CERN-TH.4367/86

CONSTITUENT QUARKS, SOFT PIONS AND MESON MASSES

M. Frank

Department of Theoretical Physics
Oxford University

and

P.J. O'Donnell*)

CERN - Geneva

ABSTRACT

In the constituent quark model, a simple mass formula is given to relate the masses of the vector and pseudoscalar mesons. Good agreement is obtained for all mesons, from the lightest ρ, π mesons to the heaviest. In the limit of vanishing pion mass, a lower limit of 230 MeV is obtained for the constituent quark mass.

---

*) On research leave from the University of Toronto, Toronto, Canada M5S 1A7.

CERN-TH.4367/86

February 1986

1. - INTRODUCTION

A long-standing problem in particle physics has been that of the light pseudoscalar (and vector) mesons. In recent years, the discovery[1] of the ι in a gluon rich process, $J/\psi \to \gamma K\bar{K}\pi$, has raised the possibility that it might be the first evidence for gluoniun, a bound state of gluons. This has underlined the need for a comprehensive understanding of the spectra and other properties of the light mesons. At the same time, more (and improved) data for the radiative decays of the light mesons have become available[2]; the existence and the masses of radial excited states are being confirmed. The $\eta(1275)$, the controversial radial excitation of $\eta(549)$ for a long time seen in only one experiment[3] has possibly been confirmed by a recent experiment at KEK[4]. In spite of the fact that the constituent quark model has masses of the order of 300-500 MeV for the light quarks, it has proved possible to account for all of these recent observations in a non-relativistic constituent quark model[5].

The presence of radial excited states complicates the simplicity of the original naive quark model. However, it has recently been shown that for the ground states, ${}^3S_1$ and ${}^1S_0$, of the mesons, the difference $m_V^2 - m_p^2$ is approximately a constant[6)-8)]. In a model in which the confining potential is linear in r with the vector-pseudoscalar mass splittings coming from hyperfine interactions, the quantity $m^2{}_V - m_p^2$ will be approximately constant for all mesons from the lightest ρ, π to the heaviest B^*, B[6].

The basic framework[9] of modern constituent quark models is based on the fundamental paper by De Rujula, Georgi and Glashow[10]. Although this paper spelled out the rôle of the underlying theory of quantum chromodynamics (QCD) in the constituent quark model, a complete connection has not as yet been given, particularly for the light hadrons. The soft pion theory[11] which arises naturally in the $SU(2)\times SU(2)$ sector of QCD does not seem to be obtained in the constituent quark model. There the pion is a Goldstone boson associated with the partially conserved axial vector current (PCAC) and the vanishing of m_π comes from the vanishing of m_u or m_d.

In the constituent model with hyperfine splitting coming from gluon exchange, the interaction is of the form $|\psi(0)|^2/m_q m_{\bar{q}}$ where $|\psi(0)|^2$ is the square of the wave function of the two-body system at the origin. This wave function plays the rôle of the pion decay constant f_π which, in the soft pion approach, is assumed not to vanish in the chiral limit, $m_\pi \to 0$. Clearly the constituent masses are not the same as the current algebra or dynamic quark masses associated with the restoration of chiral symmetry. The conventional wisdom is that some kind of "vacuum condensate" adds a constant 200-300 MeV to the dynamical quark masses to give the constituent masses[12].

In this paper we consider further our previous work[6] on the $m_V^2 - m_P^2$ relation. We give the conditions under which it can be expected that this difference of the squares of the masses is approximately constant. In terms of a dimensionless quantity x_{ij}, where $x_{ij} < 1$ and is related to the spin-spin interactions, we give a new relationship for the squares of the masses, one which fits the observed spectra rather well for a given set of quark masses. We suggest an approach to massless pions in the constituent quark model which results in the limit $m_\pi \to 0$ being associated with a lower bound of 230 MeV for the constituent quark mass. In this approach, the chiral limit and soft pion theorems would arise presumably in a pion pole dominance of the axial current (PDAC)[11].

