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D. Marquis, J. Lemaitre

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Laboratoire de Mécanique et Technologie, ENS de Cachan, CNRS, Université Paris-6, 61, Avenue du Président Wilson, 94230 Cachan, France

RESUME.- Des lois de comportement couplées sont développées pour décrire le comportement de matériaux métalliques, et utilisées dans des calculs de structures. Les différents mécanismes pouvant intervenir pendant la déformation sont associés à des variables internes. Les différents types de couplage entre ces mécanismes sont classifiés. A partir de cette classification la formulation du potentiel thermodynamique est déterminée. Ce potentiel fournit les variables associées à partir des lois d'état. Le potentiel de dissipation donne les lois d'évolution des variables internes. Les lois de comportement sont écrites dans le cas de l'élastoplasticité couplée à l'endommagement et au vieillissement. La première application concerne le calcul d'une structure comportant une entaille obtenue par déformation plastique. Les champs initiaux de plasticité et d'endommagement sont introduits à partir de mesures de microdureté. Le calcul est réalisé à l'aide du code Abaqus dans lequel les lois couplées ont été introduites. La seconde application concerne le couplage entre le vieillissement et la plasticité. Les mesures de vieillissement pour différentes déformations initiales permettent la détermination des lois de comportement couplées. Un calcul de structure avec ces lois fournit le champ de vieillissement pour une pièce entaillée.

ABSTRACT.- Coupled constitutive equations for metallic materials behavior are developed and used in structural analyses. Different mechanisms which can occur during deformation processes are associated with internal variables. The different kinds of couplings between the mechanisms are classified. From this classification, the formulation of the thermodynamical potential is determined, giving the associated variables. The potential of dissipation gives the evolution laws of the internal variables. Constitutive equations are written in the case of elasto-plasticity coupled with damage and aging. A first application is concerned with an analysis of a structure with an indented notch. The initial fields of plastic strain and damage are introduced from microhardness measurements. The calculation is performed with the Abaqus code in which coupled constitutive laws have been implemented. The second application is concerned with the coupling between aging and plastic deformation. The measurement of aging for different types of prior strainings has permitted the determination of a coupled constitutive law. A structural analysis with this law gives the aging field in a notched component.

INTRODUCTION

The development of big computers and algorithms for the integration of non-linear constitutive equations now permits a precise analysis of the failure of engineering components either by the finite element method or the boundary element method. A good accuracy of the calculations can be achieved by introducing realistic constitutive equations. For engineering materials, the loading can induce elasticity, plasticity or viscoplasticity but also progressive damage and aging. In this paper, the framework of thermodynamics of irreversible processes is used to introduce the effects of the deterioration (damage) and of the microstructural changes (aging) of the material in the constitutive equations by means of internal variables. For each microscopic mechanism corresponding to a phenomenon, a macroscopic internal variable is introduced. It represents at the scale of continuum mechanics the modification of the material behavior induced by this mechanism. For example, in the case of a continuous precipitation, the scalar variable describing aging is related to the size and volume fraction of hardening precipitates. These variables are introduced in the expression of the thermodynamical potential.

In section two, different types of couplings are described. The coupling between phenomena is described by terms of this potential in which appears the product of functions of the associated internal variables. For example, the loss of rigidity of a material induced by its deterioration is represented by the product of the elastic stiffness and a function of the damage internal variable. The kinetic laws of the internal variables are given by the partial derivative of the dissipation potential depending upon the thermodynamical forces associated to the internal variables. A non direct coupling between mechanisms can be given by the introduction of the internal variables as parameters in this potential. This induces an influence of a phenomenon on the rate of another one. For example the rate of aging is increased by the presence of dislocations so that the internal variable taking into account the dislocation density is introduced as a parameter in the aging part of the dissipation potential. Furthermore secondary couplings can exist when using

a set of constitutive equations given by the way above-described. For example, damage and aging are assumed to be uncoupled in a direct or indirect manner, nevertheless aging can modify damage evolution : for a given loading, aging can modify the plasticity behavior as these phenomena are coupled, and, as plasticity and damage are indirectly coupled, aging also modifies the damage rate.

In section three, the formulation of the thermodynamical potential is given in the case of elasticity, plasticity or viscoplasticity, damage and aging of metals by assuming some particular couplings. The associated variables are then deduced by the state laws given by the partial derivatives of this potential.

In section four, the potential of dissipation is formulated with uncoupled parts for plasticity (or viscoplasticity), damage, aging and heat conduction. The indirect couplings are obtained by the introduction of state variables as parameters. They are chosen in agreement with the main tendencies of experimental behavior of the materials. The evolution laws of the internal variables are derived from this potential by the normality rule. In the set of constitutive equations appear the different couplings between the phenomena.

The fifth section is devoted to two applications. The first one is related to the prediction of crack initiation in an indented structure. The constitutive equations are chosen to describe the phenomena involved in this problem namely elasticity plasticity and ductile damage. In this case, as the initial state of the material is not the virgin one, initial values of the internal variables are taken into account in the calculation. These values are measured by a micro-hardness technique. In the second application, the same structure is supposed to be made of an aging material. During the loading process, as the precipitation is coupled with plastic strain, the most strained zones encounter greater values of aging which modify the plastic behavior.

1. MICROMECHANISMS AND VARIABLES

In order to take into account the influence of different phenomena on the mechanical behavior, the main mechanisms involved in a material for a class of loadings are defined and associated to the state variables. The observable variables, elastic strain and temperature, are associated with the deformation of the crystal lattice and the thermal agitation. These variables and the internal variables define the state of the volume element in accordance with the method of local state [1]. In this section, some mechanisms of plasticity, damage and aging are briefly described and their corresponding macroscopic representations in the framework of thermodynamics are emphasized.

1.1. Plasticity and viscoplasticity

The mechanisms involved in plasticity and viscoplasticity have been studied very extensively. For metallic materials, the plastic straining process involves an increasing of the dislocation density along the slip systems of each single crystal. The hardening depends upon the dislocation density and the interactions between the different systems, and also on the capacity of the grain boundaries to accommodate strain incompatibilities arising from

inhomogeneous behavior in the different crystals. A lot of parameters such as the type of crystal, the chemical composition, the kind of interactions between the dislocations... have a strong influence on the plastic behavior. The deformation induced by the dislocation movements is globally represented by the plastic strain tensor which is the first internal variable.

