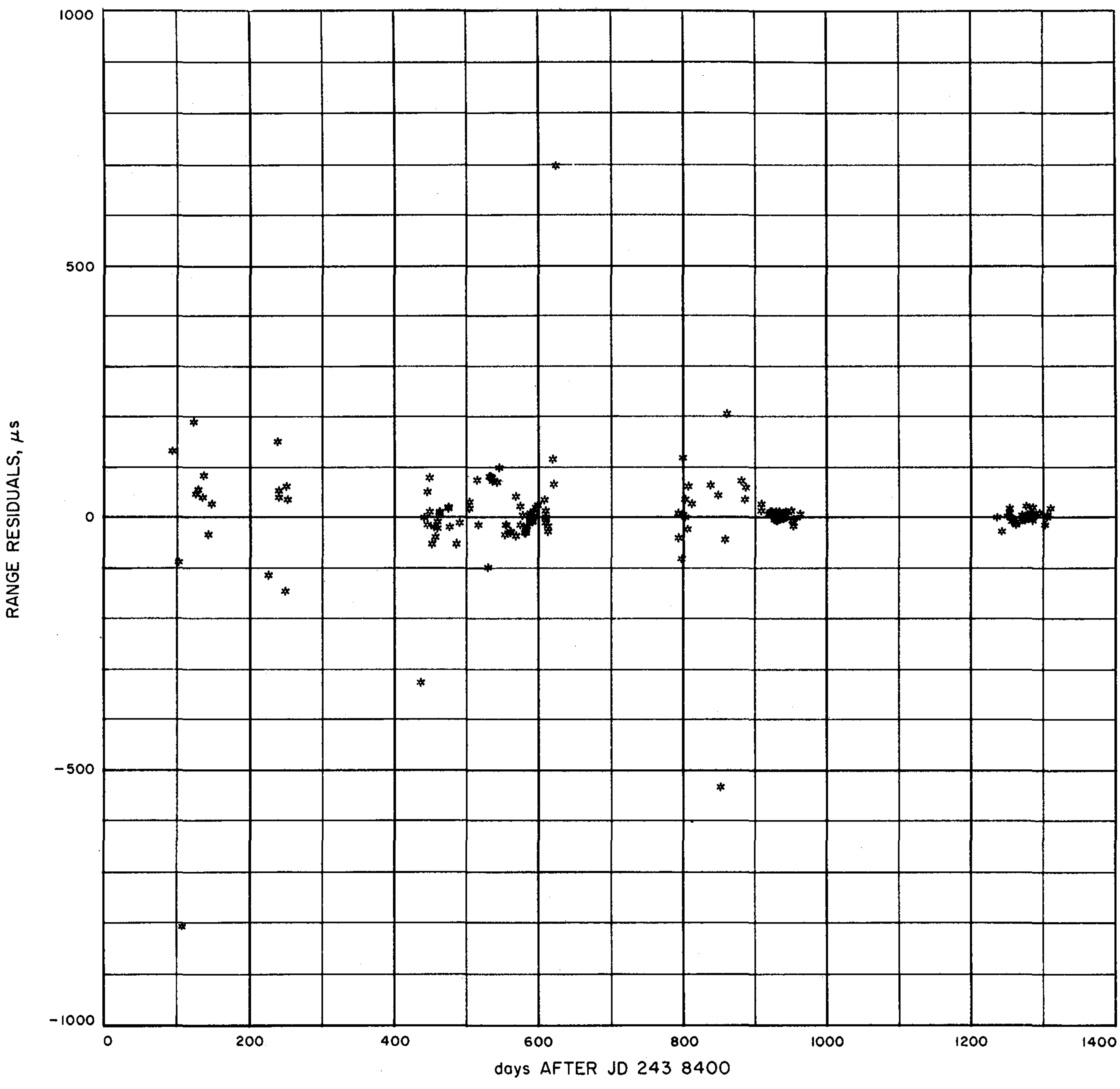

Fig. 6. Mercury radar range residuals with DE 43

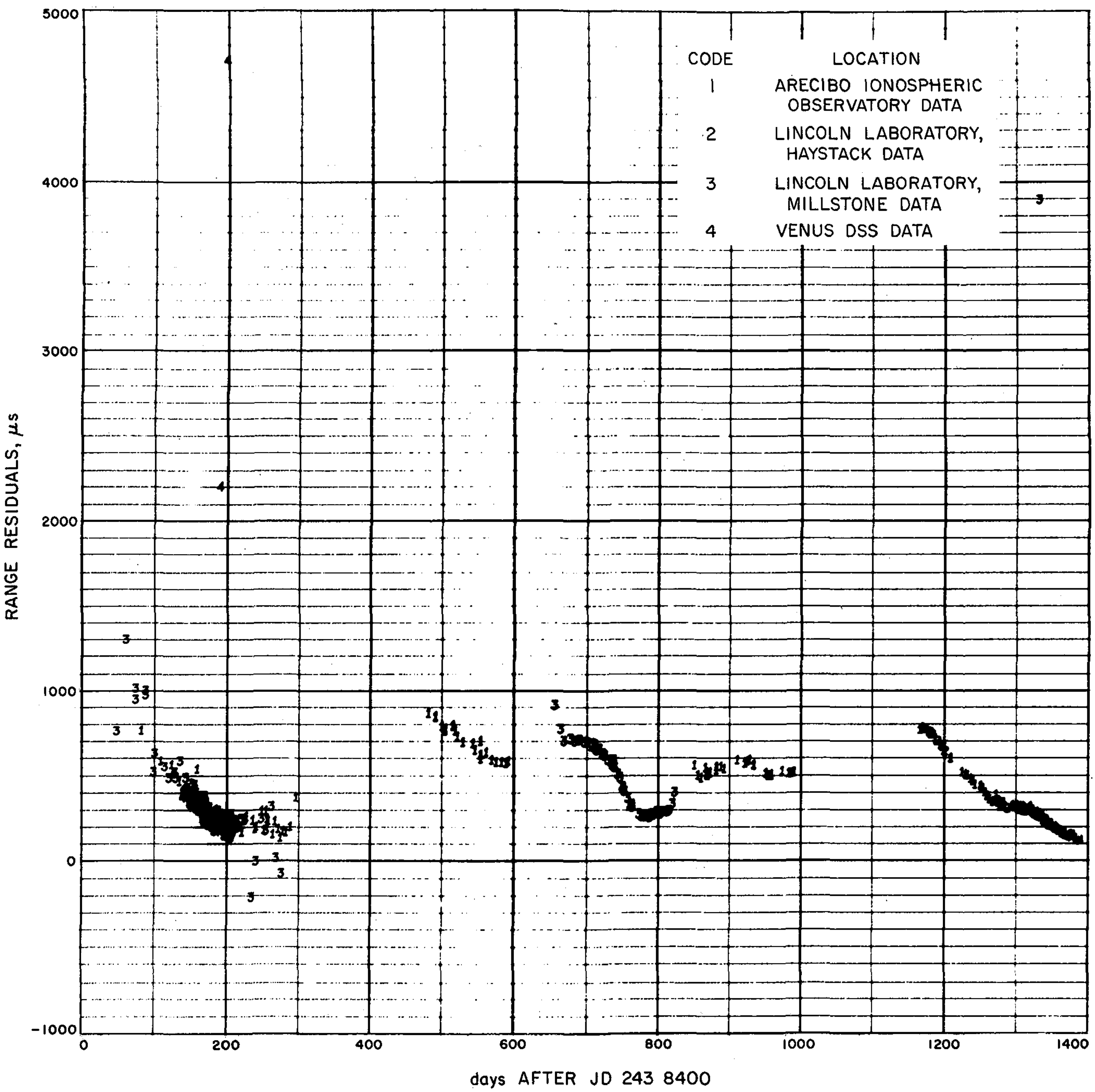

Fig. 7. Venus radar residuals with DE 35

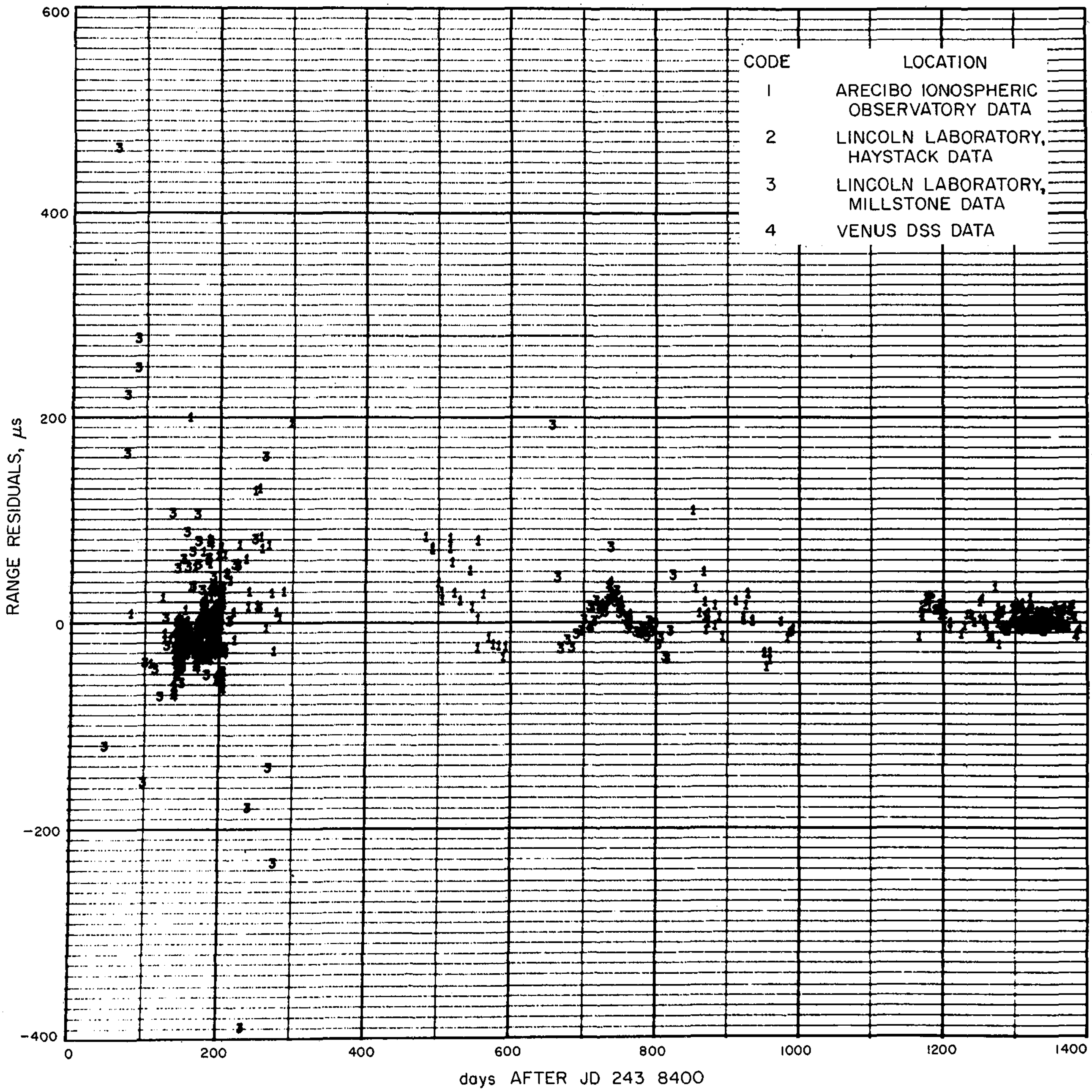

Fig. 8. Venus radar residuals with DE 43

References

1. Clarke, V. C., Jr., *Constants and Related Data for Use in Trajectory Calculations,* Technical Report 32-604. Jet Propulsion Laboratory, Pasadena, Calif., March 6, 1964.
2. *Proceedings of the Eleventh General Assembly, Berkeley 1961,* Transactions of the Int. Astron. Union, Vol. XIB, Academic Press, 1962.
3. Kaula, W., "Tesseral Harmonics of the Earth's Gravitational Field from Camera Tracking of Satellites," *J. Geophys. Res.,* Vol. 71, No. 18, Sept. 15, 1966.
4. King-Hele, D. G., and Cook, G. E., "The Even Zonal Harmonics of the Earth's Gravitational Potential," *Geophys. J.,* Vol. 10, 1965.
5. King-Hele, D. G., Cook, G. E., and Scott, D., "The Odd Zonal Harmonics in the Earth's Gravitational Potential," *Planet. Space Sci.,* Vol. 13, 1965.
6. Kozai, Y., "New Determination of Zonal Harmonic Coefficients in the Earth's Gravitational Potential," *Publ. Astron. Soc. Japan,* Vol. 16, pp. 263–284, 1964.
7. *Proceedings of the Twelfth General Assembly, Hamburg 1964,* Transactions of the Int. Astron. Union, Vol. XIIB, Academic Press, New York, 1966.
8. Guier, W. H., and Newton, R. R., "The Earth's Gravity Field as Deduced from the Doppler Tracking of Five Satellites," *J. Geophys. Res.,* Vol. 70, No. 18, Sept. 15, 1965.
