J. Phys. G: Nucl. Phys., Vol. 4, No. 3, 1978. Printed in Great Britain

LETTER TO THE EDITOR

Constrained variational results for the new Bethe homework problem

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Received 21 December 1977

Abstract. Bethe has proposed two model N–N interactions, one containing a central plus $\sigma_1.\sigma_2$ spin dependence and the other containing in addition a tensor force, to study the convergence of various many-body techniques for calculating the bulk properties of many fermion fluids. Following the success of our constrained variational calculations in describing the behaviour of the original Bethe homework problem involving a purely central interaction, we now present our results in neutron matter and nuclear matter for the new spin-dependent potentials.

Four years ago Bethe proposed that the repulsive part of the $^1S_0$ Reid soft-core potential (Reid 1968) acting in all relative partial waves should be used as a model interaction in calculating the bulk properties of a Boltzmann gas of 'neutrons' as a homework problem for the comparison of various many-body techniques. We carried out such calculations using a lowest-order constrained variational procedure (Owen *et al* 1976a) and obtained excellent agreement with the results of Chakravarty *et al* (1974) and Shen and Woo (1974), both of which involved explicit calculation of many-body terms in a cluster series. The homework problem has been extended to include neutron matter and nuclear matter calculations with full allowance for Fermi statistics. Other model central potentials have also been investigated, e.g. the full $^1S_0$ Reid soft-core potential, the central part of the $^3S_1$–$^3D_1$ Reid soft-core potential, the Iwamoto–Yamada (1957) interaction and the Ohmura–Morita–Yamada (1956) potential. These interactions have been used in calculations on both neutron matter and nuclear matter with a considerable measure of agreement between hypernetted chain (HNC) and Monte-Carlo (MC) results. Our lowest-order variational calculations with a normalisation constraint (Owen *et al* 1976b) are in satisfactory agreement with the HNC and MC results.

At the Workshop on Dense Matter organised by Pandharipande at the University of Illinois at the beginning of May 1977, Bethe announced that he considered the homework problem and its extension to fermion systems with the model central potentials as being solved. There was still considerable debate, however, about the results for spin-dependent potentials and particularly with regard to the tensor force. To help clarify this debate Bethe proposed two new model potentials

$$V_3 = V_C + V_S\sigma_1.\sigma_2 \tag{1}$$

0305-4616/78/0003-9045 $01.00

and

$$V_4 = V_3 + V_T S_{12} \tag{2}$$

where

$$V_C = (-916{\cdot}3\ e^{-4x} + 6484\, e^{-7x})/x \tag{3}$$

$$V_S = (3{\cdot}488\ e^{-x} + 263{\cdot}55\, e^{-4x})/x \tag{4}$$

and

$$V_T = -10{\cdot}463\ [(1 + 3/x + 3/x^2)e^{-x} - (3/x^2 + 12/x)e^{-4x}]/x + (351{\cdot}77\ e^{-4x} - 1673{\cdot}5\ e^{-6x})/x \tag{5}$$

in MeV and the separation distance x is in pionic units of 0·7 fm. The proposal was that V_3 should act in all channels and that V_T should act in all triplet states.

Table 1. Calculated contributions to the energy per particle in MeV for neutron matter using the potential V_3.

	k_F (fm^{-1})			
	1·5	1·7	1·9	2·1
$^1S_0$	−17·20	−20·63	−23·71	−26·36
$^3P_0$	0·31	0·54	0·91	1·55
$^3P_1$	0·92	1·61	2·74	4·66
$^3P_2$	1·56	2·69	4·58	7·80
$^3F_2$	0·16	0·24	0·31	0·36
$^1D_2$	−2·95	−5·22	−8·50	−13·07
Two-body energy	−17·20	−20·77	−23·66	−25·06
Kinetic energy	27·97	35·93	44·88	54·83
Total energy	10·77	15·16	21·22	29·77

Table 2. Calculated contributions to the energy per particle in MeV for nuclear matter using the potential V_3.

	k_F (fm^{-1})					
	1·2	1·4	1·6	1·8	2·0	2·2
$^1S_0$	−15·10	−20·49	−25·93	−30·90	−34·58	−35·76
$^3P_0$	0·21	0·39	0·72	1·34	2·48	4·42
$^1P_1$	−2·31	−4·09	−6·58	−9·85	−13·89	−18·66
$^3S_1$	5·25	8·84	14·48	23·32	36·87	56·83
$^3D_1$	0·07	0·10	0·12	0·13	0·12	0·11
$^3P_1$	0·63	1·16	2·15	4·03	7·45	13·26
$^1D_2$	−1·60	−3·24	−5·90	−9·93	−15·66	−23·48
$^3D_2$	0·08	0·11	0·14	0·15	0·16	0·19
$^3P_2$	1·07	1·97	3·66	6·82	12·56	22·30
$^3F_2$	0·10	0·19	0·30	0·41	0·50	0·55
Two-body energy	−11·61	−15·05	−16·85	−14·47	−3·99	19·76
Kinetic energy	17·91	24·37	31·85	40·31	49·76	60·22
Total energy	6·30	9·32	15·00	25·84	45·77	79·98

Table 3. Calculated contributions to the energy per particle in MeV for neutron matter using the potential V_4.

	k_F(fm^{-1})			
	1·6	1·8	2·0	2·2
$^1S_0$	−18·95	−22·22	−25·07	−27·14
$^3P_0$	0·40	0·73	1·32	2·27
$^3P_1$	−33·89	−48·69	−66·52	−87·37
$^3P_2$ }	1·27	2·29	4·55	9·02
$^3F_2$ }	3·25	5·42	8·39	12·36
$^1D_2$	−3·96	−6·72	−10·61	−15·88
Two-body energy	−51·89	−69·19	−87·94	−106·74
Kinetic energy	31·83	40·28	49·73	60·17
Total energy	−20·06	−28·91	−38·21	−46·57