The second internal variable describing the plastic strain process, is linked with the density of dislocation and their interactions, and, at a macroscopic scale with the increase of the yield stress, or the size of the yield surface. This is a scalar variable r related to the isotropic hardening. The third internal variable takes into account the microstresses due to the incompatibility between the different slip systems. It is a second order tensor α related to the translation of the center of the yield surface introduced on a phenomenological basis by PRAGER [2]. These last two variables are commonly used to develop constitutive equations of plasticity. Nevertheless they are not sufficient in the general case of complex multiaxial strainings to take into account the mechanism of latent hardening [3] at the microscopic scale which leads to changes in shape of the yield surface [4] and to additional hardening induced by nonproportional loadings [5]. In the case of viscous strain, although different types of mechanisms are involved the same macroscopic variables are commonly used to describe the creep behavior. Despite this shortcoming, in the following parts only these two internal variables r and α are considered.

1.2. Damage

The damage of materials is related to the nucleation growth and coalescence of defects. At the microscopic scale , a lot of different mechanisms have been studied and observed [6]. To take into account the density and the orientation of microcracks or microvoids, different types of variables have been introduced [7]. These variables are continuous at the scale of the elementary representative volume of mechanics although they describe at the microscopic level discontinuities of the material. This assumption is made by analogy with the definition of the plastic strain which represents the slips in crystals. The damage variable can be defined at a given point M and in a given direction n by the area of intersections of microdefects with a plane normal to n and containing M [8,9]. This area is corrected to take into account microstress concentration and interaction between microcracks or microcavities. If δS is the area of the section of a finite volume element and δS_D the resulting effective area of defects in the plane, then D is defined by (see figure 1).
The theory of isotropic damage assumes that D does not depend upon n. The anisotropy of damage can be taken into account by the introduction of a set of vectors [10], a second order tensor [11] or a fourth order tensor [12]. For simplicity in the mathematical development and as the value of the different components of damage tensors are very difficult to identify in engineering materials, an isotropic damage will be used hereafter.

1.3. Aging

The aging processes modify the microstructure of a material with time. These changes can be induced by chemical reactions inside the material with or without the intervention of an active environment.

$$D_n = \frac{\delta S_D}{\delta S}$$

D = 0 corresponds to a virgin element.

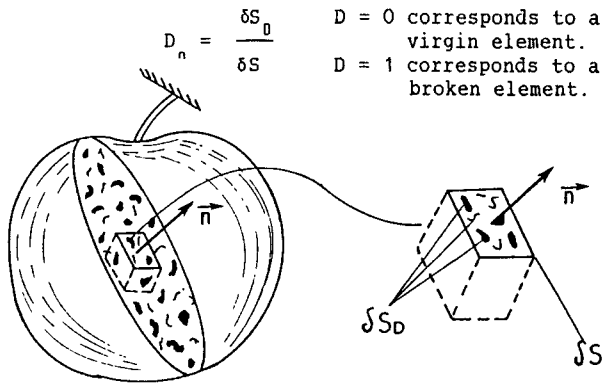D = 1 corresponds to a broken element.

Fig. 1 : Micro-macro definition of damage.

As the plastic strain or damage during a loading process can also change the microstructure, we will adopt for aging the definition given by KREMPL [13]. Aging is the modification of microstructure such that the same loading process performed in the same environment at different times (or ages of the material) gives different mechanical responses. For example, for aluminium-copper alloys after a dissolution heat treatment followed by a quench into water, diffusion of copper in the aluminium matrix occurs at room temperature. A formation of precipitates which result from this diffusion have a very strong influence on the yield stress, and different stress-strain curves are obtained depending upon the starting time of the tensile test. Internal variables are associated to this type of microstructural changes [14,15]. They take into account the state of precipitation at each point of a structure. In the case of loading at variable temperature, several internal variables are needed to describe the different types of precipitations which can induce various effects on the mechanical behavior. In the following section, a unique scalar variable is assumed. It is sufficient to give a good description of the continuous precipitation process of an aluminum-copper alloy at constant temperature.

The preceeding list is, of course, not exhaustive, and other mechanisms can be involved in a structure during a loading process. To built a constitutive equation for a material under a class of loadings in the framework of the thermodynamics of irreversible processes, the internal variables associated to the main phenomena have to be considered. Then their associated variables are deduced from the state laws, given by the partial derivation of the thermodynamic potential. The evolution laws are given by the dissipation potential. The different couplings between the mechanisms give strong arguments for the choice of the analytical functions of these two potentials.

2. COUPLINGS

During a loading process, different types of interactions and couplings can happen between the mechanisms involved in the behavior of a material.

2.1. Direct couplings

If the value of a variable V_i associated to an internal one v_i through a state law is modified by the change of the value of another variable v_j, the coupling is direct. The state laws are given by the partial derivations of the thermodynamical poten-

tial which depends upon the state variables both observable and internal. For the mechanical behavior, the total strain ε and the temperature are the two observable variables in the specific free energy Ψ chosen as potential :

$$\Psi (\varepsilon, T, v_i) = \Psi (u_i)$$

The state laws are given by :

$$U_i = \frac{\partial \Psi}{\partial u_i}$$

A direct coupling between two mechanisms characterized by u_i and u_j induces that U_i depends on u_j or :

$$\frac{\partial U_i}{\partial u_j} = \frac{\partial^2 \Psi}{\partial u_i \partial u_j} \neq 0$$

For example, as the deterioration of the material causes a loss of elastic rigidity, the internal variable associated with damage is directly coupled with the elastic strain. The corresponding part of the potential is :

$$\rho \Psi_e = \frac{1}{2} A : \varepsilon^e : \varepsilon^e (1 - D)$$

where A is the tensor of elasticity and ε^e the elastic strain. The state law of elasticity gives the influence of damage on rigidity :

$$\sigma = \rho \frac{\partial \Psi_e}{\partial \varepsilon^e} = A : \varepsilon^e (1-D)$$

2.2. Non direct couplings

This type of coupling takes into account the influence of an internal variable on the rate of another one . For example, as dislocations increase the diffusion rate of atoms, the plastic strain increases the rate of aging . This interaction is given by the introduction of the internal variables as parameters in the dissipation potential [16]. The rate of the internal variables is given by partial derivation of this potential φ^* which depends upon the thermodynamical forces V_i associated by the state laws to v_i .

$$\frac{dv_i}{dt} = \frac{\partial \varphi^* (V_1, \ldots, V_n ; v_j)}{\partial V_i}$$

The notation v_j represents the set of internal variables acting on the rate of v_i .