9. Anderle, R. J., "Geodetic Parameters Set NWL-5E-6 Based on Doppler Satellite Observations," *Proc. Sec. Int. Symp. Geod. on Use of Satellite,* edited by G. Veis, Athens, 1966, in press.
10. Wagner, C. A., *Determination of the Ellipticity of the Earth's Equator from Observations on the Drift of the Syncom II Satellite,* NASA TN D-2759, May 1965.
11. Kaula, W. M., *Theory of Satellite Geodesy,* Blaisdell, 1966.
12. Kaula, W. M., *Tests of Satellite Determinations of the Gravity Field Against Gravimetry and Their Combination,* Publication No. 508, Institute of Geophysics and Planetary Physics, UCLA, 1966.
13. Gaposhkin, E. M., "A Dynamical Solution for the Tesseral Harmonics of the Geopotential and for Station Coordinates," *Pres. to AGU Meeting,* Washington, D.C., 1966.
14. Jeffreys, Sir Harold, *The Earth,* Cambridge, 1959.
15. Cook, A. H., "The External Gravity Field of a Rotating Spheroid to the order of e^3," *Geophys. J.,* Royal Astron. Soc., Vol. 3, No. 3, Sept. 1959.
16. *Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac,* H. M. Stationery Office, London, 1961.
17. Mueller, I. I., *Introduction to Satellite Geodesy,* Ungar, 1964.

References (contd)

18. Wollard, E. W., and Clemence, G. M., *Spherical Astronomy,* Academic Press, New York, 1966.

19. Lieske, J., *Expressions for the Precession Quantities and Their Partial Derivatives,* Technical Report 32-1044, Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1967.