2. - MESON MASS FORMULAE

We consider $q\bar{q}$ bound states in a potential model as outlined in Ref. 6). The mass of an S-wave meson is given by the formula:

$$m = m_i + m_j + \frac{32\pi}{9} \frac{\alpha_s |\psi(0)|^2}{m_i m_j} \vec{S}_i \cdot \vec{S}_j \qquad (1)$$

where $m_i(m_j)$ and $\vec{S}_i(\vec{S}_j)$ represent the effective mass and spin of the quark (antiquark). The third term is the hyperfine splitting with $\psi(0)$ the wave function at the origin for the relative co-ordinate of the quark-antiquark pair.

It has been shown[6] that, for states confined by a linear potential, the quantity $\left|\psi(0)^2\right|/\mu_{ij}$, where μ_{ij} is the reduced mass of the quark-antiquark system, is approximately constant for mesons containing at least one light quark. This is consistent with the form of potential expected from lattice gauge theories for long-range interactions. The mass splittings between $^3S_1$ and $^1S_0$ states are given by the hyperfine interaction; usually this is associated with single-gluon exchange. However, it has recently been pointed out[13] that lattice gauge theory can also give rise to spin-spin interactions; these are consistent with a short-range, δ-function-like potential similar to that of the single gluon picture. Thus, to order v^2/c^2, the picture is essentially that given many years ago[14)], except for the observation about $|\psi(0)|^2/\mu_{ij} \sim$ constant. This new observation turns out to be important in explaining the regularities among the differences of the squared masses among the vector and pseudoscalar mesons.

Define the dimensionless quantity $x_{ij} = W/2(m_i+m_j)^2$ where $W \equiv 16\pi\,\alpha_s|\psi(0)|^2/9\mu_{ij}$. Then we can express the masses

$$m_p = (m_i+m_j)(1-3x_{ij}) \tag{2}$$

$$m_v = (m_i+m_j)(1+x_{ij}) \tag{3}$$

where m_p and m_v denote the masses of the pseudoscalar ($^1S_0$) and vector ($^3S_1$) mesons. From Eqs. (2) and (3) we find

$$\begin{aligned} m_V^2 - m_p^2 &= 8(m_i+m_j)^2 x_{ij}(1-x_{ij}) \\ &= 4W(1-x_{ij}) \\ &\sim 4W = \text{constant, if } x_{ij} \ll 1 \end{aligned} \tag{4}$$

Now we know that, empirically[6),15)], the quantity $m_V^2 - m_p^2 \sim 0.56$ GeV^2 is constant for all mesons that contain at least one light quark. That is, to a good approximation, $x_{ij} \ll 1$ for all of the meson spectra, so that $W \sim 0.14$ GeV^2. The worst case is that of the π, ρ system for which $x_{uu} \sim 0.16$–0.19 for $m_u \sim 300$–330 MeV. Even though one might worry about non-relativistic approximations being applied to the light π, ρ system, relation (4) holds remarkably well. Furthermore, although there could be additional terms in Eq. (1) (and indeed it has been argued that a constant term should be present[9)]) it is easy to see from Eqs. (1)-(4) that such terms must be very small on the scale of m_π, if they are to be present.

Since $x_{ij} \ll 1$ and $3x_{ij} \ll 1$ for all mesons that contain one light quark (except maybe π, ρ as we will discuss later) we obtain a square-mass relationship for mesons, correct to leading order in x_{ij},

$$m_p^2 \simeq (m_i+m_j)^2(1-6x_{ij}) = (m_i+m_j)^2 - 3W \tag{5}$$

$$m_V^2 \simeq (m_i + m_j)^2(1 + 2x_{ij}) = (m_i + m_j)^2 + W \qquad (6)$$

with W = 0.14 GeV. We calculate the values of the masses obtained from (5) and (6) and present our results in the Table.

