# Constitutive Modeling for Metals

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**Summary.** This paper reviews aspects of the plastic behavior common in metals and alloys. Macroscopic and microscopic phenomena occurring during plastic deformation are described succinctly. Constitutive models of plasticity at the micro- and macro-scales, suitable for applications to forming, are discussed in a very broad fashion. Approaches to plastic anisotropy are reviewed in a more detailed manner.

**Key words:** alloy, anisotropy, constitutive model, forming, metal, microstructure, plasticity.

## 1 Introduction

The life of a product follows the typical chronological order: material processing, product manufacturing, product service, failure, disposal or recycling. A product is a functional shape made of one or more materials, which results in a given set of properties and complies to specifications. The material selection process can be based on the designer experience or on a more analytical and comprehensive approach using databases and optimization methods (see Brechet in Lemaitre, 2001).

Today, with advanced computer hardware and software, it is possible to model material processing, product manufacturing, product performance in service, and failure. Material design is somewhat more empirical but new analytical methods are emerging for this purpose (Raabe *et al.*, 2004). Although the fine-tuning of a product manufacturing and performance is empirical, modeling can be an efficient tool to guide and optimize design, to evaluate material attributes, and to predict life time and failure. Moreover, modeling can be used as a research tool for a more fundamental understanding of physical phenomena that can result in the development of improved or new products. In any case, a constitutive model, a mathematical description of a material behavior, is needed. With more than 80000 engineered materials (see Brechet in Lemaitre, 2001) used in various possible environmental conditions and temperatures, it is impossible to derive universal constitutive models applicable to them all. Restricting the discussion on metals and alloys, even in the context of forming, is still very broad. Therefore, this paper is an attempt to look at plastic deformation for metals and corresponding constitutive models for forming in a very synthetic way. Section 2 describes common aspects of the plastic deformation of metals and alloys and Sect. 3 briefly discusses relevant constitutive models and material parameters which are important for the simulations of forming operations. A slightly more detailed description of plastic anisotropy is provided as an illustration.

# 2 Plasticity of Metals and Alloys

## 2.1 Material Processing and Forming

From its inception to its final realization, a product is subjected to a complex thermo-mechanical path that leads to a geometrical shape and a microstructure. For instance, Table 1 shows the different steps necessary to produce a sheet metal part from casting to forming. It involves a sequence of thermal and mechanical operations to which correspond a number of physical phenomena resulting in a material microstructure (Altenpohl, 1998). This table illustrates the complexity of the process. Both geometry and microstructure affect the functionality of the final product, its properties and its service performance.

The goal of materials scientists and engineers is to develop and improve materials with respect to given properties such as the elastic modulus, strength, ductility, toughness, endurance and corrosion by a careful design of the microstructure. Materials designers are concerned with lattice imperfections, solute content, second-phases, grains and grain boundary structures,

**Table 1.** Typical flow path for aluminum alloy sheet processing

| product processing | physical phenomena | microstructure |
|---|---|---|
| • melting–casting<br>• homogenization (heat-treatment)<br>• hot rolling<br>• cold rolling<br>• solution heat-treatment<br>• stretching–aging<br>• forming | • solidification<br>• phase transformation (liquid/solid) (solid/solid)<br>• plastic deformation<br>• recrystallization<br>• grain growth | • crystal structure<br>• grain and grain boundary structure<br>• crystallographic texture<br>• second-phases<br>• dislocation structure<br>• vacancies and solutes |

crystallographic texture, and the distribution of all these features throughout the material. There are many possible ways to alter a microstructure by manipulating chemical composition and material processing. A given chemical composition itself does not reflect the final physical properties of a product, which is influenced by the whole material history after casting. In fact, it is important to note that memory effects can be very persistent. For instance, with reference to Table 1, the homogenization of an ingot at an unsuitable temperature for the particular chemical composition can lead to undesirable effects on its forming and service performances, even after several deformation and recrystallization cycles occurring during the whole process.

Process engineers are usually more focused on the shape changes during forming, the appearance of geometrical defects and the existence of a macroscopic residual stress field. For instance, in forging, the material must completely fill a die. In hot rolling, a plate must achieve a certain gauge with an acceptable degree of flatness and a minimum amount of residual stresses. In sheet forming, a part is processed successfully if its final shape and dimensions fall within the dimensional tolerances after springback. These are only a few examples but in any case, the product must be achieved without fracture. Therefore, ductility is of major importance in forming.

### 2.2 Macroscopic Observations in Plasticity

Aspects of the plastic deformation and ductility of metals and alloys at low and moderate strain rates, subjected to monotonic loading or to a few load cycles, are briefly discussed here. However, it is important to remember that beside plastic deformation, microstructure transformation is the result of temperature changes as well. Moreover, at very high strain rates, dynamic effects lead to additional phenomena that are not discussed here.