2.3. Secondary couplings

From the state laws and the rate of the internal variables, the constitutive equations are obtained. For a given loading process, if two internal variables v_i and v_j are directly or non directly coupled and if v^j is coupled with another variable v_k, different initial values of v_k can give different evolutions of the mechanism associated to v_i. For example, plasticity (v_p) and aging (v_a) are directly coupled, and plasticity (v_p) and damage (v_k) are non directly coupled. For a given straining, the plastic behavior changes with different initial values of aging and, as damage is coupled with the plastic strain, the condition of rupture (D=1) depends upon aging.

3. THERMODYNAMICAL POTENTIAL

3.1. Formulation

After defining the state variables related to each mechanism and the possible couplings between the phenomena on the basis of microscopic observations and macroscopic experiments, the thermodynamics of irreversible processes introduces a thermodynamical potential in order to define the associated variables. Those variables, associated to the flux variables, or rate of the dissipative variables, give the power dissipated in all the involved processes [16].

The thermodynamic potential is a scalar function of all the state variables and contains the first type of coupling i.e. the direct coupling between the state variables

$$\Psi \ (\ \varepsilon, T, \varepsilon^{p}, r, \alpha, D, a)$$

As we consider no direct influence of total strain, plastic strain and thermoelasticity on all other phenomena, as well as no influence of plasticity and aging on elasticity, the potential Ψ may be split into three parts and only function of the elastic strain.

$$\Psi = \Psi_{e} \ (\varepsilon^{e}, T, D) + \Psi_{p} \ (r, \alpha, D, a, T) + \Psi_{T} \ (\varepsilon^{e}, T, D)$$

↑	↑	↑
Elasticity	Plasticity	Thermal
Damage	Aging	effects

The associated variables named on figure 2 are defined from this potential by partial derivatives. The accumulated plastic strain p will also be used as a function of r and D.

$$\dot{p} = (\frac{2}{3} \ \dot{\varepsilon}^{p} \ : \ \dot{\varepsilon}^{p})^{1/2} = (\frac{2}{3} \ \dot{\varepsilon}^{p}_{ij} \ \dot{\varepsilon}^{p}_{ij})^{1/2}$$

State variables		Associated variables
Observable	Internal	
ε T		σ Cauchy stress tensor s Entropy
	ε^p	-σ with ε = ε^e + ε^p
	r	R Radius of the yield surface
	α	X Translation of the yield surface
	D	Y Elastic energy density release rate
	a	Z

Fig.2 : Thermodynamical variables

The analytical particularization of the potential is a difficult task because it depends upon experimental results and may be somewhat subjective.

For linear elasticity coupled with damage by means of the principle of strain equivalence associated to the concept of effective stress [17] the elastic potential is written as :

$$\varrho \Psi_{e} = \frac{1}{2} A \ : \ \varepsilon^{e} \ : \ \varepsilon^{e}(1-D)$$

where ϱ is the density, A the elasticity tensor.
For isotropic hardening coupled to aging, combined

to kinematic hardening , the plastic potential is written as :

$$\varrho \Psi_{p} = \frac{1}{2} B\alpha \ : \ \alpha + \frac{M}{1+M} \ K(\frac{r}{1-D})^{(1+M)/M} + a(Cr-L) + L$$

where B, K and M, C and L are material constants which may vary with temperature.

For linear thermo-elasticity and thermal expansion, the thermal potential is written as :

$$\varrho \Psi_{T} = - b \ \alpha \ T \ tr \ (\varepsilon^{e}) - \varrho \ \frac{\bar{c}}{2T_{0}} \ T^{2}$$

where $b, \alpha, \bar{c}$ are material constants which may vary with temperature and T_{0} a temperature of reference.

3.2. State laws

The state laws derive from the potential Ψ as follows :
- For elasticity :

$$\sigma = \varrho \ \frac{\partial \Psi}{\delta \varepsilon^{e}} = \varrho \ \frac{\partial \Psi_{e}}{\partial \varepsilon^{e}} + \varrho \ \frac{\partial \Psi_{T}}{\partial \varepsilon^{e}} = A \ : \ \varepsilon^{e}(1-D)-b\alpha \ T \ 1$$

from which the law of isotropic thermo-elasticity coupled with damage is :

$$\varepsilon^{e} = \frac{1+v}{E} \ \frac{\sigma}{(1-D)} - \frac{v}{E} \ \frac{tr(\sigma)}{(1-D)} - \alpha \ T \ 1$$

where E is the Young 's modulus, v the Poisson's ratio and α the coefficient of thermal expansion.
For the one dimensional case

$$\varepsilon^{e} = \frac{\sigma}{E(1-D)} = \frac{\tilde{\sigma}}{E}$$

- For entropy production :

$$s = - \ \frac{\partial \Psi}{\partial T} = - \ \frac{\partial \Psi_{T}}{\partial T} = \frac{1}{\varrho} \ \frac{E \ (1-D)}{1 - 2v} \ \alpha \ tr \ (\varepsilon^{e}) + \bar{c} \ \frac{T}{T_{0}}$$

The terms $\frac{\partial \Psi_{e}}{\partial T}$ and $\frac{\partial \Psi_{p}}{\partial T}$ correspond to slow variations

of the material coefficients which are neglected.

3.3 Associated variables

The associated variables derive also from the potential Ψ.

$$R = \varrho \ \frac{\partial \Psi}{\partial r} = \varrho \ \frac{\partial \Psi_{p}}{\partial r} = K \ (\ \frac{r}{1-D})^{1/M} + Ca.$$

$$X = \varrho \ \frac{\partial \Psi}{\partial \alpha} = B \ \alpha$$

$$Y = \varrho \ \frac{\partial \Psi}{\partial D} = \varrho \ \frac{\partial \Psi_{e}}{\partial D} = - \frac{1}{2} \ A \ : \ \varepsilon^{e} \ : \ \varepsilon^{e}$$

The terms $\frac{\partial \Psi_{p}}{\partial D}$ and $\frac{\partial \Psi_{T}}{\partial D}$ are neglected.