20. Woolard, E. W., "Theory of the Rotation of the Earth Around its Center of Mass," *Astron. Papers of the American Ephemeris,* Vol. XV, Part 1, U. S. Government Printing Office, Washington, 1953.

21. Duncombe, R. L., "Motion of Venus 1750-1949," *Astron. Papers of the American Ephemeris,* Vol. XVI, Part 1, U. S. Government Printing Office, Washington, 1958.

22. Clemence, G. M., "The System of Astronomical Constants," *Annu. Rev. Astron. and Astrophys.,* Vol. 3, L. Goldberg, ed., pp. 93–112, Annual Reviews, Inc., Palo Alto, Calif., 1965.

23. Yumi, S., *Annual Report of the International Polar Motion Service for 1962,* Central Bureau of the International Polar Motion Service Observatory, Mizusawa-Shi, Iwate-Ken, Japan, 1964.

24. Markowitz, W., "Latitude, Longitude and the Secular Motion of the Pole," Reprinted by the U. S. Naval Observatory from *Methods and Techniques in Geöphysics,* edited by Runcorn, Interscience Publishers Ltd., London, 1960.

25. Chandler, C. S., "On the Variation of Latitude," *Astron. J.,* Vol. 11, p. 65, 1891.

26. Chandler, C. S., "On the Variation of Latitude (VII)," *Astron. J.,* Vol. 12, p. 97, 1892.

27. Munk, W. H., and Macdonald, G. J. F., *The Rotation of the Earth,* Cambridge, University Press, London, 1960.

28. Hattori, T., "On the Prediction of Latitude Variation," *Pub. Int. Latitude Observatory,* Mizusawa, Japan, Vol. II, No. 2, p. 113, 1956.