The stress-strain behavior of metals and alloys at low strain is almost always reversible and linear. The elastic range however, is bounded by the yield limit, the stress above which permanent or inelastic deformations occur. In the plastic range, the flow stress, described by a stress-strain curve, usually increases with the total amount of plastic dissipation or a corresponding measure of accumulated plastic strain, and becomes the new yield stress if the material is unloaded.

In general, it is considered that plastic deformation occurs without any volume change and hydrostatic pressure has virtually no influence on yielding. Experiments conducted at high confinement pressure showed that, though very small, a pressure effect is quantifiable and can explain the Strength Differential (SD) effect for high strength steels (Spitzig *et al.*, 1984). The SD effect corresponds to the difference between tension and compression yield stresses when both tests are conducted independently from an annealed state. Confinement pressure can also significantly improve ductility.

The Bauschinger effect is a common feature in metals and alloys that occurs when a material is deformed up to a given strain, unloaded and loaded in the reverse direction, typically, tension followed by compression. Its signature is that the yield stress after strain reversal is lower than the flow stress before unloading from the first deformation step. Bauschinger and SD effects are two different phenomena.

The flow stress of a material depends on the testing temperature. Moreover, at low absolute temperatures compared to the melting point, time has usually a very small influence on the flow stress and plasticity in general. However, at higher temperatures, strain rates effects are important. In fact, it has been observed that strain rate and temperature have similar effect on plasticity. Raising the temperature under which an experiment is carried out has a similar effect as decreasing the strain rate. Temperature has another influence on plasticity. When subjected to a constant stress smaller than the yield limit, a material can deform by creep. A similar phenomenon, called relaxation, corresponds to a decrease in the applied stress when the strain is held constant.

Finally, solid state transformations can occur in materials due to an applied stress. These transformations lead to phase changes under stresses that are lower than the yield stress of either phase and can induce plasticity (Taleh *et al.*, 2003).

## 2.3 Microscopic Observations in Plasticity

Commercial metals and alloys used in forming operations are polycrystalline. They are composed of numerous grains, each with a given lattice orientation with respect to macroscopic axes. At low temperature compared to the melting point, metals and alloys deform by dislocation glide or slip and by twinning on given crystallographic planes and directions, which produce microscopic shear deformations. Therefore, the distribution of grain orientations, the crystallographic texture, plays an important role in plasticity. Because of the geometrical nature of slip and twinning deformations, strain incompatibilities arise between grains and produce micro-residual stresses, which are partly responsible for the Bauschinger effect. Slip results in a gradual lattice rotation as deformation proceeds while twinning leads to abrupt changes in lattice orientation. The number of available slip systems determines the nature of the deformation mechanisms. BCC and FCC materials tend to deform by slip because of the large number of available slip systems. However, HCP materials, in which the number of potential slip systems is limited, generally tend to twin as an alternate mechanism to accommodate an imposed deformation. After slip, dislocations accumulate at microstructural obstacles and increase the slip resistance for further deformation, leading to strain hardening with its characteristic stress-strain curve.

At higher temperature, more slip systems can be available to accommodate the deformation (Perocheau *et al.*, 2002) but grain boundary sliding, which in a sense, is another type of shear, is becoming more predominant. For instance, superplastic forming occurs mainly by grain boundary sliding. In this case, the grain size and shape are important parameters. Atomic diffusion is also another mechanism that affects plastic deformation at high temperature and contributes to creep.

Commercial materials contain second-phase grains or particles. These phases are present in materials by design in order to control either the microstructure such as the grain size or mechanical properties such as strength. However, some amounts of second-phases are undesired. In any case, the presence of these non-homogeneities alters the material behavior because of their differences in elastic properties with the matrix as it happens in composite materials, or because of their interactions with dislocations. In both cases, these effects produce incompatibility stresses that contribute to the Bauschinger effect.

The mechanisms of failure intrinsic to metals and alloys are plastic flow localization and fracture. Localization tends to occur in the form of shear bands, either micro-bands, which tend to be crystallographic, or macro-bands which are not (Korbel, 1998). Necking in thin sheet occurs in plane strain deformation which, depending on the reference frame the strain is observed, is also a shear deformation mode.

In forming, ductile fracture is generally the result of the mechanisms of void nucleation, growth and coalescence. The associated micro-porosity leads to volume changes although the matrix is plastically incompressible, and hydrostatic pressure affects the material behavior. At low temperature compared to the melting point, second-phases are principally the sites of damage. The stress concentration around these phases lead to void nucleation, and growth occurs by plasticity. Coalescence is the result of plastic flow micro-localization of the ligaments between voids. At higher temperature, where creep becomes dominant, cavities nucleate at grain boundaries by various mechanisms including grain sliding and vacancy concentration (Kassner *et al.*, 2003). Generally, the materials subjected to creep and superplastic forming exhibit higher porosity levels than those deformed at lower temperature.