For isotropic elasticity, Y reduces to

$$\bar{Y} = - Y = \frac{\sigma_{eq}^2 \, R_v}{2E \, (1-D)^2}$$

with $\sigma_{eq} = (\frac{3}{2} \sigma^D : \sigma^D)^{1/2} = (\frac{3}{2} \sigma_{ij}^D \, \sigma_{ij}^D)^{1/2}$

$\sigma^D = \sigma - \sigma_H 1$

$\sigma_H = \frac{1}{3} tr(\sigma) = \sigma_{kk}$

$R_v = \frac{2}{3} (1+v) + 3(1-2v)(\frac{\sigma_H}{\sigma_{eq}})^2$

$\sigma^* = \sigma_{eq} R_v^{1/2}$ is the damage equivalent stress used

in damage mechanics as σ for plasticity with the Von Mises' criterion [18].

$$Z = \varrho \, \frac{\partial \Psi}{\partial a} = \varrho \, \frac{\partial \Psi_p}{\partial a} = C \, r - L$$

4. POTENTIAL OF DISSIPATION

4.1. Formulation

In order to ensure "a priori" the second principle of thermodynamics the rate of dissipative variables may derive from a second potential , the potential of dissipation φ . It is a scalar convex function of all the flux variables as defined on figure 3 from the dissipated power (q is the heat flux vector). An equivalent potential, function of the dual variables is obtained by means of the Legendre - Fenchel transform.

Flux variables	Dual variables
$\dot{\varepsilon}^p$	σ
$-\dot{r}$	R
$-\dot{\alpha}$	X
$-\dot{D}$	Y
$-\dot{a}$	Z
$-\vec{q}$	$\vec{gradT}$
T	s

Fig.3 : Dissipation variables.

In this potential appears the second kind of coupling : the non direct coupling on flux variables by parameters in the potential of dissipation.

$$\varphi^* \, (\sigma,R,X,Y,Z,\vec{gradT}; \, T,\varepsilon^p,r,\alpha,D,a,\ldots)$$
$$\text{variables} \qquad \text{parameters}$$

As no direct coupling is considered in this potential, it can be split into four parts with parameters chosen in accordance with qualitative characteristics of the experimental results.

$$\varphi^* = \varphi_p^*(\sigma,R,X;D,T) + \varphi_0^*(Y;\dot{r},D,T,a) + \varphi_a^*(Z;T,r,a)$$
$$\uparrow \qquad\qquad\qquad \uparrow \qquad\qquad \uparrow$$
$$\text{(Visco)Plasticity} \qquad \text{Damage} \qquad \text{Aging}$$

$$+ \varphi_T^*(\vec{gradT};T,D)$$
$$\uparrow$$
$$\text{Heat conduction}$$

As for the thermodynamical potential, the analytical particularization of φ^* depends upon experimental results often synthetized by old empirical laws.

Plasticity occurs in metals when the temperature is

below the third of the absolute melting temperature. So that it includes no effect of time or viscosity, the potential φ^* is chosen as the indicator function of a convex which is taken as the Von Mises' yield criterion coupled to damage by means of the effective stress [19].

- Plasticity $\varphi_{pp}^* = Ind.(f = \frac{J_2(\sigma-X)}{1-D} - R-\sigma_y)$

- Viscoplasticity $\varphi_{vp}^* = \frac{K}{N+1} < \frac{J_2(\sigma-X)}{(1-D)K} - \frac{R+\sigma_y}{K} >^{N+1}$

with $\langle X \rangle = X$ if $X > 0$
$\langle X \rangle = 0$ if $X < 0$

where :

$J_2 (\sigma-X) = [\frac{2}{3}(\sigma^D-X^D) : (\sigma^D-X^D)]^{1/2}$

σ_y is the yield stress, function of the temperature

N   is a Norton's coefficient which is material and temperature dependent.

For damage, many experiments show that the damage rate $\dot{D}$ is linear with Y and always linearly related to $\dot{r}$ taken here as a parameter so that φ_D is quadratic with Y.

$$\varphi_0^* = \frac{Y^2}{2 \, S_0} \frac{\dot{r}}{(1-D)^{\alpha_0+1}}$$

where S_0 and α_0 are coefficients which are material and temperature dependent.

For aging, measurements of the yield stress versus time [14] and heat production [20] allow to consider φ_a^* as a linear function of Z and also of a taken as a parameter

$$\varphi_a^* = [c-\beta \, exp \, (- \gamma \, \frac{r}{1-D} \,)][a_\infty-a]Z$$

where c,β,γ and a_∞ are material constants depending upon temperature.

For heat conduction, the Fourier's law is derived from

$$\varphi_T^* = \frac{1}{2} \frac{k}{T} \vec{gradT} . \vec{gradT}$$

4.2 Constitutive equations

The constitutive equations for all the phenomena involved derive from the potential of dissipation by the normality rule without any direct coupling by partial derivations due to the consideration of variables as parameters.

- Plasticity : if f = 0 and $\dot{f}$ = 0

$$\dot{\varepsilon}^p = -\frac{\partial f}{\partial \sigma} \, \dot{\lambda} = \frac{3}{2} \frac{\sigma^D - X}{J_2(\sigma-X)} \frac{\dot{\lambda}}{1-D}$$

$$\dot{r} = - \frac{\partial f}{\partial R} \, \dot{\lambda} = \dot{\lambda}$$

$\dot{\lambda}$ is the plastic multiplier which may be calculated from the consistency condition

$$\dot{f} = 0 \rightarrow \dot{\lambda} = \frac{M}{K} \left[\frac{J_2 (\sigma-X)}{(1-D)K} - \frac{\sigma_y + c\dot{a}}{K} \right]^{M-1} *$$

$$* \left[J_2 (\sigma-X) + \frac{J_2(\sigma-X)}{1-D} \dot{D} - (1-D)c\dot{a} \right]$$