29. Newton, I., *Mathematical Principles of Natural Philosophy,* Univ. of California Press, Berkeley, 1962.

30. Discussion of Draft Report, Commission 31 (Time), International Astronomical Union, Prague, August, 1967. *See Agenda and Draft Report, 13th General Assembly,* Int. Astron. Union, 1967.

31. Fliegel, H. F., and Hawkins, T. P., "Analysis of Variations in the Rotation of the Earth," *Astron. J.,* Vol. 72, p. 544, 1967.

32. *The American Ephemeris and Nautical Almanac for the Year 1970,* U. S. Government Printing Office, Washington, in preparation.

33. Mulholland, J. D., and Devine, C. J., "Gravitational Inconsistency in the Lunar Theory: Numerical Determination," *Science,* Vol. 160, May 24, 1968.

34. Cary, C. N., and Sjogren, W. L., "Gravitational Inconsistency in the Lunar Theory: Confirmation by Radio Tracking," *Science,* Vol. 160, May 24, 1968.

References (contd)

35. Clemence, G. M., and Szebehely, V., "Annual Variations of an Atomic Clock," *Astron. J.*, Vol. 72, p. 1324, 1967.

36. Anderson, J. D., "Inclusion of General Relativity Theory in the Representation of Spacecraft Tracking Data," Space Program Summary 37-50, Vol. III, pp. 39–47, Jet Propulsion Laboratory, Pasadena, Calif., April 30, 1968.

37. Trask, D. W., and Muller, P. M., "Timing DSIF Two-Way Doppler Inherent Accuracy Limitations," Space Program Summary 37-39, Vol. III, pp. 7–16, Jet Propulsion Laboratory, Pasadena, Calif., May 31, 1966.

38. Muller, P. M., "Timing Data and the Orbit Determination Process at JPL," Space Program Summary 37-41, Vol. III, pp. 18–24, Jet Propulsion Laboratory, Pasadena, Calif., Sept. 30, 1966.

39. Muller, P. M., "Analysis of A.1-WWV from 1955 to 1968," Space Programs Summary 37-49, Vol. II, pp. 23–32, Jet Propulsion Laboratory, Pasadena, Calif., Jan. 31, 1968.

40. Mottinger, N. A., "Status of DSS Location Solutions for Deep Space Probe Missions: III. Recent Achievement Using *Mariners IV, V* and *Pioneer VII* data." Space Programs Summary 37-54, Vol. II, Jet Propulsion Laboratory, Pasadena, Calif., Oct. 1968.

41. Lunquist, C. A., and Veis, G., "Geodetic Parameters for a 1966 Smithsonian Institution Standard Earth, 1966," Special Report No. 200, Vol. III, Table 9, p. 122, Smithsonian Astrophysical Observatory, 1966.

42. *Goddard Directory of Tracking Station Locations,* Goddard Space Flight Center, Greenbelt, Md., Aug. 1966.

43. Position of DSS-14 Antenna by Teledyne, Inc. Geotronics Divisions, Monrovia, Calif., Nov. 1966, First Order Geodetic Survey.

44. Bean, B. R., Cahoon, B. A., and Thayer, G. D., *Tables for the Statistical Prediction of Radio Ray Bending and Elevation Errors Using Surface Values of the Refraction Indes,* TN 44, National Bureau of Standards, Boulder, Colo., 1960.

45. Bean, B. R., Cahoon, B. A., and Thayer, G. D., "Methods of Predicting the Atmospheric Bending of Radio Waves," *J. Res.*, D64(1960), pp. 487–492 and D67(1963), pp. 273–285, National Bureau of Standards, Boulder, Colo.

46. Liu, A., "Range and Angle Corrections Due to the Ionosphere," Space Program Summary 37-41, Vol. III, pp. 38–41, Jet Propulsion Laboratory, Pasadena, Calif., Sept. 30, 1966.

47. Johnson, F. S., *Satellite Environment Handbook,* Stanford University Press, pp. 23–41, 1965.

48. Akin, E. L., "Determination of the Gravitational Field of the Moon by the Motion of the Artificial Lunar Satellite Luna 10," *Dokl. Akad. Nauk.*, SSSR 170(4), pp. 799–802, 1966.

49. Jeffreys, Sir Harold, "Dynamics of the Moon," *Proc. Roy. Soc. London,* Series A 296, pp. 245–247, 1967.

References (contd)

50. Lorell, J., and Sjogren, W. L., "Lunar Potential Estimate," Space Program Summary 37-50, Vol. III, pp. 47–50, Jet Propulsion Laboratory, Pasadena, Calif., April 30, 1968.

51. Tolson, R. H., and Gapcynski, J. P., "An Analysis of the Lunar Gravitational Field as Obtained from *Lunar Orbiter* Tracking Data," presented at the IQSY/COSPAR Meeting, London, England, July 17–28, 1967.

52. Koziel, K., "The Constants of the Moon's Physical Librations Derived on the Basis of Four Series of Heliometric Observations from the Years 1877 to 1915," *Icarus 7, 1–28,* Academic Press Inc., 1967.

53. Vegos, C. J., and Trask, D. W., "Ranger Combined Analysis, Part II: Determination of the Masses of the Earth and Moon from Radio Tracking Data," Space Program Summary 37-44, Vol. III, pp. 11–28, Jet Propulsion Laboratory, Pasadena, Calif., March 31, 1967.