## 3 Constitutive Modeling

### 3.1 Modeling Scale

In view of the previous section, it is obvious that it is impossible to develop constitutive models for forming applications that can capture all the macroscopic and microscopic phenomena involved in plastic deformation and ductile fracture. Plasticity can be studied at various scales but for forming applications, macroscopic models appear to be more appropriate. Because of the scale difference between the microstructure and an engineered component, the amount of microscopic material information necessary to store in a forming simulation would be enormous. It is not possible anyways to track all of the relevant microstructural features in detail. Therefore, it seems more appropriate to integrate them all in a few macroscopic variables. Microscopic models are more suitable as a guide for material design, as a tool for fundamental understanding of plasticity and for inferring suitable formulations at the macroscopic scale.

As mentioned above, memory effects in materials can be significant and, in principle, integral forms of the constitutive equations should be used to account for this history (Zhou *et al.*, 2003). However, most of the constitutive equations describing plasticity have been developed in differential forms and have assumed to contain enough of the material history. Constitutive equations have been developed in scalar forms for uniaxial or shear deformation, particularly when coupled to microstructural evolution, and in tensorial forms for multiaxial loading as observed in forming operations. The constitutive equations are often developed in the framework of thermodynamics in order to prevent violation of physical principles. The constitutive laws generally consist of a state equation, sometimes referred to as the kinetic equation, (Krausz *et al.*, 1996) and evolution equations. The state equation describes the relationship between the strain rate $\dot{\varepsilon}$, stress $\sigma$, temperature $T$ and state variables $X_i$, which represents the microstructural state of the material. This can be translated, for instance, as

$$\sigma = f\left(\dot{\varepsilon}, T, X_i\right) \tag{1}$$

The evolution equations describe the development of the microstructure through the change of the state variables and can take the form

$$\dot{X}_i = f\left(\dot{\varepsilon}, T, X_j\right) \tag{2}$$

However, the state variables do not need to be connected to a specific microstructural feature. In this case, (1) and (2) define a macroscopic model with state (or internal) variables. Since the temperature influences the kinetics of microscopic mechanisms, it has an effect on plasticity, as discussed above, similar to that of the strain rate. Therefore, in processes involving hot deformation such as rolling, forging or extrusion, these two variables are usually combined in a single quantity, the so-called Zener-Hollomon parameter $Z$ (see Hosford *et al.*, 1983)

$$Z = \dot{\varepsilon}\exp\left(-Q/RT\right) \tag{3}$$

thus reducing the number of variables in the constitutive relationships. In (3), $R$ is the gas constant and $Q$ an activation energy, which is determined experimentally for a given material.

### 3.2 Microscopic Plasticity Modeling

Because slip plays a major role in plasticity, it seems important to look at this mechanism in term of both its geometrical effect on anisotropy and its effect on strain hardening. Polycrystal description of plasticity has been very successful over the last few decades. This approach is based on the geometrical aspect of plastic deformation, slip and twinning in crystals, and on averaging procedures over a large number of grains. The crystallographic texture is the main input to these models but other parameters, such as the grain shape, can also be included. It is a multiaxial approach and involves tensors. One of the outputs of a polycrystal model is the concept of the yield surface, which generalizes the concept of uniaxial yield stress for a multiaxial stress state. Polycrystal models are also very powerful to describe plastic anisotropy. Because polycrystal models can track the lattice rotation of each individual grain, the material anisotropy is naturally evolving, which makes this approach very attractive. They can be used in multi-scale simulations of forming but they are usually expensive in time and the relevant question is to know if their benefit is worth the cost. Polycrystal modeling aspects including twinning have been treated in a large number of books and publications such as, for instance, Kocks *et al.* (1978), Gambin (2001), Kalidindi *et al.* (2001) and Staroselski *et al.* (2003).

Bishop *et al.* (1951) showed that, for a single crystal obeying the Schmid law, i.e. dislocation glide occurs when the resolved shear stress on a slip system reaches a critical value, the resulting yield surface was convex and its normal was collinear to the strain rate, i.e. that the yield surface is a convex potential. Furthermore, they extended this result for a polycrystal by averaging the behavior of a representative number of grains in an elementary volume without any assumption about the interaction mode between grains or the uniformity of the deformation gradient. Hecker (1976) reviewed a number of multiaxial experiments and did not find any significant contradiction to these assumptions about normality and convexity.