By calculation of the accumulated plastic strain rate $\dot{p} = (\frac{2}{3} \dot{\varepsilon}^p : \dot{\varepsilon}^p)^{1/2}$, these two relations show that $\dot{r} = \dot{p}(1-D)$

$$\dot{\alpha} = \frac{\partial f}{\partial X} \dot{\lambda} = \frac{3}{2} \frac{\sigma^D - X}{J_2(\sigma-X)} \frac{\dot{\lambda}}{1-D} = \dot{\varepsilon}^p$$

- Viscoplasticity

$$\dot{\varepsilon}^p = \frac{\partial \varphi^*}{\partial \sigma} = \frac{\partial \varphi^*_{pv}}{\partial \sigma} = \langle \frac{J_2 (\sigma-X)}{(1-D)K} - \frac{R+\sigma_y}{K} \rangle^N *$$

$$* \frac{3}{2} \frac{\sigma^D - X}{(1-D) J_2(\sigma-X)}$$

$$\dot{r} = -\frac{\partial \varphi^*}{\partial R} = -\frac{\partial \varphi^*_{pv}}{\partial R} = \langle \frac{J_2(\sigma-X)}{(1-D)K} - \frac{R+\sigma_y}{K} \rangle^N$$

as in plasticity, these two relations show that

$$\dot{r} = \dot{p}(1-D)$$

$$\dot{\alpha} = -\frac{\partial \varphi^*}{\partial X} = -\frac{\partial \varphi^*_{pv}}{\partial X} = \dot{\varepsilon}^p$$

- Damage

$$\dot{D} = -\frac{\partial \varphi^*}{\partial Y} = \frac{\partial \varphi^*_D}{\partial \overline{Y}} = -\frac{\overline{Y}}{S_0} \frac{\dot{r}}{(1-D)^{\alpha_0+1}} = -\frac{\overline{Y}}{S_0} \frac{\dot{p}}{(1-D)^{\alpha_0}}$$

with $\overline{Y} = -Y$.

By replacing $\dot{p}$ by its expression taken from constitutive equations for a perfectly plastic material ($\dot{p} = 0$ or ∞), a strain hardening material, a cyclically plastic material, and a visco-plastic material, it has been shown that this general damage law of evolution contains many models that have been proposed in the past for respectively brittle, ductile, low cycle fatigue, high cycle fatigue and creep fatigue damages [19].

- Aging.

$$\dot{a} = -\frac{\partial \varphi^*}{\partial Z} = -\frac{\partial \varphi^*_a}{\partial Z} = \left[c - \beta \exp (-\gamma r) \right] \left[a_\infty - a \right]$$

This evolution law has been developed for the precipitation of copper in a 2024 aluminium alloy. Just after the quench, all the copper is in solution and a is supposed to be zero. The asymptotic value a_∞ is supposed to be equal to one in a process without plastic strain. Different experimental measurements have shown that the rate of aging is increased by the plastic strain. This is obtained in the evolution law by choosing a positive value for β. In this case of large plastic strain, a_∞ the asymptotic value of a, is reduced : indeed, the dislocation structure is such that the precipitation is not complete and a_∞ is then chosen as a decreasing function of the plastic strain.

5. APPLICATIONS

Two applications are described in order to show how it is possible to calculate damage and aging fields together with strain fields by means of finite element calculations on structures.

5.1. Plasticity and ductile damage

This application concerns the behavior of a structure which contains a defect the form of which is an indented notch. This notch is assumed to be the result of an accident so that the material is strain hardened and damaged (Figure 4). Those effects may be taken into account in the calculation through initial values of the internal variables such as p, X and D if they can be measured.

Identification

Apart from these initial conditions on the component, the plastic and damage characteristics have to be identified at the considered temperature for the material, an AISI 1010 low carbon laminated steel. For this application, neither kinematic hardening nor aging are considered. Then, the constitutive equations reduce to :

$$\varepsilon = \varepsilon^e + \varepsilon^p$$

$$\varepsilon^e = \frac{1+v}{E} \frac{\sigma}{1-D} - \frac{v}{E} tr (\sigma) 1$$

$$\dot{\varepsilon}^p = \frac{M}{K} \left[\frac{\sigma_{eq}}{(1-D)K} - \frac{\sigma_y}{K} \right]^{M-1} \frac{3}{2} \frac{\dot{\sigma}^D}{\sigma_{eq}} (\frac{\dot{\sigma}_{eq}}{1-D} + \frac{\sigma_{eq}}{(1-D)^2} \dot{D})$$

$$\dot{D} = \frac{\overline{Y}}{S_0} \frac{\dot{p}}{(1-D)^{\alpha_0}}$$

The elasticity and plasticity coefficients E, v, σ_y, K and M, are identified from tensile tests on specimens in the range for which D = 0 [21].

$$\varepsilon^e = \frac{\sigma}{E} , \quad \varepsilon^p = (\frac{\sigma - \sigma_y}{K})^M$$

E = 210 000 MPa, v = 0.3
σ_y = 575 MPa, K = 552 MPa, M = 2.04

The ductile damage occurs for sufficiently large strains and it is measured also during a tensile test by means of its coupling with elasticity.

$$\varepsilon^e = \frac{\sigma}{(1-D)E} = \frac{\sigma}{\tilde{E}} \longrightarrow D = 1 - \frac{\tilde{E}}{E}$$

E is the Young's modulus of the virgin material.
$\tilde{E}$ is the elasticity modulus of the damaged material.

For many materials, the results show that α_0 = 0.

Furthermore, the strain hardening may be considered as saturated, when ductile damage occurs.

$$R = cte \text{ together with } Y = \frac{\sigma_{eq}^2 R_v}{2 E (1-D)^2} \text{ and } \frac{\sigma_{eq}}{1-D} -R-\sigma_y =0$$

allow to write the damage equation in a simple way:

$$\dot{D} = \frac{D_c}{\varepsilon_R - \varepsilon_D} R_v \dot{p}$$

where D is the critical value of damage at fracture of the volume element, ε_D and ε_R are the strain damage threshold and the rupture strain in tension. The test results give

$$\frac{D_c}{\varepsilon_R - \varepsilon_D} = 0.58$$

The initial strain hardening and damage fields are measured on the structure by a procedure developed recently and based on Vickers' microhardness tests [22].

$$D = 1 - \frac{H}{H^*} \qquad R = \sigma_y \left(\frac{H^*}{H_y} - 1\right)$$

where H is the actual micro-hardness, and H* the hardness which would - exist without any damage, obtained by an extrapolation procedure ; H is the hardness corresponding to the non hardened material.