54. Anderson, J. D., Pease, G. E., Efron, L., and Tausworthe, R. C., "Celestial Mechanics Experiment," *Science,* Vol. 154, pp. 1689–1690, Dec. 29, 1967.

55. LAC-59, Scale 1:1,000,000, 2nd Edition, Aeronautical Chart and Information Center, St. Louis, Mo.

56. *Lunar Farside Chart* (LFC-1), Scale 1:5,000,000, 2nd Edition, Aeronautical Chart and Information Center, St. Louis, Mo., Oct. 1967.

57. *The Measure of the Moon,* edited by A. Kopal and C. L. Goudas, D. Reidel Publishing Co., Dordrecht, Holland, 1967.

58. Compton, H. R., and Wells, *The Determination of the Lunar Radius Using the Data from the V/H Sensor on Lunar Orbiters I and II,* proposed NASA Technical Memorandum, Langley Research Center, Hampton, Virginia (to be published).

58a. Shapiro, A., Uliana, E. A., Yaplee, B. S., and Knowles, S. H., "Lunar Radius from Radar Measurements," presented at the IQSY/COSPAR Meeting, London, England, July 17–28, 1967.

59. *Physics and Astronomy of the Moon,* edited by Z. Kopal, Academic Press, New York, 1962.

60. Eckhardt, D. H., "Computer Solutions of the Forced Physical Libration of the Moon," *Astron. J.,* Vol. 70, pp. 466–471, 1965.

61. Goudas, C. L., "The Mechanical Ellipticities of the Moon from Luna 10," Boeing Scientific Research Laboratories, Document D1-82-0595, The Boeing Company, Seattle, Wash., Jan. 1967.

62. Melbourne, W. G., Muhleman, D. O., and O'Handley, D. A., "Radar Determination of the Radius of Venus," *Science,* Vol. 161, pp. 987, May 31, 1968.

63. Ash, M. E., Shapiro, I. I., and Smith, W. B., "Astronomical Constants and Planetary Ephemerides Deduced from Radar and Optical Observations," *Astron. J.,* Vol. 72, p. 338, 1967.

64. Melbourne, W. G., and O'Handley, D. A., "A Consistent Ephemeris of the Major Planets in the Solar System," Space Programs Summary 37-51, Vol. III, Jet Propulsion Laboratory, Pasadena, Calif., June 30, 1968.

References (contd)

65. Ash, M. E., et al., "The Case for the Radius of Venus," *Science*, Vol. 160, p. 985, May 31, 1968.

66. Kliore, A., Cain, D. L., and Levy, G. S., "Radio Occultation Measurement of Mars Atmosphere Over Two Regions by Mariner IV Space Probe," proceedings of the Seventh Int. Space Symposium COSPAR, 1966.

67. Cain, D. L., "The Implications of a New Mars Mass and Radius," Space Program Summary 37-43, Vol. IV, p. 7, Jet Propulsion Laboratory, Pasadena, Calif., Feb. 28, 1967.

68. Dicke, R. H., and Goldenberg, H. M., "Solar Oblateness and General Relativity," *Phys. Rev. Letters*, Vol. 18, p. 313, 1967.

69. Michaux, C. M., *Handbook of the Physical Properties of the Planet Mars,* NASA SP-3030, 1967.

70. McGovern, W. E., Gross, S. H., and Rasool, S. I., "Rotation Period of the Planet Mercury," *Nature,* Vol. 208, p. 375, 1965.

71. Colombo, G., and Shapiro, I. I., "Rotation of Mercury," *Astrophys. J.*, Vol. 145, pp. 296–307, 1966.

72. Pettengill, G. H., and Dyce, R., "A Radar Determination of the Rotation of the Planet Mercury," *Nature,* Vol. 206, p. 1240, 1965.

73. Camichel, H., and Dollfus, A., "La Rotation et la Cartographie de la Planète Mércure," *Icarus,* Vol. 8, pp. 216–226, 1968.

74. Carpenter, R. L., "Study of Venus by CW Radar—1964 Results," *Astron. J.*, Vol. 71, p. 142, 1966. Also JPL Technical Report 32-963.

75. Goldstein, R. M., "Radar Studies of Venus," *Moon and Planets,* North-Holland Pub. Co., Amsterdam, 1967. Also JPL Technical Report 32-1081.

76. Supplement to the A.E. 1968, in *The American Ephemeris and Nautical Almanac,* 1968, U. S. Government Printing Office, Washington.

77. De Vaucouleurs, G., "The Physical Ephemeris of Mars," *Icarus,* Vol. 3, p. 236, 1964.

78. *Le Système De Constantes Astronomique, IAU Symposium No. 21,* Gauthier-Villars et Cie., Paris, 1965.

79. Moyer, T. D., "Doppler and Range Observables," Space Programs Summary No. 37-42, Vol. III, p. 15, Jet Propulsion Laboratory, Pasadena, Calif., 1966.

80. Anderson, J. D., "Determination of the Masses of the Moon and Venus and the Astronomical Unit from Radio Tracking Data of the *Mariner II* Spacecraft," Technical Report 32-816, Jet Propulsion Laboratory, Pasadena, Calif., 1967.

81. Sjogren, W. L., Trask, D. W., Vegos, C. J., and Wollenhaupt, W. R., *Physical Constants as Determined from Radio Tracking of the Ranger Lunar Probes,* Technical Report 32-1057, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 30, 1966.