After shearing individual grains, dislocations accumulate in the material, increasing their density which, in turn, leads to strain hardening. The Kocks and Mecking approach (Estrin, 1996) has laid the foundations for many subsequent studies by connecting the dislocation density $\rho$ to the shear flow stress $\tau$ using the following state equation

$$\tau = \tau_o + \alpha\mu b\sqrt{\rho} \tag{4}$$

where $\tau_o$ is the lattice friction stress, $\mu$ is the shear elastic modulus, $b$ is the amplitude of the Burgers vector and $\alpha$ is a constant that takes dislocation interactions into account. The dislocation density represents the state of the material and its evolution, which depends on the dislocation production and annihilation rates, and can be represented for instance as (Estrin, 1996)

$$d\rho/dt = k_1 + k_2\sqrt{\rho} - k_3\rho \tag{5}$$

where $k_i$ are coefficients, possibly depending on strain rate and temperature. With this type of approach, it is possible to model time dependent behavior such as creep, and time independent behavior by applying a strain proportional to time. Moreover, parameters describing the microstructure such as grain size, second-phase and solute content can be incorporated into the formulation. In this type of approach, two or more state variables can be used such as the forest and mobile dislocation densities (Estrin, 1996). In that case, additional phenomena can be studied such as dynamic strain aging (Kubin et al., 1990; Rizzi *et al.*, 2004) which produce, for instance, serrations in the stress-strain curves. This type of models contributes to the fundamental understanding of plasticity, to microstructure optimization and to the identification of relevant parameters that need to be directly or indirectly included in constitutive models.

Dislocation models were also used to explain the small pressure effect on plastic deformation. Spitzig and Richmond (1984) showed that in high strength steels the uniaxial yield stress was larger in compression than in tension. They attributed this strength-differential (SD) effect to the sensitivity of steel to pressure. In fact, these authors conducted experiments under hydrostatic confinement and found a linear dependence between the mean stress ($\sigma_m = \sigma_{kk}/3$) and the effective stress $\sigma_e$ (associated to the square root of the second invariant of the stress tensor $\sigma_e = \sqrt{J_2}$). To a reasonable approximation, Richmond and Spitzig (1980) used the following yield function to describe their experiments

$$\phi = \sigma_e + \overline{\sigma}\left\{3\beta\sigma_m - 1\right\} = 0 \tag{6}$$

where $\overline{\sigma}$ is the uniaxial flow stress and $\beta$ is the pressure coefficient. This expression is similar to the yield condition proposed by Drucker *et al.* (1952) for soils. Spitzig and his co-workers (Spitzig 1979; Spitzig *et al.* 1976, 1984) conducted experiments for different steels and obtained approximately the same pressure coefficient, $\beta = 20\,\mathrm{TPa}^{-1}$. They also performed experiments on commercial purity aluminum and obtained $\beta = 50\,\mathrm{TPa}^{-1}$. The volume changes that they observed experimentally were negligible compared to those calculated with the classical flow rule, assuming normality between the yield surface and the strain rate.

To explain the SD effect, Jung (1981) proposed a model based on the additional work needed to induce the motion of a dislocation due to the pressure ($p$) dependence of the elastic shear modulus ($\mu$), expressing $\beta$ in (6) as

$$\beta \approx \frac{2}{3\mu_0}\frac{d\mu}{dp} \tag{7}$$

In the previous relationship, $\mu_0$ is the shear modulus at atmospheric pressure. This model leads to a pressure coefficient $\beta$ equal to about 17 and $59\,\mathrm{TPa}^{-1}$ for steel and aluminum, respectively. These values are in good agreement with the experimental values mentioned above. Bulatov *et al.* (1999) used molecular static calculations with an embedded atom potential to simulate the effect of pressure on dislocation motion. They found that this phenomenon was the result of the interaction of the transient dilatation of moving dislocation with pressure. For aluminum, they computed values for the pressure coefficient

$\beta$ of 48 and 63 TPa$^{-1}$ for screw and mixed dislocations, respectively, which are consistent with the experimental value as well. Practically, for low to medium strength materials and low confinement pressure, this departure from the classical behavior can be neglected. However, this example shows how a parameter calculated using atomistic scale simulations can be transferred to the macroscopic scale.

In multi-phase materials, the second phases, whose purpose is usually to increase strength, also contribute to plastic anisotropy. In heat-treatable aluminum alloys, precipitates are intimately linked to texture because they exhibit specific shapes and crystallographic relationships with the grains. They can influence anisotropy in a way that depends on their mode of interaction with dislocations. Wilson (1965) showed that binary Al-4%Cu alloys aged with different thermal treatments, i.e. containing different types of precipitates but the same crystallographic texture, exhibit Bauschinger effects of different magnitudes. As noted by Bate *et al.* (1981), a strong back stress builds up as deformation proceeds in alloys containing nonshearable precipitates. These authors used the results of the elastic inclusion model due to Eshelby (1957) to estimate the values of the back stress. This approach is based on the idea that dislocations accumulate around these particles and produce elastic/plastic strain incompatibilities at the precipitate interface.