Numerical implementation

The constitutive equations have been implemented in the ABAQUS code. During the "elastic predictor, plastic corrector" procedure, the stress state is iteratively relaxed on the yield surface, linearized around all the variables, and the evolution laws are integrated with an implicit algorithm [23]. This local iterative procedure to integrate the constitutive equations system is part of the global iterative procedure used to solve the equilibrium equations.
The geometry of the substructure studied is defined on figure 4 together with the initial damage field measured and the initial strain hardening field represented by the corresponding accumulated plastic strain field. It is assumed to be in plane strain conditions.
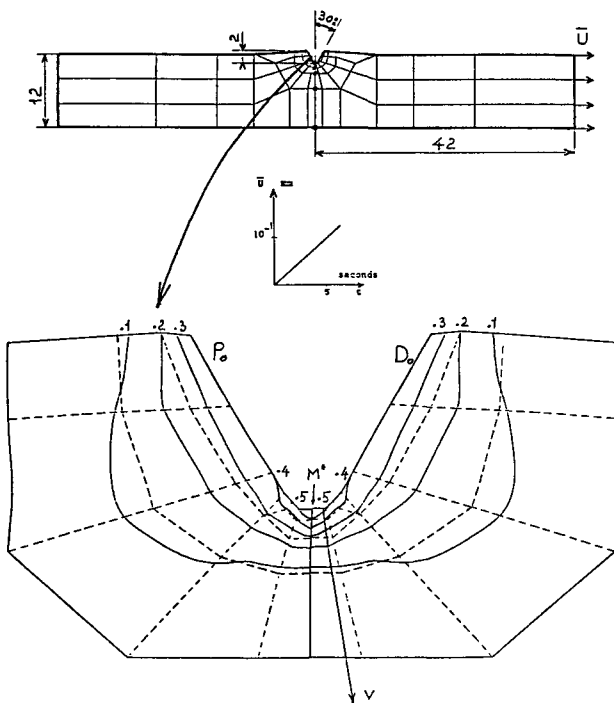

Fig.4 : Geometry and initial conditions of the structure

The initial values of p and D are introduced at each Gauss point of the mesh and are taken into account at the first step of the calculation.

Results

The calculation corresponds to a monotonic increase of the displacement U up to the time for which the damage reaches its critical value at the most loaded point.This corresponds to the initiation of a crack. The evolution of the Von Mises 'equivalent stress σ and the damage as functions of time are shown figure 5 for the critical point M*. As usual, the damage rate dramatically increases close to rupture conditions.
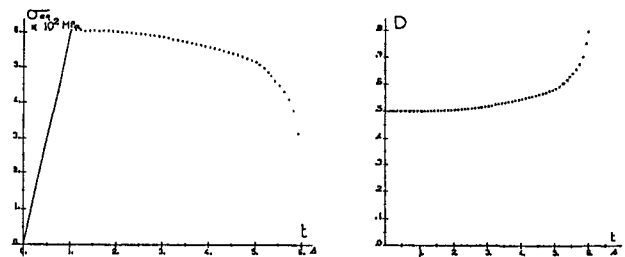

Fig.5 : Equivalent stress and damage evolutions at the critical point.

The equivalent stress and the damage along the axis v (see figure 4) are shown on figure 6 for given times. Due to the coupling with damage , the equivalent stress is not maximum at the root v = 0 but further inside. As expected, the damage gradient is very high; this is the localization of the rupture.
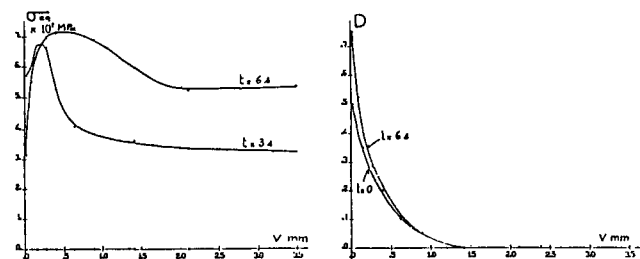

Fig.6 : Evolution of equivalent stress and damage along an axis v at crack initiation.

Finally the damaged zone at time to initiation is depicted by isodamage plots on figure 7.

If the calculation continued, it would give as a result, the evolution of a narrow zone fully damaged which represents, in damage mechanics, a crack at structural scale : this is the so-called "local approach of fracture" [24].

5.2. Plasticity and aging

This application concerns the behavior of a notched structure, the material of which is assumed to encounter plasticity and aging.

Identification [14]

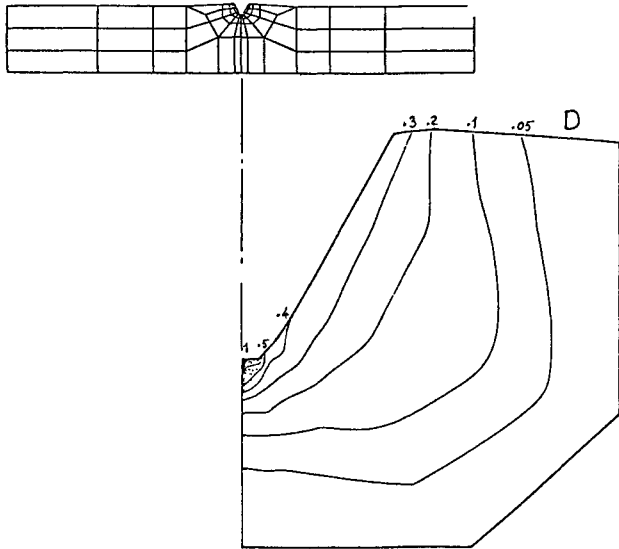The material is a 2024 aluminum alloy. It was chosen

Fig.7 : Isodomage plots at time to initiation.

because its aging occurs at room temperature in
short times. This study is rather academic and is
made to develop coupled constitutive laws easily.
This alloy is strongly hardened by Guinier-Preston
zones which are formed by copper diffusion during
the aging. Figure 8 shows the evolution of the
yield stress at room temperature after the
dissolution heat treatment (1 hour at 500° C)
followed by a quench into water. The time
corresponding to this operation is taken as the
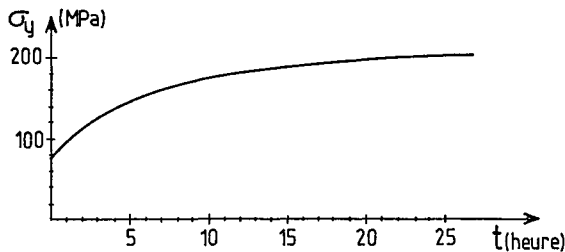origin of time scale (t=0).