References (contd)

82. Null, G. W., Gordon, H. J., and Tito, D. A., *The Mariner IV Flight Path and Its Determination from Tracking Data,* Technical Report 32-1108, Jet Propulsion Laboratory, Pasadena, Calif., Aug. 1, 1967.

83. Null, G. W., "A Solution for the Sun-Mars Mass Ratio Using Mariner IV Doppler Tracking Data," *Astron. J.*, Vol. 72, p. 1292, 1967.

84. Mulholland, J. D., "Planetary Masses for Ephemeris Development," Space Programs Summary 37-45, Vol. IV, p. 17, Jet Propulsion Laboratory, Pasadena, Calif., 1967.

85. O'Handley, D. A., "Determination of the Mass of Jupiter from the Motion of 65 Cybele," *Astron. Papers of the American Ephemeris,* Vol. 20, Part 3, United States Government Printing Office, Washington, 1968.

86. Clemence, G. M., "Masses of the Principal Planets," in *Proceedings of the Twelfth General Assembly,* Hamburg 1964, Transactions of the International Astronomical Union, Vol. XIIB, Academic Press, New York, p. 609, 1966.

87. Brouwer, D., "Relations Among Some Important Astronomical Constants," *Bull. Astron.,* Vol. 25, p. 241, 1965.

88. Anderson, J. D., Pease, G. E., Efron, L., and Tausworthe, R. C., "Determination of the Mass of Venus and Other Astronomical Constants from the Radio Tracking of Mariner V," *Astron. J.,* Vol. 73, Supplement, No. 1357, March 1968.

89. Heyl, P. R., "A New Determination of the Constant of Gravitation," *J. Res. NBS,* Vol. 29, No. 1, pp. 1–31, 1942.

90. *Handbook of Geophysics 1960,* U. S. Air Force, Geophysics Research Directorate, pp. 4–17.

91. Peabody, P. R., Scott, J. F., and Orozco, E. G., "Users' Description of JPL Ephemeris Tapes," Technical Report 32-580, Jet Propulsion Laboratory, Pasadena, Calif., March 2, 1964.

92. Devine, C. J., "JPL Development Ephemeris Number 19," Technical Report 32-1181, Jet Propulsion Laboratory, Pasadena, Calif., Nov. 15, 1967.

93. Peabody, P. R., Scott, J. F., and Orozco, E. G., "JPL Ephemeris Tapes E9510, E9511, E9512," Technical Memorandum 33-167, Jet Propulsion Laboratory, Pasadena, Calif., March 2, 1964.

94. Mulholland, J. D., and Block, N., "JPL Lunar Ephemeris No. 4," Technical Memorandum 33-346, Jet Propulsion Laboratory, Pasadena, Calif., July 15, 1967.

95. Eckert, W. J., Walker, M. J., and Eckert, D., "Transformations of the Lunar Coordinates and Orbital Parameters," *Astron. J.,* Vol. 71, pp. 314–332, 1966.

96. Eckert, W. J., and Smith, H. F., Jr., *Astron. Papers of the American Ephemeris,* Vol. 19, Part II, U. S. Nautical Almanac Office, GPO, Washington, D. C., in preparation. (For preliminary results, see Ref. 7).

References (contd)

97. Clemence, G. M., Porter, J. G., and Sadler, D. H., "Aberration in the Lunar Ephemeris," *Astron. J.*, Vol. 57, pp. 46–47, 1952.

98. Clemence, G. M., "Remarks on Current Lunar Theory," *Proceedings of the JPL Seminar on Uncertainties in the Lunar Ephemeris,* Technical Report 32-1247, Jet Propulsion Laboratory, Pasadena, Calif., edited by J. D. Mulholland, May 1, 1968.

99. Van Flandern, T. C., "A Preliminary Report on a Lunar Latitude Fluctuation," *Proceedings of the JPL Seminar on Uncertainties in the Lunar Ephemeris,* Technical Report 32-1247, Jet Propulsion Laboratory, Pasadena, Calif., edited by J. D. Mulholland, May 1, 1968.

100. Mulholland, J. D., "JPL Lunar Ephemeris Number 6," Technical Memorandum, Jet Propulsion Laboratory, Pasadena, Calif., in preparation.

101. Devine, C. J., "PLOD II: Planetary Orbit Determination Program for the IBM 7094 Computer," Technical Memorandum 33-188, Jet Propulsion Laboratory, Pasadena, Calif., April 15, 1965.

102. Clemence, G. M., "The Motion of Mercury, 1765–1937," *Astron. Papers of The American Ephemeris,* Vol. XI, Part 1, Nautical Almanac Office, U. S. Naval Observatory, U. S. Government Printing Office, Washington, D. C., 1943.

103. Newcomb, S., "Tables of the Heliocentric Motion of Venus," *Astron. Papers of the American Ephemeris,* Vol. VI, Part 3, Nautical Almanac Office, U. S. Naval Observatory, U. S. Government Printing Office, Washington, D. C., 1898.

104. Newcomb, S., "Tables of the Motion of the Earth on its Axis and Around the Sun," *Astron. Papers of the American Ephemeris,* Vol. 6, Part 1, Nautical Almanac Office, U. S. Naval Observatory, U. S., Government Printing Office, Washington, D. C., 1898.

105. Clemence, G. M., "First-Order Theory of Mars," *Astron. Papers of the American Ephemeris,* Vol. XI, Part 2, Nautical Almanac Office, U. S. Naval Observatory, U. S. Government Printing Office, Washington, D. C., 1949.