## 3.3 Macroscopic Plasticity Modeling

For time-independent plasticity, in a multiaxial stress space, plastic deformation is well described with a yield surface, a flow rule and a hardening law (Barlat *et al.*, 2004). Plastic anisotropy is the result of the distortion of the yield surface shape due to the material microstructural state. In fact, here is an example where the microstructural model, crystal plasticity, guides the development of macroscopic descriptions. The flow rule can be assumed to follow the normality property as discussed by Bishop *et al.* (1951). Strain hardening can be isotropic or anisotropic. The former corresponds to an expansion of the yield surface without distortion due to an increase of the dislocation density. It is completely defined by a single stress-strain curve. Any other form of hardening, such as kinematic hardening, which corresponds to the translation of the yield surface, is anisotropic.

Because stress states are multi-dimensional, it is necessary to describe yielding as a function of the stress tensor. Proper anisotropic plasticity formulations can be obtained if they are developed in the framework of the theory of representation for tensor functions (Boehler, 1978). In this theory, the constitutive equations are expressed such that the material symmetry conditions are automatically verified. The theorem of representation for a tensor function indicates that the constitutive equation can be expressed as an irreducible form of a set of invariants. Moreover, the theory of representation of tensor function includes the principle of isotropy of space, also called principle of material frame indifference or objectivity. A model based on this general framework was developed for materials exhibiting the orthotropic symmetry such as sheets and plates (Cazacu *et al.*, 2003). A drawback of the general approach is that it is generally difficult to check the convexity condition. However, a subset of this general theory, which is based on linear transformations of the stress tensor, is more suitable for developing convex formulations (Barlat et al., 2005). This approach is detailed below for incompressible materials for which a linear transformation is performed on the stress deviator, **s**, leading to the transformed stress deviator $\tilde{\mathbf{s}}$

$$\tilde{\mathbf{s}} = \mathbf{C}\mathbf{s} \tag{8}$$

**C**, a fourth order tensor, contains the anisotropy coefficients, accounts for the macroscopic symmetries of the material, and reduces to the identity for isotropic materials. The associated 1st, 2nd and 3rd invariants of $\tilde{\mathbf{s}}$ are (Barlat *et al.*, 2005)

$$\begin{aligned} H_1 &= (\tilde{s}_{xx} + \tilde{s}_{yy} + \tilde{s}_{zz})/3 \\ H_2 &= \left(\tilde{s}_{yz}^2 + \tilde{s}_{zx}^2 + \tilde{s}_{xy}^2 - \tilde{s}_{yy}\tilde{s}_{zz} - \tilde{s}_{zz}\tilde{s}_{xx} - \tilde{s}_{xx}\tilde{s}_{yy}\right)/3 \\ H_3 &= \left(2\tilde{s}_{yz}\tilde{s}_{zx}\tilde{s}_{xy} + \tilde{s}_{xx}\tilde{s}_{yy}\tilde{s}_{zz} - \tilde{s}_{xx}\tilde{s}_{yz}^2 - \tilde{s}_{yy}\tilde{s}_{zx}^2 - \tilde{s}_{zz}\tilde{s}_{xy}^2\right)/2 \end{aligned} \tag{9}$$

where **x**, **y** and **z** are the symmetry axes of the material. Using the variable

$$\theta = \arccos\left[\left(2H_1^3 + 3H_1H_2 + 2H_3\right)/2\left(H_1^2 + H_2\right)^{3/2}\right] \tag{10}$$

the principal values of $\tilde{\mathbf{s}}$, which are themselves invariant tensorial quantities, are

$$\begin{aligned} \tilde{s}_1 =& 2\sqrt{H_1^2 + H_2}\cos\left(\theta/3\right) + H_1 \\ \tilde{s}_2 =& 2\sqrt{H_1^2 + H_2}\cos\left(\theta/3 - 2\pi/3\right) + H_1 \\ \tilde{s}_3 =& 2\sqrt{H_1^2 + H_2}\cos\left(\theta/3 + 2\pi/3\right) + H_1 \end{aligned} \tag{11}$$

In this theory, an anisotropic yield condition is expressed with an isotropic function of $\tilde{s}_1$, $\tilde{s}_2$ and $\tilde{s}_3$. It is also possible to use two or more linear transformations, as long as the yield function is isotropic with respect to the $3n$ variables $\tilde{s}_k^{(1)}$, … $\tilde{s}_k^{(n)}$ (for $n$ linear transformations).