Fig. 8 : Evolution of the yield stress during
aging at room temperature.

To study the coupling between aging and plastic
strain, some tests have been carried out with a
monotonic or a cyclic straining in a few minutes
just after the quench , and then the evolution of
during aging. At different times the elastic domain
in tension-compression is determined using a small
offset of plastic strain (2.10^{-4}).

This gives the evolution of the kinematic hardening
X and isotropic hardening R. As a very small
variation of X is induced by aging, only the
coupling between aging and isotropic hardening is
considered in the state law given before :

$$R = K(\frac{r}{1-D})^{1/M} + Ca$$

As the loadings do not involved damage this
relation becomes

$$R = K(p)^{1/M} + Ca$$

As the accumulated plastic strain is constant, the
evolution of R during aging gives the evolution of
a. The evolution of a versus time for two monotonic
prior strainings shows that the aging rate is
increased by plastic strain, and that the saturated
level of aging is decreased by plastic strain
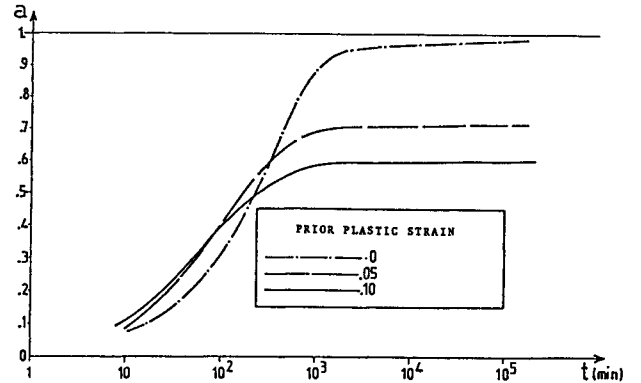(figure 9).

Fig. 9 : Evolution of aging for different prior
monotonic strainings at room temperature.

In this case of prior cyclic strainings, the aging
rate is strongly increased, but the saturated level
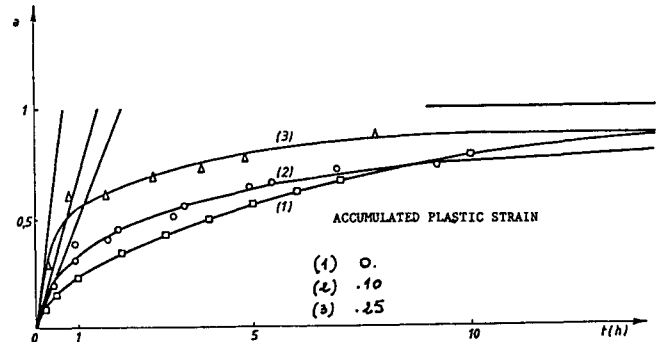is sligthly mofitied by the accumulated plastic
strain (figure 10).

Fig. 10 : Evolution of aging for different prior
cyclic strainings at room temperature.

From these test results and from monotonic and
cyclic tests on a completely aged material, the
constitutive equations are deduced by assuming a
direct coupling between aging and isotropic
hardening, and a nondirect coupling between aging
and plastic strain. As damage is not considered the
set of constitutive equations reduces to :

$$\varepsilon = \varepsilon^e + \varepsilon^p$$

$$\varepsilon^e = \frac{1+v}{E}\sigma - \frac{v}{E} tr(\sigma)\ 1$$

$$\dot{\varepsilon}^p = \lambda \frac{\sigma^0 - X}{J_2\ (\sigma - X)} \quad \text{with } \lambda \neq 0 \text{ only if}$$
$$\qquad\qquad\qquad\qquad f=0 \text{ and } \dot{f}=0$$

$$X = B\ \varepsilon^p$$

$$R = K\ p^{1/M} + Ca$$

$$\dot{a} = g(p)\left[a_\infty\ (\varepsilon^p) - a \right]$$

with $g(p) = c - \beta \exp(-\gamma p)$

and $a_\infty = A + (1-A) \exp\left[-k \, \varepsilon^p_{IImax} \right]$

$\varepsilon^p_{II} = \left[\frac{2}{3} \, tr \, (\, \varepsilon^{p2})\right]^{1/2}$

The identification of $g(p)$ and a_∞ (p) are deduced from the previous tests and the material dependent parameters are given in the following table.

E MPa	v	D MPa	K MPa	M	c s⁻¹	β s⁻¹	γ
72 500	.33	7000	500	2	.00167	.00153	1

A	k
.5	28

Results

A first application is concerned with the prediction by the previous model of a uniaxial cycling test with an evolution of aging during the straining. The loading begins at t = 0, the plastic strain range is imposed ($\Delta\varepsilon^p = \pm 0.2$ %) and the absolute value of the total strain rate is kept constant ($\varepsilon = 10^{-3}$ s⁻¹). Figure 11 shows the comparison between the predicted and the experimental stress ranges.
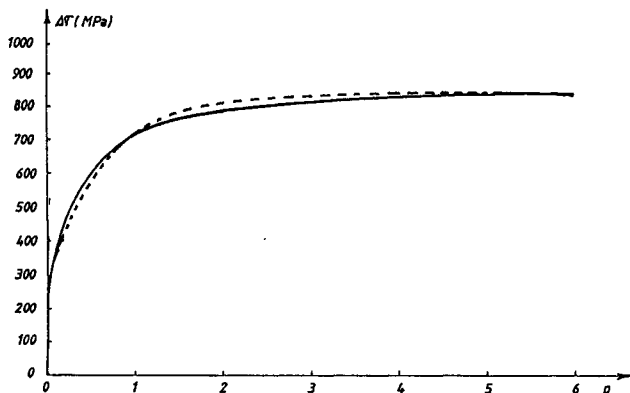
Fig. 11 : Evolution of the stress ranges versus the accumulated plastic strain.