Table 4. Calculated contributions to the energy per particle in MeV for nuclear matter using the potential V_4.

	k_F(fm^{-1})					
	1·2	1·4	1·6	1·8	2·0	2·2
$^1S_0$	−15·1	−20·47	−25·83	−30·66	−34·22	−35·43
$^3P_0$	0·22	0·41	0·77	1·43	2·57	4·49
$^1P_1$	−2·31	−4·09	−6·58	−9·84	−13·88	−18·65
$^3S_1$ }	−8·04	−9·72	−10·13	−8·39	−3·36	6·33
$^3D_1$ }	0·97	1·92	3·37	5·43	8·17	11·71
$^3P_1$	−18·18	−30·76	−47·64	−69·05	−95·07	−125·71
$^1D_2$	−1·60	−3·24	−5·90	−9·92	−15·66	−23·48
$^3D_2$	−1·87	−3·62	−6·25	−9·93	−14·83	−21·08
$^3P_2$ }	1·79	3·63	7·00	12·71	21·84	35·76
$^3F_2$ }	1·29	2·74	5·06	8·46	13·11	19·23
Two-body energy	−42·84	−63·19	−86·12	−109·78	−131·34	−148·85
Kinetic energy	17·91	24·37	31·85	40·31	49·76	60·22
Total energy	−24·93	−38·82	−54·27	−69·47	−81·58	−88·63

It was proposed that calculations on both neutron matter and nuclear matter should be performed.

Using the techniques of lowest-order constrained variational calculations described by Owen *et al* (1976b, 1977), we have carried out such calculations. In table 1 we present a partial-wave analysis of the contributions from the interaction V_3 to neutron matter and in table 2 the corresponding results for nuclear matter. In table 3 we present our neutron matter results for the interaction V_4. We note the complete lack of saturation in this case. Closer examination of table 3 shows that this is due almost entirely to the strongly attractive $^3P_1$ contribution and it becomes clear from a comparison of table 3 with table 1 that this is due almost entirely to the diagonal tensor matrix element. Usually phenomenological nucleon–nucleon potentials fitted to the scattering phase-shifts would put no tensor force in this channel.

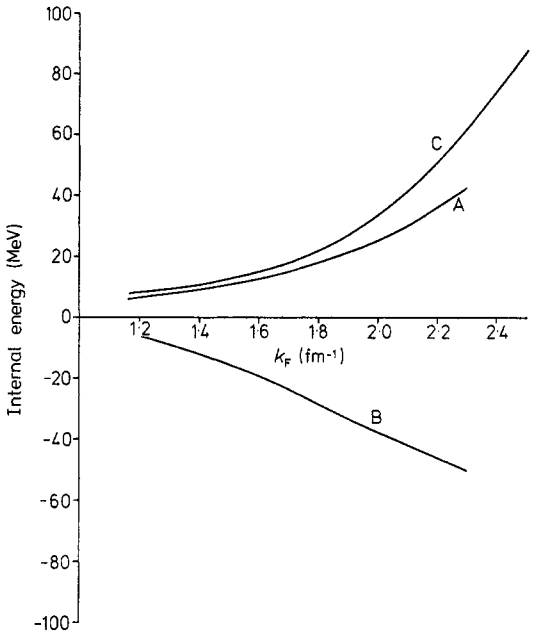

Figure 1. Calculated internal energies for neutron matter using (A) potential V_3, (B) potential V_4 and (C) potential V_3 in all channels except $^3P_2$–$^3F_2$ where V_4 is taken to act.

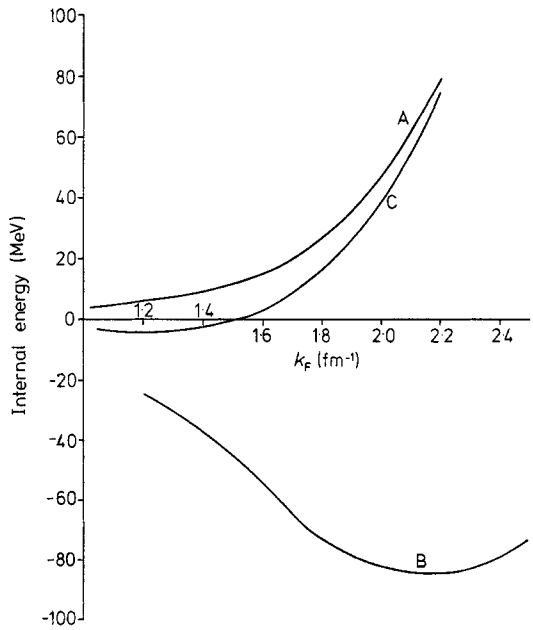

Figure 2. As figure 1, but for nuclear matter.

This result is given support in table 4, where we present our nuclear matter results for the potential V_4. In this case we do find saturation, but only at very high density and with extremely large binding energies. The overbinding is again attributable to the tensor force in the $^3P_1$ and $^3D_2$ channels. To investigate this further, we have considered the model potential V_3 acting in all channels with V_4 only acting in the tensor coupled channels, i.e. $^3P_2$–$^3F_2$ in neutron matter and $^3S_1$–$^3D_1$ and $^3P_2$–$^3F_2$ in nuclear matter. Our results are presented in figures 1 and 2.

We await with interest the results of HNC and MC calculations with which these results may be compared.

We would like to thank John Owen for his assistance with many aspects of this work and one of us (CH) wishes to acknowledge the award of an SRC Research Studentship during the course of this work.

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