In sheet forming, plastic anisotropy is an important aspect because it influences the strain distribution in a part and, consequently, the critical failure spots. In the absence of anisotropic hardening, plastic anisotropy is contained in the shape of the yield surface. For cubic metals, there are usually enough potentially active slip systems to accommodate any shape change. Moreover, compressive and tensile yield strengths are virtually identical. Yielding of such materials is usually represented adequately by an even function of the principal values of the stress deviator $s_k$, such as (Hershey, 1954)

$$\phi = |\tilde{s}_1 - \tilde{s}_2|^a + |\tilde{s}_1 - \tilde{s}_2|^a + |\tilde{s}_1 - \tilde{s}_2|^a = 2\overline{\sigma}^a \tag{12}$$

Here, $\overline{\sigma}$ is equated to $h(\overline{\varepsilon})$ a state variable, which is a function of the dissipated plastic work or the corresponding accumulated plastic strain $\overline{\varepsilon}$, and correspond to strain hardening. The exponent $a$ is connected to the crystal structure of the material, i.e. 6 for BCC and 8 for FCC (Hosford, 1993). This was established are a result of many polycrystal simulations. Therefore, although this model is macroscopic, it contains some information pertaining to the structure of the material. A recent variation of the yield function described by (12) with two linear transformations leads to a yield surface shown in Fig. 1 (Barlat et al., 2005). The general shape of the yield surface, with regions of high and low curvatures, is consistent with crystal plasticity computations. The anisotropy coefficients are calculated from mechanical tests results or polycrystal property predictions. The level of details of the anisotropic behavior captured by this model, illustrated in Fig. 2, might be excessive for many applications but in certain cases, for instance in the beverage can manufacturing industry, these details are significant.

For most hexagonal closed packed metals (e.g. Ti, Mg, Zr, etc.), at low temperatures or high strain rates, twinning plays an important role in plastic deformation. The grains cannot accommodate certain shape changes because they lack the necessary deformation systems or because these systems require high activation stresses. Unlike slip, although pressure independent, twinning is sensitive to the sign of the applied stress, which is conducive to a strength differential (SD) effect. Furthermore, the strong crystallographic

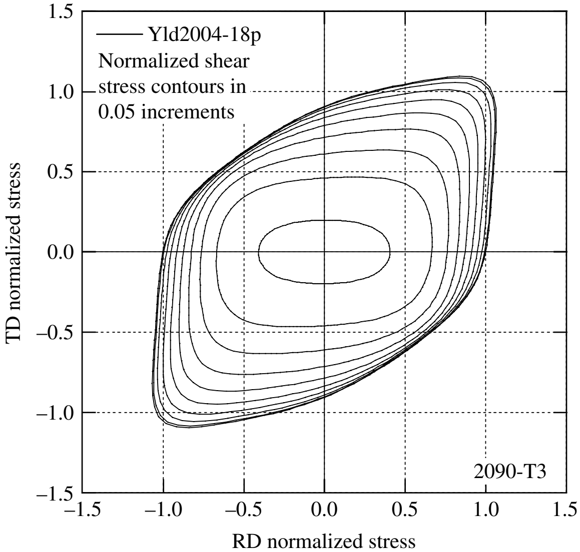

**Fig. 1.** Plane stress yield surface for a 2090-T3 Al-Li alloy sheet sample represented as contours of constant normalized shear stress $\sigma_{xy}/\overline{\sigma}$

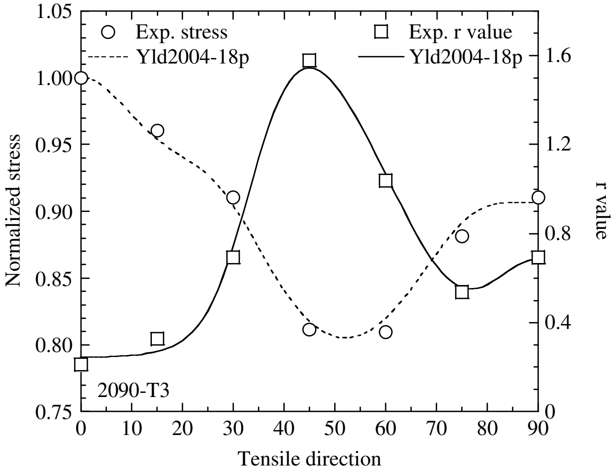

**Fig. 2.** Uniaxial flow stress and $r$ value (*width to thickness strain ratio*) directionalities with respect to the angle between tensile and rolling directions as predicted with the yield function Yld2004-18p for a 2090-T3 sheet sample

texture displayed by HCP materials leads to a pronounced anisotropy. To describe the yield asymmetry and anisotropy, Cazacu et al. (2004, 2005) proposed two yield functions, one based on the general theory of tensor representation and the other based on a linear transformation

$$\phi = ||s_1| - ks_1|^a + ||s_2| - ks_2|^a + ||s_3| - ks_3|^a \tag{13}$$

This formulation, although pressure insensitive, breaks the tension-compression symmetry and, similarly to the yield function in (12), includes anisotropy. For instance, Fig. 3 shows the corresponding yield surfaces at different strain levels predicted with this equation and applied to the case of a textured magnesium sheet. The non-isotropic hardening effect is captured by the evolution of the anisotropy coefficients and it corresponds to the rapid changes in texture due to twinning as deformation proceeds.