——— experiment   - - - - prediction

The second application is concerned with the analysis of an aging structure. The geometry of the structure is given on figure 4 of section 5.1. The displacement $\bar{U}$ is prescribed with a rate of 2.8 10^{-4} mm/s. The initial values of the internal variables are assumed to be zero.

The calculations are made with an in-house finite element code. Due to the variation of plastic strain around the notch, an aging field arises as shown on figure 12.
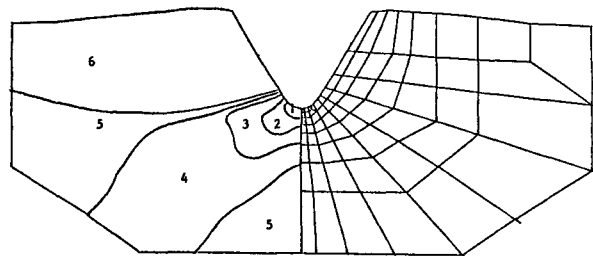
1: 0.90<a<0.97
2: 0.80<a<0.90
3: 0.70<a<0.80
4: 0.68<a<0.70
5: 0.60<a<0.68
6: 0.50<a<0.60

Fig. 12 : Aging field and mesh around the notch at time t= 9000 s.

This example shows the kind of curious effects that can arise when several phenomena may interact with one another. As a consequence, the strength may sometimes increase but it may also decrease leading to "unexplained" early failures.

REFERENCES

[1] GERMAIN P.
Mécanique des milieux continus - Tome 1, Dunod, 1973.

[2] PRAGER W.
The theory of plasticity - a survey of recent achievements, James Clayton Lecture, Proc. Instr. Mech. Engrs, 169, 1955, 41-57.

[3] FRANCIOSI P.
Plasticité à froid des monocristaux C.F.C. : étude du durcissement latent, Univ. Paris 7. Doctorat de Spécialité, 1978.

[4] ROUSSET M., MARQUIS D.
Sur la déformation des surfaces seuils en plasticité cyclique. C.R. Acad. Sc. Paris, t.301, Série II, n°11, 1985.

[5] BENALLAL A., MARQUIS D.
Constitutive equations describing non proportional effects in cyclic plasticity, Proc. of the 2nd Int. Conf. on Constitutive equations for engineering materials, Tucson USA, Desai, 1987.

[6] ENGEL L., KLINGELE H.
An atlas of metal damage, Wolf Science Books Carl Hanser Verlag, Munich-Vienna, 1981.

[7] MURAKAMI S.
Anisotropic aspects of material damage and application of continuum damage mechanics, C.I.S.M. advanced course on Damage mechanics, Udine Italy, 1986.

[8] KACHANOV L.M.
Time of the rupture process under creep condi-
tions, TVZ Akad Nauk SSR. Otd. Tech. Nauk. Vol.
8, 1958.

[9] HULT J.
CDM capabilities, limitation and promises, in
Mechanisms of Deformation and Fracture, Ed.
Easterling, Pergamon, Oxford, 1979.

[10] KRAJCINOVIC D., FONSEKA G.U.
The continuum damage theory of brittle mate-
rials, Part 1 : general theory, A.S.M.E. J. of
Appl. Mech., 48, 1981.

[11] MURAKAMl S., OHNO N.
Continuum theory of creep and creep damage, in
Creep in structures, Ed. A.R.S. Ponter and D.R.
Hayhurst, Springer, Berlin, 1981,422-444.

[12] CHABOCHE J.L.
The concept of effective stress applied to
elasticity and viscoplasticity in the presence
of anisotropic damage, in Mechanical behavior
of anisotropic solids, Ed. J.P. Boehler,
Martinus Nijhoff, The Hague, 1982, 737-760.

[13] KREMPL E.
Viscoplasticity based on total strain. The
modelling of creep with special considerations
on initial strain and aging, J. of Eng. Mat.
and Tech., Vol. 101, 1979.

[14] MARQUIS D.
Etude du couplage entre le vieillissement et la
plasticité, Internal report, Lab. de Mécanique
et Technologie, 1987.

[15] CAILLETAUD G., CHABOCHE J.L.
Macroscopic description of the microstructural
changes induced by varying temperature :
example of IN 100 cyclic behavior, Proc. of the
3rd Int. Conf. on Mechanical behavior of
materials, Ed. Miller and Smith, Pergamon,
1979.

[16] LEMAITRE J., CHABOCHE J.L.
Mécanique des matériaux solides, Dunod, 1985.

[17] LEMAITRE J., CHABOCHE J.L.
Aspects phénomènologiques de la rupture par
endommagement, J. Méca. Appl., 2, 1978,
317-365.

[18] LEMAITRE J., BAPTISTE D.
On damage criteria, Proc. Workshop N.S.F. on
Mechanics of damage and fracture, Atlanta
U.S.A., 1982.

[19] LEMAITRE J.
Formulation and identification of damage
kinetic constitutive equations, C.I.S.M.
advanced course on Damage mechanics, Udine,
Italy, 1986.

[20] CHRYSOCHOOS A.
Bilan énergétique en élastoplasticité grandes
déformations, Journal de Mécanique Théorique et
Appliquée, Vol. 4, N° 5, 1985.

[21] LEMAITRE J., DUFAILLY J.
Damage measurements, Engng Fract. Mech. Int.J.,
(to be published), 1987.

[22] LEMAITRE J., BILLARDON R., DUFAILLY J.
A procedure based on Vickers'microhardness
tests to measure damage field, C.R. Acad. Sc.
Paris, (in press), 1987.

[23] BENALLAL A., BILLARDON R., DOGHRI I.,
MORET-BAILLY L.
Crack initiation and propagation analysis
taking into account initial strain hardening
and damage fields, Proc. of the 4th Int. Conf.
on numerical methods in fracture mechanics,
San Antonio Texas U.S.A., Pineridge Swansea,
1987, 337-351.

[24] LEMAITRE J.
Local approach of fracture, Engng Fract. Mech.,
Vol. 25, No 5/6, 1986, 523-537.