We note the following.

(a) Even if the relationship $m_V^2 - m_P^2 \cong$ const = 0.56 GeV is violated for charmonium (where $m_\psi^2 - m_{\eta_c}^2$ = 0.72 GeV) and even more so for the T family, the formulae (5)-(6) hold quite well for all mesons, the error being less than a few per cent for the whole range of masses considered. Were we to allow for a small constant term to be added to Eqs. (5)-(6) for ψ and T, the agreement would improve. The meaning of such a term can be understood easily: by going to larger reduced masses μ_{ij}, we probe the region of small r_{ij}, the relative radial co-ordinate. In this region, Coulombic corrections (short range) to the confining potential become important and a constant term added would be a way of parametrizing such an effect. The fact that this term is small means that indeed the Coulomb corrections are small.

(b) The procedure we followed is important in understanding why the mass formula for mesons is quadratic, as well as giving a connection between a linear mass formula and the present one. A quadratic mass formula for mesons is seen as a result of a linear confining potential (and not of a spectra governed by a Klein-Gordon equation, as previously suggested[16)]). In a recent preprint[17)], Martin has shown that asymptotically linear Regge trajectories can be obtained from a linear potential, in agreement with our quadratic mass formulae. From our formalism we can understand why the linear mass formula also holds reasonably well. From Eq. (1), the linear mass formulae are obtained by making the assumption $\alpha_s \pi |\psi(0)|^2 \simeq$ constant = A, obtaining the following formulae:

$$m_P = m_i + m_j - \frac{8}{3}\frac{A}{m_i m_j} \qquad (7)$$

$$m_V = m_i + m_j + \frac{8}{9}\frac{A}{m_i m_j} \qquad (8)$$

There is no theoretical justification for considering $\alpha_s |\psi(0)|^2 \sim$ constant throughout the mass spectra. However, for any state containing a light quark and a heavy quark $\mu_{ij} \sim m_i$, where i is up, down or strange. Then $\alpha_s |\psi(0)|^2 \sim$ $\sim$ constant $\mu_{ij} \sim$ constant $m_i \sim$ constant insofar as a variation from 300 MeV to 500 MeV can be ignored.

Noticeable by their absence are the isoscalars. Here the problem is more complicated: we know how to handle the effects of the linear potential and obtain the flavour eigenstates η_u, η_s, ω_u and ω_s. But in addition this sector is complicated by singlet-octet (or flavour) mixing as well as gluonium and radial excitation mixing. As we know, both on experimental[2] and theoretical grounds[5], the simple constituent quark model needs to be extended to accommodate the existence of radial excitations. Nevertheless it is of interest, especially in view of the long history of the subject, to see the effect of the mass relations given here on the SU(3) singlet-octet problem. If we adopt Eqs. (5)-(6) to represent the unmixed eigenstates when $m_{ij} = m_{uu}$ or m_{ss} then the masses of the vector states ω and ϕ are simply given in terms of these quantities, that is $m_\omega = 0.76$ GeV and $m_\phi = 1.03$ GeV. For the pseudoscalars η, η' the problem is more complicated. Our formalism will favour a square-mass matrix of the form

$$m^2 = \begin{pmatrix} m_{uu}^2 + 2\Delta^2 & \sqrt{2}\,\Delta^2 \\ \sqrt{2}\,\Delta^2 & m_{ss}^2 + \Delta^2 \end{pmatrix} \tag{9}$$

on the basis of the fact that for $\Delta^2 = 0$ it connects smoothly into the other mass formulae. We have no new method to handle the flavour mixing. This mass matrix has eigenvalues $m_\eta \simeq 0.514$ GeV, $m_{\eta'} \sim 0.98$ GeV with $\Delta^2 \sim 0.226$ GeV^2 and mixing angle[18] $\alpha_p \sim -56.7°$. This is in good agreement with the new experimental determination[19] $\alpha_p \sim -55°$ (or $\theta_p \sim -20°$). The usual quadratic or linear mass formulae[2] give angles of $\alpha_p \sim -45°$ or $-58°$ respectively.