Non-isotropic hardening effects can be described more classically by kinematic hardening and are usually related to the micro-stresses resulting from strain incompatibilities between grains or by the interactions between matrix and second-phases. This type of hardening describes the Bauschinger effect very efficiently and can be represented as

$$\phi\left(\boldsymbol{\sigma} - \boldsymbol{\alpha}\right) = h\left(\bar{\varepsilon}\right) \tag{14}$$

$\boldsymbol{\sigma}$ is the applied stress tensor and $\boldsymbol{\alpha}$ is the back stress tensor, which controls the yield surfacetranslation. Evolution laws for this tensor can take many forms, including the following non-linear expression (Lemaitre *et al.*, 1990)

$$\dot{\boldsymbol{\alpha}} = \frac{C}{\bar{\sigma}}\left(\boldsymbol{\sigma} - \boldsymbol{\alpha}\right)\dot{\bar{\varepsilon}} - \gamma\boldsymbol{\alpha}\dot{\bar{\varepsilon}} \tag{15}$$

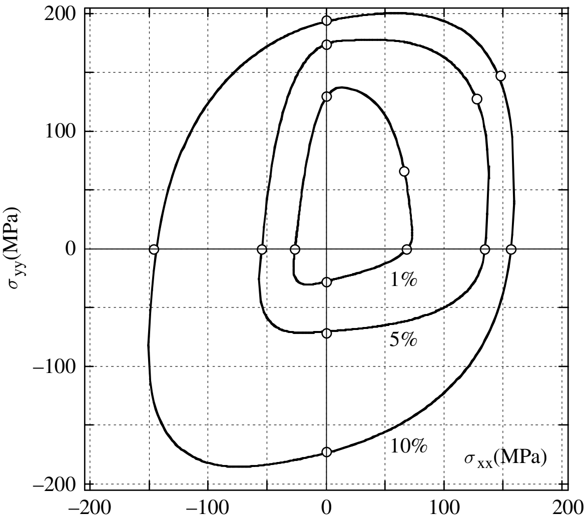

**Fig. 3.** Predicted yield locus for textured magnesium sheet. Experimental data (*open circle*) from Kelley *et al.* (*1968*)

where $C$ and $\gamma$ are the kinematic hardening coefficients. Barlat *et al.* (1998) used the non-linear kinematic hardening concept to develop a model that was able to explain the influence of non-shearable precipitates on plastic anisotropy for binary Al-Cu alloys deformed in tension and compression. In this case, the back stress was a function of the volume fraction, shape and habit planes of the precipitates, and of the crystallographic texture. Kinematic hardening can be successfully applied in forming simulations where the loading direction is changed abruptly such as, for instance, the prediction of springback in sheet forming.

Other types of anisotropic hardening formulations, which account for the Bauschinger effect, are based on multiple plasticity surfaces. In the case of two surfaces (Dafalias *et al.*, 1975; Hashiguchi, 2005), called loading and yield surfaces, respectively, the loading surface translates into stress space in a direction determined by the applied stresses or strains until it contacts the yield surface. The stress-strain relationships are determined by either surface depending whether contact between the surface is made or not. This concept appears somewhat physical since due to micro-residual stresses, the loading or reloading portion of the stress-strain curve involves the plastic contribution of only the grains that are favorably oriented for slip. A similar concept based on multiple surfaces was also proposed to model anisotropic strain hardening (see Mróz in Lemaitre, 2001). In this case, nested surfaces, each with a specific modulus, translate in stress space. The property of the active surface determines the stress-strain relationships until contact with the new active surface occurs. This type of models is suitable for the description of cyclic plasticity but its formulation might be too complex for forming simulations.

Viscoplasticity describes the time-dependent material behavior when temperature is less than typically half of the absolute melting point. Plastic deformation occurs by the motion of dislocations and the models used for plasticity are still valid. However, it is necessary to include terms containing the strain rate. Therefore, crystal plasticity and yield surface plasticity concepts can be applied in rate-dependent form. In this case, the critical resolved shear stress on the slip systems or the effective stress in the yield function formulations need to be modified with a viscous term such as

$$\overline{\sigma} = h\left(\overline{\varepsilon}\right)\left(\dot{\overline{\varepsilon}}/\dot{\varepsilon}_0\right)^m \tag{16}$$

where $\dot{\varepsilon}_0$ is a reference strain rate. Both $h\left(\overline{\varepsilon}\right)$ and $m$ depend on the temperature. $m$ is also called strain rate sensitivity parameter. Of course, other viscous terms can be used in constitutive equations.