106. Clemence, G. M., "Theory of Mars-Completion," *Astron. Papers of the American Ephemeris,* Vol. XVI, Part 2, Nautical Almanac Office, U. S. Naval Observatory, U. S. Government Printing Office, Washington, D. C., 1962.

107. Eckert, W. J., Brouwer, D., and Clemence, G. M., "Coordinates of the Five Outer Planets, 1653–2060," *Astron. Papers of the American Ephemeris,* Vol. XII, Nautical Almanac Office, U. S. Naval Observatory, U. S. Government Printing Office, Washington, D. C., 1951.

108. Clemence, G. M., "Perturbations of the Five Outer Planets by the Four Inner Ones," *Astron. Papers of the American Ephemeris,* Vol. XIII, Part 5, Nautical Almanac Office, U. S. Naval Observatory, U. S. Government Printing Office, Washington, D. C., 1954.

109. Cohen, C. J., Hubbard, E. C., and Oesterwinter, C., "New Orbit for Pluto and Analysis of Differential Corrections," *Astron. J.*, Vol. 72, pp. 973–988, 1967.

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Appendix

Notes on the IAU Constants

(1) The value given for the number of ephemeris seconds in the tropical year at 1900 is taken from the definition of the ephemeris second that was adopted by the Comité International des Poids et Mesures (Procés Vèrbaux des Séances, deuxième série, 25, 77, 1957). It is, in fact, derived from the coefficient of T, measured in Julian centuries of 36525 days, in Newcomb's expression for the geometric mean longitude of the sun referred to the mean equinox of date. In the list, 1900 refers to the fundamental epoch of ephemeris time, namely 1900 January 0 at 12^h ET, or to 1900.0, as appropriate; the values for constants 20–23 also refer to the fundamental epoch. Throughout the list and this report the term "second" must be understood to mean the "ephemeris second."

(2) The value of the Gaussian gravitational constant (k) is that adopted by the IAU in 1938, and serves to define the astronomical unit of length (AU) since the corresponding (astronomical) units of mass and time are already defined. (The unit of mass is that of the sun and the unit of time is the ephemeris day of 86400 ephemeris sec). The units of k are: $(\mathrm{AU})^{3/2}$ (ephemeris day)$^{-1}$ (sun's mass)$^{-1/2}$. To simplify the later equations, an auxiliary constant k', defined as $k/86400$, is introduced and a rounded value is given in the list.

(3) The value for the measure of the AU in meters is a rounded value of recent radar determinations.

(4) The value for the velocity of light is that recommended by the International Union of Pure and Applied Physics in September 1963.

(5) The term "equatorial radius for earth" refers to the equatorial radius of an ellipsoid of revolution that approximates to the geoid (see also note 16).

(6) The term "dynamical form-factor for earth" refers to the coefficient of the second harmonic in the expression for the earth's gravitational potential as adopted by IAU Commission 7 in 1961 (see also note 16).

(7) The geocentric gravitational constant (GE) is appropriate for use for geocentric orbits when the units of length and time are the meter and the second; E denotes the mass of the earth including its atmosphere. Kepler's third law for a body of mass M moving in an unperturbed elliptic orbit around the earth may be written:

$$GE(1 + M/E) = n^2a^3$$

where n is the sidereal mean motion in rad/sec and a is the mean distance in meters. The value of GE is based on gravity measurements and observations of satellites.

(8) Again, the mass of the earth includes the mass of the atmosphere. The reciprocal of 81.30 is 0.0123001.

(9) The value for the sidereal mean motion of the moon is consistent with the value of the tropical mean motion used in the improved lunar ephemeris, less the general precession in longitude.

(10–12) The values of the principal constants defining the relative positions and motions of the equator and ecliptic are those in current use. Secular terms and derived quantities are already tabulated elsewhere.

(13) The rounded value $8\overset{''}{.}794$ for the solar parallax should be used except where extra figures are required to ensure numerical consistency.

(14) The value of the light-time for unit distance is numerically equal to the number of light-seconds in 1 AU. Its reciprocal is equal to the velocity of light in AU/sec.

(15) Apart from the factor F_1, the constant of aberration is equal to the ratio of the speed of a hypothetical planet of negligible mass moving in a circular orbit of unit radius to the velocity of light; it is conventionally expressed in seconds of arc by multiplying by the number of seconds of arc in 1 rad. The factor F_1 is the ratio of the mean speed of the earth to the speed of the hypothetical planet and is given by:

$$F_1 = \frac{n_\odot}{k'} \frac{a_\odot}{(1 - e^2)^{1/2}}$$

where

$n_\odot$ = the sidereal mean motion of the sun in rad/sec

$a_\odot$ = the perturbed mean distance of the sun in AU

e = the mean eccentricity of the earth's orbit.

Newcomb's values for $n_\odot$, $a_\odot$, and e are of ample accuracy for this purpose. The factor F_1 and the constant of aberration take the following values:

	F_1	κ''
1800	1.0001427	20.49583
1900	1.0001420	20.49582
2000	1.0001413	20.49581

The rounded value 20".496 should be used except where the extra figures are required to ensure numerical consistency.