However, we have presented a naive approach here, valid if η and η' can be simply expressed in terms of flavour eigenstates. Recent studies of J/ψ hadronic decays[20] show this to be invalid and that about (35±18)% of the η' wave function contains other admixtures. This is a non-negligible contribution; even approximate treatments should take this into account. A more complete treatment which includes mixing from gluonium and radial excitations has been given[5] which explains successfully the masses and decays in the isoscalar region. This model predicts additional admixtures, 12% for the η and 21% for the η', consistent in magnitude with the hadronic decay measurements.

The extension of these arguments to charmonium or bottomonium is less straightforward, as noted in Ref. 6). In that reference, we estimated the "non-annihilation" part of the mass splitting by using $\tilde{m}_\phi - \tilde{m}_\eta \sim (m_u/m_s)^2(m_\rho - m_\pi) \sim$ ~ 0.29 GeV. This together with the value of $|\psi(0)|^2$ obtained from leptonic decays led to a prediction of the η_b-γ mass splitting of about 96 MeV. In view of our present results it might be more consistent to use $\sqrt{\{(2m_s)^2+W\}} - \sqrt{\{(2m_s)^2-3W\}} \sim 0.31$ GeV for $\tilde{m}_\phi - \tilde{m}_\eta$. This gives an indication of the level of uncertainty in these approaches.

3. - SOFT PIONS

In Section 2 we obtained quadratic mass formulae by taking $x_{ij} << 1$. However, in the case of the pion (and to a lesser extent for K mesons), x_{ij} is not sufficiently small to justify omission of terms of order x_{ij}^2 so that we should write $m_\pi^2 = (2m_u)^2(1-3x_{uu})^2$. We can use this fact to obtain a lower bound on the mass of the constituent quarks. The pion mass will vanish for $x_{uu} = 1/3$. Using our determination of the constant W from the complete meson spectra, we obtain for the lower limit of the quark mass, $\bar{m}$ = 230 MeV. If this value represents the contribution of vacuum condensates, say, to the effective constituent mass then we can see that the usual value of the constituent mass may arise as a difference from this minimum value, associated with the mass of the pion. That is, in the constituent quark model we can view the deviation from the massless pion limit as corresponding to the deviation of $m_u - \bar{m}$ from zero. Soft pion theorems presumably hold in the sense of pion pole dominance of the axial vector current[11)] rather than in the strong form of PCAC. Although the connection with chiral invariance is more obscure, the problem of taking the massless quark limit in the constituent model has been evaded with this approach.

4. - CONCLUSIONS

In the constituent quark model for S-wave mesons, it seems that the simple hyperfine splitting term can account for mass relationships for all of the mesons known to-date. In earlier work[5)], we showed that such a term could mix the ground and radial excited states in such a way as to account for the masses and decays of the light mesons. Now we see that for the ground states it is possible to account for the mass spectra to a good approximation for the whole range from the lightest π, ρ to the heaviest B,B* mesons.

Although the model as presented here can give correct masses for η and η' and a mixing angle consistent with the new two-photon experiments[19)] we know that it is inconsistent with the new experimental data[20] on the hadronic decays of the J/ψ since there is now known to be a non-negligible component (35±18)% of the η' wave function which is not taken into account here. We refer the reader to our previous work on models which allow for mixing with radial excitations and gluonium[5].

We also note that a generalization of our treatment of the hyperfine interaction to apply to baryons has recently been given by Lipkin[21]. He obtains new relationships between the meson and baryon mass splittings.