Another approach to viscoplasticity is to assume that there is no yield surface and that any level of stress produces some amount of inelastic deformation, possibly extremely small when the stress is small (Krempl, 1996). In crystal plasticity, this approach is very useful because it eliminates the problem of slip system ambiguity in crystals where more systems than necessary are potentially active. In the continuum approach, the inelastic strain is an increasing function of the difference between the applied stress and the kinematic stress. Here the kinematic stress is similar to what is called the back stress in the kinematic hardening theories and can be associated to the micro-residual stresses that need to be overcome to deform a material plastically.

At temperatures that are roughly higher than half of the melting point, diffusion and grain sliding mechanisms are more dominant. This is the domain of creep and superplastic deformations. The so-called unified theories such as that briefly described above for viscoplasticity do not distinguish between the different plasticity mechanisms and are therefore able to describe creep as well. For superplastic (Khaleel *et al.*, 2001) and creep (Hoh *et al.*, 2004) forming, the microporosity formed at grain boundaries is a dominant factor, which is necessary to integrate in the constitutive equations of plastic deformation (see Murakami, in Lemaitre 2001).

Constitutive models, such as the model proposed by Gurson (1977) and later extended by Tvergaard *et al.* (in Lemaitre, 2001)

$$\phi\left(\sigma_{ij}, f, \overline{\sigma}\right) = \frac{\sigma_e^2}{\overline{\sigma}^2} + 2q_1 f \cosh\left(\frac{3q_2\sigma_m}{2\sigma_e}\right) - \left(1 + q_1^2 f^2\right) = 0 \tag{17}$$

contain the porosity as an internal variable. Here, $q_1$ and $q_2$ are constant coefficients and $f$ is the void volume fraction which, along with $\overline{\sigma}$, is a second state variable. The evolution equation for $f$ does not depend on the specific form of (17). It is the same for all models containing porosity, whether void nucleation takes place or not (Leblond, 2003)

$$\dot{f} = \left(1 - f\right)\dot{\varepsilon}_{kk} \tag{18}$$

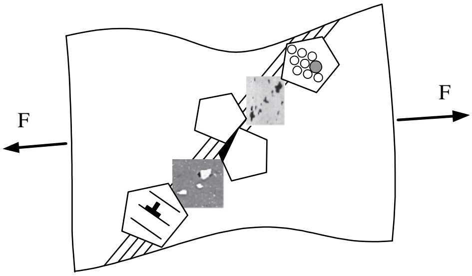

**Fig. 4.** Schematic of microstructural features within a shear localization band

Other aspects of the void growth can be included in this type of formulation, including the presence of hard inclusions in the microvoids or the coalescence of cavities (Siruguet et al., 2005a and b, respectively). To account for porosity or other form of material degradation, another approach consists in using a damage tensor $\mathbf{D}$ that modifies the applied stress tensor $\boldsymbol{\sigma}$ (see Chow in Lemaitre, 2001)

$$\tilde{\boldsymbol{\sigma}} = [\mathbf{I} - \mathbf{D}]^{-1} \boldsymbol{\sigma} \tag{19}$$

The resulting effective stress tensor $\tilde{\boldsymbol{\sigma}}$ can be used in the classical mechanics formulations and constitutive equations to describe plasticity of damaged materials.

Although discussed only briefly in this paper, strain localization and fracture are material intrinsic failure modes that are very important to consider. Shear localization is of particular interest because it can occur in any deformation state, either compressive as in rolling or tensile as in sheet forming. The modeling of shear localization is usually based on the existence of an imperfection in a material (Chien, 2004), which is physically reasonable considering the non-homogeneity of a material microstructure. All the microscopic features described above can contribute to localization through local hardening or softening (Fig. 4). In particular, porosity accounts for additional softening in the shear band. Ductile fracture can be predicted to occur with a Gurson type of constitutive equation by defining a critical porosity level $f_c$ above which fracture occurs. In the damage mechanics approach, a critical parameter $D_c$ can be defined for failure as well.

## 4 Conclusions

This paper illustrates the importance of material and process interactions. In principle, modeling of forming and microstructure evolution should be a concurrent process. However, in view of the size of forming simulations and the complexity of materials and physical phenomena occurring during plastic deformation, it seems more efficient to use macroscopic constitutive models with one or more internal variables to account for the microstructure. Constitutive models at lower scale are, of course, very important for the understanding of the microstructure evolution and to provide a basis for the development of more advanced macroscopic material models.

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