(16) The condition that the reference ellipsoid of revolution for the earth shall be an equipotential surface implies that three parameters are sufficient to define its geometrical form and external gravitational field, provided that the angular velocity (ω) of the earth and the relative mass of the atmosphere (μ_a) are assumed to be known. The variability of the rate of rotation of the earth can be ignored, and the mass of the atmosphere is only just significant; the required values are:

$$\omega = 0.000072921 \text{ rad/sec}; \mu_a = 0.000001$$

The expressions for the flattening (f) and the apparent gravity at the equator (g_e) in terms of the primary constants are, to second order:

$$f = \frac{3}{2}J_2 + \frac{1}{2}m + \frac{9}{8}J_2^2 + \frac{15}{28}J_2 m - \frac{39}{56}m^2$$

$$g_e = (GE/a_e^2)\left(1 - \mu_a + \frac{3}{2}J_2 - m + \frac{27}{8}J_2^2 - \frac{6}{7}J_2 m + \frac{47}{56}m^2\right)$$

where $m = a_e\omega^2/g_e$ is obtained by successive approximations. The new value of f is given here only for astronomical use (parallax corrections, etc).

(17) The heliocentric gravitational constant corresponds to GE, but is appropriate for heliocentric orbits when the units are the meter and the second.

(18–19) The derived values of the masses of the earth and of the earth plus moon differ from those currently in use, but will not supersede them completely until the system of planetary masses is revised as a whole (see note 24).

(20) The perturbed mean distance of the moon is the semimajor axis of Hill's variational orbit, and differs from that calculated from Kepler's law by the factor F_2, which depends on the well-determined ratio of the mean motions of the sun and moon (E. W. Brown, *Memoirs Royal Astron. Soc.*, 53, 89, 1897).

(21) The constant of sine parallax for the moon is conventionally expressed in seconds of arc by multiplying by the number of seconds of arc in 1 rad. The corresponding value of $\pi_☾$ itself is 3422".608.

(22) The constant of the lunar inequality is defined by the expression given and is conventionally expressed in seconds of arc.

(23) The constant of the parallactic inequality is defined by the expression given; the coefficient F_3 is consistent with the corresponding quantities in Brown's tables.

(24) The system of planetary masses is that adopted in the current ephemerides and the values given for the reciprocals of the masses include the contributions from atmospheres and satellites. The value for Neptune is that adopted in the numerical integrations of the motions of the outer planets; the value used in Newcomb's theories of the inner planets is 19700. In planetary theory, the adopted ratio of the mass of the earth to the mass of the moon is 81.45 (compared with 81.53 in the lunar theory), and the ratio of the mass of the sun to the mass of the earth alone is 333432. This system of masses should be revised within the next few years when improved values for the inner planets are available from determinations based on space probes.

Correction Factors and Limits

To first order, relative errors of the derived constants are given by:

$$\frac{\Delta\pi_\odot}{\pi_\odot} = \frac{\Delta a_e}{a_e} - \frac{\Delta A}{A} \qquad \frac{\Delta\tau_a}{\tau_a} = \frac{\Delta A}{A} - \frac{\Delta c}{c}$$

$$\frac{\Delta\kappa}{\kappa} = \frac{\Delta A}{A} - \frac{\Delta c}{c} = \frac{\Delta\tau_a}{\tau_a} \qquad \frac{\Delta f}{f} = \frac{\Delta J_2}{J_2}$$

$$\frac{\Delta(GS)}{GS} = \frac{3\Delta A}{A}$$

$$\frac{\Delta(S/E)}{S/E} = -\frac{\Delta(GE)}{GE} + \frac{3\Delta A}{A}$$

$$\frac{\Delta\{S/E(1+\mu)\}}{S/E(1+\mu)} = \frac{3\Delta A}{A} - \frac{\Delta(GE)}{GE} - \frac{\Delta\mu}{1+\mu}$$

$$\frac{\Delta a_☾}{a_☾} = \frac{1}{3}\frac{\Delta(GE)}{GE} - \frac{2}{3}\frac{\Delta n^*_☾}{n^*_☾} + \frac{1}{3}\frac{\Delta\mu}{1+\mu}$$

$$\frac{\Delta \sin\pi_☾}{\sin\pi_☾} = \frac{\Delta a_e}{a_e} - \frac{\Delta a_☾}{a_☾}$$

$$\frac{\Delta L}{L} = \frac{\Delta\mu}{\mu(1+\mu)} + \frac{\Delta a_☾}{a_☾} - \frac{\Delta A}{A}$$

$$\frac{\Delta P_☾}{P_☾} = \frac{2\Delta\mu}{1-\mu^2} + \frac{\Delta a_☾}{a_☾} - \frac{\Delta A}{A}$$

The true values of the primary constants are believed to lie between the following limits:

A = 149597 to 149601 $\times 10^6$ m

c = 299792 to 299793 $\times 10^3$ $m\ s^{-1}$

a_e = 6378080 to 6378240 m

J_2 = 0.0010824 to 0.0010829

GE = 398600 to 398606 $\times 10^9$ $m^3\ s^{-2}$

μ^{-1} = 81.29 to 81.31

$n^*_☾$ = correct to number of places given

p = 5026.″40 to 5026.″90

ϵ = 23°27′08.″16 to . . . 08.″36

N = 9.″200 to 9.″210

Correspondingly, the limits for the derived constants are:

$\pi_\odot$ = 8.″79388 to 8.″79434

τ_a = 499.ˢ001 to 499.ˢ016

κ = 20.″4954 to 20.″4960

GS = 132710 to 132721 $\times 10^{15}$ $m^3\ s^{-2}$

S/E = 332935 to 332968

$S/E(1+\mu)$ = 328890 to 328922

f^{-1} = 298.33 to 298.20

$a_☾$ = 384399 to 384401 $\times 10^3$ m

$\sin\pi_☾$ = 3422.″397 to 3422.″502

L = 6.″4390 to 6.″4408

$P_☾$ = 124.″984 to 124.″989