ACKNOWLEDGEMENTS

Part of this work was carried out while one of us (P.J. O'D.) was Lady Davis Visiting Professor at the Technion, Haifa, Israel. It is a pleasure to thank the Technion particle physics group for their discussions and hospitality. We also thank Professor H.C. Corben for comments and suggestions. This research has been supported in part by the Natural Science and Engineering Council of Canada, grants A3828 and T6873.

Quark Content		m_P	Experiment		m_V	Experiment
$u\bar{u}$	π°	0.135	0.135		0.76	0.769
$u\bar{s}$	K	0.496	0.494	K*	0.898	0.892
$u\bar{c}$	D	1.86	1.865	D*	2.01	2.01
$s\bar{c}$	F	2.02	1.97	F*	2.156	
$u\bar{b}$	B	5.27	5.27	B*	5.32	
$s\bar{b}$	B_s	5.43		B_s^*	5.48	
uu, dd, ss	η	0.51*	0.549		0.76	0.783
	η'	0.98*	0.958	ϕ	1.03	1.02
* (mixing angle $\alpha_P = -56.7°$ or $\theta_p = -21°$)						
$c\bar{c}$	η_c	2.98	2.98	J/ψ	3.1	3.097
$b\bar{b}$	η_b	9.364		Υ	9.46	9.46
[based on Ref. 6), Table 2]						

Table - Pseudoscalar and Vector Masses (in GeV)

These are obtained from Eqs. (4)-(6) using the following values for quark masses:

$m_u = m_d = 331$ MeV, $m_s = 485$ MeV, $m_c = 1.64$ GeV, $m_b = 4.98$ GeV.

REFERENCES

1) D.L. Scharre et al. - Phys.Lett. 97B (1980) 329;
C. Edwards et al. - Phys.Rev.Lett. 49 (1982) 259;
J. Richman - Caltech Ph.D. Thesis (1985) and Proc. XX Rencontre de Moriond (1985).

2) Particle Data Group - Rev.Mod.Phys. 56 (1984) 1.

3) N. Stanton et al. - Phys.Rev.Lett. 42 (1979) 346.

4) A. Ando et al. - Preprint, KEK, Japan (1985).

5) M. Frank and P.J. O'Donnell - Phys.Lett. 133B (1983) 253; Phys.Rev. D29 (1984) 921; Phys.Lett. 144B (1984) 451.

6) M. Frank and P.J. O'Donnell - Phys.Lett. 159B (1985) 174.

7) K. Igi and S. Ono - Phys.Rev. D32 (1985) 232.

8) H. Schnitzer - Phys.Rev. D, to be published.

9) See, e.g.: D. Gromes - in Proceedings of the Yukon Advanced Study Institute, ed. N. Isgur, G. Karl and P.J. O'Donnell (World Scientific, Singapore, 1985).

10) A. De Rujula, H. Georgi and S. Glashow - Phys.Rev. D12 (1975) 147.

11) S. Adler and R. Dashen - Current Algebra (W.A. Benjamin, 1968).

12) T.P. Cheng and L.F. Li - Gauge Theory of Elementary Particle Physics (Oxford, Clarendon Press) (1983).

13) C. Michael and P.E.L. Rakow - Nucl.Phys. B256 (1985) 640.

14) For a historical account, see: H. Lipkin - in Proceedings of the Yukon Advanced Study Institute, Ref. 9).

15) J. Lee-Franzini - in Proceedings of XX Rencontre de Moriond, Ref. 1).

16) D. Lichtenberg - Unitary Symmetry and Elementary Particles (Academic Press, New York, 1978).

17) A. Martin - CERN Preprint TH. 4259 (1985).

18) P.J. O'Donnell - Rev.Mod.Phys. 53 (1981) 673.

19) H. Kolanoski - Proceedings, Kyoto Conference (1985).

20) R.M. Baltrusaitis et al. - Phys.Rev. D32 (1985) 2883.

21) H.J. Lipkin - Argonne Preprint ANL-HEP-85-125 (1985).